

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Work out the value of $\frac{3^7 \times 3^{-2}}{3^3}$

$$3^7 \times 3^{-2} = 3^{(7+(-2))} = 3^5$$

$$\frac{3^5}{3^3} = 3^{(5-3)} = 3^2$$

(Total for Question 1 is 2 marks)

2 $v^2 = u^2 + 2as$

$$u = 12 \quad a = -3 \quad s = 18$$

(a) Work out a value of v .

$$\begin{aligned} v^2 &= (12)^2 + (2 \times -3 \times 18) \\ &= 144 + -108 \\ &= 36 \end{aligned}$$

$$\therefore v = \sqrt{36} = 6$$

(2)

(b) Make s the subject of $v^2 = u^2 + 2as$

$$\begin{aligned} v^2 &= u^2 + 2as \\ -u^2 \quad -u^2 & \\ v^2 - u^2 &= 2as \\ \div 2a \quad \div 2a & \\ \frac{v^2 - u^2}{2a} &= s \end{aligned}$$

(2)

(Total for Question 2 is 4 marks)

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3 A bonus of £2100 is shared by 10 people who work for a company.
 40% of the bonus is shared equally between 3 managers.
 The rest of the bonus is shared equally between 7 salesmen.

One of the salesmen says,

“If the bonus is shared equally between all 10 people I will get 25% more money.”

Is the salesman correct?
 You must show how you get your answer.

$$2100 \times \frac{40}{100} = 840$$

$$2100 - 840 = 1260$$

$$1260 \div 7 \Rightarrow \begin{array}{r} 180 \\ 7 \overline{) 1260} \end{array} \quad (\text{Each salesman gets } \pounds 180 \text{ bonus})$$

• If shared between 10 people equally

$$2100 \div 10 = \pounds 210$$

• Difference is $\pounds 30$

$$\frac{30}{180} \times 100 = 16.6\%$$

\therefore No, he would get 16.6% more

(Total for Question 3 is 5 marks)



4 It would take 120 minutes to fill a swimming pool using water from 5 taps.

(a) How many minutes will it take to fill the pool if only 3 of the taps are used?

$$\begin{array}{l} \times 5 \left\{ \begin{array}{l} 120 \text{ mins} = 5 \text{ taps} \\ 600 \text{ mins} = 1 \text{ tap} \end{array} \right. \div 5 \\ \div 3 \left\{ \begin{array}{l} 200 \text{ mins} = 3 \text{ taps} \end{array} \right. \times 3 \end{array}$$

..... minutes
(2)

(b) State one assumption you made in working out your answer to part (a).

- Each tap fills at the same rate.

(1)

(Total for Question 4 is 3 marks)

5 A plane travels at a speed of 213 miles per hour.

$$S = \frac{D}{T} \quad \therefore T = \frac{D}{S}$$

(a) Work out an estimate for the number of seconds the plane takes to travel 1 mile.

$$T = \frac{D}{S} = \frac{1}{213} \quad (\text{This is time in hours})$$

$$\frac{1}{213} \approx \frac{1}{200}$$

$$\frac{1}{200} \times 60 \times 60 = 18 \text{ seconds}$$

..... seconds
(3)

(b) Is your answer to part (a) an underestimate or an overestimate?
Give a reason for your answer.

Overestimate, the speed is rounded down.

(1)

(Total for Question 5 is 4 marks)



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6 Solve the simultaneous equations

$$\begin{aligned} 5x + y &= 21 \\ x - 3y &= 9 \end{aligned}$$

$$\begin{array}{r} 5x + y = 21 \\ x - 3y = 9 \quad (\times 5) \Rightarrow \\ \hline 5x + y = 21 \quad - \textcircled{1} \\ 5x - 15y = 45 \quad - \textcircled{2} \end{array}$$

Subtract $\textcircled{2}$ from $\textcircled{1}$

$$\begin{aligned} 16y &= -24 \\ \div 16 & \quad \div 16 \\ y &= -1.5 \end{aligned}$$

Substitute y into equation 1 and solve

$$\begin{aligned} 5x + (-1.5) &= 21 \\ +1.5 \quad +1.5 \\ \hline 5x &= 22.5 \\ \div 5 \quad \div 5 \\ x &= 4.5 \end{aligned}$$

