Advantage Materials Letters (AML)

First draft received FEB 2020
Accepted August 2020
PJN.3202008
SNY: 000020200828
ONLINE VERSION

Vol.40, No, 1 Aug 2020
Vbbripress.org

www.Vbbripress.org

Negative zero and infinite stiffness

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Abstract- an investigation to the principles of Zero, infinite and negative spring stiffness was carried out in this paper through applying experimental works to measure these stiffness's under static and dynamic loads conditions. By measuring these values, their effect on the rich and rich and chaotic responses were analysed. As a result, two types of testing (dynamic testing and load tester) was used to estimate the negative zero stiffness , where the results successfully achieved the zero negative stiffness.

Keywords- springs; Zero, infinite and negative spring stiffness; stiffness measurements; rich and chaotic responses

I. Introduction

Spring is an object that has elastic properties which save its mechanical energy. It absorbs the mechanical energy when loaded and releases the mechanical energy when the load is removed. Spring is used as a shock absorber in vehicles and can also be used in mechanical equipment using clamping techniques. It is also widely used in machine design purposes, maintaining stable contacts between engine components, adjusting vibrations, as a key component in measuring instruments and maintaining the stability of an equipment or machine. Spring is generally made of steel material, especially for engine components. It consists of several types that have different shapes and structures, as well as the purpose of using the spring. It is divided into three major groups namely disc spring, leaf spring and coil spring [1]. The ability of spring in storing potential energy is related to the elasticity possessed by spring. Elasticity is the ability of a material, object or object that can withstand the effect of deformation force and can return to its original form [1]. The elasticity of this spring depends on the stiffness property of spring. Stiffness is the condition of a rigid object, in which the object will resist the deformation force or load assigned to it. The physical law that applies to spring about stiffness is called Hooke's law. Hooke's law states that to press or extend the spring with a certain distance it takes force. The required force is the product of distance by the constant factor of the spring's stiffness. Stiffness that is owned by spring in general is positive. Positive stiffness is a condition where spring is given a load or applied force, then the spring is pushing the load or applied force in the opposite direction. Aside from the positive stiffness, stiffness can be negative. Negative stiffness is a condition in which an object or spring helps deformation force that occurs on it. This may happen because the object has internal stored energy.

Since the mechanism of these negative stiffness components is so unique (having internal stored energy), negative stiffness has the ability to increase acoustical and reduce vibration effects. Negative stiffness can occur in systems with negative spring constants and is made of materials with negative moduli values. One simple example of negative stiffness is a beam that has a curved shape. The structure of this curved beam has a bistable property (a system having two levels of equilibrium) and a metastable equilibrium point. With these three points of equilibrium the movement of the beam can pass through all three points.

Apart from negative stiffness, stiffness can also have a zero value or also known as zero stiffness. The concept of zero stiffness can be said to

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be the elastic ability of an object or structure capable of deformation with zero rigidity in which the changes that occur do not require an external force. Structures or objects that have zero stiffness can be said to be neutrally stable. In a stable and unstable state, structures or objects with zero stiffness can perform large displacement for both their own and constant critical stresses. In addition, zero stiffness can also be explained by the circumstances in which the structure or object is in a continuous equilibrium condition with applied force or load at a limited range of motion. Things or structures that have zero stiffness have the ability to keep energy potentials constant. The energy potential possessed by the object or structure is repeatedly distributed in the event of deformation. The stiffness of an object or structure has the potential to have an infinite value. Infinite stiffness range can be obtained by using a system that has a zero stiffness configuration.

II. *Examination of negative, zero and infinite stiffness in the spring*

There are several studies that examine negative, zero and infinite stiffness in the spring as well as on other objects or structures. One study of negative stiffness in spring was performed by C. M. Lee. Research conducted by C. M. aims to design the spring with the ability of negative stiffness on the vehicle to reduce or isolate the vibration. The design proposed by this study has the intention of obtaining vibrational frequency isolation in gravity [2]. Design spring with negative stiffness focus on the driver part of the vehicle to isolate the vibrations that occur in that part. The use of spring design with negative stiffness proposed by C. M. Lee is used to consider the design aspects of vehicle driver parts with the thin shells theory approach. Spring with negative stiffness adjusted to the existing space on the suspension of the vehicle to facilitate compatibility of the spring installation. The proposed design takes into account the added height at the part where the vibration frequency is kept to a minimum. C. M. Lee proposes an approach using a simple spring model with an iterative formula that aims to solve the nonlinear issue geometrical caused by the large amplitude of the bucking spring components. With this iterative procedure, design spring with negative stiffness can be represented optimally and can be computed. C. M. Lee uses comparative procedure and result measurement obtained as approach for validation of design spring with this negative stiffness. From that approach, M. M. Lee proposes a general spring model, which can be used on mounting cabs, cargo platforms or for suspension on the seat.

