The Tihai Clock:

A Complete Representation of North Indian Tihais

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The *tihai* is a frequently used cadential figure in the classical music of North India which consists of any musical phrase, melodic or percussive, that is repeated three times. The ternary structure of tihai creates palpable musical tension and anticipation, which is resolved with a strong feeling of catharsis or arrival at important points in the system of Hindustani rhythm cycles (known as *talas*). There are many different types of tihai, which will be examined in detail. Many previous attempts have been made to mathematically describe the structure of tihai and its many forms, yet the end results of these attempts are often either overly simplistic, only describing a limited set of tihai in very specific contexts, or overly complicated, filling pages with multitudes of dizzying formulas and variables. The former is due to insufficient flexibilty within the mathematical framework resulting in formulas lacking the descriptive power to encompass the richness of variety and context of tihai, whereas the latter is largely due to the fact that, for every new type of tihai discussed, more and more formulas and variables are introduced. This latter approach, in its attempt at completeness, misses the crucial point that, in spite of the aforementioned (and considerable) variety of tihais, in truth, they all share the same basic structure. Considering this premise, which will be elaborated upon shortly, the notion of tihai can be thoroughly described by a single yet sufficiently flexible mathematical approach. Additionally, this approach yields a powerful practical tool for musicians who want to better

comprehend and apply the multifarious notion of tihai, a tool capable of comprehensively representing the many forms and contexts of this unique rhythmic cadence: the Tihai Clock.

Brief Background of Hindustani Talas

Hindustani classical music, the classical music of North India, is a musical system that functions within a cyclic rhythm framework known as *Taal* or *Tala*. Many talas (rhythm cycles) exist, each comprised of a specific numbers of beats. The most common tala is 16 beats in length and is known as *Tintal* (or *Tintaal*, *Teentaal*, etc.). Other cycles in common practice consist of 6, 7, 10, or 12 beats (known as *Dadra*, *Rupaktal*, *Jhaptal* and *Ektal* respectively). There are also several odd-numbered talas, consisting of 11, 13, or 15 beats, to name just a few (known as *Asta Mongal*, *Jaital*, and *Pancham Sawari*). Some talas even exist that consist of fractional numbers of beats; 6 ½, 8 ½, and 10 ¼ beat cycles are examples of this kind of tala.

It is possible for several distinct talas to be comprised of the same number of beats. For example, the talas known as *Dhammar*, *Deepchandi*, and *Ada Chautal* are all 14-beat cycles. What distinguishes them, among several other things, is the specific way the 14 beats are grouped together (i.e. meter). The beats of *Dhammar* are grouped 5-2-3-4, whereas the groupings for *Dheepchandi* and *Ada Chautal* are 3-4-3-4 and 2-2-2-2-2-2-2 respectively.

The result of grouping beats together in this way gives rise to a hierarchy of beats within each tala that is also evident in specific patterns of hand claps and waving gestures associated with each cycle. By far, the most important and accented beat in this hierarchy is the first beat of the cycle which is known as the *sam* (pronounced *sum*). Two other kinds of accented beats of

secondary prominence in this hierarchy are known as the *tali* and the *khali*. With regard to the aforementioned hand clapping and waving patterns, the *sam* and the *tali* are each shown with a clap, while the *khali* is shown with a wave. In terms of notation, the *sam* is generally represented by a plus sign (+) and *khali* beats are indicated by a circle (o), while *tali* beats are given a number indicating their position in the sequence of clapped beats. For example, if the distribution of *sam*, *tali*, and *khali* beats in a particular tala is *sam*, *tali*, *khali*, *tali*, then these points would be notated as follows: +, 2, o, 3. The *tali* in this example are the numbers 2 and 3 which indicate that they are the positions of the 2nd and 3rd claps in the tala. The clapping pattern for this example would therefore be: clap, clap, wave, clap.

Below is an analysis of the basic structure of the most common tala, *Tintal* (16 beats):

+ Clap 1 Dha	2 Dhin	3 Dhin	4 Dha
2 Clap 5 Dha	6 Dhin	7 Dhin	8 Dha
o Wave 9 Dha	10 Tin	11 Tin	12 Na
3 Clap 13 Te Te	14 Dhin	15 Dhin	16 Dha

fig. 1 - Structure of Tintal

Each cell in the table above represents a beat in the tala. The syllables at the bottom of each cell (*Dha*, *Dhin*, *Tin*, *Na*, and *Te*; known as *bols*) are onomatopoeic representations of strokes played on the *tabla*, a well-known North Indian drum, and collectively they form a specific pattern known as the *theka* (pronounced *teka*). Each tala has a unique *theka* associated with it which is considered to be the most basic expression of that tala. Above these syllables in each cell are numbers (1 - 16) indicating the position of each beat in the sequence of the rhythm cycle. Finally, in the upper left-hand corner of cells 1, 5, 9, and 13 are the symbols indicating *sam*, *tali*, and *khali* beats as previously explained. Below these symbols are indications as to whether there should be a clap or a wave falling on a particular beat.

In Western notation the *theka* and clapping pattern of *Tintal* may be shown as follows:

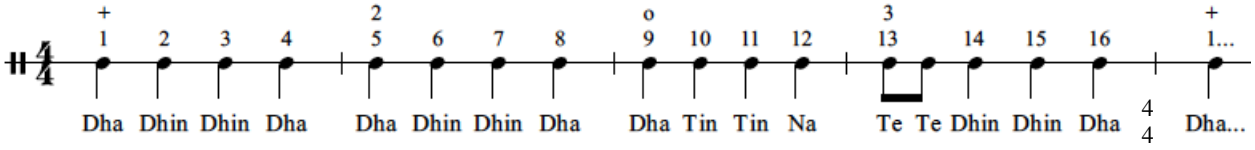

fig. 2 - Tintal Theka in Western Notiation

Whereas in Western classical music the time signature might abruptly shift from $\frac{4}{4}$ to $\frac{6}{8}$ to $\frac{7}{4}$ in a span of only a few measures, in Hindustani performance practice once a tala has been established it will usually persist for quite some time, i.e. the musicians will not jump around from tala to tala. Generally, all of the implications of a tala will be worked out over a long period of time. For example, a typical performance might begin with *Tintal* in a slow tempo (known as *vilambit laya*, ~50 bpm) and over the course of an hour might gradually accelerate to medium tempo *Tintal* (*madhya laya*, ~120 bpm) and fast *Tintal* (*drut laya*, ~200 bpm or faster). During

this hypothetical hour, the actual 16-beat framework of the tala will have been cycled through, over and over, possibly several hundred times. It is common for more than one tala to be presented in a single performance, but again, each one will be explored at length, individually.[1]

A simple analogy will be useful here in illucidating the basic nature of tala: the cyclic structure of the tala system can be compared to a multitudinous series of identically-shaped picture frames. The rectangular shape of the frames in this series remains constant, however, the paintings contained within the boundaries of the frames can be infinitely varied. For example, some of the paintings could be portraits while others might be still lifes, landscapes, or abstract paintings. Some might use only one, two, or three colors, whereas others might use all available color combinations. The themes of the images could be joyous or somber, pastoral or mythical, romantic or religious. The styles represented could be realist, impressionist, expressionist, cubist, or surrealist. The various shapes, styles, aesthetics, meanings, and creative possibilities that can occur within the frames are literally infinite, but the rectangular form of the frames themselves never varies; it is always fixed. This is the general notion of tala, i.e. a fixed rhythmical framework within which infinite melodic, rhythmic, and emotional variation can be explored. Tala is the delineation of a musical-temporal arena whereupon composition, improvisation, innovation, and inspiration interplay.

