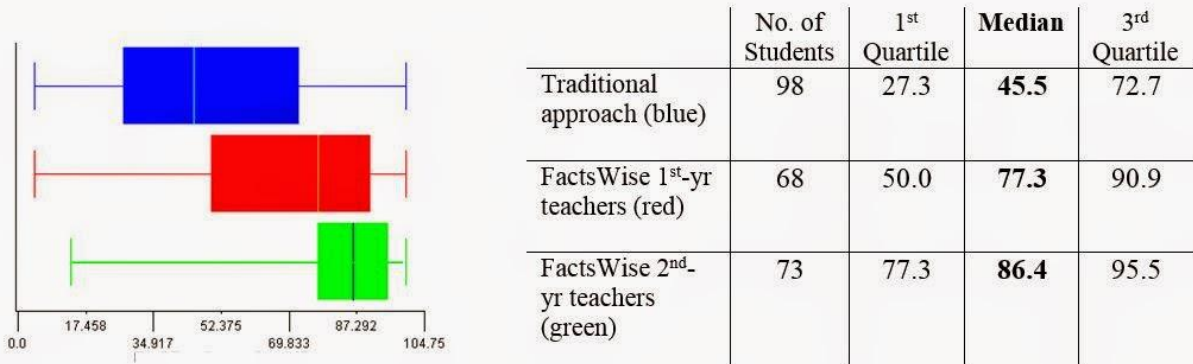


## FactsWise Addition/Subtraction Research Findings

After finding that less than 13% of sampled students were fluent with their basic facts (able to solve 80% or more of the sampled facts, without counting, in 3 seconds or less) near the end of first grade (Henry, Journal of Research in Mathematics Education, March 2008), I began designing a research-based approach to teaching and assessing basic facts. After three years of program development, results show effect sizes ranging from .80 to 1.53 for students studying with FactsWise-trained teachers when compared with those whose teachers use traditional basic fact approaches.

Percent Basic Facts Fluency (solving 10 addition and 8 subtraction facts without counting in 3 seconds or less)



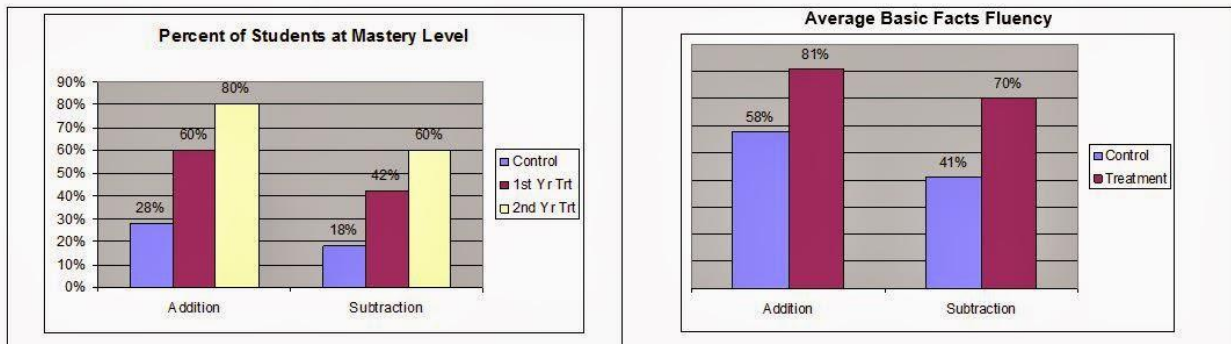
FactsWise breaks the facts up into 9 small chunks, with an early focus on 5s and 10s (see Appendix A). Consistent with the ways students from many other countries are taught their facts, students are then taught part-whole strategies using 5s and 10s to solve larger facts, including:

$$\begin{array}{c}
 8 + 5 \\
 \swarrow \quad \searrow \\
 8 + 2 + 3 \\
 \swarrow \quad \searrow \\
 10 + 3 = 13
 \end{array}
 \quad \text{or} \quad
 \begin{array}{c}
 15 - 8 \\
 \swarrow \quad \searrow \\
 5 + 10 - 8 \\
 \swarrow \quad \searrow \\
 5 + 2 = 7
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 15 \\
 - 8 \\
 \hline
 7
 \end{array}
 \rightarrow
 \begin{array}{r}
 10 + 5 \\
 - 8 \\
 \hline
 2 + 5 = 7
 \end{array}$$

