COHRINT: Cooperative Human-Robot Intelligence

GPS-Aided Navigation Approach

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Contents

1 Overview

A GPS-aided navigation filter uses high-rate inertial navigation sensors (INS) combined with correcting GPS measurements to estimate, among other states, the dynamic position of a system (e.g., vehicle, human). In this example, a Kalman Filter is used to estimate perturbations from *nominal* states. These nominal states consist of the geographic-frame position (i.e., lattitude, longitude, and height), navigation-frame velocity (i.e., velocity in the north, east, and down directions), the rotation matrix from the body to navigation frame, and the measurement corrections required to relate direct INS measurements to true inertial behaviors. The nominal states are propagated forward through mechanization equations given the accelerometer and gyroscope (i.e., INS) measurements [1]. One can think of the nominal states as the "dead-reckoning" solution. The accelerometer and gyro measurements are also used to drive the dynamics of the Kalman filter states; think of them as the system's "control inputs". The states of the filter are perturbations from the nominal state used to obtain estimated corrected states. This type of approach is used because the dynamics of the desired states (e.g., latitude, longitude) are in general nonlinear, and so they must be linearized about a nominal state and applied to small perturbations. After the new perturbation states are predicted given the INS measurements, new full position estimates can be made, which are compared to the position output by the GPS receiver. The difference between the position states predicted by the INS solution and those measured by the GPS receiver is the "measurement" in the measurement update step of the Kalman filter. Based on this difference and the uncertainty in the GPS measurement (which is also output by certain GPS receivers), the estimated perturbation states are updated. Combined with the nominal states, these estimates provide an updated belief of the true position, velocity, and orientation of the system. In the open-loop version of the navigation filter, which will be further described in this write-up, the perturbation states are *not* fed back to correct the nominal states, as in the closed-loop version.

2 Coordinate Systems

The primary coordinate systems used throughout the navigation filter are the Earth-Centered Inertial (ECI) frame, Earth-Centered Earth-Fixed (ECEF) frame, Tangential frame (the *Navigation* frame), Geographic frame, and Body frame. These coordinate systems are described below in Figures 1 - 3.

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[1]Note that *nominal* accelerometer and gyro measurement corrections remain at zero, since there is no way to directly measure these corrections and so they cannot be propagated forward.
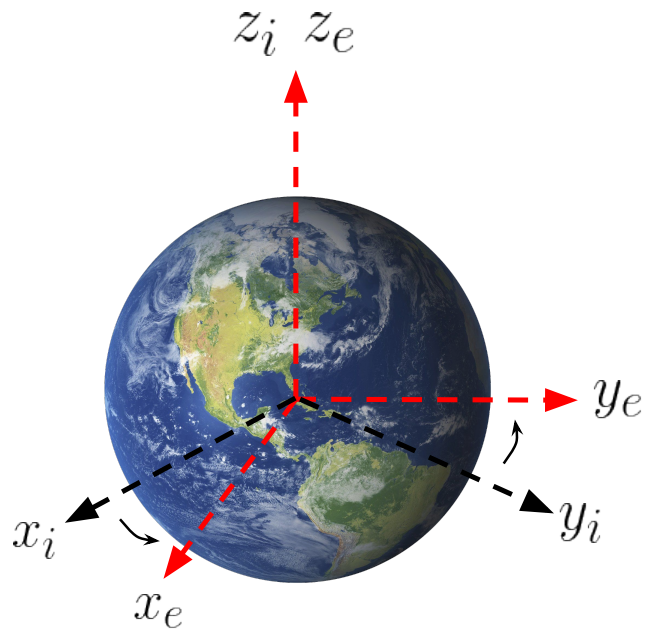
Figure 1: The Earth-Centered Inertial (ECI) and *Earth-Centered Earth-Fixed (ECEF)* frame. The ECI Frame is static, while the ECEF Frame rotates with the Earth about the z_e axis (i.e., Earth's rotation axis).
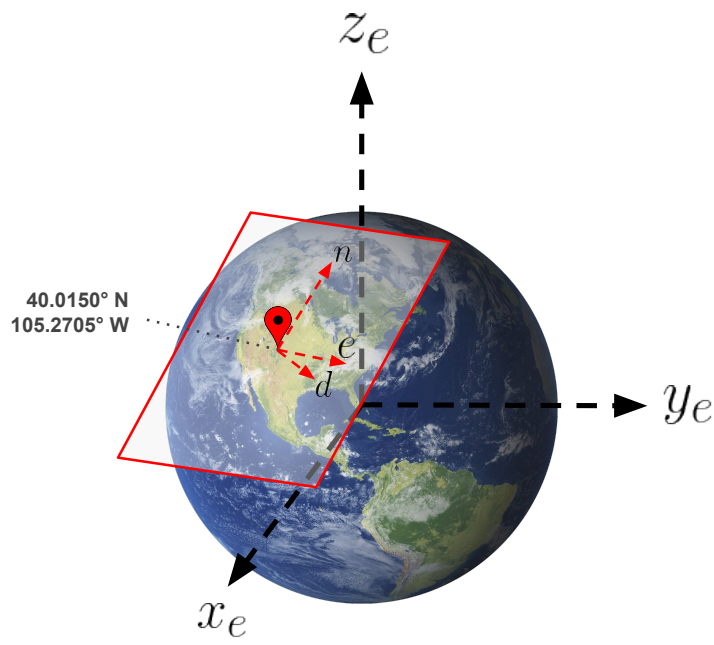

Figure 2: The ECEF and Tangential (North-East-Down) frame - i.e., the *Navigation* frame. The Tangential frame is a plane intersecting the Earth reference ellipsoid at a particular location, with basis vectors in the north/east/down directions defined at that location.
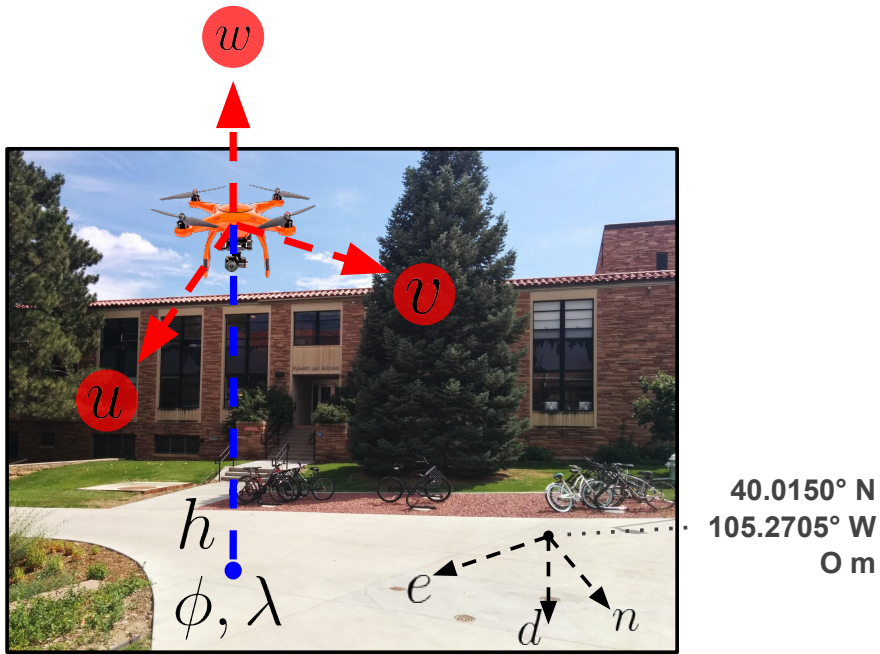
Figure 3: The Navigation, Body , and Geographic frames. The Geographic frame is the projection of the vehicle's origin onto the Earth reference ellipsoid. It has latitude, longitude, and height components.

3   Terminology

The arbitrary variable α will be used to identify key terminologies used throughout this write-up.

- The term α represents the **true value** of that particular state.

- The **nominal state** is written as $\hat{\alpha}$.

- If α described a sensor measurement, the **direct output** of the sensor would be $\tilde{\alpha}$.