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[1] The only rare exception to this general rule is the *taalmala*, or "garland of taals," where many different talas are presented in relatively rapid-fire succession, one after another. Even in this unique situation, however, each individual tala is still thoroughly established and explored before switching to the next.

The Tihai

The *tihai* (sometimes spelled *tehai*) is a ubiquitous rhythmic structure found within the tala system of North Indian classical music (and dance, most notably in *Kathak*). It is a cadential figure that consists of any musical phrase (expressed melodically or percussively) which is repeated thrice and which commonly resolves to the downbeat (the *sam*) of a particular rhythm cycle. In performance situations, the general theory is that the first repetition of the phrase presents the musical material of the tihai to the audience, while during the second and third repetitions audience members recognize that a tihai is being played and anticipate its often exciting and cathartic resolution to the *sam*.

Each of the three repetitions of the tihai phrase is known as a *palla*, therefore tihais are said to be made up of three *pallas*, where the *pallas* are equal in length. There are rare exceptions that break this rule of equally-lengthed *pallas*, where the last *palla* of a tihai is either compressed or expanded to create a sense of deception or surprise, or where the *pallas* successively expand or contract. However, this class of tihai is considered irregular, so from this point on it will be assumed that the three *pallas*, and the two gaps separating the *pallas*, of any (regular) tihai are equal in length, as this is the case with the vast majority of tihais.

Below is an example of a simple tihai in *Tintal*:

+1 Te te	2 ka ta	3 ge di	4 ge ne
5 **Dha** -	6 - -	7 Te te	8 ka ta
9 ge di	10 ge ne	11 **Dha** -	12 - -
13 Te te	14 ka ta	15 ge di	16 ge ne
+1 **Dha**	2	3	4

fig. 3 - Standard Tihai

In this example, the *palla* of the tihai consists of the phrase *"Tete kata gedi gene Dha"* (representative of a musical phrase played on the *tabla*), and is being repeated 3 times. Each individual syllable of this phrase takes up exactly half a beat of rhythmic time, hence there are 2 syllables per each beat. The first *palla* begins on beat 1 and ends on beat 5, the second *palla* begins on beat 7 and ends on beat 11, and the third *palla* begins on beat 13 and ends on beat 1 of the following cycle. Each *palla* is 4 beats in length with a punctuating note (*Dha*) on the 5th beat. There are a total of 2 gaps separating the *pallas*, each 3 half-beats long (notated by dashes). The entire tihai spans one full cycle of *Tintal*, beginning on the *sam* of the first cycle (+1) and ending on the *sam* of the second cycle (+1).

In Western notation this tihai spans a total of 4 measures of $\frac{4}{4}$ time with the final *Dha* landing on the downbeat of the 5th measure (numerals indicate beat number in Tintal):

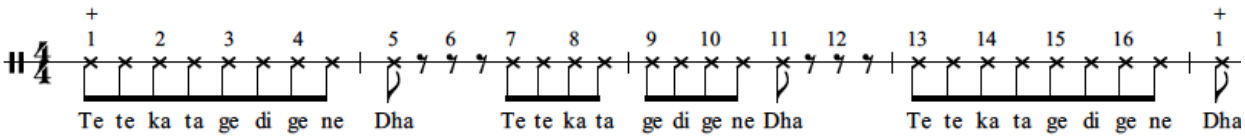

fig. 4 - Standard Tihai in Western Notation

This example demonstrates the basic structure shared by all tihais:

1st *palla*	1st gap	2nd *palla*	2nd gap	3rd *palla*

fig. 5 - Basic Structure of Tihai

Categories of Tihai

There are many different types of tihai ranging from the very short and simple to the very long and complex. Tihais do not have to start from the *sam* (indeed, they can start from any beat, or from any point in between any two beats, in any tala) nor do they have to resolve to the *sam*, although, as previously mentioned, this is by far the most common resolution point, particularly for percussionists and dancers.

Because resolutions to the *sam* are often a foregone conclusion for concert goers, two forms of deceptively resolving tihais exist that exploit this assumption. The most common of these deceptive resolutions is known as *anagat*. As a rule, the last *palla* of an *anagat* tihai must resolve before the *sam*, but by no more than half a beat. For the audience, the effect of an *anagat*-resolving tihai is similar to climbing a flight of stairs and stumbling at the top, believing that one last step remained when surprisingly, it did not, i.e. one has arrived "before *sam*" (*sam*

here meaning one's expected arrival point). The other form of deceptive tihai is known as *atit*. *Atit* tihais resolve just after the *sam*, (again, by no more than half a beat) and the effect in this case is like believing the top of the stairs has already been reached when in fact one more step still remains, i.e. arriving "after *sam*." In either case, *anagat* or *atit*, the end result is surprise.

+1 - -	2 - Te	3 te ka	4 ta ge
5 di ge	6 ne **Dha**	7 - Te	8 te ka
9 ta ge	10 di ge	11 ne **Dha**	12 - Te
13 te ka	14 ta ge	15 di ge	16 ne **Dha**
+1 -	2	3	4

fig. 6 - Anagat Tihai

Above is a modified version of the standard tihai example examined in *fig. 3*, now with an *anagat* resolution. The *palla* phrase is the same as before (i.e. *Tete kata gedi gene Dha*), but now there is only 1 half-beat of rest between the *pallas* (on beats 7 and 12) and there are 3 half-beats of rest starting from the *sam* before the tihai begins. In other words, this tihai begins at the halfway point between beats 2 and 3.

The defining factor, however, that makes this example an *anagat* tihai is the fact that its final resolution point (i.e. the punctuating *Dha* at the end of the third *palla*) lands half a beat

before the *sam* of the following cycle. Below is this *anagat*-resolving tihai in Western notation (the *sam* of the second cycle is represented by the quarter rest in the 5th measure):

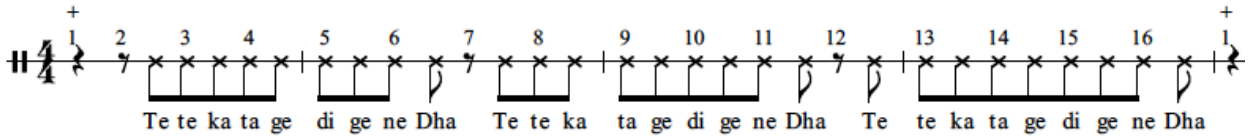

fig. 7 - Anagat Tihai in Western Notation

Next, this *anagat* tihai can be modified to create an *atit*-resolving tihai simply by adding one more half-beat of rest to the gaps between the *pallas*. Because there are always 2 gaps in a tihai (and two halves make a whole) this amounts to adding a whole beat to the entire tihai. As a result, the resolution of the tihai gets pushed forward by one beat and, instead of ending a half-beat before the *sam*, it now ends a half-beat after the *sam*.