Other study used to reduce or isolate vibrations in vehicles are done by Thanh Danh Le and Kyoung Kwan Ahn. The research conducted by them aims to perform vibration isolation in areas with excitation at low frequencies. To be able to isolate the vibrations that occur in the area, Thanh Danh Le and Kyoung Kwan Ahn use structure with negative stiffness. The focus area of their research is the seat of the vehicle. The use of structure with negative stiffness is adjusted to the condition of parallel in structure that has positive stiffness. Thanh Danh Le and Kyoung Kwan Ahn proposed a design procedure based on theoretical analysis with the aim of frequency of the peak resonance tending to the left as well as the capacity of the load support maintained. In addition, the proposed design reduces the size of the system as a whole to obtain practical and frequent use of bending that can be reduced [3]. Next, Thanh Danh Le and Kyoung Kwan Ahn use dynamic equation for the proposed system. After using the dynamic equation for the proposed system, the harmonic balance method is used to determine the characteristic of motion transfer capability in the proposed system under steady state conditions at each frequency. From the characteristics obtained, the curve of the motion transmission is predicted based on the various values of the system parameters. In this study, the time response to sinusoidal and random excitation and multi frequency conditions were studied by experiment and simulation. From the results of this test comparison of the ability of vibration isolation between systems that use structures with negative stiffness with a system without structure with negative stiffness can be known. From the research conducted by Thanh Danh Le and Kyoung Kwan Ahn this concludes that the system with negative stiffness structure more has the ability of vibration isolation which covers the greater

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frequency area compared with the system without structure with negative stiffness.

Wenjiang Wu et al. did research on the variation of the use of spring materials with negative stiffness. This research uses magnetic spring that has negative stiffness. The purpose of the study conducted by Wenjiang Wu et al. this is to analyze the ability of the vibration isolation that can be generated by using magnetic spring having negative stiffness. The use of magnetic spring with negative stiffness is collaborated with spring that has positive stiffness with characteristic stiffness of a variety of high, static, low, and dynamic stiffness. Magnetic spring with negative stiffness value is arranged with three magnetic shaped like a cube with repulsive interaction. Furthermore, the analysis is performed on the basis of the charge changes of the magnetic model, then, an estimate of the exact analysis is obtained. After that, Wenjiang Wu et al. analyze the nonlinearity of stiffness. From the results of the analysis found that magnetic spring with negative stiffness is approximately - linear for a small oscillation condition. Wenjiang Wu et al. using the analysis of the transferability capability of the vibration of the system with magnetic spring negative stiffness and the system without magnetic spring negative stiffness, for validation of the accuracy and effectiveness of magnetic spring with this negative stiffness. From the test results obtained, Wenjiang Wu et al. concluded that vibrations isolated by using magnetic spring with negative stiffness can decrease the frequency of the vibration and have good consistency [4].

In addition to the use to isolate the vibrations that occur in the vehicle, negative stiffness is also used for protection against seismic disturbances in certain structures. The research discussing this topic was conducted by A. A. Sarlis et al. Research conducted by them, using equipment that has negative stiffness. The purpose of this equipment is to reduce the seismic forces that occur in building structures and as a complement to the design of the building that has damping system. Devices with negative stiffness can surpass structural weaknesses that occur in systems without experiencing inelastic deviations as well as the occurrence of permanent deformation. Simulations performed on devices with negative stiffness indicate flexibility with displacement and by using force on the installation chassis that will interfere with the restoring force's ability of the structure. Device with negative stiffness consists of compression spring with the strengthening of double negative stiffness and gap spring mechanism that aims to delay the negative effects of stiffness until the displacement occurs in the system structure. Device with negative stiffness uses braces capable of loading by itself the vertical force required for progress on the horizontal direction of negative stiffness without channeling the force to the system structure. The development and testing of the device with negative stiffness is analyzed by and computed to determine the behavior of the device with the negative stiffness [5].

Research on zero stiffness in the spring is done by Will S. Robertson et al. This research proposes theoretically design with magnetic spring which has stiffness value near zero or quasi zero stiffness for vibration isolation purposes. Research conducted by Will S. Robertson utilizes the levitation of the magnetic system to isolate the vibrations that occur. This research uses non dimensional analysis on the magnet with ratio of force and displacement ratio. This magnetic system uses a negative stiffness component to reduce the frequency that occurs naturally in the suspension. The proposed design relates to the ability of mass loading, range of isolation frequency, resistance required by the system and the magnitude predicted by the influence of vibration [6].