$x =$

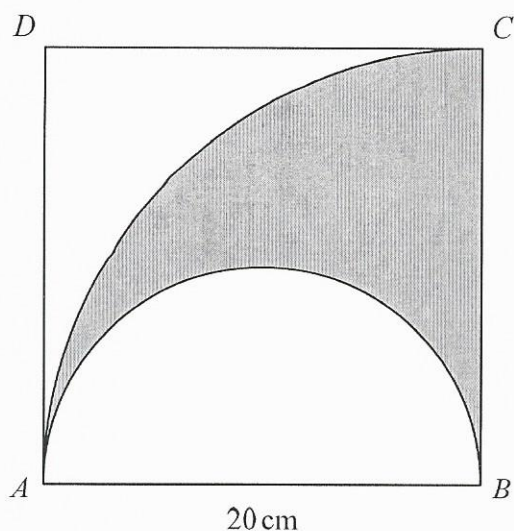
$y =$

(Total for Question 6 is 3 marks)



P 5 5 5 8 4 A 0 5 2 0

- 7 The diagram shows a square $ABCD$ with sides of length 20 cm. It also shows a semicircle and an arc of a circle.



AB is the diameter of the semicircle.
 AC is an arc of a circle with centre B .

Show that $\frac{\text{area of shaded region}}{\text{area of square}} = \frac{\pi}{8}$

$$\text{Area of sector } ABC = \frac{1}{4} \times \pi \times 20^2 = \frac{1}{4} \times \pi \times 400 = 100\pi$$

$$\text{Area of semicircle } AB = \frac{1}{2} \times \pi \times 10^2 = \frac{1}{2} \times \pi \times 100 = 50\pi$$

$$\therefore \text{Area shaded} = 100\pi - 50\pi = 50\pi$$

$$\frac{\text{Area shaded}}{\text{Area square}} = \frac{50\pi}{400} = \frac{\pi}{8}$$

(Total for Question 7 is 4 marks)



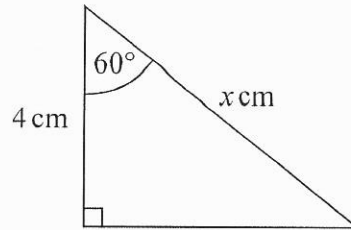
- 8 (a) Write down the exact value of $\tan 45^\circ$



$$\tan \theta = \frac{O}{A} \quad \therefore \tan 45 = \frac{1}{1} = 1$$

(1)

Here is a right-angled triangle.



$$\cos 60^\circ = 0.5$$

- (b) Work out the value of x .

