

## *Tidal Energy and Waves Interaction*

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### ABSTRACT

This project discussing the behaviour of the water waves using shallow wave theory. Three different schemes are used in this research which upwind, lax- Friedrichs and Lax-Wendroff. The processes of discretization the governing partial differential equations are taken into consideration. The wave characteristics were simulated for the different schemes using MATLAB software and the results were compared with the exact results. According to the obtained results it can be noted the high accuracy was presented for lax- Lax-Wendroff method, where the upwind, and the lax- Friedrichs method has low accuracy with increasing the step time.

**Key words:** upwind, lax- Friedrichs , Lax-Wendroff

### I. INTRODUCTION

Tidal energy can be created by using generators which have turbines. These submarine huge turbines can be placed in areas having enough tidal movements and they designed to reserve the energy from ocean tides as well as the kinetic energy of ebbing in order to produce electricity. Tidal energy has great potential of generation electricity as well as future energy because of the massive size of oceans. This project concentrates specially on the generation of tidal energy as a renewable source.

Tidal power produced from the interaction between the sun, the moon as well as the Earth's oceans. Because of gravity depends on the alignment of celestial bodies that can be predictable, the tidal power can be predictable also. The first class and businessman are increasingly looking to use technologies. The main types of renewable energies can be

extracted from the oceans are; the tidal existing energy, "Ocean Thermal Energy Conversion" (OTEC), the osmotic pressure, and the offshore wind power. It is provides an important source for generating clean renewable electricity. Dispel the natural tidal worldwide is 3.7 terawatts (TW), which is small compared with the TW 15 of the world's energy consumption [1].

Tidal energy considers as a renewable and clean source which does not produce gases emission that responsible for global warming or acid rain as well as related to the fossil fuel types. Furthermore, the usage of tidal energy could reduce the nuclear power demand as well as reduce the risks that related to radiation [3].

Tidal energy known as hydroelectricity low-head type and uses recognizable hydroelectric low-head generating tools as well as these tools were used for long years. The technologies required of the tidal energy are improved as well as the main challenge of developing the tides use is the construction costs. There are high assets cost of tidal energy project and the construction period can reach more than ten years. Moreover, the electricity cost is responded generally to reduction rate [4].

The supply of electrical energy from the tidal plant cannot probably meet the system demand. Furthermore, because of a lunar cycle as well as gravity, the tidal energy is variable, reliable as well as predictable and their energy can make important as well as a generous contribution of the specific electrical system which has different resources. Generally, tidal electricity can be used to dislocate electricity which can otherwise be generated from fossil fuel resources like coal as well as natural gas power plants thence reducing the gasses as well as greenhouse emissions.

Different models were adopted to model the flux of waves such as shallow wave modelling;

this model will be investigated in this research to assess the wave flux. The solving of wave equation model required numerical technique to solve differential equations, so the Matlab software will be used in this research for this purpose. Different numerical techniques will be used to solve these models such as Up-wind, Lax-wendroff and Lax-friedrichs.

Upwind technique is a class of numerical methods used for solving partial differential equations; the general model describes the propagation of velocity with respect to time and space for one dimensional flow. This scheme can be solved for the first order, second order and third order according to the number of data are used in finite difference analysis and the formulation of the finite difference equation( backward forward, and central) , where two data points are required to solve the first order upwind form which is also called linear upwind scheme. For the second order scheme three set points are used for solving the flux equation which can be forward, or backward set points. The third order scheme is used also three set points but in central form [5].

Also, the Lax-windroff is a scheme used for solving partial differential equations, this technique is a second order technique for the time and space and it can be used for solving the governing model at the current time. This method is effective to be used for explicit integration [6].

The Lax-friedrichs scheme is used to solve the partial differential equation. However, this scheme solves the time in forward finite difference and cantered in the space, it is also used to analyze the nonlinear problems [7].

