Memo

To: Professor Vanderbeek
From: Michael Byckovski, Brandon Porath, Max Porter, Christian Tello, Remha Yohannes
Date: December 11, 2019
Re: Air Compressor Design Review

Problem Summary: We have been tasked with finalizing the design for a small, gasoline engine powered air compressor. This includes specing the gears, both rotary shafts, couplings, and bearings. The compressor will operate with a 10 yr life of 1 shift per day, 5 days per week. This allows us to analyze the system under infinite life. The required gear ratio is 2.5:1 in a speed reduction mesh with input shaft speed of 750 RPM. Because the given output shaft torque function varies with time, special consideration is needed to determine alternating and mean torque at various crank angles.

Results summary: The finished design includes a pinion, a gear, two shafts, and 4 bearings. At a desired pressure angle and diametral pitch of 20° and 6 respectively, we sized a pinion at 24 teeth at and a gear at 60 teeth. Both the pinion and gear have standard face width, diameter, and material choice per Boston Gear. Each shaft has a shoulder to locate the gear and a keyway to secure it. Ball bearings are located on all four shaft ends to provide additional stability. Through calculation, all components are sized and rated appropriately for the torque and moment applications to each shaft. Figure 1 illustrates the completely assembled gearbox with gears, bearings and shafts of appropriate size. The final gearbox dimensions were 14.33” x 4.0” x 10.33”

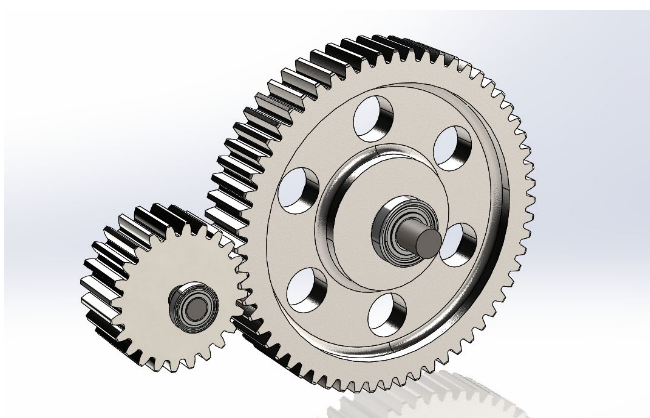


Figure 1. Gearbox Layout

Methods:

Analysis Assumptions: Design and material assumptions were made for the sake of developing a comprehensive calculation package and meeting customer specifications. Per the customer, all systems must meet minimum safety factors of 2 and maintain 95% reliability. To minimize cost, commercial grade spur gears will be used with teeth that mesh in full depth and shafts will be of standard sizes and materials. All gears will be spec’d from Boston Gears and all shielded bearings will be spec’d from SKF. Certain components such as gear bores and the shaft keyways require secondary machining prior to assembly which still offsets the cost of ordering one-off parts. The design team also assumed that there was no factor of safety built into Boston Gear's specs and that there was no torque loss in transfer through the gear mesh. When sourcing bearings, Manufacturer 2 Weibull parameters are assumed in calculations and all bearings must require double sided lubricant shielding. This allows the bearings to operate virtually maintenance free. As mentioned earlier, the desired gear mesh will have a 20° pressure angle and diametral pitch of 6 to allow for a larger gear selection for torque applications.