$$\cos \theta = \frac{A}{H}$$

$$0.5 = \frac{4}{x}$$

$$\times x \quad \times x$$

$$0.5x = 4$$

$$\div 0.5 \quad \div 0.5$$

$$x = 8 \text{ cm}$$

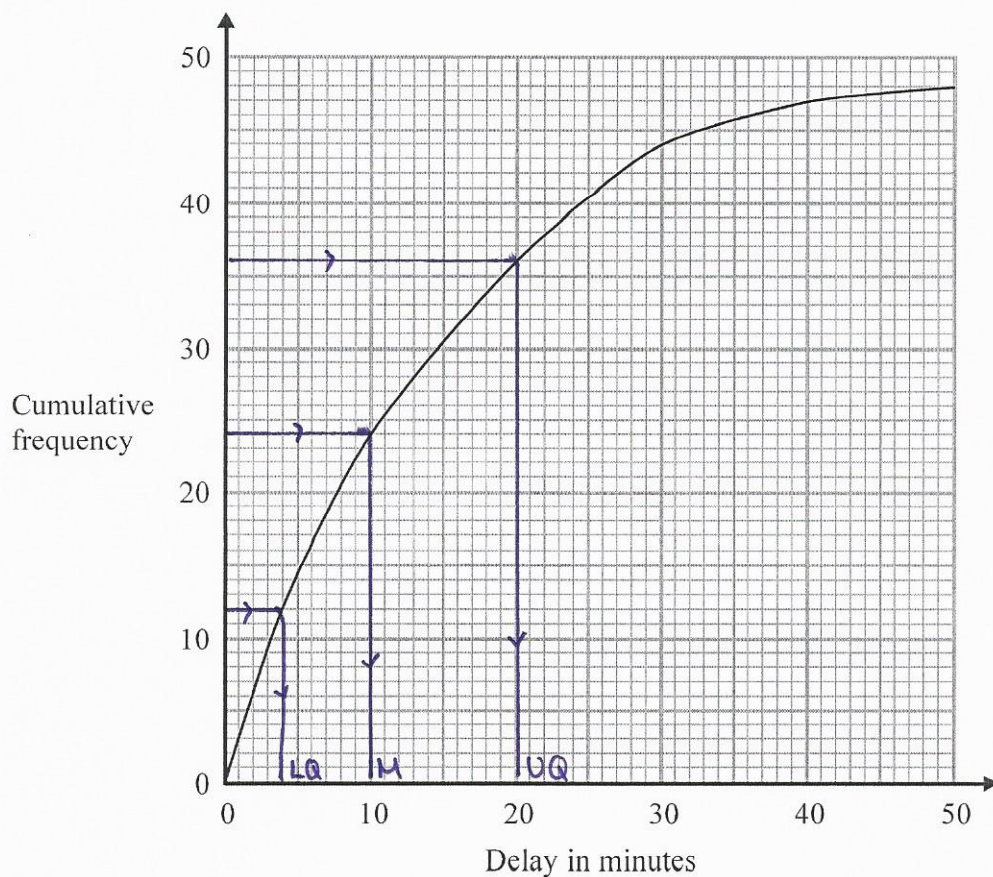
(2)

(Total for Question 8 is 3 marks)



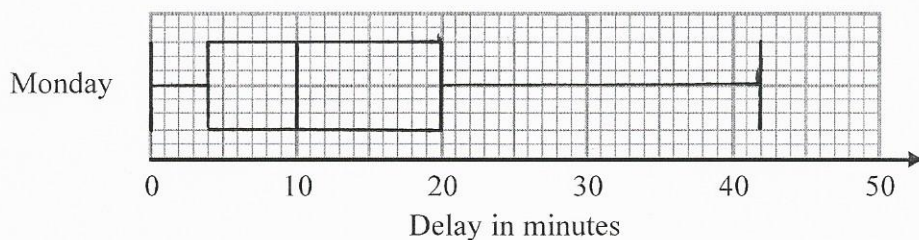
9 The times that 48 trains left a station on Monday were recorded.

The cumulative frequency graph gives information about the numbers of minutes the trains were delayed, correct to the nearest minute.



The shortest delay was 0 minutes.
The longest delay was 42 minutes.

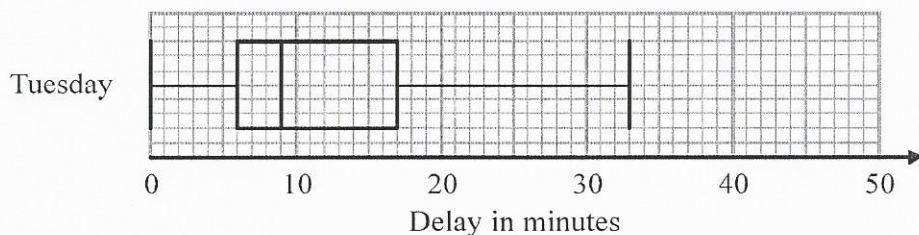
(a) On the grid below, draw a box plot for the information about the delays on Monday.



(3)

48 trains left the station on Tuesday.

The box plot below gives information about the delays on Tuesday.



(b) Compare the distribution of the delays on Monday with the distribution of the delays on Tuesday.

- Median delay on Monday is greater than on Tuesday
- Range of delays on Tuesday is lower than on Monday

(2)

Mary says,

"The longest delay on Tuesday was 33 minutes.

This means that there must be some delays of between 25 minutes and 30 minutes."

(c) Is Mary right?

You must give a reason for your answer.

No, we don't know the specific times so there could be no delays.