## II. LITERATURE REVIEW

Power generation from the rise and fall of the tide refers to the Middle Ages, perhaps, to the Roman period. A tide mill made of a storage tanks that filled by the incoming tide and unloads by the outgoing ebb as well as flow through water wheel. The tidal barrage is the new version of this technology, which it operates in a manner similar to the traditional generation of hydroelectric power. Tidal barrages paid great attention in the middle of the 20th century. Even though, globally, there were only three sites ready to be used because of the effects of the environmental and capital cost of these high-tech. recently, the development is focusing on the kinetic energy of the fast-moving tidal currents. This Technique has the ability to generate energy from the tides with

less environmental effect or economic challenges [2].

Tidal energy is generated by converting the kinetic energy of the water to electricity. The tidal energy machine consist of turbine that revolving in the horizontal axis and a rectangular space for the vertical axis. The cubic speed of water and, and efficiency can be described mathematically as in equation [2].

$$p = \frac{1}{2} \rho U^3 A \eta \quad (1)$$

Where P is the energy produced by the turbine,  $\rho$  is the density of seawater (approx.1024 kg/m<sup>3</sup>), U is the speed of wave, A is the field of water intercepted by the machine and  $\eta$  is the device efficiency.

Generally, there are three main types of wave energy technologies. The first type uses buoys or pitching devices using the height and descent of ocean swells to push hydraulic pumps. The second model uses Oscillating Water Column (OWC) system devices to generate electricity at the beach using the height and descent of water inside a cylindrical shaft. The height water pushes the air outside of the top of the shaft, Operates the turbine-driven air. The third type is a tapered channel, which can be located on the beach or the outside beach. On this type trapping device concentrate waves and push them inside an elevated reservoir, where the energy is created using turbines to generate hydroelectric power which has been generated from the water. The vast majority of the wave energy project that has been proposed recently to use offshore floats, buoys or pitching devices [3].

At the beginning of 2007 it has been established in Portugal's first commercial organization of wave energy at the beach of the Atlantic coast in Portugal. The first phase of the project, work on ocean energy harvesting and includes three devices work on the conversion of wave energy and this combination generates of about 22.5MWof electricity, and the company has worked to expand the facility to produce 22.5MW.

According to [8] the global demand of energy is growing as well as the devices that generate wave and tidal energy can provide an important renewable energy source. Technological developments indicate that there are rising in the traditional energy cost that means the resources of offshore energy are economic in the future. There is huge growing now on the data body that related to environmental impacts

of wind farms. Moreover, the scientific information on which making a correct decision of the environmental impact of different offshore energy sources was lacked. The barrages of Tidal waves had the ability to make sever ecological effects especially on the areas of bird feeding when they constructed the coastal bays or estuaries. Offshore tidal wave energy as well as stream energy collectors give the developments scope at different scales. They have also the potential of changing the homeland. A design variety is exist and includes seabed mounted devices, mid-water column as well as floating with a diversity of the moving part components resulting in a unparalleled complexity of environmental effects of all types that discussed. The results illustrate that the collectors of wave energy had the ability to alter sea bed as well as water column habitats when changing the environment of wave that causes a lot of changes over wide are of the installation. Generally, the impacts scale can scale with development of the size as well as change depending on the location nature. The effects can be fairly rapidly as well as reversible, when an installation is removed. Moreover, scientific studies could be escort with the commercial scale licensing of devices installations.

The numerical models Review were used to determine the tidal waves interaction as well as their properties. Wave energy that occurs in ocean is very promising source because it is considered important resource of renewable energy. The potential of the ocean wave energy can be found by a numerical model that can provide guidelines for more researches on the field of renewable energy.

According to [9] there is a numerical model created in black sea by MIKE21 SW that used in order to calculate the wave energy. Moreover, this model illustrate that MIKE21 SW provided better reads of wave altitudes than the periods of waves in the black sea as well as the annual wave energy average potentials have been calculated for various locations. There are another work had been done by [2]. Furthermore, the authors are used the model WAM, in the enhanced version which allowing for different two nesting ways, for generation wave as well as covering most of North Atlantic, where SWAN model is used to transform the waves among the coastal regions. The energy of wave can be calculated easily from the components of energy transport.