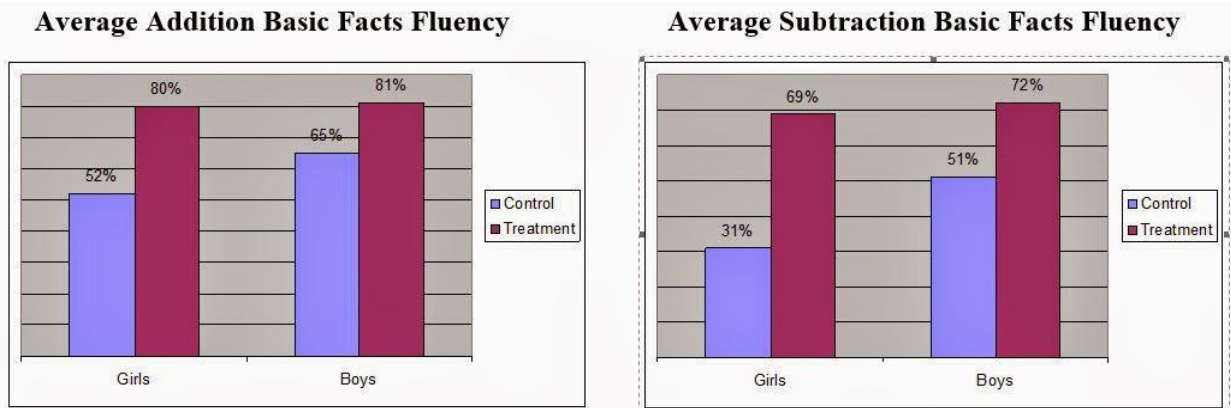
With this approach, students are encouraged to work on both memorization of facts and part-whole thinking strategies that are extremely important when solving more complex numeric and algebraic problems. Teachers were taught to use short one-on-one assessments (instead of timed tests) to learn how their students were actually solving problems (i.e., counting, recalling from memory, or using part-whole strategies), and were provided with a variety of instructional strategies, games, and on-line activities specifically designed to focus on each of the 9 goals. Another difference between this approach and other approaches teachers tend to use is that once students master a small set of addition facts, they move immediately to the related subtraction facts.

Students who studied in FactsWise classrooms showed much stronger memorization and part-whole thinking than students in classrooms using more traditional approaches. After three

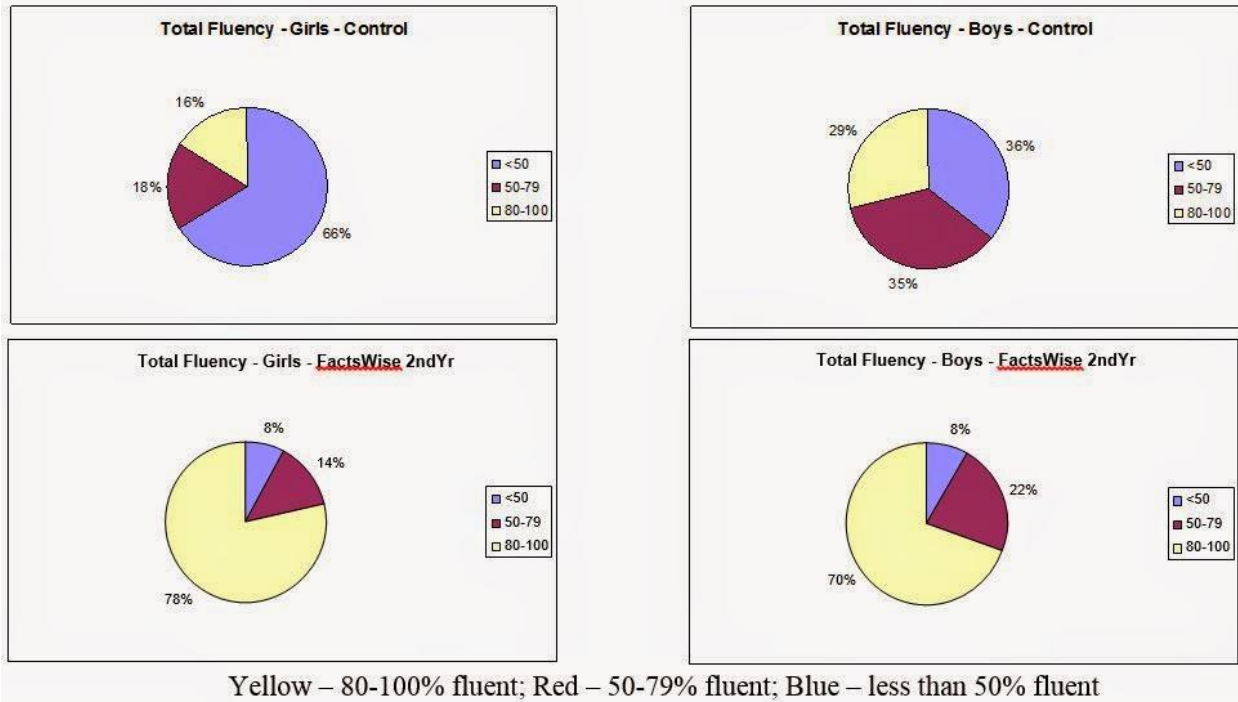
years of program development and implementation, first-grade students from 14 “treatment” and 10 “control” classrooms were given one-on-one assessments just a few weeks before the end of the school year. While only 28% of the control students demonstrated mastery of their addition facts (80% or more correctly solved without counting), 60% of the students working with 1<sup>st</sup>-year FactsWise teachers were at mastery. Students who studied with 2<sup>nd</sup>- and 3<sup>rd</sup>-year FactsWise teachers showed even stronger results – 80% were at mastery with their addition facts. Subtraction shows the same trend, with more than three times as many students working with experienced FactsWise teachers at mastery as control students.