. In a common spring system, when decreasing the stiffness of the support system it will increase the static deflection. In addition, the lower limit of stiffness is affected by the permissible displacement limits. Zero stiffness is likely to occur in the local region. Parallel connection of vertical and inclined spring is one example of case of behavior of the system. Parallel connections of the vertical and inclined spheres have an approximate characteristic of displacement with cubic force, with zero stiffness occurring in the local region experiencing zero deflection, which is referred to as quasi-zero stiffness. To get the quasi-zero stiffness condition it can use buckling beam which acts as negative stiffness component.

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Research conducted by Robertson et al. testing a system that shows the occurrence of zero stiffness in the local region which occurs in fixed magnets that support the mass of gravity and also holds the mass from the top. The relationship between force and displacement characteristics of this system is estimated by quadratic polynomial, which is tested on small variations of the gap between the magnets. This study using a more practical cubic force curve to create a more stable inflection point with zero stiffness in the local region. This is in contrast to that experienced by quadratic spring, whose stability is limited to quasi-zero stiffness conditions and can't be executed under these conditions. Preparation of this magnet spring system can produce a mechanism with a low stiffness value, can be said to achieve zero stiffness. This is because the magnetic spring system reduces the value of stiffness that occurs at three points of freedom on the translational degree. In addition, the magnetic spring system with quasi-zero stiffness can be used to design mounting with low vibration isolation frequency. With this idea it can be used on a structure that can be reduced its stiffness to overcome the vibrations that occur after the structure is created. The concept of non-direct contact possessed by a magnet can be an easy option in its installation of the existing structure.

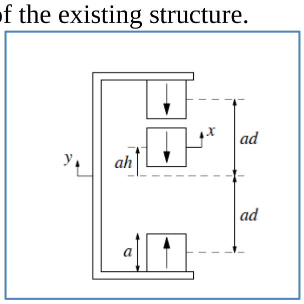

Figure 1: Magnetic spring system [7]

Figure 1 is a schematic of the magnetic spring system proposed by Robertson using quasi-zero stiffness. In this schematic, h = 0 for the purpose of isolating the displacements shown by x from the effect of the vibrational noise indicated by y. Large arrows on the schematic show the direction of polarization of magnets.. The positive value h is an upward movement with respect to unstable equilibrium and negative h is a stable downward movement.

Research conducted by Robertson uses an equation for the forces that occur between the two magnets. The forces that occur in the magnets are calculated using the formula from Akoun and Yonnet. In addition, for the geometry of the magnet, Robertson uses the equation of Bancel with an easier algorithmic capability for complex magnetic geometry and with multiple pole arrays. Magnetic spring with quasi-zero stiffness consists of magnets that are opposite each other. The gap parameter between the center of the magnet (ad) has a quasi-zero stiffness condition as well as a static displacement of the floating mass (x = ah) between the two magnets. The parameters h and d represent the normalization state of the magnetic gap and the normalization of the magnet displacement.

Research conducted by Robertson managed to analyze the use of magnetic spring that aims as a load bearing with a low stiffness value. The estimates and certainty of this research are based on a cubic magnet to determine the behavior of this system. Estimates of this system are so simple and can produce accurate results of large range displacements. In addition, the use of this magnetic spring system has the possibility to be used with magnetic size variations. Robertson proposed four design standards with two variable parameters: the gap distance between the fixed magnets and the size of the magnets. Research conducted by Robertson shows a clear technique to know the limits of these parameters. The design proposed by the research can be optimized based on the availability of magnetic size and allowable stroke, with the frequency condition of resonance and load bearing.

This magnetic spring system has a weak, nonlinear condition with a distorted level compared to a linear system. The variations that occurs show that the peak of the resonance is more inclined at lower frequency conditions. This nonlinear condition seems to occur only in vibrations with large amplitude under equilibrium conditions nearing quasi-zero stiffness conditions. This causes the system to remain stable, nonlinear conditions do not cause conditions that harm the system or disrupt the stability of the system to the frequency that occurs in the system. The magnetic spring system acting as a vibration isolator can be used for precision related

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applications where low frequencies are required as parameters in the system. The design proposed by Robertson has opportunities that can be used in parallel on a variety of sizes with the availability of magnetic sizes, especially as bearings for larger loads. At high frequencies, the reduced noise performance of this system produces good results. As for the low frequency interference, the resulting disturbance is quite large due to the high resonance peak. This disturbance can be overcome by using a sky hook damping on the system.