(1)

(Total for Question 9 is 6 marks)

10 (a) Simplify $\frac{x-1}{5(x-1)^2}$

$$\frac{\cancel{(x-1)}}{5(x-1)\cancel{(x-1)}} = \frac{1}{5(x-1)}$$

(1)

(b) Factorise fully $50 - 2y^2$

- Factorise out 2

$$2(25 - y^2)$$

- Bracket is now difference of two squares

$$2(5+y)(5-y)$$

(2)

(Total for Question 10 is 3 marks)



P 5 5 5 8 4 A 0 9 2 0

11 Jack and Sadia work for a company that sells boxes of breakfast cereal.

The company wants to have a special offer.

Here is Jack's idea for the special offer.

Put 25% more cereal into each box and do **not** change the price.

Here is Sadia's idea.

Reduce the price and do **not** change the amount of cereal in each box.

Sadia wants her idea to give the same value for money as Jack's idea.

By what percentage does she need to reduce the price?

Make an assumption . i.e. 100g of cereal = £2

Jacks idea
 $\therefore 125\text{g of cereal} = \pounds 2$
 $\therefore \downarrow \div 1.25$
 $\therefore 100\text{g of cereal} = \pounds 1.60$

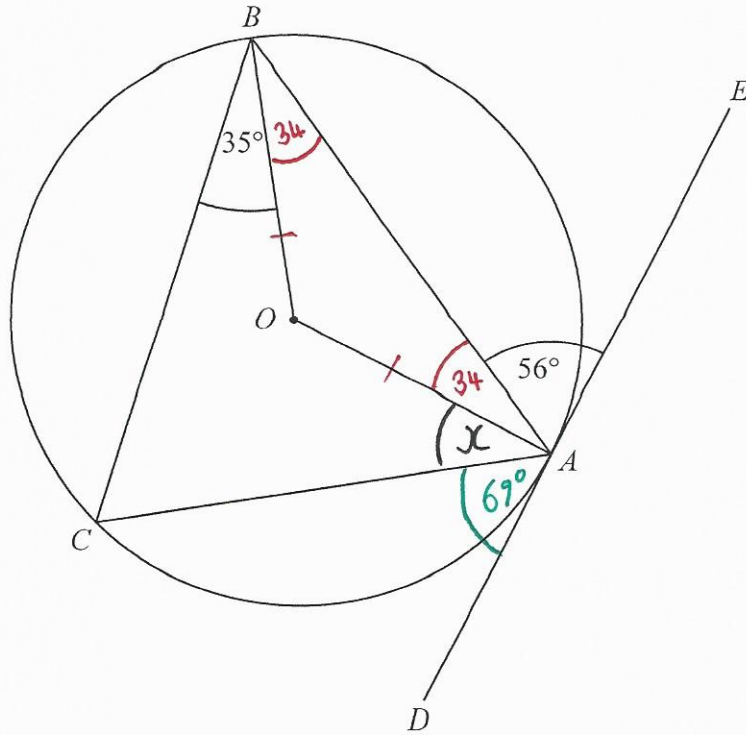
\therefore Sadia needs to reduce the price by 40p.

$$\frac{40}{200} \times 100 = 20\%$$

..... %

(Total for Question 11 is 3 marks)





A , B and C are points on the circumference of a circle, centre O .
 DAE is the tangent to the circle at A .

Angle $BAE = 56^\circ$

Angle $CBO = 35^\circ$

Work out the size of angle CAO . \leftarrow Call this x
 You must show all your working.

$$\angle BAO = 90 - 56 = 34^\circ$$

{ radius & tangent meet at 90° }

$$\angle OBA = 34^\circ$$

{ OBA is an isosceles triangle }

$$\angle CAD = 69^\circ$$

{ Angles in alternate segment }

$$\begin{aligned} \angle CAO &= 180 - (34 + 56 + 69) \\ &= 21^\circ \end{aligned}$$

{ Angles on a straight line = 180° }

(Total for Question 12 is 3 marks)



