NCERT Solutions for Class 12 Physics

Chapter 13 Nuclei

Question 1.

(a) Two stable isotopes of lithium $^{6}_{3}\mathrm{Li}$ and $^{7}_{3}\mathrm{Li}$ have respective abundance of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u respectively. Find the atomic weight of lithium.

(b) Boron has two stable isotopes $^{10}_{5}\mathrm{Li}$ and $^{11}_{5}\mathrm{Li}$ Their respective masses are 10.01294 u and 11.00931 u and the atomic weight of boron is 10.811 u. Find the abundances of $^{10}_{5}\mathrm{Li}$ and $^{11}_{5}\mathrm{Li}$

Answer:

(a) Atomic weight of lithium

$$= \frac{6{\cdot}01512 \times 7{\cdot}5 + 7{\cdot}01600 \times 92{\cdot}5}{100} = \frac{45{\cdot}1134 + 648{\cdot}98}{100}$$

$$= 6{\cdot}9409\ u$$

(*b*) Let $x\%$ and $y\%$ be the abundances of $^{10}_{5}\mathrm{B}$ and $^{11}_{5}\mathrm{B}$ respectively.

$$\therefore \quad x + y = 100 \qquad \ldots(i)$$

Also, Atomic weight

$$= \frac{x \times 10{\cdot}01294 + y \times 11{\cdot}00931}{100}$$

i.e. $10{\cdot}811 \times 100 = 10{\cdot}01294x + 11{\cdot}00931y$

i.e. $10{\cdot}01294x + 11{\cdot}00931y = 1081{\cdot}1 \qquad \ldots(ii)$

Using equation (*i*), we get

$10{\cdot}01294(100 - y) + 11{\cdot}00931y = 1081{\cdot}1$

i.e. $1001{\cdot}294 + 0{\cdot}99637y = 1081{\cdot}1$

i.e. $y = 80{\cdot}09\%$

Then $x = 100 - 80{\cdot}09 = 19{\cdot}91\%$.

Question 2.

The three stable isotopes of neon :$^{20}_{10}\mathrm{Ne}$ and $^{22}_{10}\mathrm{Ne}$ have respective abundance of 90.51%, 0.27% and 9.22%. The atomic masses of three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Answer:

Average atomic mass of neon

$$= \frac{90 \cdot 51 \times 19 \cdot 99 + 0 \cdot 27 \times 20 \cdot 99 + 9 \cdot 22 \times 21 \cdot 99}{100}$$

$$= \frac{1809 \cdot 29 + 5 \cdot 67 + 202 \cdot 75}{100} = 20 \cdot 18 \text{ u.}$$

Question 3.

Obtain the binding energy of a nitrogen nucleus (147N) from the following data :

m_H = 1.00783 u

m_n = 1.00867 u

m_n = 14.00307 u

Give your answer in MeV.

Answer:

Mass defect, $\Delta m = Zm_H + (A - Z)\, m_n - m_N$

Here $A = 14$, $Z = 7$

$$\therefore \quad \Delta m = 7 \times 1 \cdot 00783 + (14 - 7) \times 1 \cdot 00867 - 14 \cdot 00307$$

$$= 7 \cdot 05481 + 7 \cdot 06069 - 14 \cdot 00307$$

$$= 0 \cdot 011243 \text{ u}$$

$$\therefore \quad \text{B.E.} = \Delta m \times 931 \text{ MeV}$$

$$= 0 \cdot 11243 \times 931 = 104 \cdot 7 \text{ MeV}$$

Question 4.

Obtain the binding energy of the nuclei 5626Fe and

in units of 20983Bi from the following data:

m_H =1007825u

m_n =1008665u

m (5626Fe)= 55.934939 u

m (20983Bi)

Which nucleus has greater binding energy per nucleon?

Answer:

For $^{56}_{26}Fe$, mass defect

$$\Delta m = Z\, m_H + (A - Z)\, m_n - m\left(^{56}_{26}Fe\right)$$
$$= 26 \times 1.007825 + (56 - 26)$$
$$\times 1.008665 - 55.934939 = 0.528461 \text{ u.}$$

$\therefore$ B.E. $= \Delta m \times 931.5$ MeV

$= 492.0$ MeV.

$\therefore$ Binding energy per nucleon

$$= \frac{B.E.}{A} = \frac{492.0}{56} = 8.79 \text{ MeV}$$

For $^{209}_{83}Bi$, mass defect is

$$\Delta m = Zm_H + (A - Z)\, m_n - m\left(^{209}_{83}Bi\right)$$
$$= 83 \times 1.007825 + (209 - 83)$$
$$\times 1.008665 - 208.980388 = 1.760877 \text{ u.}$$

$\therefore$ Binding energy $= \Delta m \times 931.5$ MeV

$= 1.760877 \times 931.5$

$= 1640.26$ MeV

$\therefore$ Binding energy per nucleon

$$= \frac{B.E.}{A} = \frac{1640.26}{209}$$
$$= 7.85 \text{ MeV}$$

Clearly, $^{56}_{26}Fe$ has more Binding energy per nucleon compared to $^{209}_{83}Bi$.

Question 5.

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of 6329Cu atoms (of mass 62.92960 u). The masses of proton and neutron are 1.00783 u and 1.00867 u, respectively.

Answer:

Mass defect $\Delta m = Zm_p + (A - Z)\, m_n - M$

Here A = 63, Z = 29

$\therefore$ (A – Z) = 63 – 29 = 34

Also $m_p = 1{\cdot}00783$ u, $m_n = 1{\cdot}00867$ u, and M = 62·92960 u

$$\therefore \Delta m = 29 \times 1{\cdot}00783 + 34 \times 1{\cdot}00867 - 62{\cdot}92960 = 0{\cdot}59225 \text{ u.}$$

$$\therefore \quad \text{Binding energy} = \Delta m \times 931 \text{ MeV} = 0{\cdot}59225 \times 931 = 551{\cdot}38 \text{ MeV.}$$

Now 1 gram mole of copper contains $6{\cdot}02 \times 10^{23}$ atoms.

$\therefore$ mass of 63 g has $6{\cdot}02 \times 10^{23}$ atoms.

$\therefore$ mass of 3 g of copper has $\dfrac{6{\cdot}02 \times 10^{23} \times 3}{63}$ atoms.

Now Binding energy of one atom is 551·38 MeV

$\therefore$ Binding energy of $\dfrac{6{\cdot}02 \times 10^{23} \times 3}{63}$ atoms

$$= \frac{551{\cdot}38 \times 6{\cdot}02 \times 10^{23} \times 3}{63}$$

$= 1{\cdot}58 \times 10^{25}$ MeV.

Question 6.