+1 - -	2 - Te	3 te ka	4 ta ge
5 di ge	6 ne **Dha**	7 - -	8 Te te
9 ka ta	10 ge di	11 ge ne	12 **Dha** -
13 - Te	14 te ka	15 ta ge	16 di ge
+1 ne **Dha**	2	3	4

fig. 8 - Atit Tihai

Below is this *atit* tihai in Western notation:

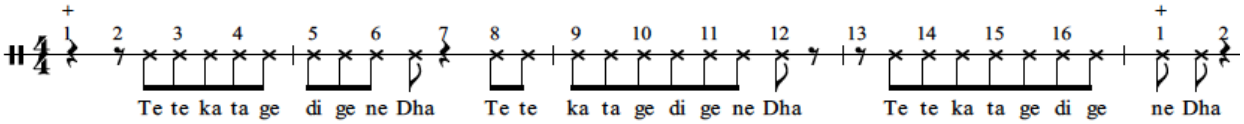

fig. 9 - Atit Tihai in Western Notation

There is one other common resolution point besides the *sam*, the half-beat before *sam* (*anagat*), and the half-beat after the *sam* (*atit*). It is at the beginning of what is known as the *mukra* (or *mukda*). The *mukra* is a melodic lead-in that is considered the beginning of the main melodic theme in a Hindustani classical performance. Although the main melodic theme (known as the *gat* or *bandish*) usually accentuates the *sam*, it does not necessarily begin on the *sam*, rather, it generally begins several beats before the *sam*. It is this melodic lead-in to the *sam* which is known as the *mukra* and which propels the melodic momentum of the *gat* or *bandish* towards the *sam*. Therefore, the beginning of the *mukra* is a structurally important point in the tala, particularly for melodic instrumentalists, and many tihais are designed to resolve to the beginning point of this melodic figure. *Mukras* in *Tintal* commonly begin on the 12th beat, though theoretically the *mukra* can begin at any point in the tala.

Additionally, the number of tihai categories can be expanded by considering the amount of space separating the three *pallas*. In this regard there are two types, *dumdar* (lit. 'with breath') and *bedum* or *bedumdar* (lit. 'without breath'). Tihais that have discernable spaces or rests separating the *pallas* are known as *dumdar* while tihais where these spaces are so small it they seem nonexistent are known as *bedum*. In truth, however, no matter how miniscule, there is always some distance between the last note of one *palla* and the first note of the next, so

although the notion of *bedum* implies that there are no gaps, the reality is that the gaps are indeed present, even though they are so fleeting they are not perceptible.

The standard tihai analyzed in *fig. 3* is a prime example of a *dumdar* tihai because the 3 half-beats of rest audibly separate the *pallas*. It can be modified, however, to be considered a *bedum* tihai by adding 2 more punctuating *Dha*s to the end of each *palla*. The resultant *palla* phrase now becomes: *"Tete kata gedi gene Dha Dha Dha."*

+1 Te te	2 ka ta	3 ge di	4 ge ne
5 **Dha Dha**	6 **Dha** Te	7 te ka	8 ta ge
9 di ge	10 ne **Dha**	11 **Dha Dha**	12 Te te
13 ka ta	14 ge di	15 ge ne	16 **Dha Dha**
+1 **Dha**	2	3	4

fig. 10 - Bedum Tihai

There are now no (perceivable) gaps between the *pallas*. However, as mentioned before, even if the last note of one *palla* is sustained until the first note of the following *palla* is played and there is no musical silence between the *pallas*, there will still always be some measurable distance between the two notes, i.e. there is a definite rhythmic interval (half a beat) between the *"Dha"* on beat 6 and the *"Te"* immediately following it. Therefore, although this space is not

perceived musically, it is certainly present, and consequently *bedum* tihais do not violate the basic structure of all tihais diagramed in *fig. 5*.

There also exists a large and complex class of tihai, known as *chakradar*, where the individual *pallas* themselves contain smaller tihai (which will be referred to as sub-tihai), giving the fractal effect of tihais within a tihai, or of a large tihai made up of smaller tihais. The sub-tihai are usually preceded by a short introductory phrase. The basic structure of a typical *chakradar* is as follows:

Main-*palla*: [introductory phrase {sub-*palla*}x3]x3

Chakradar tihais are sometimes known as *nauhar* tihais because the sub-*palla* is repeated a total of 9 (*nau*) times. Below is an expanded view of this basic structure:

1st main-*palla*: introductory phrase / 1st sub-*palla* / 2nd sub-*palla* / 3rd sub-*palla*

2nd main-*palla*: introductory phrase / 1st sub-*palla* / 2nd sub-*palla* / 3rd sub-*palla*

3rd main-*palla*: introductory phrase / 1st sub-*palla* / 2nd sub-*palla* / 3rd sub-*palla*

Two unique subgroups of the *chakradar* category exist that are set apart from typical *chakradars*. These two subgroups, known as *farmaishi chakradar* and *kamali chakradar*, retain the same structural principle as standard *chakradars* (i.e. that each *palla* contains a sub-tihai made up of sub-*pallas*). What sets them apart, however, is the way that the sub-*pallas* are arranged in reference to the tala.

During the first main-*palla* of a *farmaishi chakradar*, the first sub-*palla* will resolve to the *sam*. During the second main-*palla*, the second sub-*palla* will resolve to the *sam.* During the third and final main-*palla*, the third and final sub-*palla* will resolve to the *sam*.

Below is an example of a *farmaishi chakradar* in *Rupaktal*, a 7 beat rhythm cycle. The complete main-*palla* consists of the phrase:

Tete kata gedi gene

KATA GEDI GENE Dha -

KATA GEDI GENE Dha -

KATA GEDI GENE Dha - - -

The sub-*palla* in this example consists of the phrase *KATA GEDI GENE Dha* (which has been capitalized to visually set it apart) and will be repeated a total of 9 times:

+1 Te te	2 ka ta	3 ge di	4 ge ne	5 KA TA	6 GE DI	7 GE NE
+1 **Dha -**	2 KA TA	3 GE DI	4 GE NE	5 **Dha -**	6 KA TA	7 GE DI
+1 GE NE	2 **Dha -**	3 - -	4 Te te	5 ka ta	6 ge di	7 ge ne
+1 KA TA	2 GE DI	3 GE NE	4 **Dha -**	5 KA TA	6 GE DI	7 GE NE
+1 **Dha -**	2 KA TA	3 GE DI	4 GE NE	5 **Dha -**	6 - -	7 Te te
+1 ka ta	2 ge di	3 ge ne	4 KA TA	5 GE DI	6 GE NE	7 **Dha -**
+1 KA TA	2 GE DI	3 GE NE	4 **Dha -**	5 KA TA	6 GE DI	7 GE NE
+1 **Dha**	2	3	4	5	6	7

fig. 11 - Farmaishi Chakradar

This *farmaishi chakradar* spans exactly seven full cycles of *Rupaktal*. As mentioned before, during first main-*palla*, the punctuating *Dha* of the first sub-*palla* lands on the *sam* of the cycle, as does the punctuating *Dha* of the second sub-*palla* during the second main-*palla*, and the punctuating *Dha* of the third sub-*palla* during the third main-*palla*.