Other studies related to quasi-zero stiffness were performed by Lingshuai Meng et al. Research conducted by Lingshuai Meng et al. focus on the theoretical design and characteristic analysis of insulators with quasi-zero stiffness using disc spring components as negative stiffness. Lingshuai Meng et al. proposed an isolator design with quasi-zero stiffness by combining vertical linear spring and spring disk. In this study, investigation is done for the characteristics of the quasi-zero stiffness isolator and disk spring. The study used an optimum combination of parameters determined to obtain a large displacement range at the equilibrium position in which the stiffness value was low and slightly changed. Lingshuai Meng et al. using dynamic equation for displacement and force based on underload and overload condition calculations. This study uses the harmonic balance method to obtain the frequency response curve and validated with numerical simulation. Floquet theory was used in the study to analyze the stability of steady state conditions. Evaluation of the insulation performance of this system based on acceleration transmissibility, absolute displacement and force. In addition, a study conducted by Lingshuai Meng et al. It also studies the effects of damping ratio, excitation amplitude and offset displacement on quasi-zero stiffness isolator system and equivalent system. From the research, it is found that quasi-zero stiffness isolator on overload and underload conditions shows different stiffness characteristics based on excitation amplitude change. Quasi-zero stiffness isolator system can produce better vibration isolation performance compared to its equivalent system at low frequency range, if loaded with the right mass, and does not receive an amplitude that is too large and has a larger damper [8].

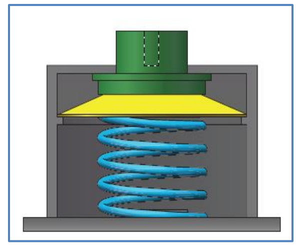
Figure 2 Isolator with quasi-zero stiffness using disk spring and vertical linear spring

Research conducted by Lingshuai Meng et al. suggested an isolator system consisting of a disc (negative stiffness component) and a vertical linear spring as shown in Figure 2. The selection of spring disk as a negative stiffness component because the spring disk has the ability to withstand a large load with a small deflection. In addition, the spring disk can store styles on flat conditions. The design of the quasi-zero stiffness isolator can be used in some applications and can be used with limited space conditions. Quasi-zero stiffness of this isolator can achieve quasi-zero stiffness condition with nonlinear axial restoring force at static equilibrium position. To study the condition of suggested stiffness the conditions are shown in Figure 3. The curve 1 in Figure 3 shows the general relation between load and motion for vertical linear spring. The curve showed by number 2 shows the general relation between force and displacement on the spring disk, while the third curve is a combination the first and second curve..

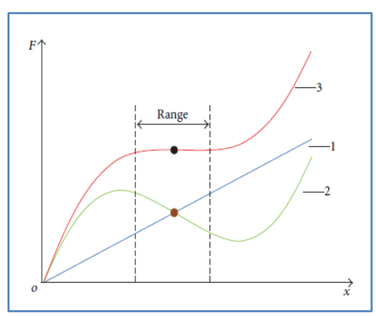

Figure 3 Principle of quasi-zero stiffness curve [9]

Research conducted by Lingshuai Meng et al. aims to produce designs with configurative

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parameters. The study describes the characteristics of spring disc under static conditions as well as quasi-zero stiffness isolator system. In addition, the optimization of the quasi-zero stiffness isolator system is also carried out with static equilibrium conditions where stiffness has a lower value and undergoes little change. This quasi-zero stiffness isolator system optimization is done to obtain a larger displacement range. In addition, dynamic modeling, numerical simulation, stability analysis and solution were also conducted in the study. Underload and overload conditions transmissibility of force and frequency response curves are also investigated. The study also discusses the effect of damping ratio on the quasi-zero stiffness isolator system.