14 (a) Work out the value of $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

$$16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$$

$$\frac{8}{27}$$

(2)

$$3^a = \frac{1}{9} \quad 3^b = 9\sqrt{3} \quad 3^c = \frac{1}{\sqrt{3}}$$

(b) Work out the value of $a + b + c$

$$3^a = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$$

$$3^b = 9\sqrt{3} = 3^2 \times 3^{1/2} = 3^{(2+1/2)} = 3^{2.5}$$

$$3^c = \frac{1}{\sqrt{3}} = \frac{1}{3^{1/2}} = 3^{-1/2}$$

$$\therefore a = -2 \quad b = 2.5 \quad c = -0.5$$

$$a + b + c = -2 + 2.5 - 0.5$$

$$0$$

(2)

(Total for Question 14 is 4 marks)



15 Three solid shapes A, B and C are similar.

The surface area of shape A is 4 cm^2

The surface area of shape B is 25 cm^2

The ratio of the volume of shape B to the volume of shape C is $27:64$

Work out the ratio of the height of shape A to the height of shape C.

Give your answer in its simplest form.

A : B

Area scale factor = $4:25$

$$\therefore \text{length scale factor} = \sqrt{4} : \sqrt{25} \\ = 2 : 5$$

B : C

Volume scale factor = $27:64$

$$\therefore \text{length scale factor} = \sqrt[3]{27} : \sqrt[3]{64} \\ = 3 : 4$$

A : B : C

2 : 5 $(\times 3)$

3 : 4 $(\times 5)$

6 : 15 : 20

$$A : C = 6 : 20 \downarrow \div 2 \\ = 3 : 10$$

(Total for Question 15 is 4 marks)

16 Prove algebraically that $0.2\dot{5}\dot{6}$ can be written as $\frac{127}{495}$

$$x = 0.2\dot{5}\dot{6} \quad (2 \text{ recurring dots} \rightarrow \times 100)$$

$$100x = 25.6\dot{5}\dot{6}$$

$$\hline 99x = 25.4$$

$$\therefore x = \frac{25.4}{99} = \frac{254}{990} = \frac{127}{495}$$

$\xrightarrow{\times 10} \quad \xrightarrow{\div 2}$

(Total for Question 16 is 3 marks)

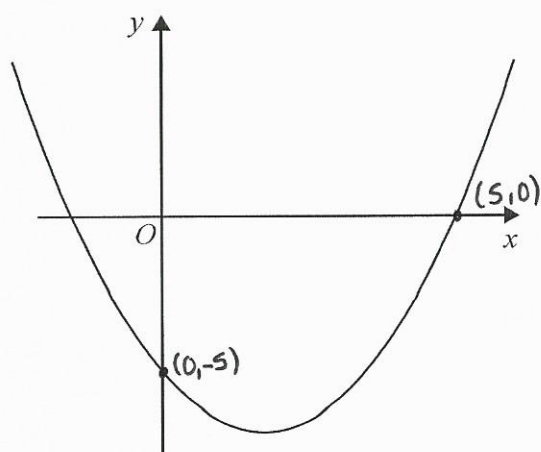


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17 Here is a sketch of a curve.



The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

$$(0, -5) \text{ is the } y \text{ intercept } (x=0) \quad \therefore \quad y = x^2 + ax + b$$

$$-5 = (0)^2 + (0)^2 + b$$

$$b = -5$$

$$\textcircled{a} \quad x = 5, y = 0$$

$$0 = 5^2 + 5a - 5$$

$$5a = -20$$

$$a = -4$$

$$\therefore y = x^2 + 4x - 5$$

$$(x+2)^2 - 9$$

↓ Complete the square

Minimum point when $(x+2) = 0$

$$\therefore (2, -9)$$

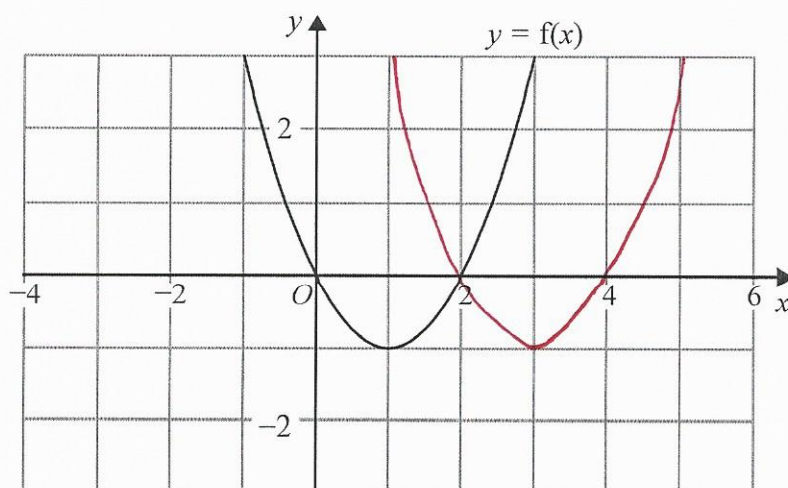
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(Total for Question 17 is 4 marks)