Write nuclear equations for :

(a) the α-decay of 22686Ra

(b) the β--decay of 3215p

(c) the β+-decay of 116p

Answer:

(a) $^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$ (α particle)

(b) $^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^{-}$ (β particle) + $\bar{\nu}$

(Antineutrino)

(c) $^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^{+}$ (position) + ν

(neutrino)

Question 7.

A radioactive isotope has a half-life of T years. After how much time is its activity reduced to 3.125% of its original activity (b) 1% of original value ?

Answer:

(*a*) $\frac{A}{A_0} = \frac{N}{N_0} = \frac{3.125}{1000_2} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$

Now $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\therefore \quad n = 5$ or $t = 5$ T years

(*b*) $\frac{A}{A_0} = \frac{N}{N_0} = \frac{1}{100}$

$\therefore \quad t = \frac{2.303}{\lambda} \log \frac{N_0}{N} = \frac{2.303\,T}{0.093} \log 100$

$= \frac{2.303\,T \times 2}{0.693} = 6.65\,T$ years

Question 8.

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive 146C present with the stable carbon isotope

612C When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of 146C , and the measured activity, the age of the specimen can be approximately estimated. This is the principle of 146C dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Answer:

Here $A_0 = 15$ decays/minute/g;

$A = 9$ decays/minute/g

$\therefore$ Using $A = A_0 e^{-\lambda t} \Rightarrow 9 = 15 e^{-\lambda t}$

or $e^{\lambda t} = \frac{15}{9} = \frac{5}{3}$

Taking natural log of both sides

$$\log_e e^{\lambda t} = \log_e \left(\frac{5}{3}\right)$$

or $\lambda t = 2 \cdot 303 \left(\log \frac{5}{3}\right)$

$= 2{\cdot}303 \, [\log 5 - \log 3]$

$\therefore \lambda t = 2{\cdot}303 \, [0{\cdot}6990 - 0{\cdot}4771]$

$= 2{\cdot}303 \times 0{\cdot}2219 = 0{\cdot}511$

$\therefore t = \frac{0 \cdot 511}{\lambda} = \frac{0 \cdot 511}{0 \cdot 693} \times T_{1/2}$

$$\left[\because \lambda = \frac{0 \cdot 693}{T_{1/2}}\right]$$

$= \frac{0 \cdot 511}{0 \cdot 693} \times 5730 = 4225$ years.

Half-life formula is the time required for the amount of something to fall to half its initial value.

Question 9.

Obtain the amount of 6027Co necessary to provide a radioactive source of 8.0 mCi strength. The half life of 6027Co is 5.3 years.

Answer:

1 Ci = 3.7×10^{10}

disintegrations per second.

$\therefore 8.0$ mCi = $8 \times 10^{-3} \times 3.7 \times 10^{10}$

= 2.96×10^{8} disintegrations/s.

Also, T = 5.3 years = $5.3 \times 365 \times 24 \times 60 \times 60$ s
= 1.67×10^{8} s

$$A = \lambda N = \frac{0.693}{T} N$$

$$N = \frac{AT}{0.693}$$

$$= \frac{2.96 \times 10^{8} \times 1.67 \times 10^{8}}{0.693}$$

$$= 7.13 \times 10^{16}$$

Now 6.023×10^{23} atoms of cobalt have mass = 60 g

$\therefore$ 7.13×10^{16} atoms of cobalt have mass

$$= \frac{60 \times 7.13 \times 10^{16}}{6.023 \times 10^{23}} = 7.1 \times 10^{-6} \text{ g}$$

Question 10.

The half-life of 9038Sr is 28 years. What is the disintegration rate of 15 mg of this isotope?

Answer:

Number of atoms in 15 mg $^{90}_{38}Sr$

$$= \frac{6.02 \times 10^{23} \times 15 \times 10^{-3}}{90} = N$$

Disintegration constant, λ

$$= \frac{0.693}{T} = \frac{0.693}{28 \times 365 \times 24 \times 3600} s^{-1}$$

$$\therefore \text{Rate of disintegration } \left(\frac{dN}{dt}\right) = \lambda N$$

$$= \frac{0.693 \times 6.02 \times 10^{23} \times 15 \times 10^{-3}}{28 \times 365 \times 24 \times 3600 \times 90}$$

= 7.87×10^{10} Bq.

Question 11.

Obtain approximately the ratio of the nuclear radii of the gold isotope 19779Au and silver isotope 10747Au.

Answer:

$R \propto A^{1/3}$

$$\therefore \frac{R_{Au}}{R_{Ag}} = \left(\frac{A_{Au}}{A_{Ag}}\right)^{\frac{1}{3}} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = (1 \cdot 84)^{\frac{1}{3}}$$

$$= 1 \cdot 23.$$

Question 12.

Find Q-value and kinetic energy of the emitted α-particle in the α-decay of (*a*) $^{226}_{88}Ra$ (*b*) $^{220}_{86}Rn$. Given, $m(^{226}_{88}Ra) = 226 \cdot 02540\ u$;

$$m(^{222}_{86}Rn) = 222 \cdot 01750\ u$$

$$m(^{220}_{86}Rn) = 220 \cdot 01137\ u;$$

$$m(^{216}_{84}Po) = 216 \cdot 00189\ u;\ M(_2He^4) = 4 \cdot 00260\ u.$$

Answer:

(*a*) $^{226}_{88}Ra \longrightarrow ^{222}_{86}Rn + {}_2He^4$

∴ Q-Value

$= M(^{226}_{88}Ra) - [M(^{222}_{86}Rn) + M(_2He^4)]$

$= 226{\cdot}02540 - 226{\cdot}0201$

$= 0{\cdot}0053\ u = 0{\cdot}0053 \times 931\ MeV$

$(\because 1u = 931\ MeV)$

$= 4{\cdot}93\ MeV$

K.E. of α-particle

$$= \left(\frac{A-4}{A}\right)Q = \left(\frac{222-4}{222}\right) \times 4{\cdot}93$$

$= 4{\cdot}84\ MeV$

(*b*) $^{220}_{86}Rn \longrightarrow ^{216}_{82}Po + {}_2He^4$

∴ Q-value

$= m(^{220}_{86}Rn) - [m(^{216}_{82}Po) + m(^4_2He)]$

$= 220{\cdot}011373 - [216{\cdot}00189 + 4{\cdot}00260]$

$= 220{\cdot}011373 - 220{\cdot}00449 = 0{\cdot}006883\ u$

$= 0{\cdot}006883 \times 931\ MeV = 6{\cdot}41\ MeV$

K.E. of α-particle

$$= \left(\frac{A-4}{4}\right)Q = \left(\frac{216-4}{216}\right) \times 6{\cdot}41$$

$= 6{\cdot}29\ MeV.$

Question 13.