Research conducted by Lingshuai Meng et al. it produces an analysis of theoretic design as well as characteristic analysis of the quasi-zero stiffness isolator system using spring and vertical linear spring disks. The characteristics of the disc spring in static conditions with the thickness variations were tested and compared with the constant spring disk thickness. Zero stiffness obtained on the quasi-zero stiffness isolator system in static equilibrium is obtained from the stiffness relationship specified and the parameters are configurative. The optimization is aimed to get a larger displacement range with stiffness value which is slightly changed and low stiffness value. Underloaded and overloaded conditions result in a displacement overlap at the static equilibrium position of the zero stiffness state.. The maximum value of steady state conditions is used as a function of displacement and amplitude. Floquet theory is used as a reference to study the stability and multiple jumps that occur in the quasi-zero stiffness isolator system. The frequency response curve obtained from the ideal condition isolator and the disturbed condition detector is illustrated by a different combination of excitation amplitude and offset displacement. The results of this frequency response curve indicate that the disturbed insulator can exhibit a linear condition, a mixture of softening and hardening, softening and hardening stiffness at the expense of amplitude excitation. Unbound response can occur in excitation displacement. The decrease in offset displacement and amplitude can extend the area frequency of the quasi-zero stiffness isolator system isolation. The isolation capability of quasi-zero stiffness is assessed using acceleration transmissibility, force transmissibility, absolute displacement and by comparison to the equivalent system with two types of excitation. The disturbed quasi-zero stiffness isolator system has a damping ratio effect on the transmissibility studied in this study. If the load on the quasi-zero stiffness isolator system is less than its support capability then the amplitude excitation that occurs will not be large and damping is of great value. Quasi-zero stiffness isolator system has better insulation performance at low frequency range when compared with equivalent system. Proper damper upgrades can be a great choice in avoiding jumps on the Quasi-zero stiffness isolator system.

Stefan Groothuis et al. conducting research on the mechanism of infinite stiffness range and unlimited motion by using leaf spring as one of the main components. The mechanism proposed by Groothuis et al. is to use leaf spring and two supports that can be moved. It aims to know the output variables of stiffness. In this study Groothuis et al. using leaf spring model by deriving Euler-Bernoulli theory and confirmed by using experimental data. By forming the shape of leaf spring, the stiffness characteristics obtained can be changed for other purposes of use. In addition, Groothuis et al. also proposed another alternative design using one leaf spring and one supportable pin, with the same behavior as the previously proposed design [10].

Groothuis et al. doing research using stiffness variable actuator. Variable stiffness actuator is a mechanism that aims to set the stiffness of the actuator to meet the needs of stiffness with changes in operating and environmental conditions. The basic concept of this actuator stiffness variable can be set to achieve higher energy efficiency. This variable stiffness actuator was developed by Groothuis et al. to improve the movement of robotic components to be more safety and compliant. In the study, Groothuis et al. proposed a variable stiffness mechanism with the transmission conditions between spring and load changed. This stiffness mechanism variable has a zero stiffness configuration with output separated from the input which is a rotor. From this

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configuration, unlimited motion output is likely to occur. The behavior of this output can only be generated from the dynamics of the output load. The proposed actuator stiffnes variable can achieve the infinite stiffness condition of the zero stiffness condition to infinite stiffness. The design concept of the stiffness variable of this mechanism can be seen in Figure 4.

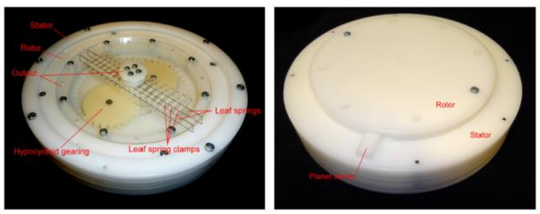

Figure 4: Variable stiffness mechanism with leaf spring [11]

Figure 4 shows that the stiffness mechanism variable consists of rotor, stator and output. In this stiffness mechanism variable, it can be seen that two leaf springs adjust the rotor and output, with two leaf springs connected to the rotor with support pins mounted on fixed condition with gear mechanism. The pins that support the leaf springs can be moved in a straight direction, determined by the gear mechanism. By being able to move the pins it can adjust the variable between output and rotor. The two planet gears contained in the variable stiffness mechanism are connected and driven by the gear carrier. The working principle of this variable mechanism is based on the transmission changes that occur between load and spring. The working principle which is only related to one leaf spring can be seen in Figure 5.

Two forces having opposite directions as shown in Figure 5 are used in an undeflected leaf spring causing the deflected leaf spring. The position of the movable support pin (shown in Figure 5 by x_0) affects the deflection shape in leaf spring. With the increase of x0 transmissions that occur between the spring and the load changes. This results in a stiffness value experienced in a position where the applied force increases. Infinite stiffness can occur when pin supports are placed at both ends of leaf spring. Zero stiffness can occur when the position of the supports pin is in the middle of the leaf spring, in other words the distance from $x_0 = 0$. This can cause the spring to rotate freely against its center. For the stiffness mechanism variable, this can mean that the output is separated from the rotor and the output shows the dynamics affected by the load. The stiffness received in this system is determined by the length of the leaf spring.