According to [10], a model of numerical wave techniques is used. Because of the powerful computers advent, as well as the development of different numerical methods to solve the

coastal issues as well as problems by using numerical techniques were found in order to be more reliable, time saving as well as cost effective tools. Moreover, numerical techniques were used to forecast as well as hind cast the parameters of wave that helping for coastal structures design. Thus good wave parameters are very important. Numerical techniques are good powerful tool that can be easily used to calculate the climate of wave. Numerical models can help to reduce the rises difficulties for wave prediction because of the ocean random nature. Numerical models had been divided into four main categories named first then second and third as well as improved third numerical models of generation. Numerical models are practical when designing different coastal as well as ports structures, by using accurate parameters. Averaged phase model cannot simulate the diffraction phenomenon that leading to use the phase resolving models. Furthermore, the numerical method is helpful for forecasting the ocean wave, which very important to determine the heights of peak wave during tornado as well as acts as tools for lifesaving during natural environmental hazards. The Comparison between waves models show that the fine grid methods are very suitable at the coastal areas because the bathymetry is varying while coarser grid in the deep water can produce accurate results also with low computational time. Numerical techniques are very useful for renewable energy improvement. With the numerical models help, the possibility of high energy regions in ocean can be identified that can help to make more research and studies on these zones for economically extraction of energy from waves. The main numerical model inputs are the wind data, when the wind data are inaccurate, the results of model will be incorrect. Steps such as the assimilation of data can be taken in order to improve and develop the data as well as to increase also the numerical model accuracy. By forecasting the error, the numerical methods accuracy can be improved. Different attempts had been done for coupling the wave techniques with the models of ocean circulation for better understanding of the numerical models phenomenon, the uses as well as steps are taken in order to increase the numerical models accuracy.

[11]An improvement of mathematical methods is done in (2009) for the cross-flow maritime vertical-axis present turbine. An expectation of the performance of the rotor as well as the hydrodynamic loads help to get good design computations. The survey is firstly applied methods for double as well as multiple stream

tubes theoretically, monitored also by physical testing methods which mainly were a scaled-down turbine as well as mathematical simulations. Mathematical testing for the used full-scale turbine type that was (flow behavior, blade loads as well as power coefficient) which accepted the utilizing of advanced computational methods. The turbine design was applying a precise time merchant solver (RANS). The transient-rotor-stator model with the animated procedure mesh had been applied in order to get an alteration in the flow field in a specific period. The modes of disturbance shear stress-transport had been applied for modelling strong flow properties. The mathematical results very perfect convention during the investigational measurements as well as the theoretic of double-multiple-stream tube model. Turbine sensitivity to parameters differences was confirmed by full-scale study. Furthermore, this indicates that the advanced techniques could be successfully expecting the hydrodynamic performance as well as the vertical-axis present turbine maritime essential of blade loads design.

### III. UPWIND THEORY FOR SHALLOW WATER EQUATIONS

According to (Kurganov and Petrova, 2008) "a simple one-dimensional (1-D) toy model" has been investigated in order to study landslides-generated water waves. Also, the landslide has been built as a rigid bump transforming down the bottom side when the motion of water is also modelled using the Saint-Venant system, which is applied in order to solve shallow water equations problem. The output system has been solved numerically utilizing positive balanced in order to maintain the central system upwind. Digital results have been obtained in new agreement with each of the numerical simulation two-dimensional flow of an incompressible in addition to experimental data [12].

As mentioned by (Delestre, 2010), Rounding numerical equations in shallow water with the topography is flat. At which it provides to individualize the new topography, which makes each scheme able to be strong and balanced. In contrast with the reconstruction hydrostatic known, the suggested numerical action does not include any cutting process. Furthermore, the plan that has been obtained is capable to cope with the dry regions. Many numerical benchmarks have been performed in order to confirm the benefits of techniques [13].