Kamali chakradars are similar to *farmaishi chakradars* except that the nine sub-*palla*s have three punctuating *Dha*s each. It is the way that these three *Dha*s are arranged in reference to the tala that distinguishes *kamali chakradars* from other forms of *chakradar*. Specifically, during the first main-*palla*, the first punctuating *Dha* of the first sub-*palla* lands on the *sam* of the cycle. During the second main-*palla*, the second punctuating *Dha* of the second sub-*palla* lands on the *sam*. During the third and final main-*palla*, the third and final and punctuating *Dha* of the third and final sub-*palla* will land on *sam*.

In the following example of a *kamali chakradar* (again in *Rupaktal*) the main-*palla* now consists of the phrase:

Tete kata gedi gene kata

GEDI GENE <u>Dha</u> Dha Dha

GEDI GENE Dha <u>Dha</u> Dha

GEDI GENE Dha Dha <u>Dha</u> - - -

The relevant *sam* resolving *Dha*s are underlined.

+1 Te te	2 ka ta	3 ge di	4 ge ne	5 ka ta	6 GE DI	7 GE NE
+1 **Dha Dha**	2 **Dha** GE	3 DI GE	4 NE **Dha**	5 **Dha Dha**	6 GE DI	7 GE NE
+1 **Dha Dha**	2 **Dha** -	3 - -	4 Te te	5 ka ta	6 ge di	7 ge ne
+1 ka ta	2 GE DI	3 GE NE	4 **Dha Dha**	5 **Dha** GE	6 DI GE	7 NE **Dha**
+1 **Dha Dha**	2 GE DI	3 GE NE	4 **Dha Dha**	5 **Dha** -	6 - -	7 Te te
+1 ka ta	2 ge di	3 ge ne	4 ka ta	5 GE DI	6 GE NE	7 **Dha Dha**
+1 **Dha** GE	2 DI GE	3 NE **Dha**	4 **Dha Dha**	5 GE DI	6 GE NE	7 **Dha Dha**
+1 **Dha**	2	3	4	5	6	7

fig. 12 - Kamali Chakradar

A Unified Mathematical Approach

To summarize what has been discussed up to this point, the primary musical function of a tihai is to create a sense of rhythmic tension and anticipation that resolves with a strong sense of arrival or catharsis when it concludes. Tihais come in many varieties. They may start on any beat, or at any point in between any two beats, in any tala, and their *pallas*, though usually equal in length, may be of any arbitrary length. They may resolve to the *sam*, to the half-beat before or after the *sam*, or to other structurally significant points in the tala. There may be a discernable

gap separating the *pallas*, or this space may be so small as to be considered absent. They may also be of a particularly complex, fractal variety, some with internal structural points intricately falling on important beats at designated times. However, despite this rich variety, all tihais share a common structure and can therefore be described by a simple, yet comprehensive mathematical approach that, in turn, yields a powerful practical tool for musicians who want to better understand the notion of tihai: the Tihai Clock.

The first step in realizing this approach requires one to understand what all tihais, regardless of their type, have in common, i.e. that they consist of 3 *pallas* and 2 gaps (see *fig. 5*). Considering that each of the three *pallas* has a starting point and an ending point within the tala, it becomes clear that there are six points of primary importance when mathematical describing a tihai: the beginning and ending points for each of the three *pallas*. These points will be referred to as P_{1a} (1st *palla*'s starting point), P_{1b} (1st *palla*'s ending point), P_{2a} (2nd *palla*'s starting point), P_{2b} (2nd *palla*'s ending point), P_{3a} (3rd *palla*'s starting point), and P_{3b} (3rd *palla*'s ending point).

However, because the three *pallas* in all regular tihais are identical in length, as are the two gaps, only the first three points (P_{1a}, P_{1b}, and P_{2a}) are actually needed to describe *palla* and gap length. In other words, the difference between P_{1a} and P_{1b} represents the length of each of the three *pallas* and the difference between P_{1b} and P_{2a} represents the length of each of the two gaps. Mathematically these statements are: $P_{1b} - P_{1a}$ = *palla* length and $P_{2a} - P_{1b}$ = gap length.

It is through the mathematical relationships between these first three points that tihais can be formulaically described. For instance, consider the standard tihai example that was analyzed earlier in *fig. 3.* Because the first *palla* begins on the *sam*, i.e. beat 1, and ends on beat 5, the values for P_{1a} and P_{1b} would be 1 and 5, respectively. Thus, through the aforemetioned

relationship P_{1b} - P_{1a} (in this case 5 minus 1), a length of 4 beats per *palla* can be calculated and, because there are 3 *pallas* in every tihai, that means there are a total of 12 beats (4 x 3 = 12) of musical material in this example. The amount of gap can be calculated using the same method. Because the second *palla* of this example begins on beat 7, the value for P_{2a} would be 7. Thus, through the relationship P_{2a} - P_{1b} (in this case 7 minus 5) a length of 2 beats per gap can be calculated and, because there are always 2 gaps in a tihai, that means there are a total of 4 beats of gap (2 x 2 = 4) in this example. Adding up the total number of beats of musical material and gaps (12 + 4 = 16) yields the number of same number of beats in tala, which is what one would expect, as the tihai in *fig. 3* spans exactly 1 16-beat cycle of *Tintal*.

However, these first three points (P_{1a}, P_{1b}, and P_{2a}) alone are not sufficient to describe all tihais. One last point, point P_{3b}, is required. P_{3b}, which represents the third *palla*'s ending point, and hence the final point of the entire tihai, is needed in order to include the tihai's resolution into the mathematical description. It is the term P_{3b} that reconciles *sam*, *anagat*, *atit*, and *mukra*-resolving tihais into a single formulaic framework.