- A **correction** term for $\tilde{\alpha}$ to achieve the true values acting on the system would be $\Delta\alpha$ (e.g., $\mathbf{f}^b = \tilde{\mathbf{f}}^b - \Delta\mathbf{f}^b$).

- The **perturbation** of the variable of interest $\{\alpha\}$ is written as $\delta\alpha = \{\alpha\} - \{\hat{\alpha}\}$ (e.g., $\delta\alpha = \alpha - \hat{\alpha}$ or $\delta\alpha = \Delta\alpha - \Delta\hat{\alpha}$).

- The **estimation error** is the difference between the true value, α, and the estimated corrected value, $\hat{\alpha} + \delta\alpha$.

4 Variable Definitions

System state variables: The following states represent the actual behavior of the system.

- $\mathbf{p} = [\theta, \lambda, h]^T \to$ geographic frame position vector, consisting of the latitude, longitude, and height of vehicle

- $\mathbf{v}_e^n = [v_n, v_e, v_d]^T \to$ earth-relative velocity vector of vehicle, represented in the navigation frame (i.e., in "east-north-down" coordinates)

- $\mathbf{R}_b^n \to$ rotation matrix bringing vectors in the vehicle body frame to the navigation frame

- $\mathbf{f}^b \to$ accelerometer reading (i.e., force), represented in the vehicle body frame

- $\omega_{ib}^b \to$ gyroscope reading (i.e., angular velocity of the vehicle body frame w.r.t. the inertial frame), represented in the body frame

- $\Delta \mathbf{f}^b \to$ accelerometer reading error in the body frame

- $\Delta \omega_{ib}^b \to$ gyroscope reading error in the body frame

- $\mathbf{x}_a \to$ modeled accelerometer error states, related to the accelerometer reading correction by $\Delta \mathbf{f}^b = \mathbf{F}_{va} \mathbf{x}_a$

- $\mathbf{x}_g \to$ modeled gyroscope error states, related to the gyroscope reading correction by $\Delta \omega_{ib}^b = \mathbf{F}_{\rho g} \mathbf{x}_g$

Direct Sensor Readings: Raw measurement readings directly outputted by sensors.

- $\tilde{\mathbf{f}}^b \to$ direct accelerometer reading, $\hat{\mathbf{f}}^b = \tilde{f}^b - \Delta \hat{\mathbf{f}}^b$.

- $\tilde{\omega}_{ib}^b \to$ direct gyroscope reading, $\hat{\omega}_{ib}^b = \tilde{\omega}_{ib}^b - \Delta \hat{\omega}_{ib}^b$.

- $\tilde{\mathbf{y}}_{K+1}^{GPS} = [\tilde{\theta}, \tilde{\lambda}, \tilde{h}]^T \to$ direct GPS reading of vehicle's latitude, longitude, and height

- $\tilde{\mathbf{P}}_{k+1}^{GPS} \to$ direct GPS measurement noise covariance (i.e., measurement uncertainty), reported to user through GPS module

INS Error States: states being estimated through navigation filter, representing deviations of the nominal states from the true states.

- $\delta \mathbf{p} = [\delta\theta, \delta\lambda, \delta h]^T \to$ position perturbation states (in geographic frame), $\delta \mathbf{p} = \mathbf{p} - \hat{\mathbf{p}}$

- $\delta \mathbf{v} = [\delta v_n, \delta v_e, \delta v_d]^T \to$ velocity perturbation states (relative to Earth, in navigation frame), $\delta \mathbf{v} = \mathbf{v}_e^n - \hat{\mathbf{v}}_e^n$

- $\rho = [\epsilon_N, \epsilon_E, \epsilon_D] \rightarrow$ *small-angle* rotations bringing true body-to-navigation rotation matrix to belief[2], $\hat{\mathbf{R}}_b^n = (\mathbf{I} - [\rho\times])\mathbf{R}_b^n$

- $\delta\mathbf{f}^b \rightarrow$ perturbation states of accelerometer reading correction term, $\delta\mathbf{f}^b = \Delta\mathbf{f}^b - \Delta\hat{\mathbf{f}}^b$

- $\delta\omega_{ib}^b \rightarrow$ perturbation states of gyroscope reading correction term, $\delta\omega_{ib}^b = \Delta\omega_{ib}^b - \Delta\hat{\omega}_{ib}^b$

- $\delta\mathbf{x}_a = [\mathbf{x}_{b_a}^T, \mathbf{x}_{A_a}^T, \mathbf{x}_{ka}^T] \rightarrow$ perturbations of modeled accelerometer error states, consisting of bias, scale factor and misalignment, and nonlinearity terms, respectively. $\delta\mathbf{f}^b = \mathbf{F}_{va}\delta\mathbf{x}_a + \nu_a$. Note that $\hat{\mathbf{x}}_{b_a}^T, \hat{\mathbf{x}}_{A_a}^T$, and $\hat{\mathbf{x}}_{ka}^T = \mathbf{0}$.

- $\delta\mathbf{x}_g = [\mathbf{x}_{b_g}^T, \mathbf{x}_{A_g}^T, \mathbf{x}_{kg}^T] \rightarrow$ perturbations of modeled gyroscope error states, consisting of bias, scale factor and misalignment, and g-sensitivity terms, respectively. $\delta\omega_{ib}^b = \mathbf{F}_{\rho g}\delta\mathbf{x}_g + \nu_g$. Note that $\hat{\mathbf{x}}_{b_g}^T, \hat{\mathbf{x}}_{A_g}^T$, and $\hat{\mathbf{x}}_{kg}^T = \mathbf{0}$.

- $\delta\mathbf{x} = [\delta\mathbf{p}, \delta\mathbf{v}, \rho, \delta\mathbf{x}_a, \delta\mathbf{x}_g] \rightarrow$ state vector used in navigation filter (33 states total), $\dot{\delta\mathbf{x}}(t) = \mathbf{F}(t)\delta\mathbf{x}(t) + \Gamma\mathbf{q}$

- $\nu_a \rightarrow$ accelerometer reading process noise

- $\nu_g \rightarrow$ gyroscope reading process noise

- $\omega_a \rightarrow$ process noise of modeled accelerometer perturbation states, $\dot{\delta\mathbf{x}}_a = \mathbf{F}_{aa}\delta\mathbf{x}_a + \omega_a$

- $\omega_g \rightarrow$ process noise of modeled gyroscope perturbation states, $\dot{\delta\mathbf{x}}_g = \mathbf{F}_{gg}\delta\mathbf{x}_g + \omega_g$

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[2]Note that $\hat{\rho} = \mathbf{0}$ since the nominal state of these small-angle rotations remain zero (i.e., they aren't propagated forward in the mechaniation equations), so $\delta\rho = \rho - \hat{\rho} = \rho$.
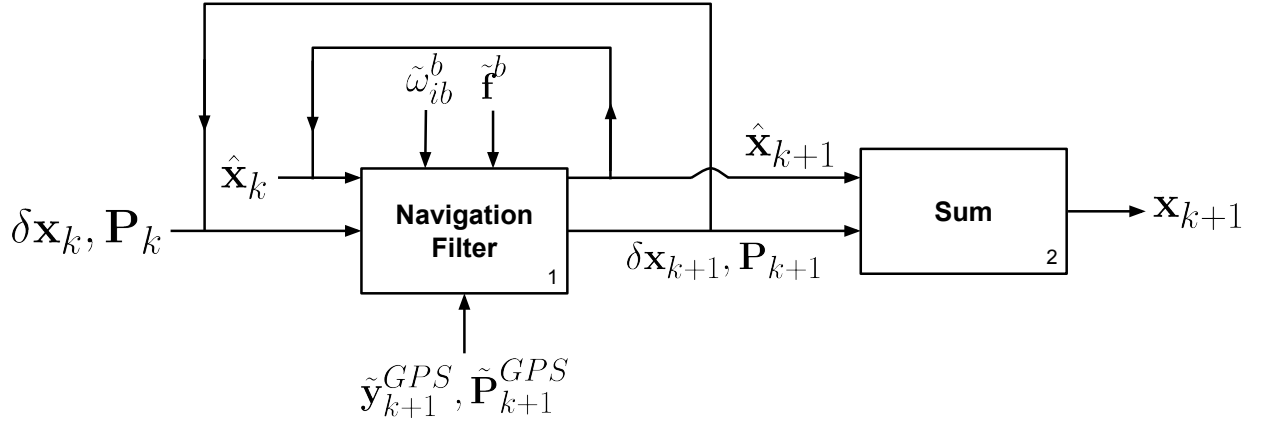
5 Block Diagrams