The radionuclide ^{11}C decays according to

$$^{11}_6C \to ^{11}_5B + e^+ + \nu\ ;\ T_{1/2} = 20{\cdot}3\ min.$$

The maximum energy of the emitted positron is 0·960 MeV.

Given the mass values :

$m\left(^{11}_6C\right) = 11{\cdot}011434\ u,$

$m\left(^{11}_5C\right) = 11{\cdot}009305\ u,\quad m_e = 0{\cdot}000548\ u$

Calculate Q and compare it with the maximum energy of the positron emitted.

Answer:

The equation of the decay process is

$${}^{11}_{6}C \longrightarrow {}^{11}_{5}B + {}_{+1}e^{0} + \nu + Q$$

$$\therefore Q = [m_N({}^{11}_{6}C) - m_N({}^{11}_{5}B) - m_e] \times 931{\cdot}5 \text{ MeV}$$

Here m_N stands for the nuclear mass of the element or particle. In order to express the Q value in terms of the atomic masses, 6 m_e mass has to be subtracted from the atomic mass of 116Au and

subtracted from atomic mass of ${}^{11}_{5}B$ (Because their atoms contain outside electrons). Therefore, in terms of the atomic mass m [*i.e.* $m({}^{11}_{6}C)$ and $m({}^{11}_{5}B)$]

$$Q = \left[m({}^{11}_{6}C) - 6m_e - m({}^{11}_{5}B) + 5m_e - m_e\right] \times 931{\cdot}5 \text{ MeV}$$

$$= \left[m({}^{11}_{6}C) - m({}^{11}_{5}B) - 2m_e\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [11{\cdot}011434 - 11{\cdot}009305 - 2 \times 0{\cdot}000548] \times 931{\cdot}5$$

$$= 0{\cdot}001033 \times 931{\cdot}5 = 0{\cdot}962 \text{ MeV}.$$

This energy is comparable to actual energy released in the decay process.

5 m_e mass has to be

Question 14.

The nucleus 2310Ne decays by β~ emission. Write down the p-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

m(2310Sr) = 22.994466 u

m(2311Sr) = 22.989770 u.

Answer:

The decay process of $^{23}_{10}\text{Ne}$ is

$$^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{\nu} + Q$$

If whole of energy is carried by β particle, then

$$Q = \left[m_N\left(^{23}_{10}\text{Ne}\right) - m_N\left(^{23}_{11}\text{Na}\right) - m_e\right] \times 931{\cdot}5 \text{ MeV}$$

where $m_N\left(^{23}_{10}\text{Ne}\right)$ and $m_N\left(^{23}_{11}\text{Na}\right)$ are nuclear masses of $^{23}_{10}\text{Ne}$ and $^{23}_{11}\text{Na}$ respectively. If $m\left(^{23}_{10}\text{Ne}\right)$ and $m\left(^{23}_{11}\text{Na}\right)$ are atomic masses of $^{23}_{10}\text{Ne}$ and $^{23}_{11}\text{Na}$ respectively, then

$$m_N\left(^{23}_{10}\text{Ne}\right) = m\left(^{23}_{10}\text{Ne}\right) - 10m_e$$

(∵ atom of Ne contains 10 electrons)

and $m_N\left(^{23}_{11}\text{Na}\right) = m\left(^{23}_{11}\text{Na}\right) - 11m_e$

(∵ atom of Na contains 11 electrons)

$$\therefore Q = \left[m\left(^{23}_{10}\text{Ne}\right) - 10m_e - m\left(^{23}_{11}\text{Na}\right) + 11m_e - m_e\right] \times 931{\cdot}5$$

$$= \left[m\left(^{23}_{10}\text{Ne}\right) - m\left(^{23}_{11}\text{Na}\right)\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [22{\cdot}994466 - 22{\cdot}989770] \times 931{\cdot}5 \text{ MeV}$$

$$= 0{\cdot}004689 \times 931{\cdot}5 = 4{\cdot}37 \text{ MeV}.$$

Question 15.

The Q value of a nuclear reaction A + b ⇒ C + d is defined by [Q = m_A + m_b-m_c− m_d] c^2 where the masses refer to nuclear rest masses. Determine from the given data whether the following reactions are exothermic or endothermic.

(*i*) $^{1}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H}$

(*ii*) $^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$

Atomic masses are given to be

$m\left(^{1}_{1}\text{H}\right) = 1{\cdot}007825$ u, $m\left(^{2}_{1}\text{H}\right) = 2{\cdot}014102$ u, $m\left(^{3}_{1}\text{H}\right) = 3{\cdot}016049$ u,

$m\left(^{12}_{6}\text{C}\right) = 12{\cdot}000000$ u

$m\left(^{20}_{10}\text{Ne}\right) = 19{\cdot}992439$ u,

$m\left(^{4}_{2}\text{He}\right) = 4{\cdot}002603$ u

Answer:

(*i*) The given reaction is

$$ {}_1^1\mathrm{H} + {}_1^3\mathrm{H} \longrightarrow {}_1^2\mathrm{H} + {}_1^2\mathrm{H} $$

$$ \therefore\ Q = [m_N({}_1^1\mathrm{H}) + m_N({}_1^3\mathrm{H}) - m_N({}_1^2\mathrm{H}) - m_N({}_1^2\mathrm{H})] \times c^2 \quad \ldots(1) $$

where m_N refers to nuclear masses.

$\therefore$ if m refers to atomic masses, then

$$ m\left({}_1^1\mathrm{H}\right) = m_N\left({}_1^1\mathrm{H}\right) + m_e $$

$$ \Rightarrow\quad m_N\left({}_1^1\mathrm{H}\right) = m\left({}_1^1\mathrm{H}\right) - m_e $$

Similarly, $m_N\left({}_1^3\mathrm{H}\right) = m\left({}_1^3\mathrm{H}\right) - m_e$,

$m_N\left({}_1^2\mathrm{H}\right) = m\left({}_1^2\mathrm{H}\right) - m_e$

$$ \therefore\ m_N\left({}_1^1\mathrm{H}\right) + m_N\left({}_1^3\mathrm{H}\right) - m_N\left({}_1^2\mathrm{H}\right) - m_N\left({}_1^2\mathrm{H}\right) $$

$$ = m\left({}_1^1\mathrm{H}\right) - m_e + m\left({}_1^3\mathrm{H}\right) - m_e $$

$$ - m\left({}_1^2\mathrm{H}\right) + m_e - m\left({}_1^2\mathrm{H}\right) + m_e $$

$= m\left({}_1^1\mathrm{H}\right) + m\left({}_1^3\mathrm{H}\right) - m\left({}_1^2\mathrm{H}\right) - m\left({}_1^2\mathrm{H}\right)$

$= 1{\cdot}007825 + 3{\cdot}016049 - 2{\cdot}014102 - 2{\cdot}014102$

$= -0{\cdot}00433$ amu

$= -0{\cdot}00433 \times 1{\cdot}67 \times 10^{-27}$ kg

$= 7{\cdot}23 \times 10^{-30}$ kg

$\therefore$ Using equation (1),

$Q = -7{\cdot}23 \times 10^{-30} \times (3 \times 10^8)^2$

$= -6{\cdot}51 \times 10^{-13}$ J

$$= -\frac{6{\cdot}51\times10^{-13}}{1{\cdot}6\times10^{-13}}\text{MeV} = -4{\cdot}06\text{ MeV}.$$

Since Q value is negative, the reaction is endothermic.