With these four points then (P_{1a}, P_{1b}, P_{2a} and P_{3b}), the requisite information that is needed to create a simple but complete mathematical description of tihai is now almost all accounted for, however, one final consideration must be made, that of multi-cycle tihais. A hypothetical example will help illustrate this next point; a tihai in *Tintal* begins on *sam*, but each of its *pallas* is 20 beats long. In this scenario P_{1a} would be given the value of 1, but P_{1b} would be given the value of only 5. This is because the *palla*, being 20 beats long, would span an entire 16-beat cycle, plus an additional 4 beats (20 = 16 + 4), to end on the 5^{th} beat of the second cycle. If the previous mathematical relationship were used to calculate *palla* length (i.e. P_{1b} - P_{1a}), then a

length of 4 beats would be the result (5 - 1 = 4). This result is clearly incorrect because the initial premise was that the *palla* is 20 beats long, not 4 (a discrepancy of 16 beats, or one cycle of the tala).

In order to resolve this problem and to be able to accurately describe tihais that span more than one cycle, it becomes necessary to introduce two additional terms: T, which represents the number of beats in the *tala* (hence the designation "T"), and C_{1b}, which simply represents the number of *completed cycles* (hence the designation "C") of the tala that occur *before* P_{1b} appears (counted from the *sam* of the cycle in which the tihai begins).[2]

P_{1b} (which, in this hypothetical 20-beat long *palla*, has the value of 5) can now be combined with the terms T and C_{1b} to accurately reflect its position in the second cycle in the following way: $TC_{1b} + P_{1b}$. The value of T in this scenario is 16 because there are 16 beats in *Tintal*. The value of C_{1b} is 1 because there is one complete cycle of the tala occurring before P_{1b} appears. The value of P_{1b} is still 5 because that is its position within the tala. This produces the statement (16 x 1) + 5, which gives a value of 21, i.e. 20 beats from beat 1. Thus, the previous relationship used to calculate *palla* length, $P_{1b} - P_{1a}$, is made more accurate by adding the term TC_{1b} in the following way: $TC_{1b} + P_{1b} - P_{1a}$ = *palla* length. This gives the result 16 x 1 + 5 - 1 = 20 beats, i.e. the actual *palla* length.[3]

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[2] Because C_{1b} represents *completed* cycles, its numerical value will always be in the form of a whole integer.

[3] In situations where P_{1b} falls in the first cycle, as in the tihai example of *fig. 3*, the value of C_{1b} will be 0 because 0 *completed* cycles precede P_{1b}. As a result, the term TC_{1b} also becomes 0 because T x 0 = 0. Therefore, in tihais that span only a single cycle, the method of calculating *palla* length just described, i.e. $TC_{1b} + P_{1b} - P_{1a}$, is still valid because the term TC_{1b} simply cancels itself out.

This method of adding TC_{1b} to P_{1b} when calculating *palla* length must also be used when thinking about gap length as well as the final resolution, because the points P_{2a} and P_{3b} may also fall in subsequent cycles beyond the first.[4] In other words, TC_{2a}, which represents the number of completed cycles that occur before P_{2a} appears, must be added to P_{2a} (i.e. $TC_{2a} + P_{2a}$), and TC_{3b}, which represents the number of completed cycles that occur before P_{3b} appears, must be added to P_{3b} (i.e. $TC_{3b} + P_{3b}$), to reflect that these points may fall in subsequent cycles beyond the first.[5] Therefore, to calculate gap length, the previous relationship, $P_{2a} - P_{1b}$, in order to describe multi-cycled tihais, must be changed in the following way: $(TC_{2a} + P_{2a}) - (TC_{1b} + P_{1b})$ = gap length.

At last, with the four points P_{1a}, P_{1b}, P_{2a} and P_{3b}, as well as the terms T, C_{1b}, C_{2a}, and C_{3b}, it is now possible to mathematically describe every possible regular tihai, of any length *palla*, starting from any beat, resolving to any beat, with any amount of gap, and in any tala, with the following formula:

Tihai Formula

$$\mathbf{B + 3P + 2G = F - 1}$$

Where:

B = $\mathbf{P_{1a} - 1}$ = # of beats in the tala from *sam* **B**efore $\mathbf{P_{1a}}$ appears

$\mathbf{P_{1a}}$ = 1st palla's starting **P**oint

$\mathbf{P_{1b}}$ = 1st palla's ending **P**oint

$\mathbf{C_{1b}}$ = # of completed **C**ycles before $\mathbf{P_{1b}}$ appears (always a whole integer)

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[4] It is not necessary to use this method when thinking about point P_{1a} because, as this point is the first in the tihai, it will always fall in the first cycle and hence there will always be zero *completed* cycles before it appears. Thus, the term TC_{1a} will always have a value of zero and is therefore never needed.

[5] The numerical value of C_{2a} and C_{3b} (just as with C_{1b}) will always be in the form of a whole integer.

P_{2a} = 2nd palla's starting **P**oint

C_{2a} = # of completed **C**ycles before **P_{2a}** appears (always a whole integer)

P_{3b} = 3rd palla's ending **P**oint

C_{3b} = # of completed **C**ycles before **P_{3b}** appears (always a whole integer)

T = # of beats in the **T**ala

P = **$TC_{1b} + P_{1b} - P_{1a}$** = # of beats in each **P**ala

G = **$TC_{2a} + P_{2a} - TC_{1b} + P_{1b}$** = # of beats in each **G**ap

F = **$TC_{3b} + P_{3b}$** = **F**inal resolution point of the tihai

The term "B" in this formula (defined as P_{1a} - 1) indicates the distance (if any) between the *sam* of the cycle in which the tihai begins and the beginning of the tihai itself. In other words, this term represents the number of beats from *sam before* the actual tihai begins (hence the designation "B"). For example, if a tihai were to begin on beat 5, then "B" would have a value of 5 minus 1, i.e. there would be 4 beats, from the *sam*, before the tihai starts. If another tihai were to begin directly on *sam*, then the value of P_{1a} in would be 1 and "B" would have a value of 1 minus 1, i.e. zero beats between the *sam* and P_{1a}.

The terms "P" and "G" in this formula represent *palla* and gap length respectively and are calculated as previously explained, i.e. $TC_{1b} + P_{1b} - P_{1a}$ = *palla* length and $(TC_{2a} + P_{2a}) - (TC_{1b} + P_{1b})$ = gap length. The last remaining term to be described is "F" which simply stands for the tihai's *final* resolution point (hence the designation "F") and is defined as $TC_{3b} + P_{3b}$.

The Tihai Clock

What is the purpose of this relatively lengthy discussion of the mathematics of tihai? When students of Hindustani music begin to learn the art of improvisation and start attempting their own tihais, are they thinking in these abstract mathematical terms? Are they juggling algebraic terms like $(TC_{2a} + P_{2a}) - (TC_{1b} + P_{1b})$ in their heads as they try to determine how much gap there should be between *pallas*? Certainly not. Instead, the focus for students at the beginning of this process is probably on tihais that are already known to them, templates that have been handed down by teachers, tried and true phrases and structures that are familiar and comfortable. The next step in this process might be to experiment by making alterations to these well-known tihai in order to come up with something original. The practical process of mastering the art of improvising tihai is necessarily a process of trial and error, so for a student in the midst of such a process what practical value is there in the preceding discussion on tihai mathematics? If the discussion were to simply end at this point, the answer would probably be relatively little, perhaps beyond a purely theoretical novelty. However, the very fact that there is an underlying mathematical unity to the many forms, rich variety, and myriad contexts of tihai allows for the creation of a powerful practical tool for musicians interested in deepening their knowledge of tihai: the Tihai Clock.