As mentioned by (Xing and Shu, 2005) the Second-class semi-detached Central upwind planner for one in addition to two-dimensional systems of equations two shallow water. Scheme contained is balanced in the sense, which is considered fixed solutions steady-state as well as perfectly preserved through the system. In addition, there is a new method that is utilized in order to treat the non-governor of products and the momentum exchange between the existed layers. Also, the performance of the suggested technique is demonstrated on some numerical examples that are obtaining interfaces for publication as well as successfully steady state solutions [14].

Upwind scheme is depending on the idea of discretizing spatial derivatives. Hence, a propagation sense is implied and these procedures are also well adjusting to advection dominated problems. The method is considered as an extension to a non-linear system for example

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

Exploits formula is illustrated below;

$$\frac{\partial U}{\partial t} + J \frac{\partial U}{\partial x} = 0$$

Also, the Jacobean property demonstrated as the following;

$$j = p \wedge p^{-1}$$

Where that  $\Lambda$  is equal to diagonal matrix in addition to the eigenvalues in the diagonal and also the matrix right eigenvectors can be expressed by (P). When the system is linear, it would be possible to decouple it in an easy way in order to implement the procedure of scalar for all equations as illustrated below [12]:

$$U_i^{n+1} = U_i^n - \frac{\Delta}{\Delta x_i} (p \wedge p^{-1})_{i+0.5}^n (U_{i+1}^n - U_i^n) + ((p \wedge p^{-1})_{i-0.5}^n) (U_i^n - U_{i-1}^n)$$

### IV. LAX-FRIEDRICHS SCHEME

In general, this is considered as the first place in time and space Central Technology teams are limited, system conservation homogeneous are illustrated in the next equation [12, 13];

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

The technique in order to update one time step  $\Delta t$  the interior point 2. N - 1 of a uniform grid is depending on a nodal updating:

$$U_i^{n+1} = \alpha U_i^n + \frac{1-\alpha}{2} (U_{i+1}^n + U_{i-1}^n) - \frac{\Delta t}{2\Delta x} (F_{i+1}^n - F_{i-1}^n) \quad 0 \leq \alpha < 1$$

The value of  $\alpha$  is equal to 1 renders so that the scheme can be considered unstable. Greater stability in addition to more numerical diffusion is entered as  $\alpha$  oncoming zero. Also, the value of  $\alpha$  is equal to 0.1 is typically adopted. Conservative scheme, which recognizes the next numerical flow:

$$F_{i+0.5}^* = 0.5 \left( F_{i+1} + F_i - (1-\alpha) \frac{\Delta x_i}{\Delta t} (U_{i+1} - U_i) \right)$$

Also, the scheme of Lax–Friedrichs is generally derived using finite different for example the spatial discretization. The scheme is considered very strong; this is because of the added numerical dissipation that tends to destroy stopping point. A formal analysis of Taylor for time step offers that the truncation scheme of error is existed for two discretization factors.

Instead of truncation errors, it is necessary to determine a condition when error is occur at a fixed time (using an increasing number of time steps), and also it is important to divide time to slices using  $\Delta t$ , giving order single error, such as solving the calculated error for example the  $L^1$  -and  $L^\infty$  norm therefore reduces in a linear shape as the discretization factors.

## V. SCHEME OF THE LAX–WENDROFF

Forward Euler that used integration should be changed by the central difference method in order to investigate precision of formal second-order, as illustrated in next formula.

$$Q_i^{n+1} = Q_i^n - r[F_{i+1}^{n+0.5} - F_{i-0.5}^{n+1}]$$

Also, the fluxes  $-F_{i-0.5}^{n+1}$  should be identified, which is related to all values located at the center points at a specified time can be calculated using  $t^{n+0.5} = (n + 0.5)\Delta t$  formula. Also, it is a clear that all of these values can be detected with acceptable accuracy when scheme of the lax\_friedrich is used on the grid with grid \_spacing equal to  $0.5\Delta t$ , the following is scheme.