(*ii*) The given reaction is

$$^{12}_{6}C + ^{12}_{6}C \longrightarrow ^{20}_{10}Ne + ^{4}_{2}He$$

$$\therefore Q = [m_N(^{12}_{6}C) + m_N(^{12}_{6}C) - m_N(^{20}_{10}Ne) - m_N(^{4}_{2}He)]c^2 \quad ...(2)$$

where m_N refers to the nuclear mass. If m refers to the atomic mass, then

$$m_N\left(^{12}_{6}C\right) = m\left(^{12}_{6}C\right) - 6m_e$$

$$\left(^{12}_{6}C \text{ has 6 electrons in its atom}\right)$$

$$m_N\left(^{20}_{10}Ne\right) = m\left(^{20}_{10}Ne\right) - 10m_e$$

and $$m_N\left(^{4}_{2}He\right) = m\left(^{4}_{2}He\right) - 2m_e$$

$$\therefore m_N(^{12}_{6}C) + m_N(^{12}_{6}C) - m_N(^{20}_{10}Ne) - m_N(^{4}_{2}He)$$

$$= m\left(^{12}_{6}C\right) - 6m_e + m\left(^{12}_{6}C\right) - 6m_e$$

$$-m\left(^{20}_{10}Ne\right) + 10m_e - m\left(^{4}_{2}He\right) + 2m_e$$

$$= m\left(^{12}_{6}C\right) + m\left(^{12}_{6}C\right) - m\left(^{20}_{10}Ne\right) - m\left(^{4}_{2}He\right)$$

$= 12{\cdot}000000 + 12{\cdot}000000 - 19{\cdot}992439 - 4{\cdot}002603 = 0{\cdot}004958$ u

$= 0{\cdot}004958 \times 1{\cdot}67 \times 10^{-27}$ kg

$= 8{\cdot}28 \times 10^{-30}$ kg

$\therefore$ from eqn. (2),

$Q = 8{\cdot}28 \times 10^{-30} \times (3 \times 10^8)^2$

$= 7{\cdot}45 \times 10^{-13}$ J

$$= \frac{7{\cdot}45\times10^{-13}}{1{\cdot}6\times10^{-13}}\text{MeV} = 4{\cdot}66\text{ MeV}$$

Since Q value is positive, the reaction is exothermic.

Question 16.

Suppose, we think of fission of a 5626Fe nucleus into two equal fragments, if 2813Al. Is the fission energetically possible ? Argue by working out Q of the process. Given, m (5626Fe) = 55.93494 u and m (2813Al)= 27.98191

Answer:

$$^{58}_{26}\text{Fe} \longrightarrow {}^{28}_{13}\text{Al} + {}^{28}_{13}\text{Al}$$

$$\text{Q-value of process} = m(^{58}_{26}\text{Fe} - 2m(^{28}_{13}\text{Al})$$

$$= 55{\cdot}93494 - 2 \times 27{\cdot}98191$$

$$= -0{\cdot}02888 \text{ u}$$

$$= -0{\cdot}02888 \times 931$$

$$(\because 1 \text{ u} = 931 \text{ MeV})$$

$$= -26{\cdot}89 \text{ MeV}.$$

Since Q-value is negative, so the fission is not possible.

Question 17.

The fission properties of 23994Pu are very similar to those of 23592uu. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure
23994Pu undergo fission ?

Answer:

239 g of $^{239}_{94}\text{Pu}$ contains

$= 6{\cdot}023 \times 10^{23}$ nuclei

1000 g of $^{239}_{94}\text{Pu}$ contains

$$= \frac{6{\cdot}023 \times 10^{23}}{239} \times 1000$$

$= 2{\cdot}52 \times 10^{24}$ nuclei.

Energy released per fission of each nucleus

$= 180$ MeV

$\therefore$ Total energy released

$= 180 \times 2{\cdot}52 \times 10^{24}$ MeV

$= 4{\cdot}54 \times 10^{26}$ MeV.

Question 18.

A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much
23592u did it contain initially ? Assume that all the energy generated arises from the fission
of 23592u and that this nuclide is consumed by the fission process.

Answer:

Power P = 1000 MW

$= 1000 \times 10^6$ W $= 10^9$ W

Also, time t = 5·00 year

$= 5 \times 365 \times 24 \times 60 \times 60$ seconds

$= 1{\cdot}577 \times 10^8$ s

∴ Energy consumed = Pt

$= 10^9 \times 1{\cdot}577 \times 10^8 = 1{\cdot}577 \times 10^{17}$ J

We know energy generated per fission of ${}^{235}_{92}U$ = 200 MeV

$= 200 \times 1{\cdot}6 \times 10^{-13}$ J $= 3{\cdot}2 \times 10^{-11}$ J

∴ Number of total fissions occurred in five years

$$= \frac{1{\cdot}577 \times 10^{17}}{3{\cdot}2 \times 10^{-11}} = 4{\cdot}93 \times 10^{27}$$

Now $6{\cdot}023 \times 10^{23}$ atoms (fissions) are produced by 235 g of ${}^{235}_{92}U$

∴ Mass of ${}^{235}_{92}U$ consumed in five years

$$= \frac{235}{6{\cdot}023 \times 10^{23}} \times 4{\cdot}93 \times 10^{27} \text{ g}$$

= 1923543 g ≈ 1924 kg

∴ Initial mass of ${}^{235}_{92}U$ = 2 × 1924

= 3848 kg

Question 19.

How long an electric lamp of 100 W can be kept glowing by fusion of 2.0 kg of deuterium ? The fusion reaction can be taken as

$${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + n + 3{\cdot}2 \text{ MeV}.$$

Answer:

Power of lamp, P = 100 W and mass of deuterium = 2 kg

From the reaction given in the exercise, we find that 2 nuclei of ^2_1H combine to give 3·2 MeV of energy.