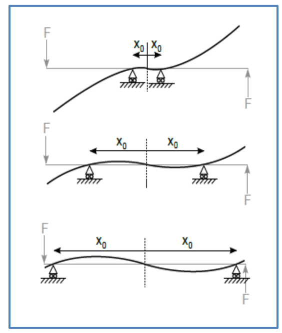

Figure 5 Variable stiffness mechanism working principle [12]

From research conducted by Groothuis et al. obtained variable stiffness mechanism that has ability that can reach infinite stiffness range. In addition, the study can also generate unlimited output movements apart from the rotor to create passive safety behavior. One of the important capabilities of this stiffness mechanism variable is its ability to change the stiffness value based on the shape change of leaf spring. The study used Euler-Bernoulli beam theory for modeling one of the leaf spring used in the variable stiffness mechanism. This leaf spring model is validated by experiment with PETG copolyester. The measurement of the stiffness of two leaf springs is adjusted to the measurement with one leaf spring, it aims to measure the effect of coupling leaf spring by clamping it on the other leaf spring. The effect is estimated and applied to adjust the model data according to the measurement of two leaf springs. In this way approximate estimates of the model are obtained. Variable stiffness mechanism has an opportunity for design development using one leaf spring and one support.

III. Theory

By pressing on a spring, a displacement in the spring can be observed where the ratio between the applied forces to the resulted displacement is known as spring stiffness. As any other objects, when it's subjected to a force it tries to resist deformation

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through what called a restoring force. Three possible stiffness's may take place in the spring which are; positive, negative and zero stiffness.

For a positive stiffness, it takes place when the deformation in a direction similar to the exerted forces direction, corresponding to a restoring force that returns the object to its original shape after deformation. For a negative stiffness, the object like spring supported the imposed deformation. It comprises reversal of the common directional relationship among force and displacement in deformed body. This type of stiffness can be occurred in systems with a preloading condition. For the zero stiffness, it refers to a decoupling among force and displacement for two disparate bodies in space.

For this project, it must be noted that negative stiffness is observed in structures that have snap during their motions, and is a main characteristic of structures that are bistable - ie. Stable in two dissimilar shapes. When it is in a parallel configuration with a traditional positive stiffness, the negative stiffness can be utilized to produce a spring with zero stiffness which can be effectively applied in vibration isolation applications. In the other hand, when the negative stiffness is at a series configuration with a positive one; in this case an infinite stiffness can be created.

For this study, in key interest is to experimentally produce and measure some simple negative, zero and infinite stiffness springs under the effect of dynamic and static loading in order to examine the rich and chaotic responses.

A. Experimental work

The researcher started with experimental work to assembly the suggested models , figure 6 shows the starting with assembling the system in the

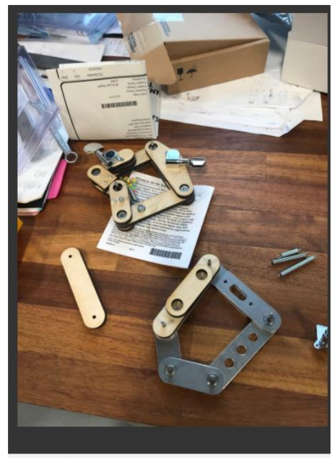

Figure 6 starting with assembling the system in the lab

Where two tests are suggested to be used in this research which are dynamics test and load tester test , the data for these tests was analyzed in the next section.

IV. Results of experimental work

the suggested model is look like figure 6 but with adding a spring between the two horizontal links , where links are connected with a spring as shown

when the lower link move x distance the two side links will be contracted which will cause a compression on the spring the mathematical modeling for this case can be obtained using basic math of triangles whereas following :

$$L = 2a - \frac{b}{c}(2a - y) \qquad (1)$$

Where $x^2 + \left(\frac{y}{2}\right)^2 = a^2$ (2)

By subsisting equation 1 into 2 these equations itcan be produces the following equation

$$l = 2a - \frac{b}{c}(2a - 2(a^2 + x^2)^{\frac{1}{2}} \qquad (3)$$

For the attached spring the forces applied can be calculated as

$$F = f_0 + kd \qquad (4)$$

In general the storage potential force of the spring cane be given as linear relation between the

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displacement and the force where the slope of the curve is the energy stored by the spring and its depends on the stiffness of the material . In equation (4) (*Fo*) is the initial force stored in the spring , for unloaded spring its to be zero but in the suggested case in this research it has value .
the displacement of the spring(d) under the subjected force can be written using the following equation

$$d = l - l_o - \propto \qquad (5)$$

Where∝: is the adjacent of the spring .
The potential energy of the spring can be derived according to the integration of the of equation (4) from zero to d. and can be written as shown in equation 5 .

