

**General Theory For REITs Amidst Inflation and Interest Rate Changes**  
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**Overview:** It is useful to understand the impact of changes in inflation rates and interest rates on a Real Estate Investment Trust (REIT). This is a topic often raised by investors old or informed enough to know that the rates of the present era are at historic lows and likely to increase at some point. Here we develop the mathematics necessary for a general treatment of such problems, and implement one specific model.

Important features of this problem are as follows.

- Inflation impacts most costs on an immediate basis
- Revenue may fully respond to inflation only as leases renew, depending on lease escalators.
- Interest costs filter through to actual costs as debt is rolled or as new debt is added.

Here we are not considering the growth of the revenue of the REIT by means other than increased rents. This is a quite pessimistic assumption, as successful REITs also grow by redevelopment, development, capital recycling, mergers, and accretive equity issuance. REITs who manage these types of growth will substantially outperform the models developed here.

**The Inflationary Context**

Our REIT operates within an economy where general prices,  $P$ , and a rate of inflation,  $R$ , are functions of time,  $t$ . These are related by

$$\frac{dP}{dt} = RP, \tag{1}$$

so that the cumulative inflationary price increase,  $P_{cum}$ , from time  $t_1$  to  $t_2$  is

$$P_{cum}(t_1, t_2) = e^{\int_{t_1}^{t_2} R(t)dt}. \tag{2}$$

One can use the equation above to find  $I_{cum}$  for any history of price inflation described by  $R(t)$ .

Some readers may find this confusing relative to typical ways of thinking about compounding, one year at a time. If  $t_1 = 0$  and  $R(t)$  is constant and small, then Eq. 2 gives a factor of  $(1 + R)$  after one year.

For our specific model, we will treat the next cycle of inflation as a Gaussian modeled after the inflationary spike that peaked in 1980. Thus we take

$$R(t) = R_o + (R_m - R_o)\text{Exp}\left[-\frac{(t - t_o)^2}{W}\right], \tag{3}$$

where the initial rate is  $R_o$ , the maximum is  $R_m$ , the time of the peak is  $t_o$ , and  $W$  establishes the width. Matching a current inflation rate approximated as 2% and the 9.5% increase produced in the spike, one has  $t_o = 10$ ,  $W = 25$ , and  $R_o = 1.8\%$  and  $R_m = 11.5\%$ .

For this case, the cumulative inflation from time zero (the present) to a later time  $t$  is

$$P_{cum}(0, t) = \exp\left[R_o t + (R_m - R_o)\frac{\sqrt{\pi W}}{2}\left(\text{Erf}\left[\frac{t_o}{\sqrt{W}}\right] - \text{Erf}\left[\frac{t_o - t}{\sqrt{W}}\right]\right)\right]. \tag{4}$$

Figure 2 shows  $R(t)$  and the cumulative price inflation for this model. Here  $R(t) = 2\%$  for  $t < 0$ . Inflation spikes to just over 10% and the overall price increase is a factor of about 3.5.

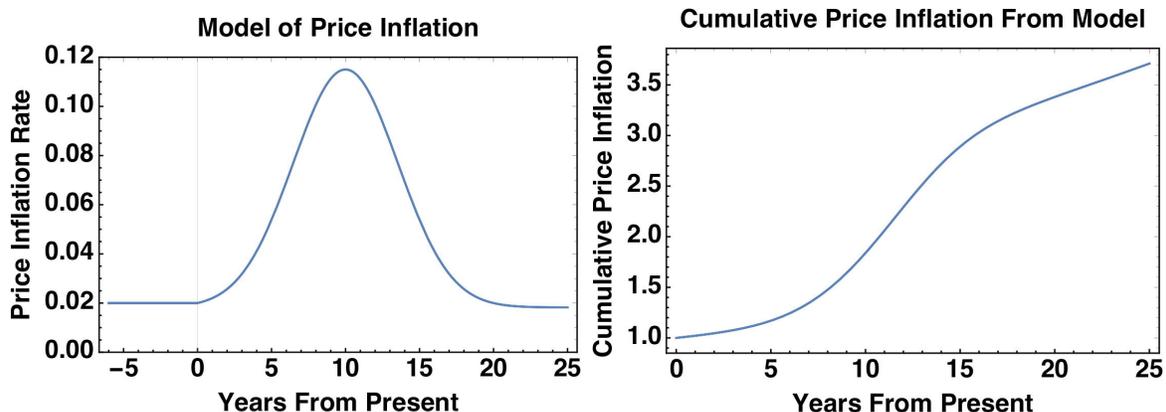


Figure 1: The model of an inflationary spike and its cumulative price increase.