Now 2 g of ^2_1H is equivalent to $6{\cdot}023 \times 10^{23}$ nuclei

$$\therefore \text{ 2 kg of } ^2_1H \text{ is equivalent to } \frac{6{\cdot}023 \times 10^{23}}{2} \times 2000$$

$$= 6{\cdot}023 \times 10^{26} \text{ nuclei}$$

∴ The energy released in the fusion of $6{\cdot}023 \times 10^{26}$ nuclei,

$$E = \frac{3{\cdot}2}{2} \times 6{\cdot}023 \times 10^{26}$$

$$= 9{\cdot}64 \times 10^{26} \text{ MeV}$$

$$= 9{\cdot}64 \times 10^{26} \times 1{\cdot}6 \times 10^{-13} \text{ J}$$

$$= 1{\cdot}54 \times 10^{14} \text{ J.}$$

$$\therefore \text{ Using } P = \frac{E}{t} \text{ or } t = \frac{E}{P}, \text{ we get}$$

$$t = \frac{1{\cdot}54 \times 10^{14}}{100}$$

$$= 1{\cdot}54 \times 10^{12} \text{ s}$$

$$= \frac{1{\cdot}54 \times 10^{12}}{365 \times 24 \times 60 \times 60} \text{ years}$$

$$= 4{\cdot}89 \times 10^{4} \text{ years.}$$

Question 20.

Calculate the height of Coulomb barrier for the head on collision of two deuterons. The effective radius of deuteron can be taken to be 2.0 fm.

Answer:

The initial mechanical energy E of the two deutrons before collision is given by

E = 2 K.E.

(K.E. = kinetic energy)

When the two deutrons stop, their energy is totally potential energy U given by

$$U = \frac{1}{4\pi\epsilon_0}\frac{e.e}{2R} = \frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{2R}.$$

∴ Using law of conservation of energy,

$$2\text{ K.E.} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{2R}$$

$$\therefore\ \text{K.E.} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{4R}$$

But $R = 2\text{ fm} = 2 \times 10^{-15}$ m,

$e = 1{\cdot}6 \times 10^{-19}$ C

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$$

$$\therefore\ \text{K.E.} = \frac{9 \times 10^9 \times (1{\cdot}6 \times 10^{-19})^2}{4 \times 2 \times 10^{-15}}$$

$$= 2{\cdot}88 \times 10^{-14}\ \text{J.}$$

$$\therefore\ \text{Coulomb barrier} = \frac{2{\cdot}88\times10^{-14}}{1{\cdot}6\times10^{-19}}\ eV$$

$$= 180000\ eV = \mathbf{180\ keV}$$

Question 21.

From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that nuclear matter density is nearly constant (i.e. independent of A)

Answer:

$$\rho = \frac{M}{V} = \frac{Am}{\frac{4}{3}\pi R^3} \quad (m = \text{mass of a nucleon})$$

$$= \frac{Am}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Thus, nuclear density is independent of A.

Question 22.

For the β⁺ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K- shell, is captured by the nucleus and a neutrino is emitted).

$$e^- + {}^{A}_{Z}X \rightarrow {}_{Z-1}^{\;\;A}Y + \nu$$

Show that if β⁺ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa

Answer:

For the positron emission, we have

$${}^{A}_{Z}X \rightarrow {}_{Z-1}^{\;\;A}X + {}^{0}_{+1}e + \nu + Q_1 \quad ...(1)$$

And for Electron Capture, we have

$${}^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}_{Z-1}^{\;\;A}Y + \nu + Q_2 \quad ...(2)$$

If m_N represents the nuclear mass and m represents the atomic mass, then for m_e as the mass of ${}^{0}_{-1}e$ or ${}^{0}_{+1}e$, we have

Nuclear mass of ${}^{A}_{Z}X$

$$= m_N\left({}^{A}_{Z}X\right) = m\left({}^{A}_{Z}X\right) - Z m_e$$

Nuclear mass of ${}_{Z-1}^{\;\;A}Y$

$$= m_N\left({}_{Z-1}^{\;\;A}Y\right) = m\left({}^{A}_{Z}Y\right) - (Z-1) m_e$$

$$\therefore Q_1 = \left[m_N\left({}^{A}_{Z}X\right) - m_N\left({}_{Z-1}^{\;\;A}Y\right) - m_e\right]c^2$$

$$= \left[m\left({}^{A}_{Z}X\right) - Zm_e - m\left({}_{Z-1}^{\;\;A}Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$\Rightarrow Q_1 = \left[m\left({}^{A}_{Z}X\right) - m\left({}_{Z-1}^{\;\;A}Y\right) - 2m_e\right]c^2 \quad ...(3)$$

$$\text{Also } Q_2 = \left[m_N\left({}^{A}_{Z}X\right) + m_e - m_N\left({}_{Z-1}^{\;\;A}Y\right)\right]c^2$$

$$= \left[m\left({}^{A}_{Z}X\right) + Zm_e + m_e - m\left({}_{Z-1}^{\;\;A}Y\right) + (Z-1)m_e\right]c^2$$

$$\Rightarrow Q_2 = \left[m\left({}^{A}_{Z}X\right) - m\left({}_{Z-1}^{\;\;A}Y\right)\right]c^2 \quad ...(4)$$

From eqns. (3) and (4), we find that if $Q_1 > 0$, Q_2 is necessarily greater than zero. However, if $Q_2 > 0$, Q_1 does not always necessarily greater than zero.

Question 23.

In a Periodic Table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on Earth. The three isotopes and their masses are 2412Mg (23.98504u), ? 2512Mg (24.98584) and 2612Mg (25.98259u). The natural abundance of 2412Mg is 78.99% by mass. Calculate the abundances of the other two isotopes.

Answer:

Let the natural abundance of $^{25}_{12}\text{Mg}$ is $x\%$. Then natural abundance of $^{26}_{12}\text{Mg}$ is $[100 - (x + 78{\cdot}99)]\%$

$\therefore$ 24·312

$$= \frac{78{\cdot}99 \times 23{\cdot}98504 + x \times 24{\cdot}98584 + (100 - x - 78{\cdot}99) \times 25{\cdot}98259}{100}$$

$$\Rightarrow 24{\cdot}312 \times 100 = 1894{\cdot}58 + 24{\cdot}98584\,x + 2598{\cdot}259 - 25{\cdot}98259\,x - 2052{\cdot}36$$

or $25{\cdot}98259\,x - 24{\cdot}98584$

$= 1894{\cdot}58 + 2598{\cdot}259 - 2052{\cdot}36 - 2431{\cdot}2$

or $\quad 0{\cdot}99675x = 9{\cdot}279$

$$\therefore \quad x = \frac{9{\cdot}279}{0{\cdot}99675} = 9{\cdot}31$$

$\therefore$ Natural abundance of $^{25}_{12}\text{Mg}$ is $9{\cdot}31\%$

Also, natural abundance of $^{26}_{12}\text{Mg}$ is $(100 - 9{\cdot}31 - 78{\cdot}99)\% = 11{\cdot}7\%$

Question 24.