Analysis Approach: The analysis was primarily performed in MathCAD. Firstly, gear and pinion tooth numbers were justified by finding the minimum number of teeth acceptable using design parameters and assumptions. The mesh was verified by calculating an appropriate gear teeth contact ratio. Generally, a contact ratio of at least 1.4 is sufficient and ours was 1.7. Moving forward, the team began designing the shafts. Force diagrams in Appendix [A] illustrate the resultant gear forces on the centers of both shafts and the reaction forces at bearing locations. The associated moment diagrams helped us identify critical locations along the shaft that are prone to failure. In addition to large radial forces at gear and bearing locations, stress concentrations from shaft keyways and shoulders were inspected for failure likelihood. Assigning standard material to our shafts, we took a conservative approach selecting size factors, surface factors, endurance strengths, and material strengths used in the DE-Goodman equation. A minimum shaft diameter with a safety factor of 2 was calculated, sized up to the next standard imperial dimension, and used to recalculate safety factors in the shaft. Knowing the critical locations for yielding and a keyway, we then calculated material yielding safety factor and the required keyway length to resist crushing and shear failure. Shaft #2 followed a similar iterative approach to calculate minimum diameters, critical dimension safety factors, and ultimately, the likelihood of failure. Because Shaft #2 experiences higher torques, we also chose a higher strength steel in order to keep the shaft diameter economical and maintain a safety factor greater than 2.0. After shaft design verification, bearings were selected. Knowing desired life and reaction forces at the bearings, we calculated our desired load rating enabling us to select suitable deep groove ball bearings. The final task was selecting an appropriate coupling solutions for the input and output shafts. Using the calculated torques at both the inputs and the outputs, couplings of appropriate shaft diameters and torque rating were selected from McMaster Carr and able to adequately handle the sustained load. Because we identified critical stress locations, FEA was not required and therefore saved us time and money. The MathCAD calculations and important values are illustrated and highlighted in the Appendix [C].

Results:

There are numerous components and dimensions to report on. A summary of this can be seen in Appendix [B]. Firstly, the gear mesh was selected from Boston Gear using a 24 tooth pinion and 60 tooth gear. Both gears are above the minimum tooth count of 15. Final center to center distance of the mesh was 7.0". This will be a driving factor in our overall gearbox dimensions. Next, gear forces on the shafts were determined so that shaft analysis could be performed. Using a conservative DE-Goodman approach, shaft #1 is stepped with a minor diameter of 0.625" and major diameter of 0.75", machined from 1020 CD steel. Shaft #2 is also a stepped shaft with diameters of 0.8125" and 1.0", machined from 1050 CD steel. Both shafts are machined to standard diameters and use standard shaft steel, minimizing the fabrication costs. Furthermore, the factors of safety on shaft #1 all sit above the minimum 2.0 specification. They are 3.57 at the shoulder, 2.123 at the keyway with a minimum length of 0.298" and 5.851 in material yielding. Because shaft #2 experiences greater torques, its machined from higher strength steel and has slightly larger diameters. This allows all safety factors associated with the shaft to be greater than the minimum 2.0. They were 2.774 at the shoulder, 2.087 at the keyway with a minimum length of 0.382”, and 4.623 in material yielding. For the selected bearings on shaft #1, we calculated a load rating of 631.557 lbf which is below the 1,111 lbf rating on the SKF Bearings. One shaft #2, the bearing load rating was calculated to be 465.335 lbf which is below the 3,147 lbf rating on SKF Bearings. The infographic in Figure 2 highlights a couple of these characteristic dimensions.

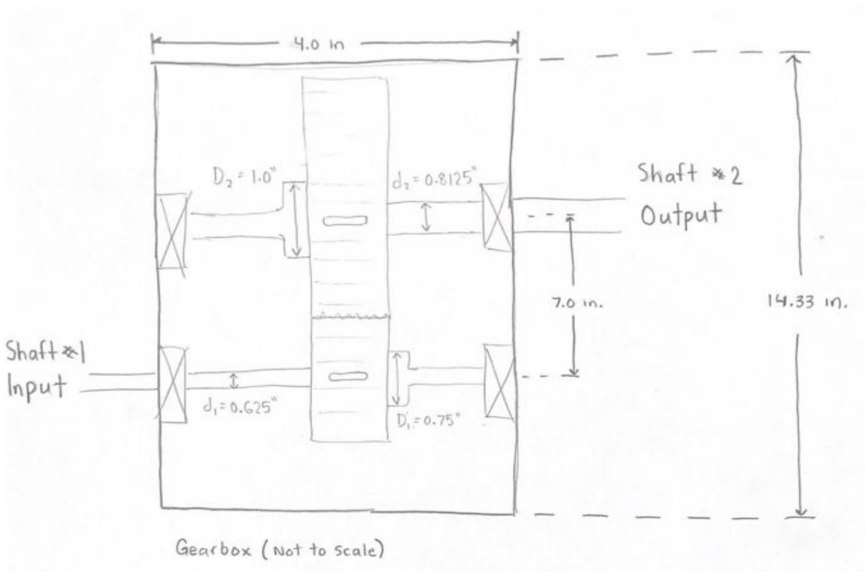


Figure 2. Gearbox Shaft Dimensions

Conclusions:

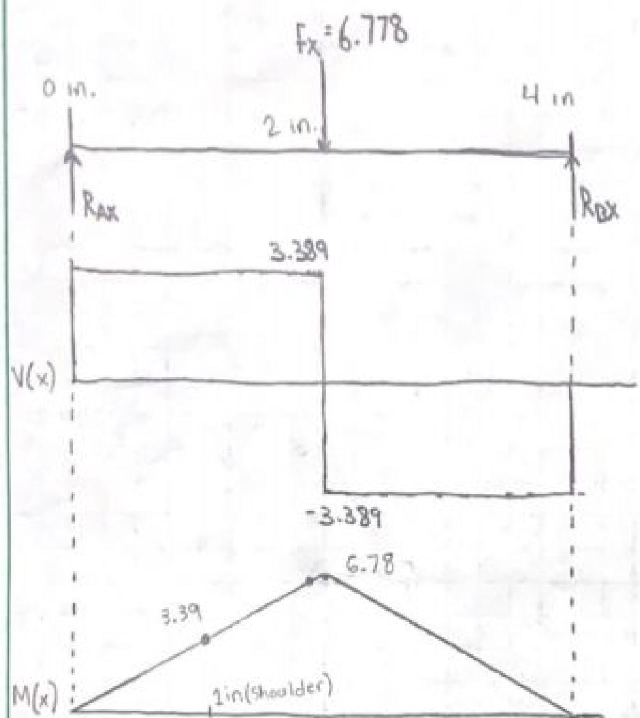
The resulting gearbox design and calculations make logical sense given our engineering intuition. All factors of safety fall within appropriate ranges. They are above the minimum 2.0 but do not venture much above in an effort to minimize fabrication and material costs. It is also important to note that the keyways will fail first on the shaft, protecting more valuable components. The gearbox is designed without overhanging shafts (with the exceptions of where the coupling solutions are implemented). However, there are a couple unknowns that would inhibit this gearbox from proceeding into immediate production. There aren’t any CAD- Finite Element Analysis simulations that would otherwise be able to verify all of our design considerations. The force exerted on the gearbox is also unknown, so there isn’t verification on the required wall thickness to support both shafts for the duration of the compressor’s life, and there hasn’t been AGMA calculations to determine gear failure in either tooth bending or surface pitting. The current factors of safety, although calculated in a conservative manner, are suitable given material properties and available application information, but lack prototype testing. To further increase confidence a physical model could be tested and validated.

After conducting MathCAD calculations, bearings, shafts, and gear types have been selected and verified for the air compressor operation. It is a robust, single mesh design with double sealed bearings to minimize maintenance and downtime. The large pinion increases the number of teeth in engagement and the configuration has minimally overhung shafts that aren’t required to take any axial load. The downsides to this design implementation are a lack of testing, and an overly large size (since the widths of the gears haven’t been tuned to just within the factor of safety). Prior to implementation, it is recommended that more testing be done.

Appendix A:

Shaft Force Diagrams

SHAFT Force Diagrams | Design Project #3



$\Sigma F_x = 0 = R_{Ax} + R_{Bx} - F_x$

$R_{Ax} + R_{Bx} = F_x$

$\Sigma M_{Ax} = 0 = -F_x(2) + R_{Bx}(4)$

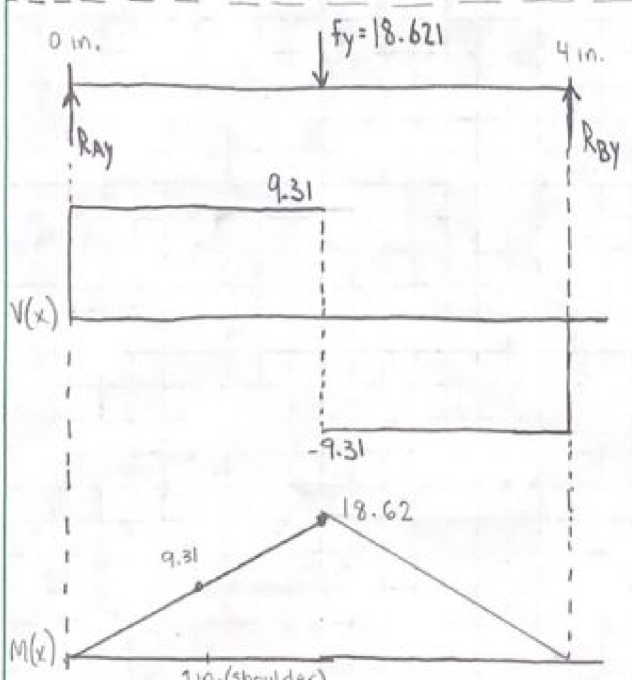
$\boxed{R_{Bx} = 3.39 \text{ lb·f}}$

$R_{Ax} = 6.778 - 3.39$

$\boxed{R_{Ax} = 3.39 \text{ lb·f}}$

M_{max} @ 2in. $= (3.389)(2)$

$\boxed{M_{max} = 6.78 \text{ lbf·in}}$



$\Sigma F_y = 0 = R_{Ay} + R_{By} - F_y$

$R_{Ay} + R_{By} = F_y$

$\Sigma M_{Ay} = 0 = -F_y(2) + R_{By}(4)$

$\boxed{R_{By} = 9.31 \text{ lb·f}}$

$R_{Ay} = 18.621 - 9.31$

$\boxed{R_{Ay} = 9.31 \text{ lb·f}}$

M_{max} @ 2 in $= (9.31)(2)$

$\boxed{M_{max} = 18.62 \text{ lbf·in}}$

$M_a = \sqrt{6.78^2 + 18.62^2} = \boxed{19.82 \text{ lbf-in}}$

$\boxed{M_m = 0 \text{ lbf-in}}$

Appendix B:

Component Selection

Component:	Description:
Standard (Shafts)	
1020 CD Steel	Sy = 57ksi, Sut = 68ksi, d = .625in, l = 4in.
1050 CD Steel	Sy = 84ksi, Sut = 100ksi, d = .8125in, l = 4in.
Boston Gears (Gears)	
YJ24A	OD:4.33 in, 24 teeth, DP=6, PA=20
YJ60B	OD:10.33 inches, 60 teeth, DP=6, PA=20
SKF (Bearings)	
RLS 7-27	Bore=.875in., C10=3147 lb*f, Speed Rating 26000 rpm
D/W R10-27	Bore=.625, C10=1111lb*f, Speed Rating 40000 rpm
McMaster Carr (Shaft Couplings)	
61005K155	⅞”, 3600 in.-lbs, 4000RPM

Appendix C:
MathCAD Calculations

Machine Design Project #3
Michael Byckovski, Christian Tello, Max Porter, Remha Yohannes, Brandon Porath
12/11/19

Given:

$Phi := 20 \cdot deg$	Desired Pressure Angle
$P_d := 6 \cdot \left(\frac{1}{in}\right)$	Desired Diametral Pitch
$n_i := 750 \cdot \left(\frac{1}{min}\right)$	Input Shaft Speed
$m := 2.5$	Gear Ratio
$T_f := 585 \cdot lbf \cdot in$	Max Output Shaft Torque
$n_f := \frac{n_i}{m} = 300 \ \frac{1}{min}$	Output Shaft Speed
$H := T_f \cdot n_f = 0.443 \ hp$	Horsepower
$T_i := \frac{H}{n_i} = 234 \ lbf \cdot in$	Input Shaft Torque

Gear Sizing and Interference Analysis

$k := 1$ Full Depth Contact

Minimum number of teeth

$$N_p := \frac{(2 \cdot k)}{(1 + 2 \cdot m) \cdot (\sin(Phi))^2} \cdot \left(m + \sqrt[2]{m^2 + (1 + 2 \cdot m) \cdot (\sin(Phi))^2}\right) = 14.637$$

$N_2 := 24$ Pinion Teeth Number

$N_3 := 60$ Gear Teeth Number

$d_2 := \frac{N_2}{P_d} = 4\ in$ $\quad r_2 := \frac{d_2}{2} = 2\ in$ Pinion Pitch Diameter/ Radius

$d_3 := \frac{N_3}{P_d} = 10\ in$ $\quad r_3 := \frac{d_3}{2} = 5\ in$ Gear Pitch Diameter/ Radius