### Inflationary Revenue Increases

REITs lease their properties with various durations. These range from 1 day for a hotel to 99 years for a ground lease. The actual lease rates reflect markets with many influences, in addition to changes in the desirability of the location. Inflation is only one of these influences, but here we will consider only its effects.

There is an average lease term for the REIT,  $T$ . For modeling purposes, we will assume that leases expire uniformly over a period that is twice the average lease term. The lease expiration rate, as a fraction, is thus  $1/(2 \times T)$ .

Most REIT leases include escalators of some sort. A rent increase of 2% per year or of 10% every five years is common. In some cases, the lease escalators are based on the CPI. Preliminary modeling, as part of this study, found an unsurprising result. In the absence of lease escalators, plausibly rapid inflation could push the REIT into a position of generating net losses.

Our approach to revenues is as follows. For the past, we assume that the escalators have matched the price inflation, so that at the moment the revenues are evenly distributed over the full lease term,  $T$ . Going forward, we assume the leases have annual escalators at a rate  $r_e$ .

Beyond this, when a lease expires, there is an adjustment. If the general amount of price inflation is larger than has been matched by the escalators, the rent is adjusted upward by the difference. Otherwise, there is no adjustment.

We describe the distribution of revenue from rents as  $V(t)$ , having units of dollars per year. We assume that the escalators have kept up with inflation in recent years. Then, for a total, initial revenue of  $L_o$  dollars, the constant distribution of revenue is

$$V_0 = \frac{L_o}{T} \text{ for } -T < t < 0, \quad (5)$$

In the absence of sufficient price inflation, and treating the increasing rent as continuous in time, the distribution later in time would be

$$V(t) = V_0 e^{r_e t}. \quad (6)$$

Note that the small argument expansion of  $e^{r_{esc} t}$  can be written as  $(1 + r_{esc})^t$ , so this is equivalent to the usual formulation so long as  $r_{esc} t$  remains small.

Now comes the complicated part. When a lease expires, the rent will be increased to make up for any inflationary shortfall (all else equal). In actuality, once inflation becomes significant more of the rent escalators are likely to become CPI based. Of interest here, though, is cases where they do not.

As time goes on, a given lease will be adjusted at intervals of  $T$ . This introduces a cadence to the adjustments. We end up with

$$V(t) = V_0 e^{adj(t)}. \quad (7)$$

where

$$\begin{aligned}
j_t &= \text{IntegerPart} \left[ \frac{t}{T} \right] \\
adj(t) &= \exp \left[ \text{Max} \left[ Tr_e, \int_{t-T}^t R(\tau) d\tau \right] \right. \\
&\quad \left. + \sum_{j=1}^{j_t} \text{Max} \left[ Tr_e, \int_{t-(j+1)T}^{t-jT} R(\tau) d\tau \right] \right. \\
&\quad \left. - Tr_e \left( 1 - \frac{(t-j_t T)}{T} \right) \right],
\end{aligned} \tag{8}$$

noting that  $\sum$  taken from 1 to 0 gives 0. An aspect of this product that may seem unusual is that the first term is the one that finds the most recent adjustment. Larger values of  $j$ , when they exist, cover the adjustments for progressively older lease terms. Evidently, in application  $adj$  will have to be found numerically. The final term in the argument of the exponent corrects for the fraction of  $T$  being sampled at negative times.