Girls in the control classrooms were significantly less fluent than boys in both addition and subtraction. This disparity was virtually eliminated in FactsWise classrooms.

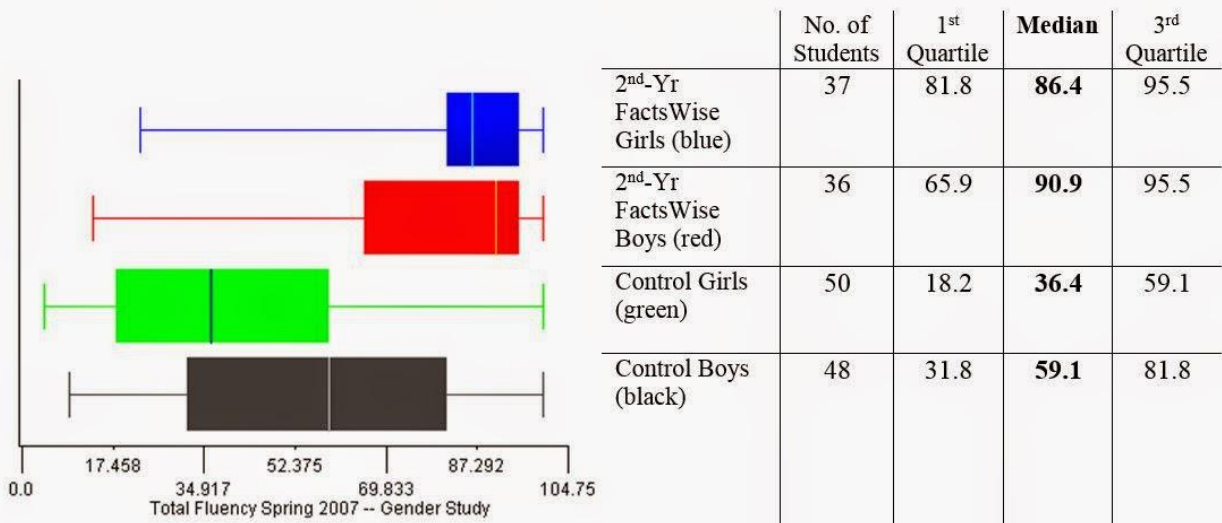


Results were even more pronounced in classrooms with teachers who were implementing FactsWise for the second-year. As the yellow portions of the graphs below show, only 16% of the girls and 29% of the boys in the control classrooms demonstrated mastery of basic facts (80% or more memorized or correctly solved using part-whole thinking in 3 seconds or less). In 2<sup>nd</sup> year FactsWise classrooms, 78% of the girls and 70% of the boys demonstrated mastery.



The boxplots below illustrate the striking results for girls working with experienced FactsWise teachers -- their first quartile result (81.8%) is substantially larger than the control girls' 3<sup>rd</sup> quartile result (59.1%).