$$V = fd + f_o d + \frac{1}{2} k d^2 \qquad (6)$$

Where this equation can be solved by substituting the equation 3,in 4 then equation 4 in 5 .
However experimentally the load vs. the displacement was plotted for different samples , where figure 7 shows the results for sample No.12 where the relation between the standard force and strain is semi-linear and was fitted as shown in the following figure , where the relation between the stress and the strain using can be fitted as shown in equation 7.

$$F(N) = 0.3899x + 0.7623 \qquad (7)$$

The value of R2 for this fitting is 0.9782
To find the potential energy storage by this sample the equation was integrated from 0 to maximum strain .

$$V = \frac{0.3899}{2} x^2 + 0.7623\, x \qquad (8)$$

= 265 N.mm

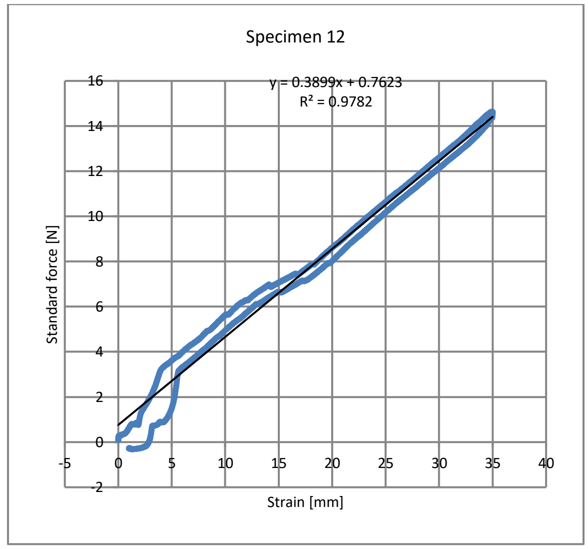

Figure 7 results of the load tester for sample 12

The following figure shows the force vs. the strain for sample 13 . the results show the relation losing its linearity in a region between 15-18 mm approximately, where the fitting parameter R^2 was reduced to 0.95 comparing with sample 12 which was 0.9782. where the potential energy about 278 N.mm

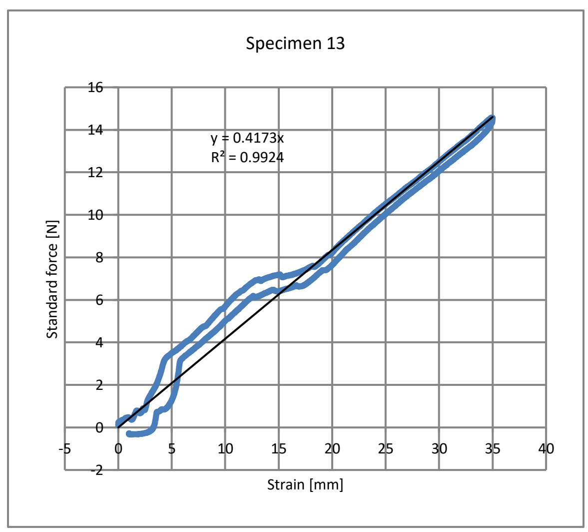

Figure 8 results of the load tester for sample 13

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The following figure shows the force vs. the strain for sample 14 . as shown the sample behaves as vibration isolator in range of 12-18 mm. where the effect of stiffness is not appear . the potential energy for the sample about 291 N.mm

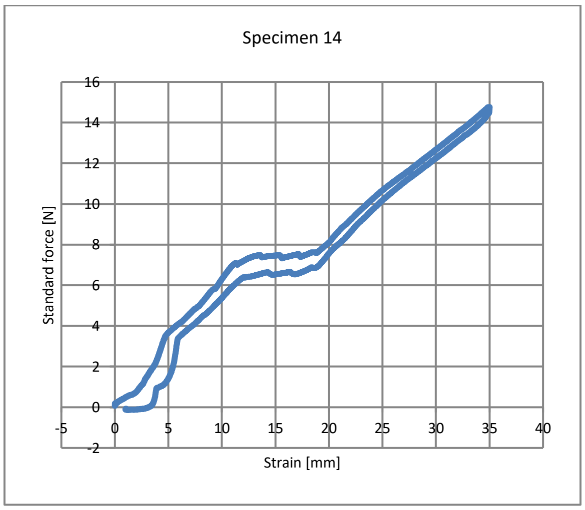

Figure 9 results of the load tester for sample 14

Regarding sample 19 the relation between the force and displacement is shown in the following figure where the strain increases suddenly at force 8 N from 10 to 20 N, the slope at this region is zero which means the negative stiffness can be obtained at the force value for this sample