The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei 2412Ca and 2713Al from the following data :

$$m_n = 1{\cdot}008665\text{ u},$$

$$m\left(^{40}_{20}\text{Ca}\right) = 39{\cdot}962591\text{ u},$$

$$m\left(^{41}_{20}\text{Ca}\right) = 40{\cdot}962278\text{ u},$$

$$m\left(^{26}_{13}\text{Al}\right) = 25{\cdot}986895\text{ u},$$

$$m\left(^{27}_{13}\text{Al}\right) = 26{\cdot}981541\text{ u}.$$

Answer:

For $^{41}_{20}\text{Ca}$, the process of neutron separation is

$$^{41}_{20}\text{Ca} + \text{E} = ^{40}_{20}\text{Ca} + ^{1}_{0}n$$

$$\therefore \text{E} = \left[m\left(^{40}_{20}\text{Ca}\right) + m_n - m\left(^{41}_{20}\text{Ca}\right)\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [39{\cdot}962591 + 1{\cdot}008665 - 40{\cdot}962278] \times 931{\cdot}5 \text{ MeV}$$

$$= 0{\cdot}008978 \times 931{\cdot}5 = 8{\cdot}36 \text{ MeV}$$

For $^{27}_{13}\text{Al}$, the process of neutron separation is

$$^{27}_{13}\text{Al} + \text{E} = ^{26}_{13}\text{Al} + ^{1}_{0}n$$

$$\therefore \text{E} = \left[m\left(^{26}_{13}\text{Al}\right) + m_n - m\left(^{27}_{13}\text{Al}\right)\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [25{\cdot}986895 + 1{\cdot}008665 - 26{\cdot}981541] \times 931{\cdot}5 \text{ MeV}$$

$$= 0{\cdot}014019 \times 931{\cdot}5 = 13{\cdot}06 \text{ MeV}.$$

Question 25.

A source contains two phosphorus radionuclides $^{32}_{15}\text{P}$ ($T_{1/2}$ = 14·3d) and $^{33}_{15}\text{P}$ ($T_{1/2}$ = 25·3d). Initially, 10% of the decays come from $^{33}_{15}\text{P}$. How long one must wait until 90% do so ?

Answer:

Initially, source has 90% of $^{32}_{15}P$ and 10% of $^{33}_{15}P$. Let

after t days, source has 10% of $^{32}_{15}P$ and 90% of $^{33}_{15}P$.

$\therefore$ Initial number of $^{32}_{15}P$ $= 9x$

Initial number of $^{33}_{15}P$ $= x$

Final number of $^{32}_{15}P$ $= y$

Final number of $^{33}_{15}P$ $= 9y$

Now $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

$\therefore \quad N = N_0(2)^{-t/T_{1/2}}$

For first isotope,

$$y = 9x\,(2)^{-t/14\cdot3} \quad ...(1)$$

For second isotope,

$$9y = xe^{-t/25\cdot3} \quad ...(2)$$

Dividing (2) by (1)

$$9 = \frac{1}{9}(2)^{t\left(\frac{1}{14\cdot3}-\frac{1}{25\cdot3}\right)}$$

$$81 = 2^{\frac{11t}{14\cdot3\times25\cdot3}}$$

$$\log 81 = \frac{11t}{14\cdot3\times25\cdot3}\log 2$$

$$1\cdot9085 = \frac{11t\times0\cdot3010}{14\cdot3\times25\cdot3}$$

or $\quad t = 208\cdot5$ days.

Question 26.

Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes :

$$^{223}_{88}Ra \rightarrow \,^{209}_{82}Pb + \,^{14}_{6}C \;;\quad ^{223}_{88}Ra \rightarrow \,^{219}_{86}Rn + \,^{4}_{2}He$$

(a) Calculate the Q values for these decays and determine that both are energetically possible.

(b) The Coulomb barrier height for α-particle emission is 30·0 MeV. What is the barrier height for $^{14}_{6}C$? The required data is

$m\left(^{223}_{88}Ra\right) = 223{\cdot}01850\ u$;

$m\left(^{209}_{82}Pb\right) = 208{\cdot}98107\ u$;

$m\left(^{219}_{86}Rn\right) = 219{\cdot}00948\ u$

$m\left(^{14}_{6}C\right) = 14{\cdot}00324\ u$;

$m\left(^{4}_{2}He\right) = 4{\cdot}00260\ u$

Answer:

(*a*) The given decay process for $^{223}_{88}Ra$ is

$$^{223}_{88}Ra \rightarrow\ ^{209}_{82}Pb + ^{14}_{6}C + Q$$

$$\therefore Q = \left[m_N\left(^{223}_{88}Ra\right) - m_N\left(^{209}_{82}Pb\right) - m_N\left(^{14}_{6}C\right)\right] \times 931{\cdot}5\ MeV$$

where m_N denotes the nuclear mass. If m represents the atomic mass, then

$$Q = \left[m\left(^{223}_{88}Ra\right) - 88m_e - m\left(^{209}_{82}Pb\right) + 82m_e - m\left(^{14}_{6}C\right) + 6m_e\right] \times 931{\cdot}5\ MeV$$

$$= \left[m\left(^{223}_{88}Ra\right) - m\left(^{209}_{82}Pb\right) - m\left(^{14}_{6}C\right)\right] \times 931{\cdot}5\ MeV$$

$$= [223{\cdot}01850 - 208{\cdot}98107 - 14{\cdot}00324] \times 931{\cdot}5$$

$$= 31{\cdot}85\ MeV.$$

Also, another decay for $^{223}_{88}Ra$ is

$$^{223}_{88}Ra = ^{219}_{86}Rn + ^{4}_{2}He + Q'$$

$$\therefore Q' = \left[m_N\left(^{223}_{88}Ra\right) - m_N\left(^{219}_{86}Rn\right) - m_N\left(^{4}_{2}He\right)\right] \times 931{\cdot}5\ MeV$$

$= \left[m\left({}^{223}_{88}\text{Ra}\right) - m\left({}^{209}_{82}\text{Pb}\right) - m\left({}^{14}_{6}\text{C}\right)\right] \times 931{\cdot}5$ MeV

$= [223{\cdot}01850 - 208{\cdot}98107 - 14{\cdot}00324] \times 931{\cdot}5$

$= 31{\cdot}85$ MeV.

Also, another decay for ${}^{223}_{88}\text{Ra}$ is

$${}^{223}_{88}\text{Ra} = {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He} + Q'$$

$$\therefore Q' = \left[m_N\left({}^{223}_{88}\text{Ra}\right) - m_N\left({}^{219}_{86}\text{Rn}\right) - m_N\left({}^{4}_{2}\text{He}\right)\right] \times 931{\cdot}5 \text{ MeV}$$

$= \left[m\left({}^{223}_{88}\text{Ra}\right) - m\left({}^{219}_{86}\text{Rn}\right) - m\left({}^{4}_{2}\text{He}\right)\right] \times 931{\cdot}5$ MeV

$= [223{\cdot}01850 - 219{\cdot}00948 - 4{\cdot}00260] \times 931{\cdot}5$ MeV

$= 5{\cdot}98$ MeV

Since in both cases, $Q > 0$, therefore, both decays are possible.