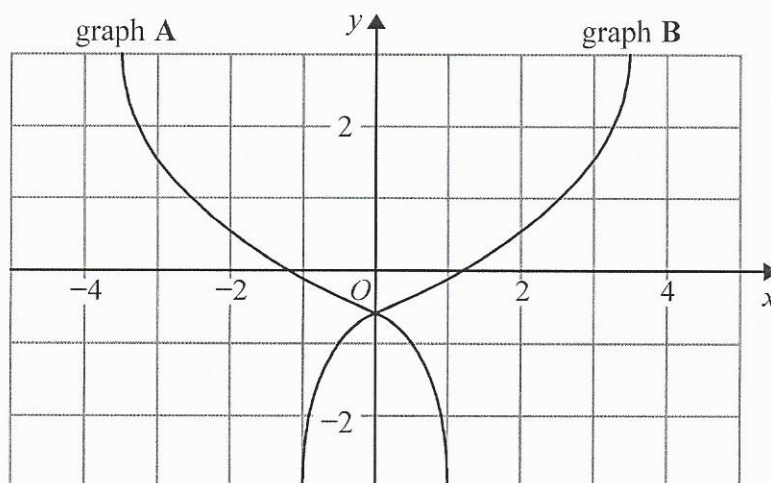
P 5 5 5 8 4 A 0 1 5 2 0

18 The graph of $y = f(x)$ is shown on the grid below.



(a) On the grid above, sketch the graph of $y = f(x - 2)$ ← Moves 2 to the right

(1)



On the grid, graph A has been reflected to give graph B.

The equation of graph A is $y = g(x)$

(b) Write down the equation of graph B.

$$y = g(-x)$$

(1)

(Total for Question 18 is 2 marks)



19 For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that $gf(x) = 2x(x + 2)$

Substitute $f(x)$ into $g(x)$

$$\begin{aligned} & 2((x + 1)^2 - 1) \\ & 2(x^2 + 2x + 1 - 1) \\ & 2(x^2 + 2x) \\ & 2x(x + 2) \end{aligned}$$

(b) Find $g^{-1}(7)$

$$\begin{aligned} g(x) &= 2(x - 1) \\ \frac{1}{2}x + 1 &= g^{-1}(x) \\ g^{-1}(7) &= \frac{1}{2}(7) + 1 \\ &= 3.5 + 1 \\ &= 4.5 \end{aligned}$$

TRICK

Treat it like a rearranged equation

$$y = 2(x - 1)$$

$$\frac{y}{2} = x - 1$$

$$x = \frac{y}{2} + 1$$

Then just switch x & y

(2)

(2)

(Total for Question 19 is 4 marks)



P 5 5 5 8 4 A 0 1 7 2 0

20 Show that $\frac{(\sqrt{18} + \sqrt{2})^2}{\sqrt{8} - 2}$ can be written in the form $a(b + \sqrt{2})$ where a and b are integers.

$$\begin{aligned}\sqrt{18} &= \sqrt{9\sqrt{2}} \\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{8} &= \sqrt{4\sqrt{2}} \\ &= 2\sqrt{2}\end{aligned}$$

$$\frac{(\sqrt{18} + \sqrt{2})^2}{\sqrt{8} - 2} = \frac{(3\sqrt{2} + \sqrt{2})^2}{2\sqrt{2} - 2} = \frac{(4\sqrt{2})^2}{2\sqrt{2} - 2} = \frac{32}{2\sqrt{2} - 2} = \frac{16}{\sqrt{2} - 1}$$