$$Q_{i+0.5}^{n+0.5} = 0.5(Q_{i+1}^n + Q_i^n) - 0.5r[F_{i+0.5}^{n+0.5} - F_{i-0.5}^{n+0.5}]$$

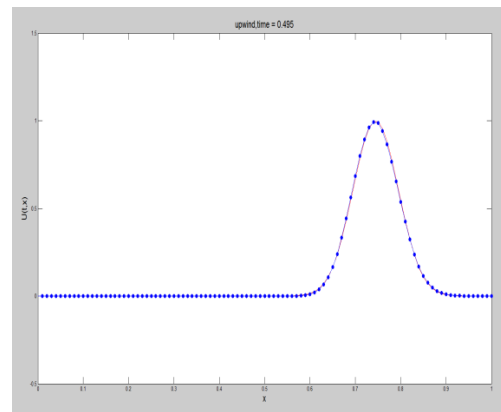
Perfectly, this provide a second \_order, scheme of the predictor \_corrector that called the scheme of lax \_wendroff for instance the scheme of lax \_friedrichs, as illustrated in next formula

$$Q_{i+0.5}^{n+0.5} = 0.5(Q_{i+1}^n + Q_i^n) - r0.5[F_{i+1}^n - F_i^n]$$

Generally, the planner is considered as a non-dissipative when the value of J is constant, in addition to show oscillations near strong gradients that called shocks. Also, it can cause numerical problems near sonic or critical points. The majority of researchers in these fields recommended that there is a need to add extra dissipative terms in these situations, such as the scheme of Lax-Wendroff's that can be considered as one of the most frequently investigated in the literature that related to standard schemes of shock-capturing. In addition, all problems have been listed and solved when trying in order to precision keep of the second order as well as contain terms of a source in the discretization [13,14].

## VI. RESULTS

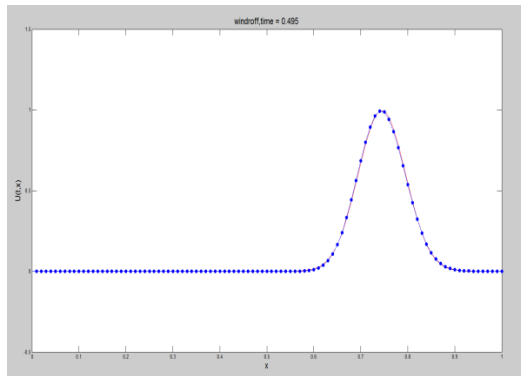
The three methods which are mentioned before are used to solve the flow equation, where the matlab code was built, the results to these methods are shown in the following figures, the main assumption was considered are; the flow rate at initial is zero, the flow rate profile at the cross section is constant at each time step, the calculation of the finite difference depends on the central finite difference scheme. Where Figure (1) shows the results for the upwind scheme



**Figure (1) : The flow vs. time for upwind scheme**

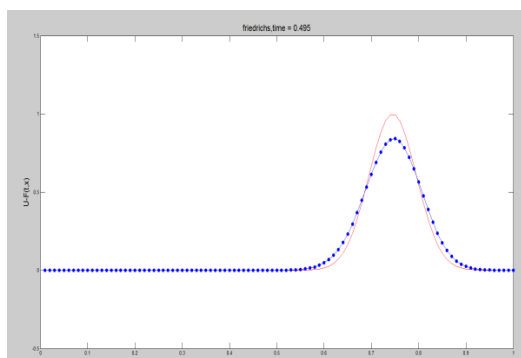
Figure (2) also shows the results for the Wendroff Scheme, where this method depends on the forward scheme of finite difference. The results of exact solution are closely to results of simulation. Where the results of exact solution

are plotted in red colour and the results of numerical solution are plotted in blue colour.



**Figure (2) : The flow vs. time for Wendroff scheme**

Figure (3) shows the results when Lax–Friedrichs scheme are implemented where the results shows there are different between the exact solution and the numerical results for the flow velocity. Where the results of exact solution are plotted in red colour and the results of numerical solution are plotted in blue colour.