(*b*) The barrier height for a particle is given by

$$U = \frac{Z_1 Z_2 e^2}{4\pi \epsilon_0 (r_1 + r_2)}$$

where Z_1 is the atomic no. of the particle and r_1 is its radius. Similarly Z_2 is the atomic number of daughter nucleus and r_2 is its radius.

But $r_1 = r_0 A_1^{1/3}$ and $r_2 = r_0 A_2^{1/3}$ where r_0 is nuclear unit radius.

$$\therefore \quad U = \frac{Z_1 Z_2 e^2}{4\pi \epsilon_0 r_0 [A_1^{1/3} + A_2^{1/3}]}$$

$\therefore$ For $\alpha-$ particle,

$$U(\alpha) = \frac{2 \times 86 \times (1{\cdot}6 \times 10^{-19})^2}{4\pi \epsilon_0 r_0 [4^{1/3} + 219^{1/3}]}$$

$$= \frac{1}{4\pi \epsilon_0 r_0} \times 5{\cdot}78 \times 10^{-37}$$

$$= \frac{5{\cdot}78 \times 10^{-37}}{4\pi \epsilon_0 r_0} \text{ J}$$

For ${}^{14}_{6}\text{C}$, the barrier height

$$U\left({}^{14}_{6}C\right) = \frac{6\times 82\times(1.6\times 10^{-19})^2}{4\pi\epsilon_0 r_0[14^{1/3}+209^{1/3}]}$$

$$= \frac{1.51\times 10^{-36}}{4\pi\epsilon_0 r_0} J$$

$$\therefore \frac{U\left({}^{14}_{6}C\right)}{U(\alpha)} = \frac{\frac{1.51\times 10^{-36}}{4\pi\epsilon_0 r_0}}{\frac{5.78\times 10^{-37}}{4\pi\epsilon_0 r_0}} = 2.61$$

$$\therefore U\left({}^{14}_{6}C\right) = 2.61 \times U(\alpha)$$

$$= 2.61 \times 30 \text{ MeV} \quad (\because U(\alpha) = 30 \text{ MeV})$$

$$= 78.3 \text{ MeV}.$$

Question 27.

Consider the fission of 23992u by fast neutrons. In one fission event, no neutrons are emitted and the final stable end products, after the beta-decay of the primary fragments, are 14058Ce and 9944Ru. Calculate Q for this fission process. The relevant atomic and particle masses

$$m\left({}^{238}_{92}U\right) = 238.05079 \text{ u};$$

$$m\left({}^{140}_{58}Ce\right) = 139.90543 \text{ u}$$

$$m\left({}^{99}_{44}Ru\right) = 98.90594 \text{ u}; \; m_n = 1.00867 \text{ u}$$

Answer:

The fission process may be expressed as

$${}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{140}_{58}Ce + {}^{99}_{44}Ru + Q$$

$$\therefore Q = \left[m\left({}^{238}_{92}U\right) + m_n - m\left({}^{140}_{58}Ce\right) - m\left({}^{99}_{44}Ru\right)\right] \times 931.5 \text{ MeV}$$

$$= [238.05079 + 1.00867 - 139.90543 - 98.90594] \times 931.5$$

$$= 231.1 \text{ MeV}.$$

Question 28.

Consider the D-T reaction (deuterium-tritium-fusion) given in eqn. :

$${}^2_1H + {}^3_1H \rightarrow {}^4_2He + n + Q$$

(a) Calculate the energy released in MeV in this reaction from the data :

$$m\left({}^2_1H\right) = 2{\cdot}014102 \text{ u};$$

$$m\left({}^3_1H\right) = 3{\cdot}016049 \text{ u}$$

$$m\left({}^4_2He\right) = 4{\cdot}002603 \text{ u}; \; m_n = 1{\cdot}00867 \text{ u}$$

(b) Consider the radius of both deuterium and tritium to be approximately 1.5 fm. What is the kinetic energy needed to overcome the Coulomb repulsion? To what temperature must the gases be heated to initiate the reaction?

Answer:

From the equation given in the question,

$$Q = \left[m_N\left({}^2_1H\right) + m_N\left({}^3_1H\right) - m_N\left({}^4_2He\right) - m_n\right] \times 931{\cdot}5 \text{ MeV}$$

m_N refers to the nuclear mass of the element given in the brackets and m_n = mass of the neutron.
If in represents the atomic mass, then

$$m_N\left({}^2_1H\right) = m\left({}^2_1H\right) - m_e$$

$$m_N\left({}^3_1H\right) = m\left({}^3_1H\right) - m_e$$

$$m_N\left({}^4_2He\right) = m\left({}^4_2He\right) - 2e$$

$$\therefore Q = \left[m\left({}^2_1H\right) - m_e + m\left({}^3_1H\right) - m_e - \right.$$

$m\left({}^4_2\text{He}\right) + 2m_e - m_n\Big] \times 931{\cdot}5 \text{ MeV}$

$$= \left[m\left({}^2_1\text{H}\right) + m\left({}^3_1\text{H}\right) - m\left({}^4_2\text{He}\right) - m_n\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [2{\cdot}014102 + 3{\cdot}016049 - 4{\cdot}002603 - 1{\cdot}00867] \times 931{\cdot}5 \text{ MeV}$$

$$= 17{\cdot}585 \text{ MeV}.$$

(*b*) Repulsive potential energy is to be provided to the particles in doing so.

∴ K.E. needed

= Repulsive potential energy

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{2r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{2r}$$

$[q_1 = q_2 = e]$

$$= \frac{9 \times 10^9 \times (1{\cdot}6 \times 10^{-19})^2}{2(1{\cdot}5) \times 10^{-15}}$$

[distance between particle $= 2r$]

$$= 7{\cdot}68 \times 10^{-14} \text{ J}$$

$$\text{K.E.} = \frac{3}{2}kT$$

$$T = \frac{2\text{K.E.}}{3k} = \frac{2 \times 7{\cdot}68 \times 10^{-14}}{3 \times 1{\cdot}38 \times 10^{-23}}$$

$$= 3{\cdot}7 \times 10^9 \text{ K}.$$

Question 29.