The Tihai Clock is a device, with both mechanical and digital embodiments, that consists of a clock-like interface that translates the mathematical inter-*palla* relationships previously discussed into the movements of six clock hands around a circular dial. It is capable of graphically representing every single possible regular tihai, in any tala, of any length *palla*, with

any amount of gap, starting from any point, and resolving to any point. Its original, purely mechanical embodiment is the result of the realisation by the author that the mathematical relationships of tihai *palla* points can be expressed mechanically, using planetary gear ratios. In the digital embodiment that followed, which allows for auditory representation of tihai in addition to the graphical representation of the clock interface, these relationships are written into the programming code.

The cyclic nature of tala in this device is represented by circular dials. Numerals, evenly-spaced around the dials, represent the individual beats of the tala. For example, if one were using the Tihai Clock to analyze tihais in *Tintal*, then the numbers 1 - 16 would be evenly-spaced around the circular dial with the *sam* at the "12 o'clock" position and beat 9 at the "6 o'clock" position. If one were to use the clock to analyze tihais in *Jhaptal*, a 10-beat cycle, then the numbers 1 - 10 would be evenly spaced around the dial with the *sam* at the "12 o'clock" position and beat 6 at the "6 o'clock" position. In the original wooden, mechanical version of this device, there are multiple dials for the various talas that can be physically switched out. In the digital version, the various talas are chosen from a menu and the beats appear evenly-spaced around the dial thanks to the program code.

By using a circular dial to represent the rhythm cycle, it is possible to use a singular method to represent tihais in all the different talas because the mathematics of measuring 3 equal parts within a circle never changes, regardless of how many numbers (i.e. beats) are indicated around the dial. As long as they are evenly-spaced, then regardless of whether the numbers around the dial are 1 - 7, or 1 - 10, or 1 - 16, the method utilized by the Tihai Clock to describe tihais works faithfully in every tala.

The six hands that revolve around the center of the clock face represent the 6 points that define *palla* starting and ending points, just as described before, i.e. P_{1a} (1st *palla*'s starting point), P_{1b} (1st *palla*'s ending point), P_{2a} (2nd *palla*'s starting point), P_{2b} (2nd *palla*'s ending point), P_{3a} (3rd *palla*'s starting point), and P_{3b} (3rd *palla*'s ending point). However, for the sake of simplicity, these points, and the clock hands associated with them, will henceforth be known simply as 1A, 1B, 2A, 2B, 3A, and 3B.

Any one of these six hands can be a point of manipulation (i.e. input) for a user of the Tihai Clock. In other words, a user could manipulate hand 1A to set the starting point of a tihai and (thanks to the gear ratios in the mechanical clock and the programming code in the digital version) the rest of the hands would move according to the mathematical relationships governing tihai (i.e. output). Likewise, the user could then manipulate hand 1B to set the *palla* length and again the other hands would move according to the proportions of tihai. No matter which of the six hands is being manipulated, nor by how much, the Tihai Clock will always show tihais with 3 equally-lengthed *pallas* separated by 2 equally-lengthed gaps. The relative motions of the six hands will be discussed shortly.

Regardless of tala, the default starting position for the hands of the Tihai Clock is as follows: hands 1A, and 1B are at the “12 o’clock” position; hands 2A, and 2B in the “6 o’clock” position; hands 3A, and 3B in the “12 o’clock” position. This position represents the simplest possible tihai spanning exactly 1 cycle of a tala.

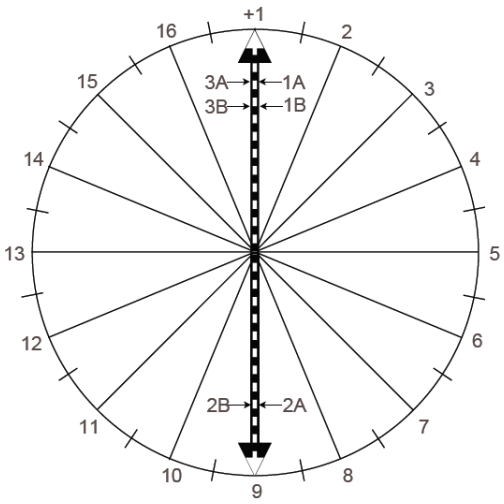

fig. 13 - Tihai Clock Default Starting Position

When, as in this default position, the starting and ending hands of a *palla* are superimposed and pointing at the same beat, it is an indication that the *palla* has the shortest possible length, i.e. it is just a single point. These single-pointed *pallas* can be musically represented by the note *Dha* that has been punctuating the *pallas* in all of the previous tihai examples discussed thus far. For example, if the tala were set to *Tintal*, the tihai represented by the default position of the Tihai Clock can be diagramed by the table below:

+1 **Dha -**	2 - -	3 - -	4 - -
5 - -	6 - -	7 - -	8 - -
9 **Dha -**	10 - -	11 - -	12 - -
13 - -	14 - -	15 - -	16 - -
+1 **Dha**	2	3	4

fig. 14 - Default Tihai in Tintal

Hand Movement Ratios

The way the six hands/*palla* points move in relation to one another is governed by three sets of ratios:

1. “A Motion” (only when manipulating hands 1A, 2A, and 3A):

Δ1A : Δ2A : Δ3A

2. “B Motion” (only when manipulating hands 1B, and 2B):

Δ1B : (-½)Δ2A : (½)Δ2B : (-1)Δ3A

3. “C Motion” (only when manipulating hand 3B):

Δ1A : Δ1B : Δ2A : Δ2B : Δ3A : Δ3B

The ratios here are in reference to motion, i.e. change of position, and so the symbol delta (**Δ**) simply means "change in." Therefore, "A motion," which is defined as Δ1A : Δ2A : Δ3A means that when manipulating hand 1A (or 2A, or 3A) any change in position of that hand will be applied equally to the other 2 hands. Simple examples of these three kinds of *palla* hand motions are diagramed below (all of the following examples will be in *Tintal*, although the same exact ratios governing hand motions will always apply, regardless of tala).

The first is an example of "B motion," which is only used when either hand 1B or hand 2B are being manipulated. In this example, a user moves hand 1B from the default position at the *sam* to beat 5. This means that Δ1B (i.e. change in the position of 1B) is 4 beats. Because of the ratios defining "B Motion," Δ1B : (-½)Δ2A : (½)Δ2B : (-1)Δ3A, this means that Δ3A is minus 4 beats and thus hand 3A will move backward (counter-clockwise) by 4 beats. Finally, according to "B motion," Δ2A and Δ2B will be minus 2, and 2, respectively, meaning that hand 2A will move counter-clockwise by 2 beats, and hand 2B will move clockwise by 2 beats. Hands 1A and 3B remain unaffected by "B motion" so both continue to point towards the *sam* (solid black arrows indicate individual hands, whereas black and white dotted arrows indicate multiple hands pointing to the same position).