Percent Basic Facts Fluency (solving 10 addition and 8 subtraction facts without counting in 3 seconds or less)



These results are a strong indication that students can attain much higher levels of basic facts fluency when their teachers are provided with research-based strategies for helping students move beyond counting to part-whole thinking and memorization. Six broad principles informed the development of this project:

1. Focus on accuracy, fluency, and part-whole thinking (encouraging students to move from counting to part-whole strategies as well as memorization)

2. Teach new material in small chunks using a hierarchical order (other than +0, +1, +2, ...)
3. Teach subtraction facts just after related addition facts
4. Ongoing whole-class instruction & practice with immediate feedback (every 24-48 hours)
5. Ongoing one-on-one assessment
6. Systematic homework to provide reinforcement of individual progress

This study was conducted in high-performing California schools with a wide range of language and ethnic backgrounds (39% of the sampled students were from English-only backgrounds). The effect sizes for the Spanish-speaking students in these classrooms are suggestive that purposeful, teacher-directed basic facts instruction may be effective in a wide range of language and SES classrooms.

**Mean Differences – Independent Samples T-Test (FactsWise and Control Classrooms)**

Percent Basic Facts Fluency (solving 10 addition and 8 subtraction facts without counting in 3 seconds or less)

Students in FactsWise Classes : Control Students	Effect Size (Hedge's g)	Percent Difference	Pooled Standard Deviation
<b>Addition Fluency</b>			
All Students (n=141:98)	0.90	22% (81:58)**	24.8
English Only Students (n=53:41)	0.80	23% (79:56)**	28.1
Spanish-Speaking Students (n=12:12)	1.44	35% (79:44)**	24.2
<b>Subtraction Fluency</b>			
All Students (n=141:98)	0.98	29% (70:41)**	29.8
English Only Students (n=53:41)	1.11	32% (69:37)**	28.7
Spanish-Speaking Students (n=12:12)	1.39	42% (67:25)**	30.0
<b>Total Fluency</b>			
All Students (n=141:98)	1.00	26% (75:49)**	25.9
English Only Students (n=53:41)	1.01	27% (74:46)**	27.0
Spanish-Speaking Students (n=12:12)	1.53	38% (73:35)**	25.1

\*\* Statistically significant at  $p < .01$  based on independent samples t-test

## Appendix A

### NINE GOALS FOR BASIC FACTS SUCCESS

Once a child is fluent with the addition facts in each goal, begin work on the related subtraction facts.

#### Goal 1 - Within 4s & 5s

Add: 1+3, 2+2, 3+1, 1+4, 2+3, 3+2, 4+1

Sub: 4-1, 4-2, 4-3, 5-1, 5-2, 5-3, 5-4

#### Goal 2 - With 5s (part 1)

Add: 1+5, 2+5, 3+5, 4+5, 5+5

Sub: 6-1, 6-5, 7-2, 7-5, 8-3, 8-5, 9-4, 9-5, 10-5

#### Goal 3 - Within 10s

Add: 0+10, 1+9, 2+8, 3+7, 4+6

Sub: 10-0, 10-10, 10-1, 10-9, 10-2, 10-8, 10-3, 10-7, 10-4, 10-6

#### Goal 4 - With 10s

Add: 10+1, 10+2, 10+3, 10+4, 10+5, 10+6, 10+7, 10+8, 10+9, 10+10

Sub: 11-1, 11-10, 12-2, 12-10, 13-3, 13-10, 14-4, 14-10, 15-5, 15-10,  
16-6, 16-10, 17-7, 17-10, 18-8, 18-10, 19-9, 19-10, 20-10

#### Goal 5 - With 5s (part 2)

Add: 5+6, 5+7, 5+8, 5+9

Sub: 11-5, 11-6, 12-5, 12-7, 13-5, 13-8, 14-5, 14-9

#### Goal 6 - Doubles

Add: 3+3, 4+4, 6+6, 7+7, 8+8, 9+9

Sub: 6-3, 8-4, 12-6, 14-7, 16-8, 18-9

#### Goal 7 - Under Tens

Add: 2+4, 2+6, 2+7, 3+4, 3+6

Sub: 6-2, 6-4, 8-2, 8-6, 9-2, 9-7, 7-3, 7-4, 9-3, 9-6

#### Goal 8 - With 9s

Add: 2+9, 3+9, 4+9, 6+9, 7+9, 8+9

Sub: 11-2, 11-9, 12-3, 12-9, 13-4, 13-9, 15-6, 15-9, 16-7, 16-9, 17-8, 17-9

#### Goal 9 - With 7s & 8s

Add: 4+7, 6+7, 3+8, 4+8, 6+8, 7+8

Sub: 11-4, 11-7, 13-6, 13-7, 11-3, 11-8, 12-4, 12-8, 14-6, 14-8, 15-7, 15-8