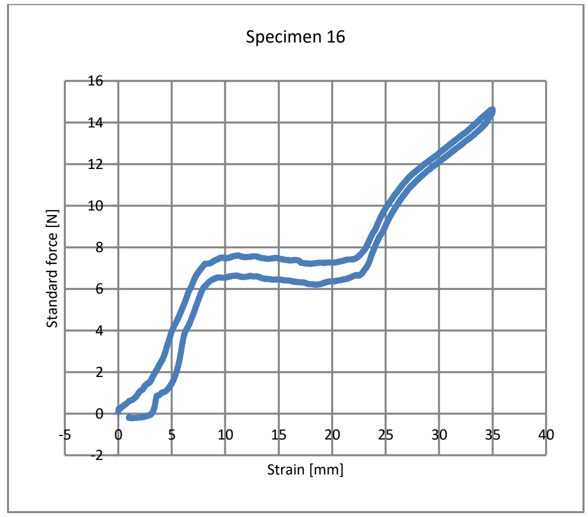

Figure 10 results of the load tester for sample 16

The following figure shows a sample of results for the dynmamics testing where the waveform was plotted for the different recods, it is worth to mention that the size of provided data per run was huge so the dealing with these data ws bit diffectult due to smalll step time . the wave form which is shown in the following figure shows three types of waveform where zero stiffness behavour is presented for Y3 , this can be explained according to zero oscilation was provided by this sample comparing with the other samples .

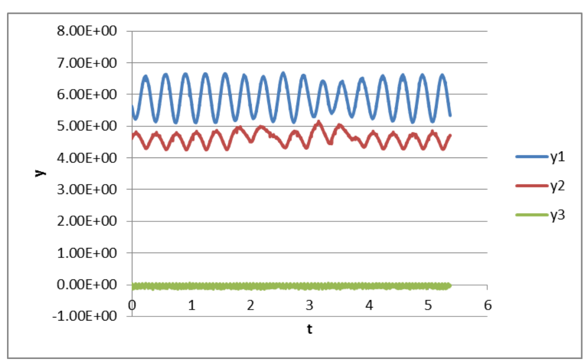

Figure 11 results of dynamics testing No.1

Advantage Materials Letters (AML)

First draft received FEB 2020
Accepted August 2020
PJN.3202008
SNY: 000020200828
ONLINE VERSION

Vol.40, No, 1 Aug 2020
Vbbripress.org

www.Vbbripress.org

The second set of data for case 2 was provided the same behavior where the results was plotted as sample time period, the results validate the negative zero stiffness for the waveform No.3.

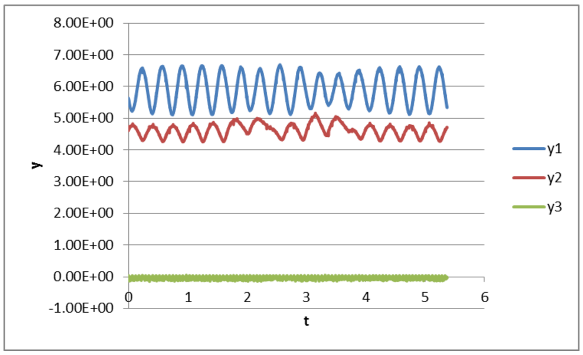

Figure 12 results of dynamics testing No.2

V. Conclusion

In this research the negative zero stiffness was investigated experimentally where a model of different links was assembled in the lab to check its stiffness , the created model was tested using two testing rigs which are the dynamic testing and the load tester , the results of load tester present the curve of load vs. the strain , where the negative zero stiffness was noted for sample 16 clearly. The negative zero stiffness region includes no osculation comparing with and the effect of stiffness cannot be noted, in other words the relation between the force and strain is linear according to hooks law , where the slope of this relation is the stiffness , so if the slope is zero or negative the material has zero stiffness as presented in sample 16 in region 10-20mm .
The dynamics testing was obtained for three different wave form , where the results of this testing shows the zero stiffness is presented to sample y3 , where this sample dose not oscillate with the time comparing with positive stiffness waveforms y1 and y2.

Acknowledgement
I would like to thank the supers our for his grateful help and support to achieve this project.

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