The total revenues at time  $t$ ,  $L(t)$ , are then

$$L(t) = \int_{t-T}^t V(t) dt = \frac{L_o}{T} \int_{t-T}^t e^{adj(t)} dt \tag{9}$$

Some rent escalators are CPI based, usually with a CPI floor. We will ignore the floor here, as it matters little during an inflationary spike. In this case, the function that adjusts the rents is simpler to calculate. It becomes

$$adj(t) = \exp \left[ \int_0^t R(\tau) d\tau \right], \tag{10}$$

Now, however, this adjustment applies to all active leases. The result is that the revenues increase directly with the cumulative inflationary price increase. We have

$$L(t) = L_o \exp \left[ \int_0^t R(\tau) d\tau \right] \tag{11}$$

### Inflationary Interest Rate Increases

In actual history, interest rates do not perfectly track inflation rates. They often lag, but not always. In addition, the spreads between interest rates paid by REITs and the Treasury (i.e., risk-free) rate varies with time. Here we will work with the idealized model that the interest rate is some fixed spread  $I_{po}$  above the inflation rate. Then the interest rate paid by the REIT on debt rolled over at time  $t$ ,  $I_p(t)$ , is  $I_p(t) = I_{po} + R(t)$

Recently, many REITs with good credit have been issuing 30-year debt at fixed and low rates. This has driven the weighted average term to maturity of REITs overall up to about 7 years. We will not try to capture such long debt, but will use a simpler model that will provide pessimistic results if applied to any REIT with a significant amount of very long debt. We also will ignore the lack of maturities for the next year or two that many REITs maintain today.

We assume that the debt maturity ladder is perfectly uniform. Then the fraction of debt coming due each year is  $1/(2M)$ , where the average maturity is  $M$ . We take the initial interest rate to be uniform across the ladder and to equal  $I_{po}$ . The weighted average interest rate,  $I_{av}$  at some time is then

$$I_{av}(t) = \frac{1}{2M} \int_{t-2M}^t I_p(t) dt = I_{po} + \frac{1}{2M} \int_{t-2M}^t R(t) dt. \tag{12}$$

Since one's primary interest will be in the impact of a cycle of increasing interest rates, it may make sense to simplify the current era as having had a fixed interest rate across the existing debt maturity ladder. Then one wants  $R(t) = R_{neg}$  for  $t < 0$  and some defined function for  $t > 0$ .

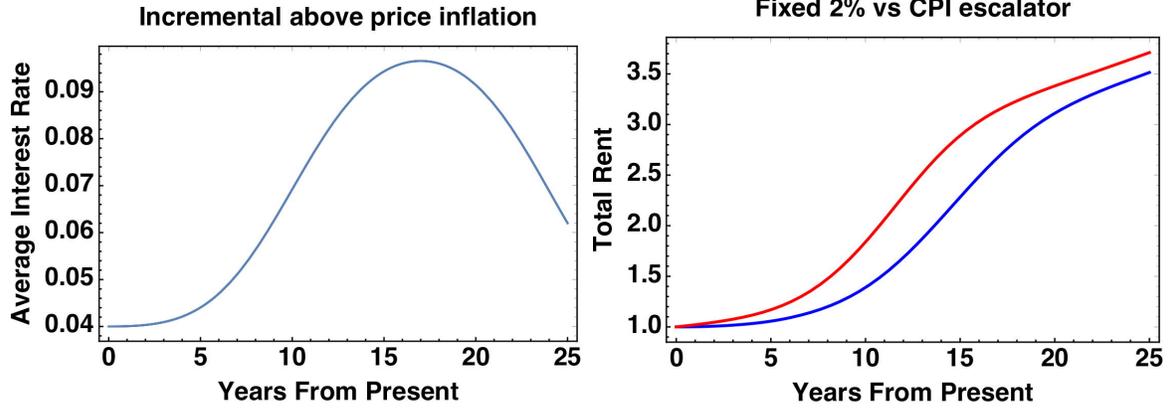


Figure 2: The interest rate and rent revenue. The upper rent curve is from leases having a CPI based escalator.