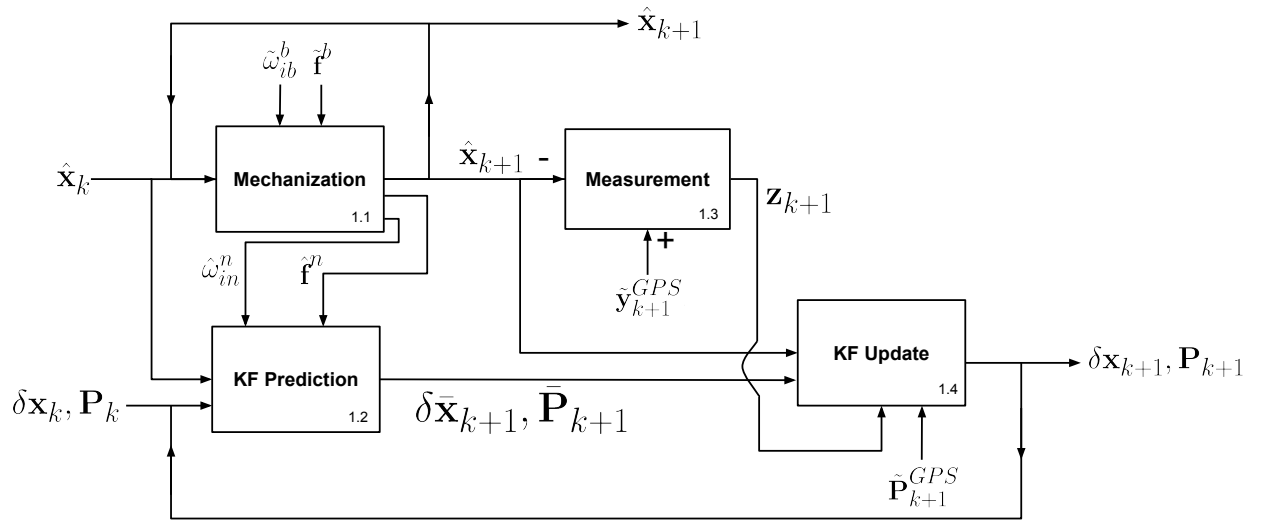*Figure 4: Navigation filter high-level block diagram.*

Figure 5: Navigation filter lower-level block diagram (expansion of Block 1).
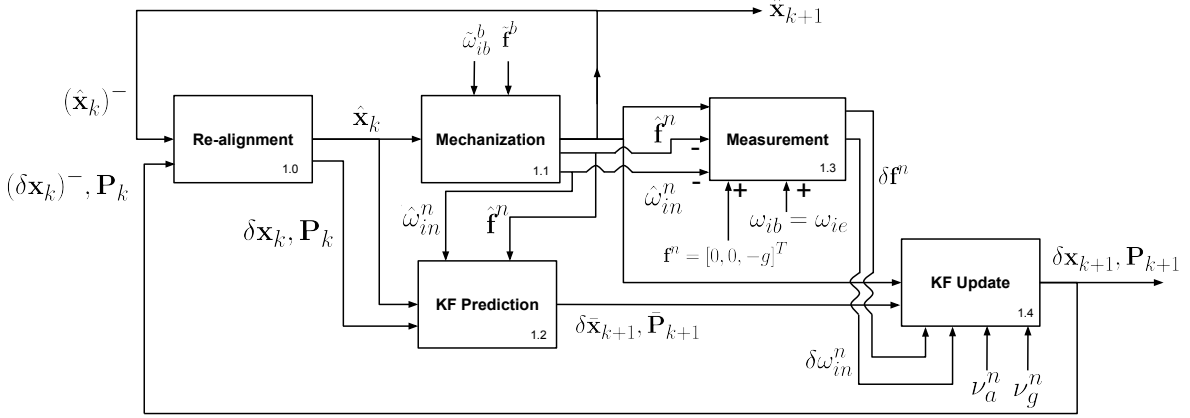
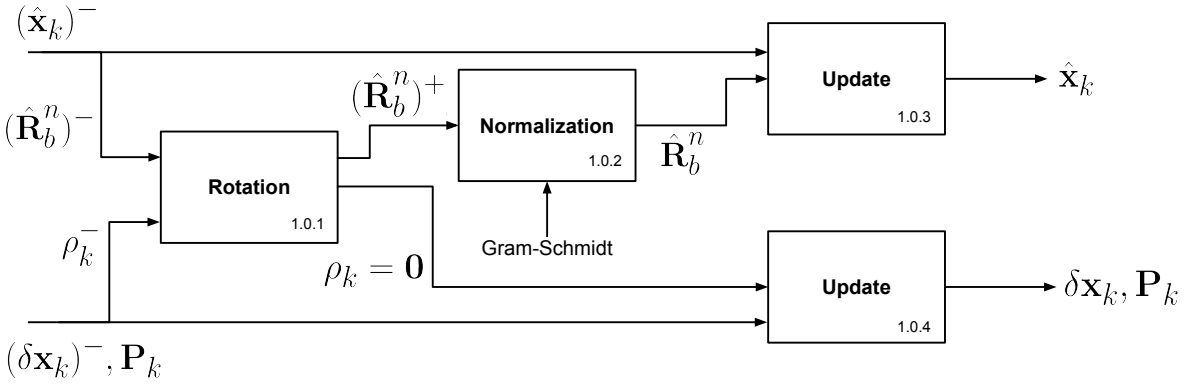*Figure 6: Self-Alignment filter block diagram.*

Figure 7: Self-Alignment lower-level block diagram (expansion of Block 1.0).

6   Detailed Description

6.1   Mechanization (Strap-Down) Equations

Position vector:

$$
\begin{bmatrix} \dot{\hat{\phi}} \\ \dot{\hat{\lambda}} \\ \dot{\hat{h}} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_m + \hat{h}} & 0 & 0 \\ 0 & \frac{1}{\cos(\hat{\phi})(R_n + \hat{h})} & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_e \\ \hat{v}_d \end{bmatrix} \tag{1}
$$

Velocity vector:

$$\begin{bmatrix} \dot{\hat{v}}_n \\ \dot{\hat{v}}_e \\ \dot{\hat{v}}_d \end{bmatrix} = \begin{bmatrix} \hat{f}_n \\ \hat{f}_e \\ \hat{f}_d \end{bmatrix} + \mathbf{g}^n + \begin{bmatrix} 0 & -\omega_d & \omega_e \\ \omega_d & 0 & -\omega_n \\ -\omega_e & \omega_n & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_e \\ \hat{v}_d \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \omega_n \\ \omega_e \\ \omega_d \end{bmatrix} = (\omega_{en}^n + 2\omega i_e{}^n) = \begin{bmatrix} (\dot{\hat{\lambda}} + 2\omega_{ie})cos(\hat{\phi}) \\ -\dot{\hat{\phi}} \\ -(\dot{\hat{\lambda}} + 2\omega_{ie})sin(\hat{\phi}) \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \hat{f}_n \\ \hat{f}_e \\ \hat{f}_d \end{bmatrix} = \hat{\mathbf{f}}^n = \hat{\mathbf{R}}_b^n \hat{\mathbf{f}}^b \tag{4}$$

