Generative Adversarial Networks for Partial Differential Equations

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Abstract

In this paper, GAN is considered for a possibility of resolving image based resolution given trained set to use for test for a given heat conduction equation with Dirichlet and Neumann boundary conditions and conditioning the image resolution and pix2pix fine tuning the image resolution with the GAN image based model image resolution training fitting iterative minimization equation and compared with image generated numerical solution of partial differential equation and GANs perform with 6.6% MSE error for Dirichlet boundary conditions and 2.03% MSE error for Neumann boundary conditions for using the present GANs as Deep Learning models. An improvement in the image based resolution of GANs from a partial image equation (PIE) formulation with color mathematics based set of color choices formulation may prove to improve GANs for DL prediction with high DL prediction to use for engineering match with experiment match.

1. Introduction

A partial differential equation (PDE) is a mathematical equation that involves two or more independent variables, an unknown function (dependent on those variables), and partial derivatives of the unknown function with respect to the independent variables. A solution to a PDE is a function that solves the equation[1-3]. PDEs are used to mathematically formulate, and thus aid the solution of, physical and other problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity, electrostatics, electrodynamics, etc[1-3].

The multidimensional heat conduction equation and its variants are used in diverse fields, such as explaining Brownian motion in probability theory, wave function characteristics in quantum mechanics, and solving the Black Scholes PDE in financial mathematics. Over the last four hundred years, PDEs are solved by analytical methods like separation of variables, Fourier series, finding an integral form of the solution, change of variable method to transform the equation to something easily solvable. With the advent of numerical methods and high-end computer architecture, PDE mathematics were solved using three potent and highly successful approaches like finite element method (FEM), finite difference method

(FDM), and computational fluid dynamics (CFD) or finite volume method (FVM). But, in these approaches, it would take days for a high-performance computing facility to solve the complex and exhaustive matrix inversion calculations involved in mesh with extremely fine resolution as shown in Fig. 1.

Non-physics based, deep learning models using RNN-LSTM[2] after being trained can not only predict in a matter of milliseconds but can reduce the computing resources to a laptop. Fig. 2 shows the Deep Learning models involves two steps, first training the DL model[4] and second using the trained model to predict the required extrapolated new user needs output.

In this paper, we examine the use of deep learning models using GANs through image-based data. The GAN architecture consists of a generator model from a series of experiments of images being trained to generate a new image with efficacy for the next set of experiment in the image or in the next experiment to know the image. GANs have seen their use majorly when the required output matrix is much larger than that of the input, making computer facility bigger scale and generated till date. However, more often than not, in analysing complex physical, biological or engineering systems, the cost of data acquisition is limiting. In this paper, GANs efficacy is low for such small data training while DANN in the same set of data training proves maximum efficacy[2]. Here, a sample data set size of images of 15 samples of numerical/analytical is considered for training.

Figure 1: Traditional Approaches

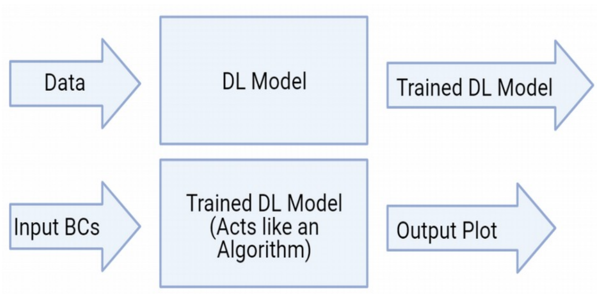

Figure 2: Deep Learning Pipeline

2. Problem Statement

Here, we solve heat conduction given by,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

subjected to Dirichlet and Neumann BCs as two problem set to solve over the geometry (x,y) given in (1).

$0 \le x \le a;$
$0 \le y \le b;$

15 Ansys and data images were used for training while 5 data images to be tested for were solved using DL models.

3. Deep Learning Models

GAN model is given by,

$$\min_G \max_D V(D,G) = \mathbb{E}_{\boldsymbol{x} \sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z} \sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))]. \tag{2}$$

The objective of the GAN model is to minimize logD(X), where X is G(z), G(z) is the image pixel of an experiment which is minimized from iterative D and G(z) minimization that image based model efficacy with mathematical formulation of the image based partial image equation is given in equation (2).