Adapting the Gaussian model given above, we can write  $R(t)$  as

$$R(t) = (1 - H[t])R_{neg} + H(t) \left( R_o + (R_m - R_o) \text{Exp} \left[ -\frac{(t - t_o)^2}{W} \right] \right), \quad (13)$$

with  $H(t)$  being the Heaviside step function and  $R_{neg}$  the value in recent years. Then one has

$$I_{av}(t) = I_{po} + \frac{1}{2M} \left[ R_{neg} \text{Min}[t - 2M, 0] + \int_{\text{Max}[0, t-2M]}^t R(t) dt \right]. \quad (14)$$

Figure 2 shows interest rates based on the Gaussian spike model of inflation defined above. It shows the  $I_{av}$  for debt with an average maturity of 7 years, distributed evenly in time.  $I_{po} = 2\%$  so that the starting interest rate is 4%, as is the case for quality REITs today. It also shows the Revenue through the inflationary spike, for leases with a 6 year term. The Revenue starts at 1 (i.e., 100%). One can see how the lag in adjusting for inflation reduces revenue for the case with the escalator.

### Application to a REIT

An investor should be concerned primarily with the Adjusted Funds From Operations (AFFO) generated by a REIT. AFFO represents the funds available to reward the investor through dividends, growth, or buybacks. We will represent AFFO as

$$\text{AFFO} = L(t) - C(t) - I_{av}(t)D, \quad (15)$$

in which the costs subject to price inflation are  $C(t)$  and the debt, which we take to be fixed, is  $D$ .

The costs include property operating expenses, general & administrative expenses, and maintenance capital expenditures. A detail is that GAAP rent is not actual cash rent. When drawing the numbers from GAAP accounting, as is typical, there is also an adjustment to obtain cash rent. This is most often small, so only small errors are introduced by including this with the other costs. Then

$$C(t) = C_o P_{cum}(t). \quad (16)$$

Then

$$\text{AFFO} = L(t) - C_o P_{cum}(t) - I_{av}(t)D, \quad (17)$$

In actual cases the funds available are subject to a variety of adjustments, for example from the purchase and sale of properties. We are not focused here on that aspect.

Figure 3 shows the AFFO from this model. This would be AFFO/share if share count were constant or if shares outstanding increased in proportion to  $L_o$ ,  $C_o$ , and  $D$  with no change in their ratios. Here  $L_o = 100\%$  and  $C_o = 35\%$ .

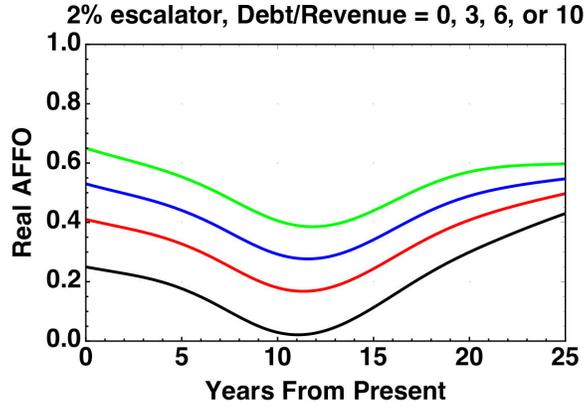


Figure 3: The real, inflation adjusted AFFO per share, across the inflationary spike for a fixed, 2% escalator and various debt levels. Here the initial revenue is 1.0

The four curves in the figure show debt of zero, 3, 6, and 10 times revenue. This leads to initial interest costs of 0%, 12%, 24%, and 40% of revenues. This is a bit unrealistic because any REIT with the higher levels of debt would be paying higher interest, making things even worse.

One can see two effects from the figure. First, the lag in catching rent up to general price inflation causes AFFO to drop in every case. Second, the drop becomes proportionally larger as debt increases. The maximum decline in AFFO is 39%, 46%, 57%, and 89% for Debt of 0, 3, 6, and 10 times Revenues, respectively.

### Use of the Model

To use the model, one must do the following. It may be convenient to define  $L_o$  as 100% and use percentages of revenues for the other quantities.

- Define the price inflation rate  $R(t)$ , for times dating back to the earliest lease or debt origination date.
- Define the total revenues at present  $L_o$  and the lease term  $T$ .
- Define the nature of the rent escalators.
- Define the average debt maturity  $M$  and evaluate the average interest rate,  $I_{av}(t)$
- Define the initial costs,  $C_o$ , and debt,  $D$ .

This lets one evaluate the cumulative inflation,  $P_{cum}(t)$ , the revenue over time,  $L(t)$ , and the AFFO over time.