Rotation matrix:

$$\dot{\hat{\mathbf{R}}}_b^n = \hat{\mathbf{R}}_b^n([\hat{\omega}_{ib}^b \times] - [\hat{\omega}_{in}^b \times]) \tag{5}$$

$$\hat{\omega}_{in}^b = \hat{\mathbf{R}}_n^b \hat{\omega}_{in}^n \tag{6}$$

$$\hat{\omega}_{in}^n = \begin{bmatrix} (\dot{\hat{\lambda}} + \omega_{ie})cos(\hat{\phi}) \\ -\dot{\hat{\phi}} \\ -(\dot{\hat{\lambda}} + \omega_{ie})sin(\hat{\phi}) \end{bmatrix} \tag{7}$$

6.2   Navigation Filter State Dynamics Equations

$$\begin{bmatrix} \delta\dot{\mathbf{p}} \\ \delta\dot{\mathbf{v}} \\ \dot{\rho} \\ \delta\dot{\mathbf{x}}_a \\ \delta\dot{\mathbf{x}}_g \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{pp} & \mathbf{F}_{pv} & \mathbf{F}_{p\rho} & \mathbf{0} & \mathbf{0} \\ \mathbf{F}_{vp} & \mathbf{F}_{vv} & \mathbf{F}_{v\rho} & -\hat{\mathbf{R}}_b^n\mathbf{F}_{va} & \mathbf{0} \\ \mathbf{F}_{\rho p} & \mathbf{F}_{\rho v} & \mathbf{F}_{\rho\rho} & \mathbf{0} & \hat{\mathbf{R}}_b^n\mathbf{F}_{\rho g} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{aa} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{gg} \end{bmatrix} \begin{bmatrix} \delta\mathbf{p} \\ \delta\mathbf{v} \\ \rho \\ \delta\mathbf{x}_a \\ \delta\mathbf{x}_g \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\hat{\mathbf{R}}_b^n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{R}}_b^n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \nu_a \\ \nu_g \\ \omega_a \\ \omega_g \end{bmatrix} \tag{8}$$

The submatrices $\mathbf{F}_{ij}$ above are found by linearized the state derivatives about the cooresponding nominal states. For example, the derivation of $\mathbf{F}_{pp}, \mathbf{F}_{pv}$, and $\mathbf{F}_{p\rho}$ is provided below by linearizing $\dot{\mathbf{p}}$ about $\hat{\mathbf{p}}$:

$$\dot{\mathbf{p}} = \begin{bmatrix} \frac{1}{R_m+\hat{h}} & 0 & 0 \\ 0 & \frac{1}{\cos(\hat{\phi})(R_n+\hat{h})} & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_e \\ \hat{v}_d \end{bmatrix} = \begin{bmatrix} \frac{\hat{v}_n}{R_m+\hat{h}} \\ \frac{v_e}{\cos\hat{\phi}(R_m+\hat{h})} \\ -\hat{v}_d \end{bmatrix} = f_p(\hat{\mathbf{p}}, \hat{\mathbf{v}}) \tag{9}$$

$$\dot{\mathbf{p}} = f_p(\hat{\mathbf{p}}, \hat{\mathbf{v}}) + \left(\frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \mathbf{p}}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}\right)\delta\mathbf{p} + \left(\frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \mathbf{v}}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}\right)\delta\mathbf{v} + \left(\frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \rho}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}\right)\rho \tag{10}$$

$$\delta\dot{\mathbf{p}} = \dot{\mathbf{p}} - \dot{\hat{\mathbf{p}}} = \mathbf{F}_{pp}\delta\mathbf{p} + \mathbf{F}_{pv}\delta\mathbf{v} + \mathbf{F}_{p\rho}\rho \tag{11}$$

$$\mathbf{F}_{pp} = \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \mathbf{p}}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}} = \left[\frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \phi}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}, \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \lambda}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}, \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta h}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}\right] \tag{12}$$

$$= \begin{bmatrix} 0 & 0 & \frac{-\hat{v}_n}{(R_m+\hat{h})^2} \\ \frac{\hat{v}_e \sin(\hat{\phi})}{\cos^2(\hat{\phi})(R_n+\hat{h})} & 0 & \frac{-\hat{v}_e}{\cos(\hat{\phi})(R_n+\hat{h})^2} \\ 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$\mathbf{F}_{pv} = \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \mathbf{v}}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}} = \left[\frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta v_n}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}, \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta v_e}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}, \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta v_d}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}}\right] \tag{14}$$

$$= \begin{bmatrix} \frac{1}{R_m+\hat{h}} & 0 & 0 \\ 0 & \frac{1}{\cos(\hat{\phi})(R_n+\hat{h})} & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{15}$$

$$\mathbf{F}_{p\rho} = \frac{\delta f_p(\mathbf{p}, \mathbf{v})}{\delta \rho}\Big|_{\hat{\mathbf{p}}, \hat{\mathbf{v}}, \hat{\rho}} = \mathbf{0} \tag{16}$$

The derivation and solutions of all other submatrices $\mathbf{F}_{ij}$ in Equation 8 can be found in Sections 11.4 (pp. 392-396) and 11.6 (pp. 406-413) of Jay Farrell's Aided Navigation [1]. The solutions are provided below in terms of the variables defined in Figure 8. Note the approximation $R_n = R_m = R_e$ and that all *hat* symbols are omitted for further simplification. Also, the submatrices associated with accelerometer and gyro perturbations vary depending on the modeled INS error states; those provided below neglect accelerometer nonlinearity terms ($\delta\mathbf{x}_{ka}$) and gyroscope g-sensitivity terms ($\delta\mathbf{x}_{kg}$), and assume that variations in the error states are independent of the previous state.

$\Omega_N = \omega_{ie} \cos(\phi)$	$F_{41} = -2\Omega_N v_e - \frac{\rho_N v_e}{\cos^2(\phi)}$
$\Omega_D = -\omega_{ie} \sin(\phi)$	$F_{43} = \rho_E k_D - \rho_N \rho_D$
$\rho_N = \frac{v_e}{R_e}$	$F_{51} = 2(\Omega_N v_n + \Omega_D v_d) + \frac{\rho_N v_n}{\cos(\phi)^2}$
$\rho_E = \frac{-v_n}{R_e}$	$F_{53} = -\rho_E \rho_D - k_D \rho_N$
$\rho_D = \frac{-v_e \tan(\phi)}{R_e}$	$F_{54} = -(\omega_D + \Omega_D)$
$\omega_N = \Omega_N + \rho_N$	$F_{55} = k_D - \rho_E \tan(\phi)$
$\omega_E = \rho_E$	$F_{56} = \omega_N + \Omega_N$
$\omega_D = \Omega_D + \rho_D$	$F_{63} = \rho_N^2 + \rho_E^2 - 2\frac{g}{R_e}$
$k_D = \frac{v_d}{R_e}$	$F_{91} = \Omega_N + \frac{\rho_N}{\cos(\phi)^2}$

Figure 8: Definition of variables used to simplify state dynamics transition matrices.