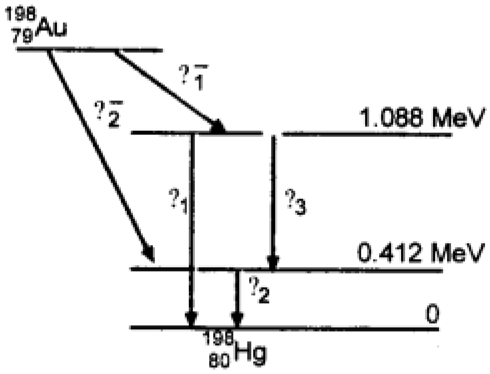
Obtain the maximum kinetic energy of p-particles and the radiation frequencies to y decay in the following decay scheme. You are given that

m (^{198}Au) = 197.968233 u

m (^{198}Hg) = 197.966760 u

Answer:

The total energy released for the transformation of 19879Au to 19880u can be found by considering the energies of Y-rays. We first find the frequencies of the Y-rays emitted.



For γ_1, the frequency

$$\nu_1 = \frac{E_1}{h} = \frac{1{\cdot}088 \times 1{\cdot}6 \times 10^{-13}}{6{\cdot}626 \times 10^{-34}}$$

$= 2{\cdot}63 \times 10^{20}$ Hz

For γ_2, the frequency is

$$\nu_2 = \frac{E_2}{h} = \frac{0{\cdot}412 \times 1{\cdot}6 \times 10^{-13}}{6{\cdot}626 \times 10^{-34}}$$

$= 9{\cdot}95 \times 10^{19}$ Hz.

For γ_3, the energy $E_3 = 1{\cdot}088 - 0{\cdot}412$

$= 0{\cdot}676$ MeV $= 0{\cdot}676 \times 1{\cdot}6 \times 10^{-13}$ J

$= 1{\cdot}082 \times 10^{-13}$ J

$$\therefore \nu_3 = \frac{E_3}{h} = \frac{1{\cdot}082 \times 10^{-13}}{6{\cdot}626 \times 10^{-34}}$$

$= 1{\cdot}63 \times 10^{20}$ Hz

Now for β_1^- decay,

$$^{198}_{79}\text{Au} \longrightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}e + Q(\beta_1^-) + Q(\gamma_1)$$

Maximum kinetic energy

$= \left[m\left(^{198}_{79}\text{Au}\right) - m\left(^{198}_{80}\text{Hg}\right)\right] \times 931{\cdot}5 - 1{\cdot}088$ MeV

$= [197{\cdot}968233 - 197{\cdot}966760] \times 931{\cdot}5 - 1{\cdot}088$ MeV

$= 1{\cdot}372 - 1{\cdot}088 = 0{\cdot}294$ MeV

For β_2^- decay,

Maximum kinetic energy

$= \left[m\left(^{198}_{79}\text{Au}\right) - m\left(^{198}_{80}\text{Hg}\right)\right] \times 931{\cdot}5 - 0{\cdot}412$

$= (197{\cdot}968233 - 1{\cdot}97{\cdot}966760) \times 931{\cdot}5 - 0{\cdot}412$ MeV

$= 1{\cdot}372 - 0{\cdot}412 = 0{\cdot}96$ MeV

Question 30.

Calculate and compare the energy released by (a) fusion of 1.0 kg of hydrogen deep within the sun and (b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Answer:

(*a*) In sun fusion takes place according to the equation

$$4\,{}_1^1H \longrightarrow {}_2^4He + 2\,{}_{+1}^{0}e + 26\text{ MeV}$$

∴ 4 hydrogen atoms combine to produce 26 MeV of energy.

Now 1 g of hydrogen contains

$= 6{\cdot}02 \times 10^{23}$ nuclei

∴ 1000 g of hydrogen contains

$= 6{\cdot}02 \times 10^{23} \times 1000 = 6{\cdot}02 \times 10^{26}$ nuclei

∴ Energy released by 1 kg of hydrogen

$$= \frac{26\text{ MeV}}{4} \times 6{\cdot}02 \times 10^{26}$$

$= 3{\cdot}913 \times 10^{27}$ MeV

(*b*) Fission of one ${}_{92}^{235}U$ nucleus gives energy $= 200$ MeV.

Now 235 g of ${}_{92}^{235}U$ has no. of atoms

$= 6{\cdot}02 \times 10^{23}$

∴ 1 kg (1000 g) of ${}_{92}^{235}U$ has

$$= \frac{6{\cdot}02 \times 10^{23}}{235} \times 1000 \text{ atoms}$$

$= 2{\cdot}56 \times 10^{24}$ atoms

∴ Total energy released

$= 200 \times 2{\cdot}56 \times 10^{24} = 5{\cdot}12 \times 10^{26}$ MeV.

$$\therefore \frac{\text{Energy released by 1 kg of fusion of } {}_1^1H}{\text{Energy released by 1 kg of fission of } {}_{92}^{235}U}$$

$$= \frac{3{\cdot}913 \times 10^{27}}{5{\cdot}12 \times 10^{26}} = 7{\cdot}6$$

Question 31.

Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plant. Suppose we are given that, on average, the efficiency of utilisation (i.e., conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium did our country need per year by 2000 ? Take the heat energy per fission of ^{235}U to be about 200 MeV. Avogadro's number = 6.023 x 10^{23} mol^{-1}.

Answer:

Total power target = 200000 MW

$= 200000 \times 10^6 = 20^{11}$ W

$\therefore$ Nuclear power target

$$= 10\% = \frac{10}{100} \times 20^{11} = 20^{10} \text{ W}$$

$$\text{Efficiency } \eta = \frac{\text{Total useful power}}{\text{Total power generated}}$$

$\therefore$ Total power generated

$$= \frac{\text{Total useful power}}{\eta}$$

$$= \frac{20^{10}}{\frac{25}{100}} = 8 \times 10^{10} \text{ W}$$

$\therefore$ Total energy required for the year 2020, is

$E = P \times t = 8 \times 10^{10} \times 366 \times 25 \times 60 \times 60$ (2020 is a leap year)

$= 2{\cdot}265 \times 10^{18}$ J.

Now 1 fission of ${}^{235}_{92}U$ produces 200 MeV of energy

$= 200 \times 1{\cdot}6 \times 10^{-13}$ J $= 3{\cdot}2 \times 10^{-11}$ J

$\therefore$ Number of fission required for the generation of energy E

$$= \frac{2{\cdot}265 \times 10^{18}}{3{\cdot}2 \times 10^{-11}} = 7{\cdot}90 \times 10^{28}.$$

Now $6{\cdot}023 \times 10^{23}$ nuclei of ${}^{235}_{92}U$ have mass = 235 g.

$\therefore$ Mass required to produce $7{\cdot}90 \times 10^{28}$ nuclei

$$= \frac{235}{6{\cdot}023 \times 10^{23}} \times 3{\cdot}95 \times 10^{28} \text{ g}$$

$\approx 3{\cdot}084 \times 10^4$ kg.