**Figure (3) : The flow vs. time for Friedrichs scheme**

The dam break theory was used also to simulate the velocity and the height variation with time as will be shown in the following analysis, the wave solution can be expressed according to the height and the velocity of the wave, where the water falls from the end of the wall, the height of the wall (depth 0 was assumed as 1m, where the critical height was assumed as 0.2, with zero initial velocity as shown in the following assumptions

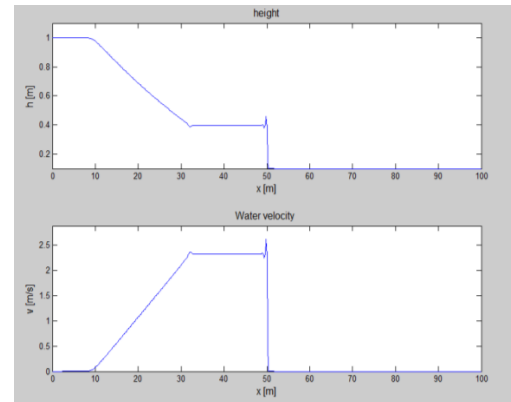
$$h(0) = 1$$

$$h_r = 0.2$$

$$v(0) = 0$$

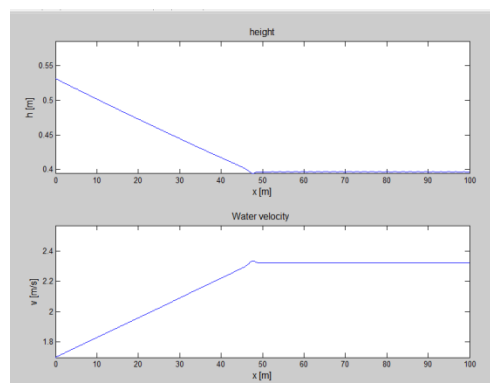
Figure (4) shows The values of the velocities and the height by the time at x=50 m when Lax-

Wendroff is applied , where this simulation was obtained for distance (100 m) in x-axis as shown the height is reduced with distance and time because the water falls down , where the velocity increases up to 2.5 m/s



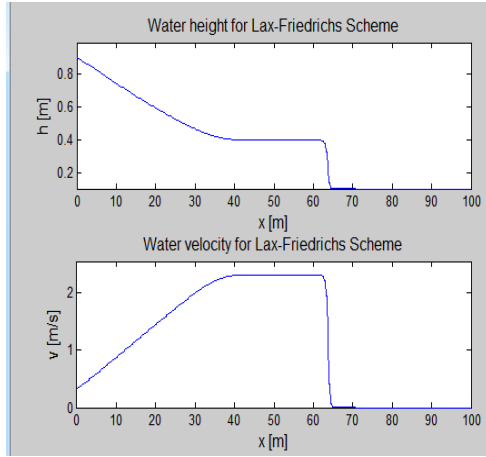
**Figure (4) : The velocity and the height variation for Dam break theory (x=50 m).**

Figure (5) shows other plot for the values of the velocities and the height with the time at x=100 m, when also Lax-Wendroff is applied as shown the height is reduced with distance and time because the water falls down , where the velocity increases up to 2.4 m/s.



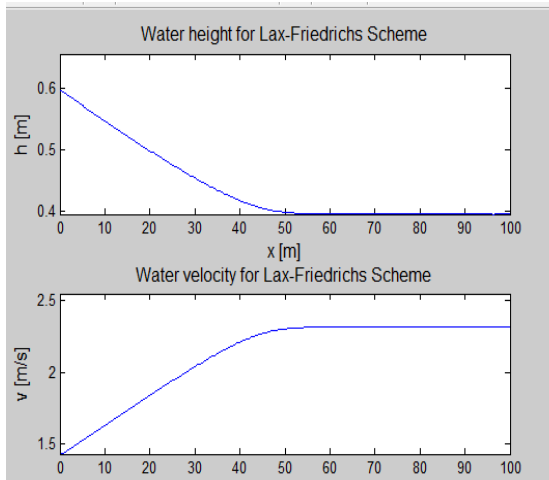
**Figure (5) : The velocity and the height variation for Dam break theory(x=100 m).**