6.3 Navigation Filter Measurement Equation

The "measurement" used in the Kalman Filter measurement update equation is the difference in ECEF position between the GPS measurement and predicted INS nominal states:

$$\mathbf{z}_{k+1} = \delta\mathbf{p}^e = [\delta x, \delta y, \delta z] \tag{17}$$
$$= \tilde{\mathbf{y}}_{k+1}^{GPS} - (\hat{\mathbf{p}})^e \tag{18}$$

This measurement is related to the geographic-frame perturbation states as follows:

$$\delta\mathbf{p}^e = \mathbf{R}_n^e \mathbf{D} \delta\mathbf{p}, \tag{19}$$

where, upon linearizing the relationships between ϕ, λ, h and ECEF x, y, z, it can be shown that

$$\mathbf{D} = \begin{bmatrix} (R_m + h) & 0 & 0 \\ 0 & \cos(\phi)(R_n + h) & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{20}$$

Given the entire state space $\delta\mathbf{x} = [\delta\mathbf{p}, \delta\mathbf{v}, \rho, \delta\mathbf{x}_a, \delta\mathbf{x}_g]$, the measurement matrix $\mathbf{H}$ used in the Kalman Filter update equations is

$$\mathbf{H} = [\mathbf{R}_n^e \mathbf{D}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}] \tag{21}$$

6.4 Self-Alignment Filter Equations

The states used for the self-alignment filter are exactly the same as those of the navigation filter: $\delta\mathbf{x} = [\delta\mathbf{p}, \delta\mathbf{v}, \rho, \delta\mathbf{x}_a, \delta\mathbf{x}_g]$. Therefore the mechanization and state dynamics

equations are identical, besides the closed-loop nature of the rotation matrix correction; after each iteration, the estimated small-angle rotations ρ^- are applied to correct the nominal body-to-navigation rotation matrix, $(\hat{\mathbf{R}}_b^n)^-$, before the next filter iteration:

$$\hat{\mathbf{R}}_b^n = (\mathbf{I} + [\rho\times])(\hat{\mathbf{R}}_b^n)^- \tag{22}$$

After correcting the rotation matrix, ρ is reset to zeros for the next iteration. The measurements used during self-alignment are the exact, known zero position (if the system is at a surveyed location) and/or velocity (if the system remains still). Therefore, the position measurement equations are identical to those used in the ordinary navigation filter,

$$\delta\mathbf{p}^e = \mathbf{R}_n^e \mathbf{D}\delta\mathbf{p}, \tag{23}$$

while the velocity measurement equations are simply:

$$\mathbf{0} - (\hat{\mathbf{v}}) = \delta\mathbf{v} \tag{24}$$

Therefore, the measurement matrix $\mathbf{H}$ used in the self-alignment Kalman Filter update equation is

$$\mathbf{H} = [\mathbf{R}_n^e \mathbf{D}, \mathbf{I}, \mathbf{0}, \mathbf{0}, \mathbf{0}] \tag{25}$$

7   Simulation Example Results

A simulation was ran for a vehicle with constant radial acceleration and angular velocity (i.e., a spiral motion). Refer to Figures 9, 10, 11 for the truth trajectory (in the north-east plane) and simulated GPS measurements, simulated accelerometer measurements, and simulated gyroscope measurements, respectively.

The navigation filter results in the north-east plane are provided below in Figure 12.

Last, the estimation errors of all 33 states are provided below in Figures 13 - 20.
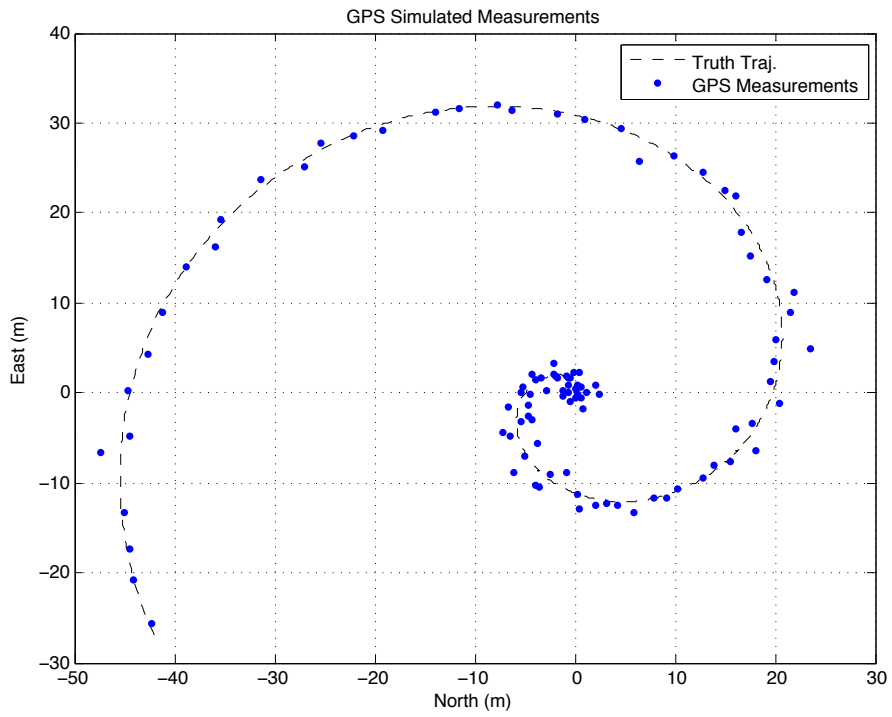
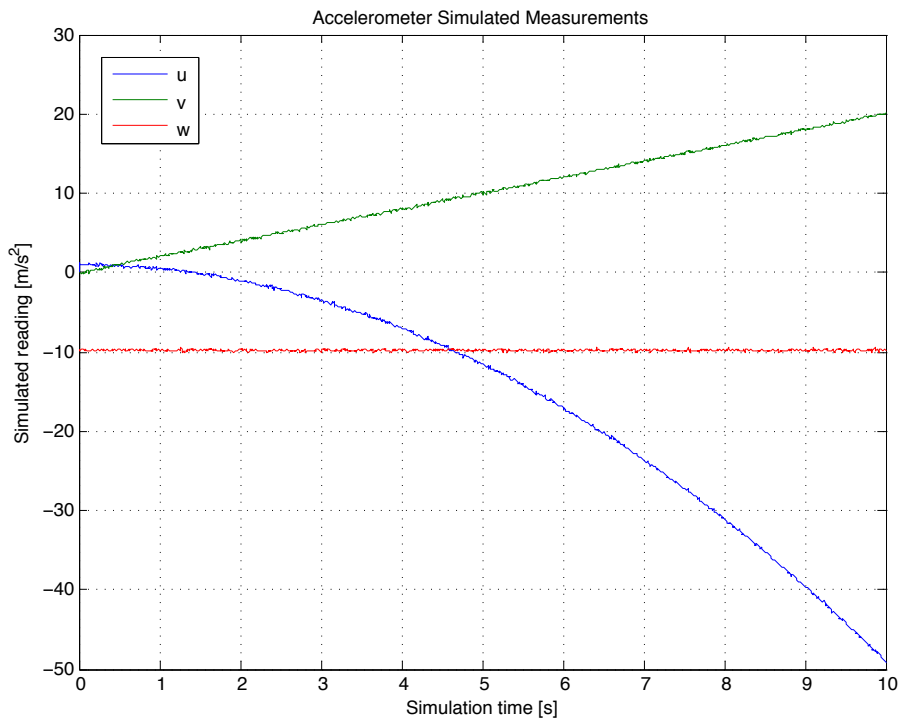*Figure 9: Truth motion with simulated GPS measurements.*

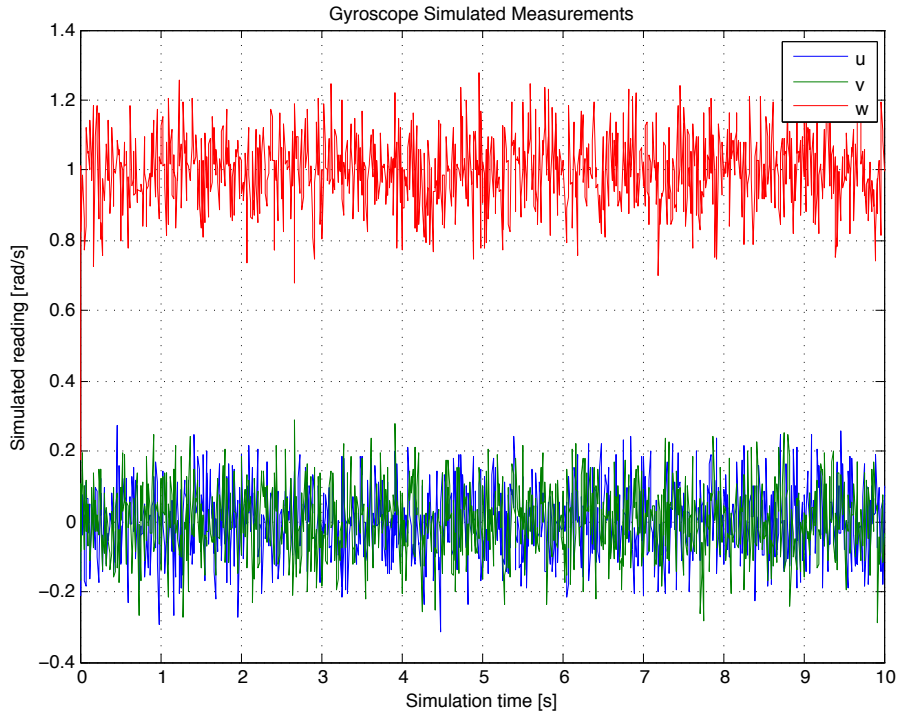*Figure 10: Simulated accelerometer measurements.*