Rationalise the surd

$$\frac{16}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{16\sqrt{2} + 16}{1}$$

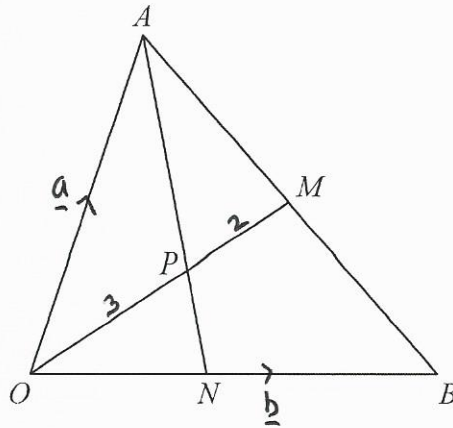
$$= 16\sqrt{2} + 16$$

$$= 16(\sqrt{2} + 1)$$

$$= 16(1 + \sqrt{2})$$

(Total for Question 20 is 3 marks)





OAB is a triangle.

OPM and APN are straight lines.

M is the midpoint of AB .

$$\vec{OA} = \underline{a} \quad \vec{OB} = \underline{b}$$

$$OP:PM = 3:2$$

Work out the ratio $ON:NB$

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -\underline{a} + \underline{b} \end{aligned}$$

$$\begin{aligned} \vec{OM} &= \vec{OA} + \frac{1}{2}(\vec{AB}) \\ &= \underline{a} + \frac{1}{2}(-\underline{a} + \underline{b}) = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} = \frac{1}{2}(\underline{a} + \underline{b}) \end{aligned}$$

$$\begin{aligned} \vec{AP} &= \vec{AO} + \frac{3}{5}(\vec{OM}) \\ &= -\underline{a} + \frac{3}{5} \times \frac{1}{2}(\underline{a} + \underline{b}) \\ &= -\underline{a} + \frac{3}{10}\underline{a} + \frac{3}{10}\underline{b} \\ &= -\frac{7}{10}\underline{a} + \frac{3}{10}\underline{b} = \frac{7}{10}(\underline{a} + \frac{3}{7}\underline{b}) \end{aligned}$$

$$\vec{AN} = -\underline{a} + k\underline{b}$$

As APN are on a straight line, \vec{AP} and \vec{AN} are multiples

$$\therefore k = \frac{3}{7}$$

$$\text{This gives } \vec{ON} = \frac{3}{7} \times \vec{OB}$$

$$\begin{aligned} \therefore \vec{ON} &: \vec{OB} \\ 3 &: 7 \end{aligned}$$

(Total for Question 21 is 5 marks)



22 There are only green pens and blue pens in a box.

There are three more blue pens than green pens in the box.

There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

Conditional

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box.

Green pens $\rightarrow x$

Blue pens $\rightarrow x+3$

$$\therefore P(\text{Green}) = \frac{x}{2x+3}$$

$$P(\text{BLUE}) = \frac{x+3}{2x+3}$$

$$P(G,G) + P(B,B) = \frac{27}{55}$$

$$\left\{ \frac{x}{2x+3} \times \frac{x-1}{2x+2} \right\} + \left\{ \frac{x+3}{2x+3} \times \frac{x+2}{2x+2} \right\} = \frac{27}{55}$$

$$\frac{x^2 - x}{(2x+3)(2x+2)} + \frac{x^2 + 5x + 6}{(2x+3)(2x+2)} = \frac{27}{55}$$

$$\frac{2x^2 + 4x + 6}{(2x+3)(2x+2)} = \frac{27}{55}$$

$$55(2x^2 + 4x + 6) = 27(2x+3)(2x+2)$$

$$110x^2 + 220x + 330 = 108x^2 + 270x + 162$$

$$2x^2 - 50x + 168 = 0$$

$$\therefore x^2 - 25x + 84 = 0$$

$$(x-21)(x-4) = 0$$

$$\therefore x = 21$$

$$x = 4$$

Told that $x > 12$

(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 80 MARKS





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(Total for Question 13 is 2 marks)

Describe fully the single transformation that maps triangle A onto triangle B.
Enlargement, SF = $\frac{1}{3}$, Centre (2, 2)