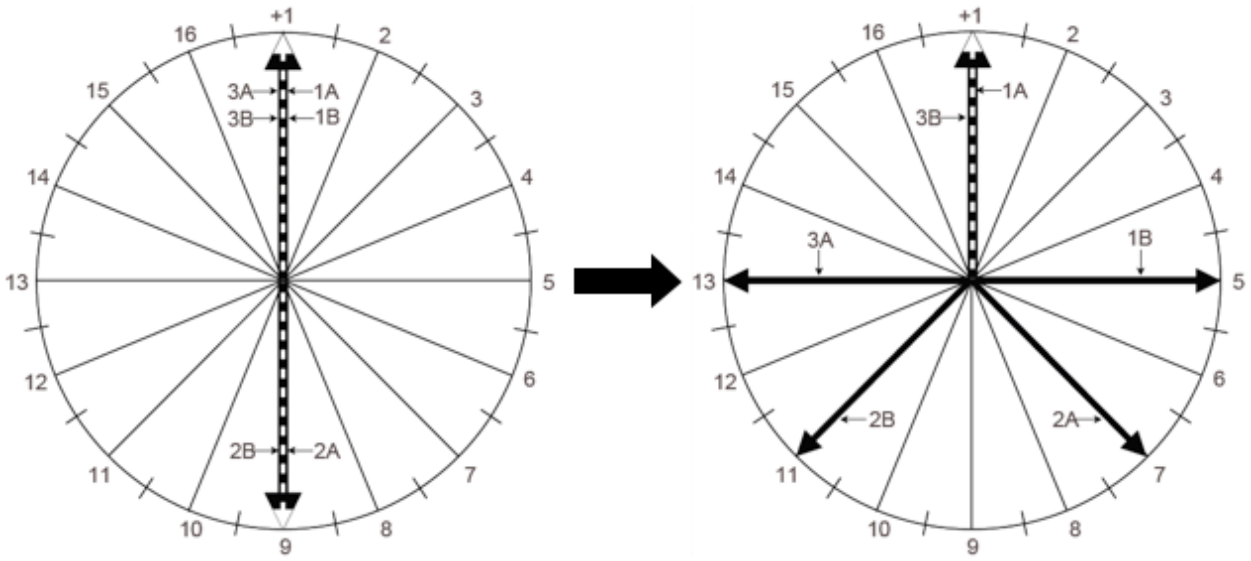

*fig. 15 - "**B Motion**"*: from the default position, user moves hand 1B to beat 5.

Δ1B : (-½)Δ2A : (½)Δ2B : (-1)Δ3A

4 : -2 : 2 : -4

This new position of the hands is a description of the very same tihai that was discussed in *fig. 3.* From this new position, "A motion," which is only used when manipulating hands 1A, 2A, or 3A, will be demonstrated next. The user now moves hand 1A from its position at the *sam* to beat 3, so Δ1A will be 2 beats. Because of the ratios defining "A Motion," Δ1A : Δ2A : Δ3A, only hands 1A, 2A, and 3A will move (hands 1B, 2B, and 3B are unaffected by "A motion"), and they will all move by the exact same amount. Thus hand 2A moves forward 2 beats from beat 7 to beat 9, and hand 3A moves forward 2 beats from beat 13 to beat 15.

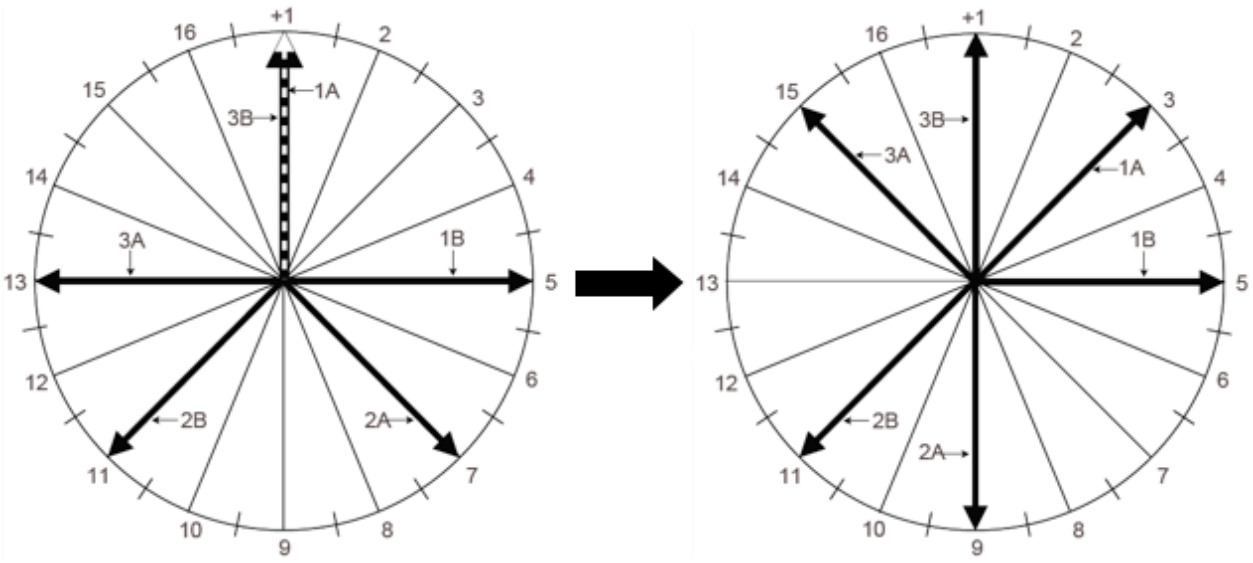

fig. 16 - "**A Motion**": from Example 1 position, user moves hand 1A to beat 3.

Δ1A : Δ2A : Δ3A

2 : 2 : 2

This new position of the hands now represents a tihai that starts on beat 3, that has a *palla* length of 2 beats, that has a gap length of 4 beats, and that resovles to *sam*. From this position, "C motion" will mow be demonstrated. "C motion" is only used when manipulating hand 3B, so this is the kind of motion that is used to affect tihai resolutions, i.e. *anagat*, *atit*, *mukra*, etc. In this next example, the user moves hand 3B from its position at the *sam* counter-clockwise by ½ a beat to beat 16 ½. Thus, Δ3B is minus ½ a beat, and because of the ratios governing "C motion," Δ1A : Δ1B : Δ2A : Δ2B : Δ3A : Δ3B, all of the other 5 hands will move in exactly the same way, i.e. counter-clockwise by ½ a beat.