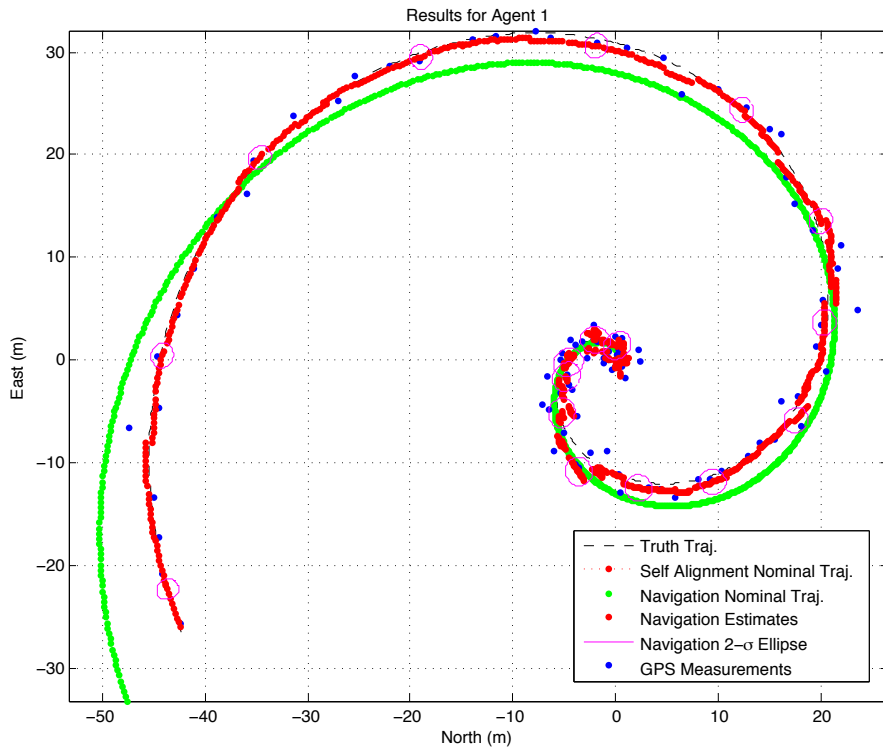*Figure 11: Simulated gyroscope measurements.*

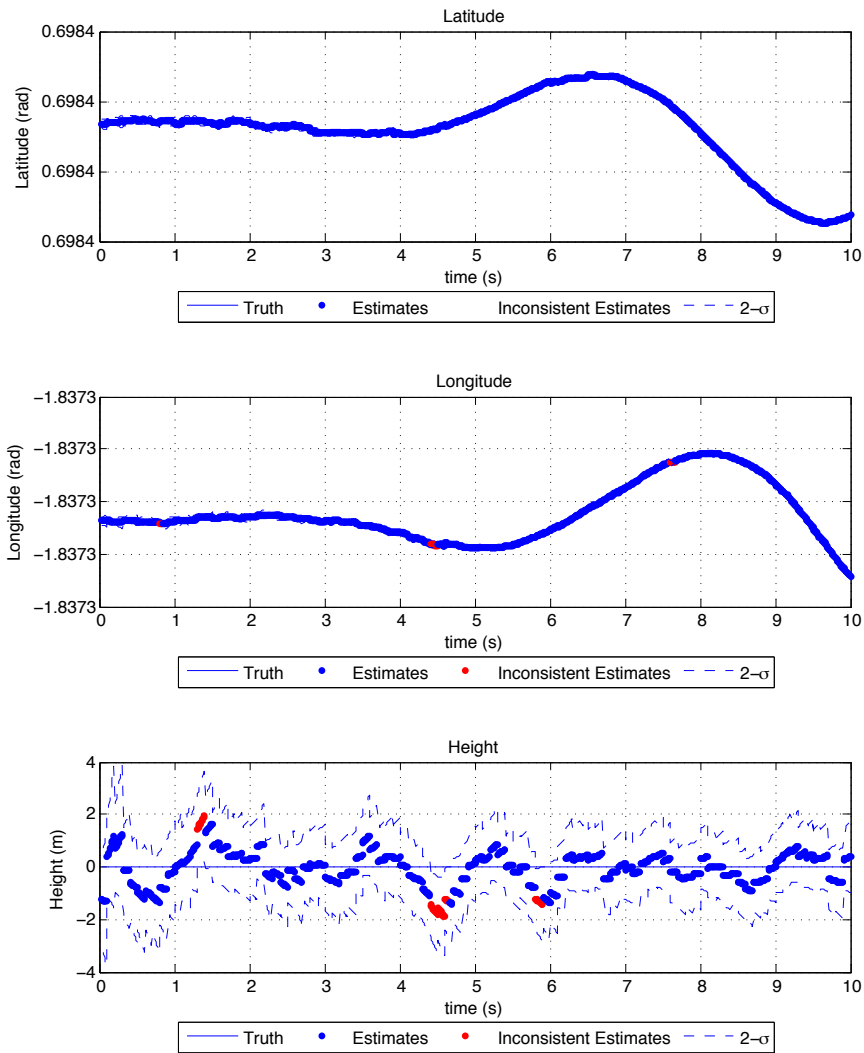*Figure 12: Navigation filter results in the North-East plane.*