Other scheme was used to simulate the wave equations using lax- Friedrichs, with keeping the same assumptions, where Figure (6) shows the values of the velocities and the height with the time for x=50 m when lax- Friedrichs is applied, as shown the height is reduced with distance and time because the water falls down, where the velocity increases up to 2.5 m/s



**Figure (6) : The velocity and the height variation for Dam break theory( $x=50$  m).**

Figure (7) shows the values of the velocities for lax- Friedrichs at  $x=100$  m when, as shown the height is reduced with distance and time because the water falls down, where the velocity increases up to 2.4 m/s. the velocity profile for Lax-Wendroff considered the instantaneous velocity where velocity profile for the lax- Friedrichs doesn't include this velocity.



**Figure (7) : The velocity and the height variation for Dam break theory( $x=100$  m).**

Comparing between the three methods the following points can be concluded the lax - Friedrichs scheme has low accuracy comparing with the Wendroff scheme this point can be explained according to the order of differential equations can be solved by each of them. For the lax windroff high accuracy can be presented because more terms are included in the numerical iterations[15] , so the error is faster decreases by increasing the number of iterations, where this scheme is the most suitable to be used for all orders of differential

equations[1]. The lax Friedrichs method gives accurate results for first order differential equation because it depend on the upwind – downwind procedure [17]. The upwind scheme depends on the forward difference method so it's accurate for the first order differential equations only. The following table shows a comparison between the three difference schemes.

Table.1: comparison between the different schemes

	Upwind	lax Friedrichs	lax windroff
Order	First	First	Any
Difference method	Forward	Forward-backward	Forward-backward Up to five terms

Figure (8) shows the difference in the velocity calculation for both the lax Friedrichs (ufd) and lax windroff (uwe), where the difference increases with increasing the step times.

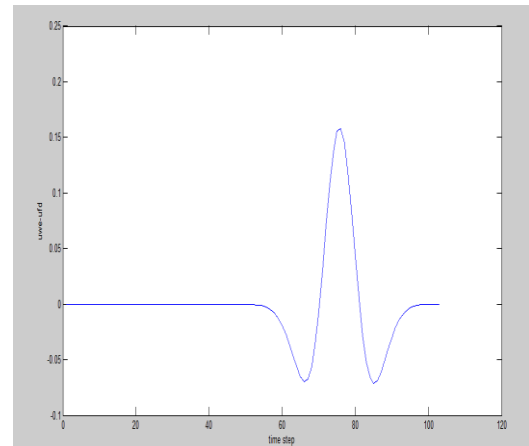


Figure (8): the difference in velocities between lax Friedrichs (ufd) and lax windroff (uwe),

## VII CONCLUSION

Tidal waves have potential energy can be used to extract a useful energy so in this project the behaviour of the water waves was investigated using shallow wave equations . The shallow wave equation was solved in this project using different schemes which are upwind, lax-Friedrichs and Lax-Wendroff. The wave velocity was simulated for each case of them



using Matlab software and the results were compared with the exact results. By comparing between the three different methods can be noted that; the upwind and the lax- Friedrichs are suitable to be used for the first order equations where the Lax-Wendroff method is a suitable to be used for the high order equations. According to the dam break theory, the results shows the height decreases by increasing the time, also the velocity of the water increases , where the velocity increases according to the theory of energy conservation. The potential energy converts to kinetic energy at the water fail, so the velocity increases with decreases the height.

By changing the angle of the water failing, it was noted that; the velocity of the water was not changed due to energy conservation theory, where the velocity of failing is a function of height, in the other words the potential energy of the water will be converted to kinetic energy, so, to increase the velocity of failing the height of dam should be increased. it's worth to mention that; the useful energy which can be extracted from waves interaction is a function of water velocity and height.

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