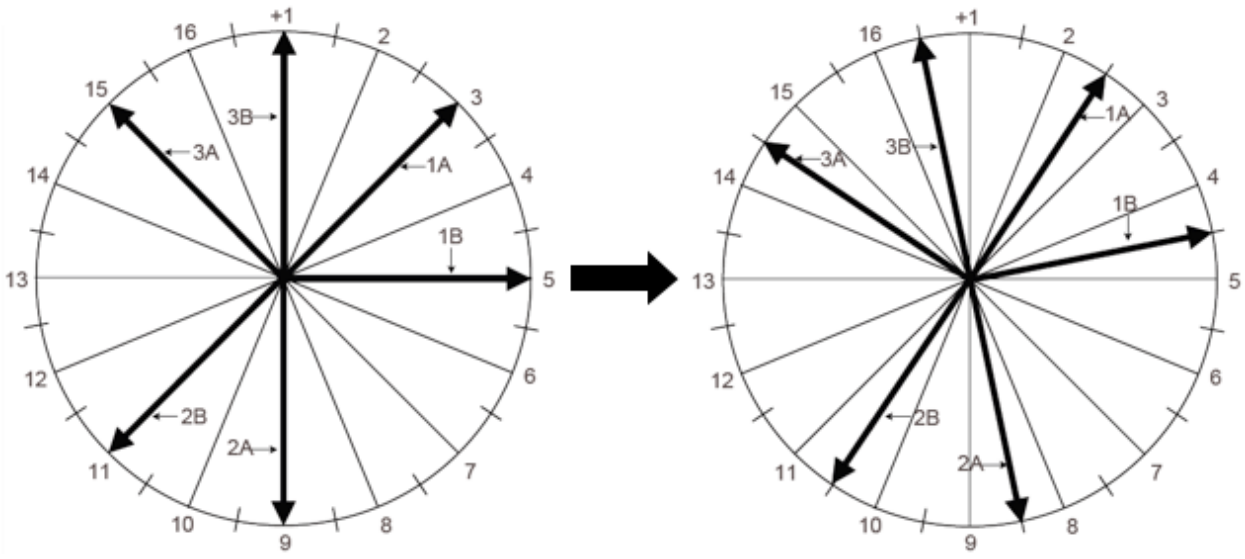

fig. 17 - "**C Motion**": from Example 2 position, user moves hand 3B to beat 16 ½.

Δ1A : Δ1B : Δ2A : Δ2B : Δ3A : Δ3B

-½ : -½ : -½ : -½ : -½ : -½

Now the Tihai Clock is showing a tihai which begins on beat 2 ½, that has a *palla* length of 2 beats, that has a gap length of 4 beats, and that resovles to the half-beat before the *sam*, an *anagat* resolution.

There is one more kind of motion that the Tihai Clock employs to generate tihai. This kind of motion only affects hands 2A and 2B, and represents an inversion of the 2nd *palla* with respect to the tala. In other words, if the user were to employ the invert 2nd *palla* function of the Tihai Clock, then hands 2A and 2B would each revolve exaclty 180º from their original positions. In the example below, which starts from the first tihai example after the discussion of the Tihai Clock default position, the user employs the invert 2nd *palla* function and as a result, hand 2A, which was pointing at beat 7, rotates 180º to point at beat 15, and hand 2B, which was pointing at beat 11, rotates 180º to point at beat 3. In effect, the 2nd *palla* has been inverted from

the bottom of the circle to the top. As mentioned before, the remaining 4 hands are not affected by this movement.

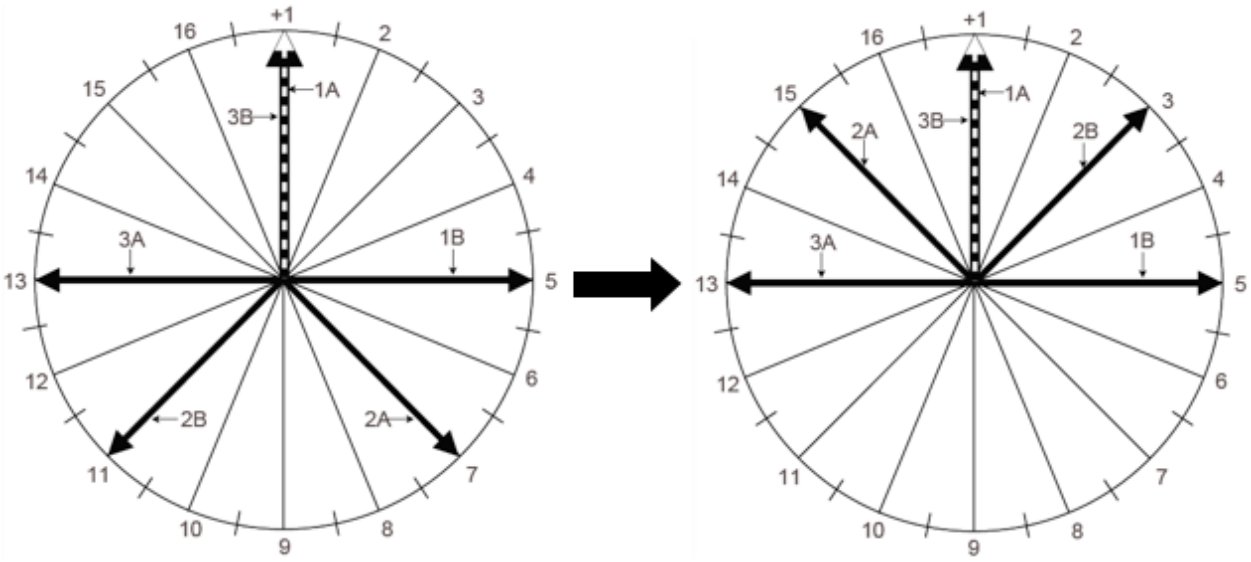

fig. 18 - "**Invert 2nd *palla***": from Example 1 position, hands 2A and 2B each revolve 180° around the clock.

This position of the Tihai Clock's hands represents a tihai that begins on *sam*, has a *palla* length of 4 beats, a gap of 10 beats, and resolves to the *sam* (this tihai spans not 1, but 2 full cycles of the tala).

Through the 3 types of motion discussed, "A," "B," and "C," as well as with the 2nd *palla* inversion function, the Tihai Clock is a complete graphic representation of every possible regular tihai, regardless of type or context. In addition to this visual representation, the digital version of the clock will provided several more features which are of great value to students of Indian music, including audio functions that enable users to actually hear the tihais via samples stored in the program, while also watching the passage of musical time in real-time via indicators

traveling along the dial of the interface.

Conclusion

The tihai and it various forms, exciting to hear as rendered by masters of the tradition, present students of Hindustani music attempting to acquire the skill of improvising tihais, and its many forms, with a daunting task. However, an analysis of the fundemental mathematics of tihai, though at first glance seemingly arcane and esoteric, ultimately yeilds an elegant and immensely practical tool for comprehending both the variety and unity of tihai: the Tihai Clock.

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