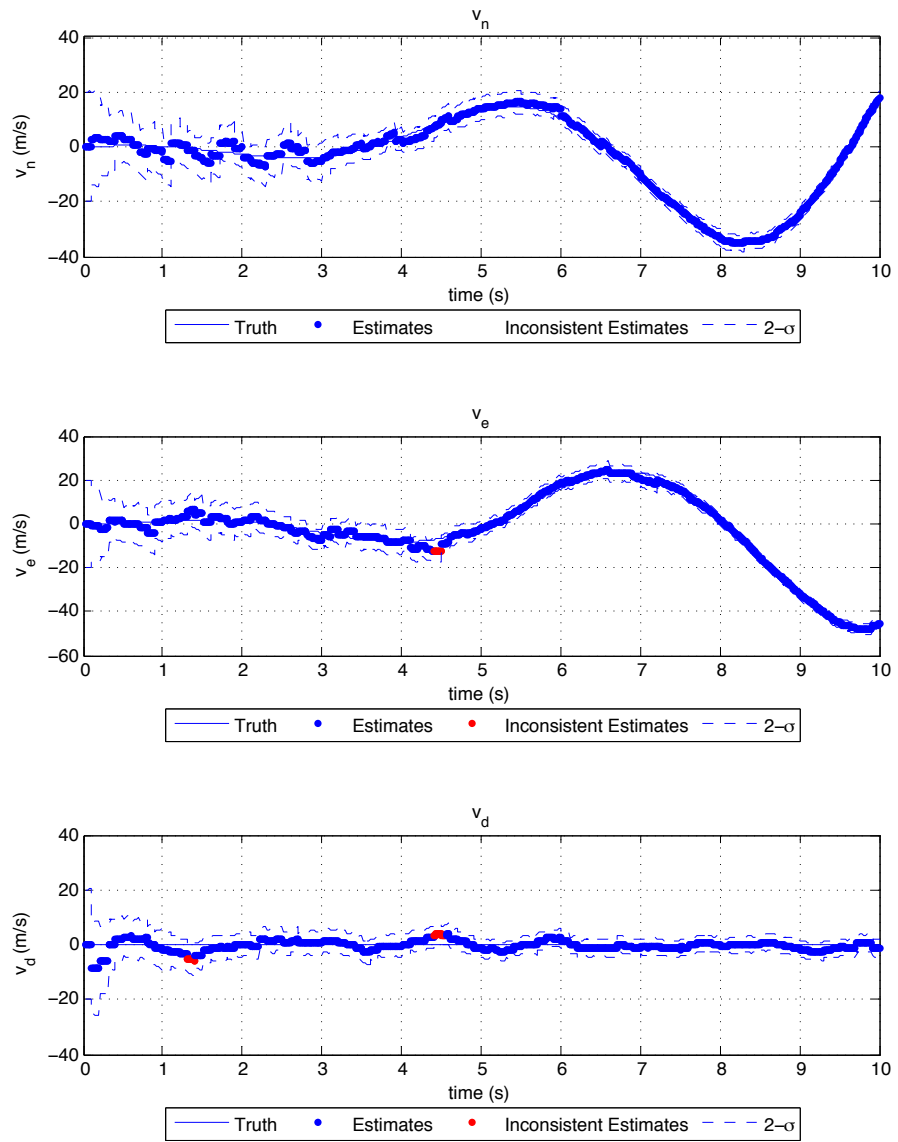Figure 13: Position state estimation errors.
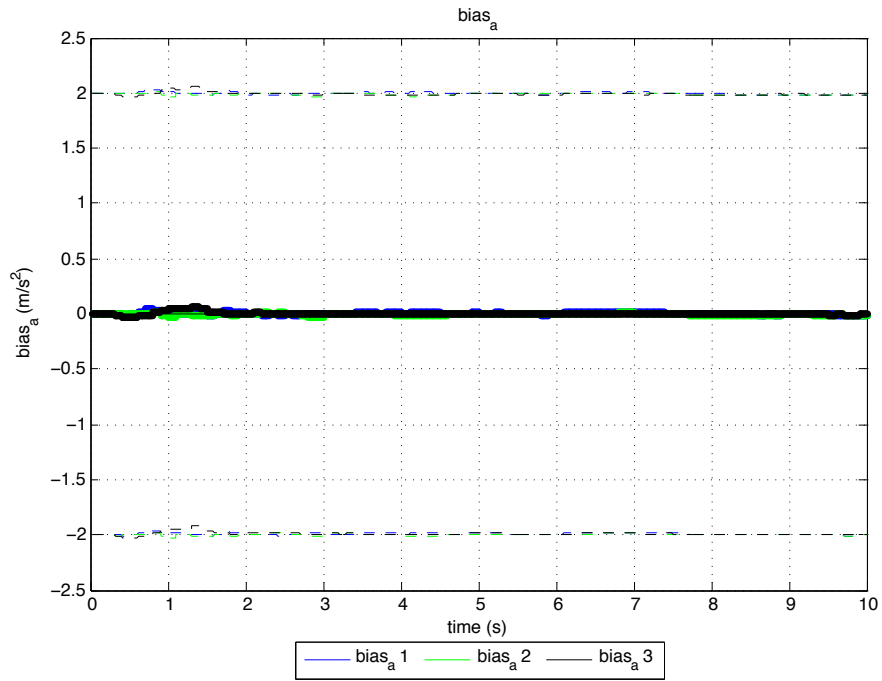
Figure 14: Velocity state estimation errors.
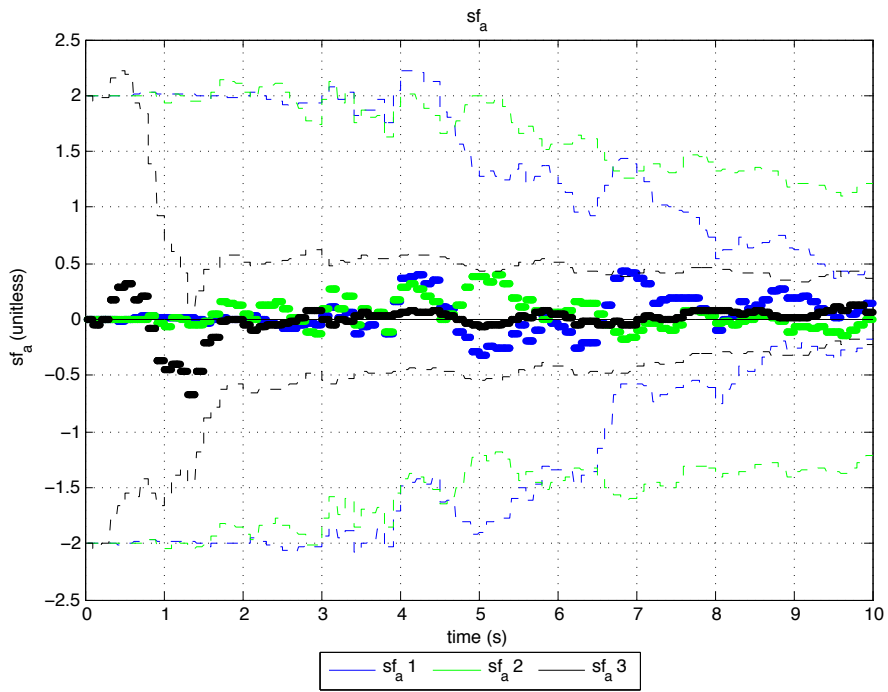
Figure 15: Accelerometer bias state estimation errors.

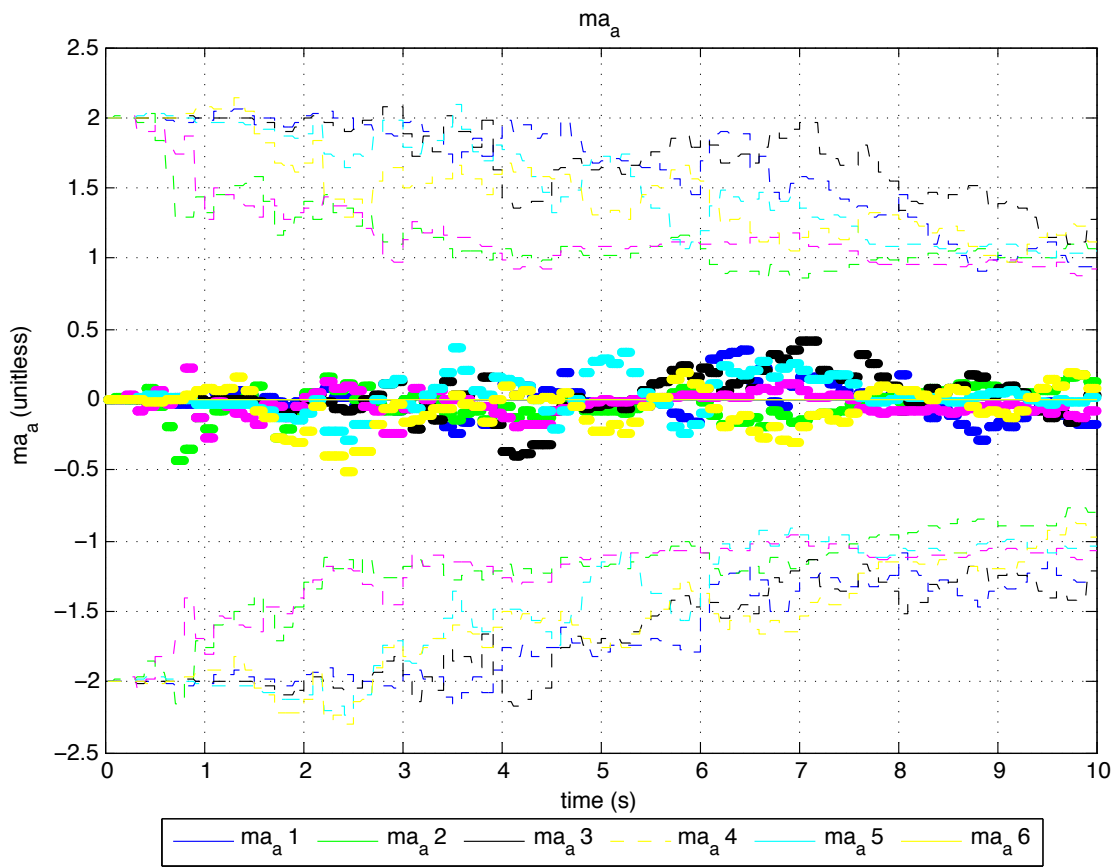Figure 16: Accelerometer scale factor state estimation errors.