4. Results and Comparison

GANs with improved conditioning, and added pix2pix conditioning improved models during training, taking into picture mode collapsing, diminished gradient, overfitting. MSE error calculations of temperature of square plate under two boundary models, with image pix2pix as data provides the present efficiency of DL models. MSE error of 6.6% for Dirichlet boundary condition and 2.03% for Neumann boundary conditions is obtained for the cGANs models as shown in Fig. 3 and Fig. 4 respectively.

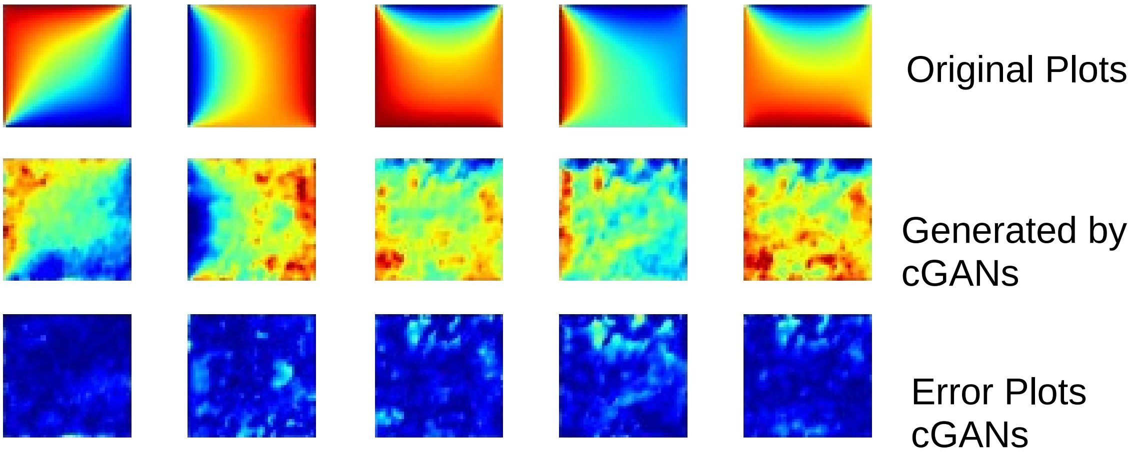

Figure 3: cGAN comparison of losses to Numerical solution for Dirichlet BCs

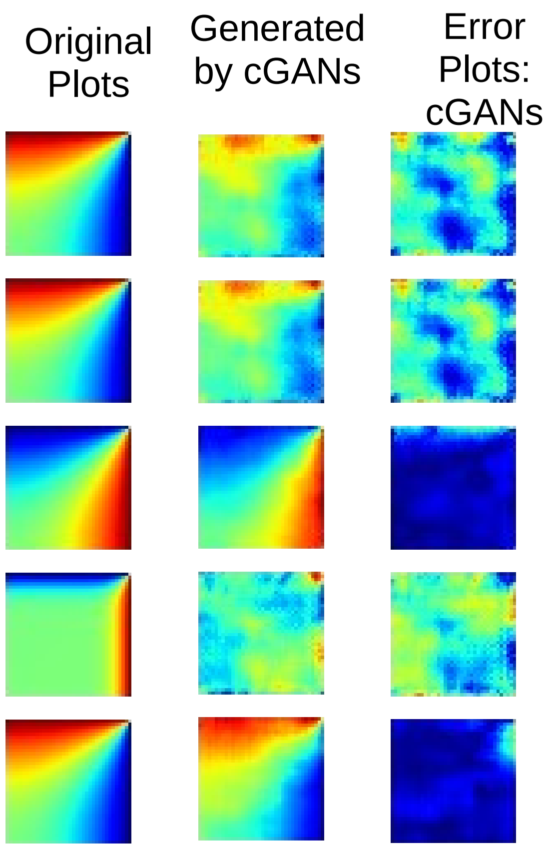

Figure 4: cGAN comparison of losses to Numerical solution for Neumann BCs

5. Conclusions

cGANs models with image based models DL for heat conduction over genearlized materials with temperature BCs only and temperature gradient of large scale cluster of materials with Neumann BCs demonstrated in this paper, provides improved efficacy of image models with first hand of quick data log book for material selection with image based log book as a software with less back and forth of theoretical physics confirmation for new materials today of 2021 era.

6. Conflict of interest

The authors provide no conflict of interest.

7. Acknowledgements

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8. References

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