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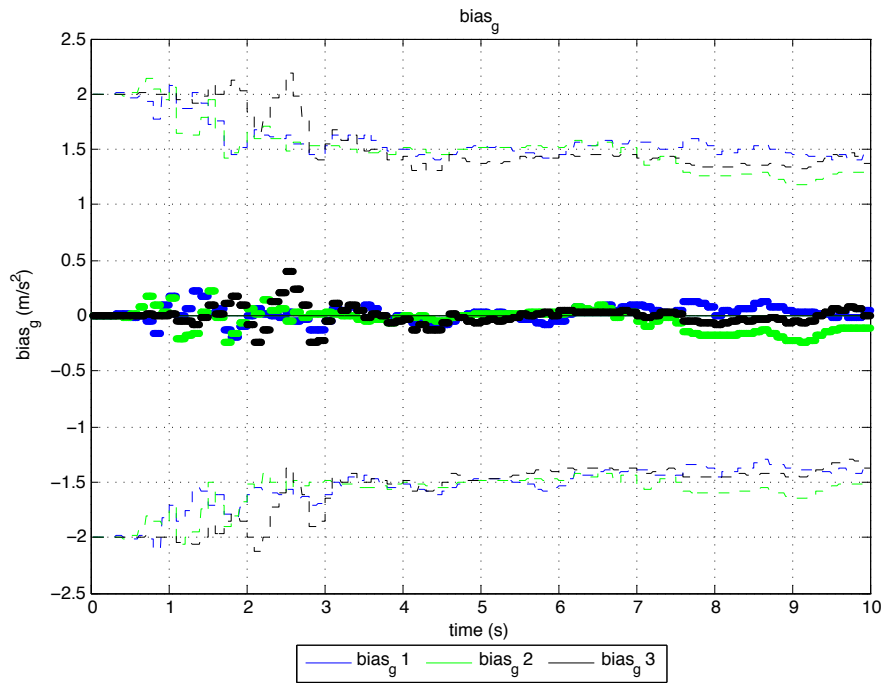*Figure 17: Accelerometer misalignment state estimation errors.*

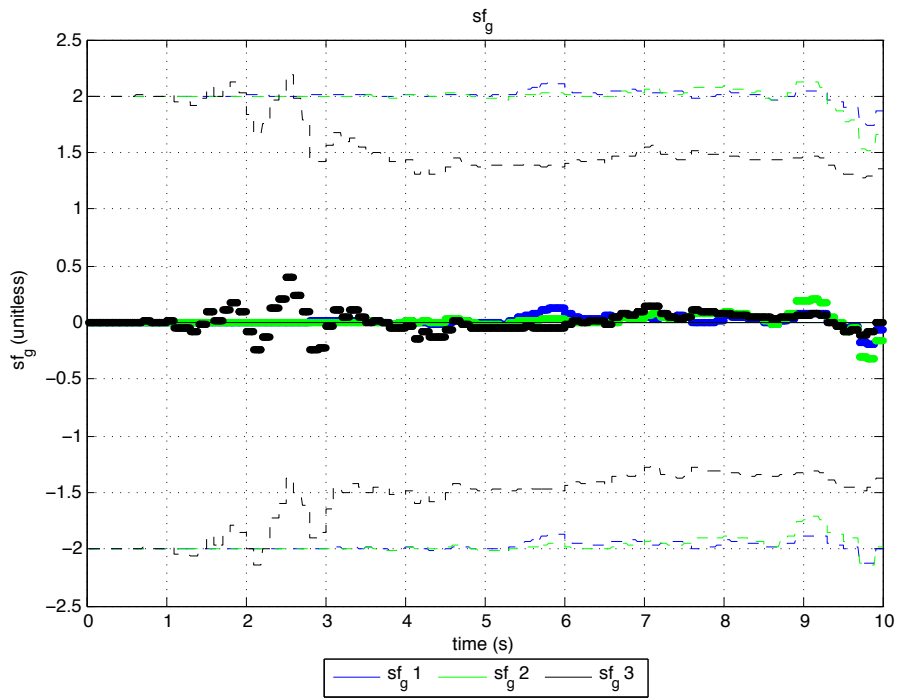*Figure 18: Gyroscope bias state estimation errors.*

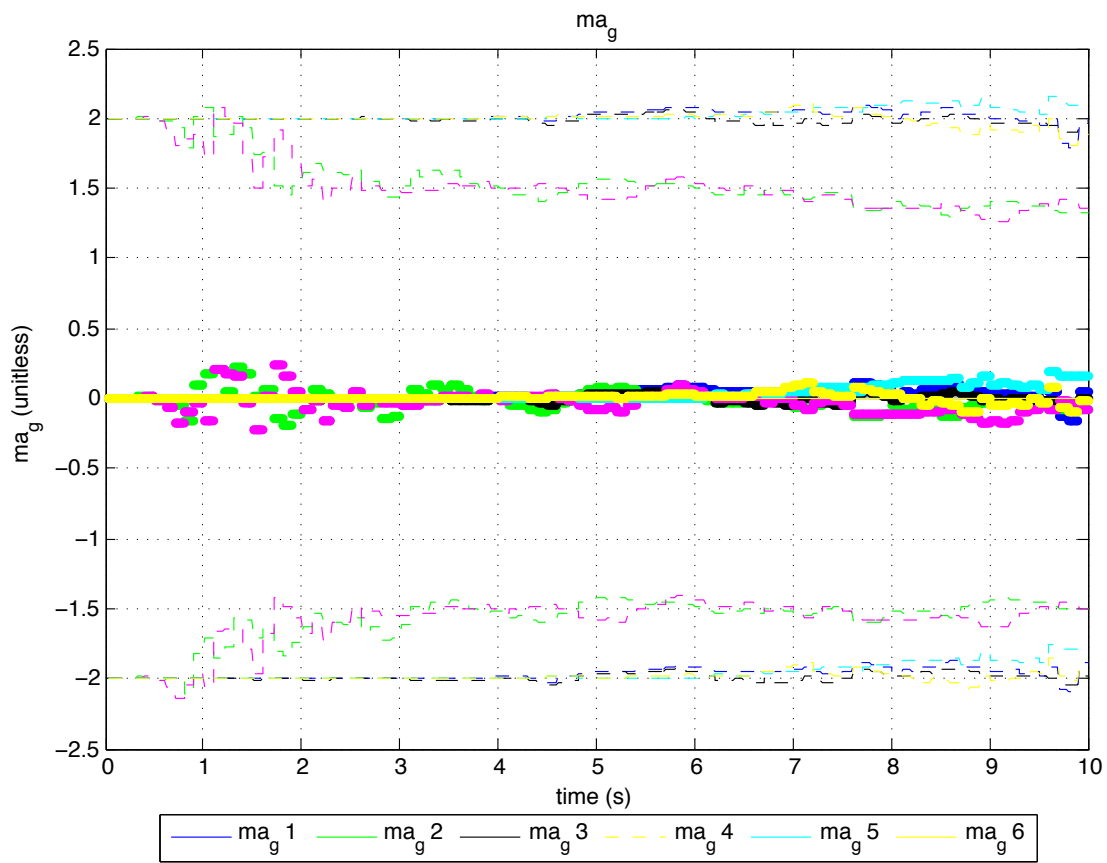*Figure 19: Gyroscope scale factor state estimation errors.*

Figure 20: Gyroscope misalignment state estimation errors.

References

[1] Farrell, Jay A., *Aided Navigation: GPS with High Rate Sensors*, McGraw-Hill Companies, 2008.