$C := r_2 + r_3 = 7\ in$ Center to Center Distance

$$a_p := \frac{1}{P_d} = 0.167\ in$$

$$Z := \sqrt{(r_2 + a_p)^2 - (r_2 \cdot \cos(Phi))^2} + \sqrt{(r_3 + a_p)^2 - (r_3 \cdot \cos(Phi))^2} - C \cdot (\sin(Phi))$$

$Z = 0.833\ in$ Length of Action

$p_c := \frac{\pi}{P_d} = 0.524\ in$ Circular Pitch

$p_b := p_c \cdot \cos(Phi) = 0.492\ in$ Base Pitch

$m_p := \frac{Z}{p_b} = 1.693$ Contact Ratio

$m_p \geq 1.4$ Appropriate Contact Ratio

Shaft Sizing and Analysis

$V := (\pi \cdot d_3 \cdot n_f) = 785.398 \ \frac{ft}{min}$ Pitch-line Velocity

$W_t := \frac{H}{V} = 18.621 \ lbf$ Tangential Transmitted Load

$F_{23.t} := W_t = 18.621 \ lbf$ Pinion to Gear Tangential Load

$F_{23.r} := F_{23.t} \cdot \tan(Phi) = 6.778 \ lbf$ Pinion to Gear Radial Load

$F_{a3} := \sqrt[2]{(F_{23.t})^2 + (F_{23.r})^2} = 19.816 \ lbf$ Shaft 1 Reaction to Gear

$F_{a4} := F_{a3} = 19.816 \ lbf$ Shaft 2 Reaction to Pinion

Shaft 1 Design

Material: 1020 CD Steel

$S_y := 57 \ ksi$ Yielding Strength

$S_{ut} := 68$ Ultimate Tensile Strength

$S'_e := 0.5 \cdot S_{ut}$ Test Specimen Endurance Limit

Critical Location: Shoulder

$n := 2$ Assume a Factor of Safety of 2

$K_t := 1.7$ $K_{ts} := 1.5$ $d := 1.0$ Conservative First Guess (Table 7-1)

$K_f := K_t$ $K_{fs} := K_{ts}$ Assume Kf=Kt, Kfs=Kts

$k_a := 2.7 \cdot S_{ut}^{-0.265}$ Surface Factor

$k_b := \left(\frac{d}{0.3}\right)^{-0.107}$ Size Factor for Assumed Diameter

$S'_e := 34 \cdot ksi$ $\quad S_{ut} := 68 \cdot ksi$ Restating Test Specimen Endurance Limit

$S_e := k_a \cdot k_b \cdot S'_e = 26.38\ ksi$ Endurance Limit at Shoulder

$M_a := 9.91 \cdot lbf \cdot in$ Alternating Moment at Shoulder
(Found via Moment Diagram)

$M_m := 0 \cdot lbf \cdot in$ Mean Moment

$T_a := \frac{(234 + 70)}{2} \cdot lbf \cdot in = 152\ lbf \cdot in$ Alternating Torque

$T_m := \frac{(-70 + 234)}{2} \cdot lbf \cdot in = 82\ lbf \cdot in$ Mean Torque

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = 396.342\ lbf \cdot in$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 213.042\ lbf \cdot in$$

$$d := \left(\frac{(16 \cdot n)}{\pi} \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right)^{\frac{1}{3}} = 0.57\ in$$ Conservative DE-Goodman

$d := 0.625 \cdot in$ Resize Shoulder Diameter to
Next Largest Size

$D := (1.2) \cdot d = 0.75\ in$ Typical Shoulder D-d Relationship

$r := d \cdot (0.1) = 0.063\ in$ Shoulder Fillet Radius

$\frac{D}{d} = 1.2 \quad \frac{r}{d} = 0.1$ Typical Size Relationships

$K_t := 1.6$ Stress Concentration due to Bending (Figure A-15-9)

$q := 0.75$ Notch-Sensitivity to Bending (Figure 6-26)

$K_{ts} := 1.19$ Stress Concentration due to Torsion (Figure A-15-8)

$q_s := 0.79$ Notch-Sensitivity to Torsion (Figure 6-27)

$K_f := 1 + q \cdot (K_t - 1) = 1.45$ Combined Bending Effect

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 1.15$ Combined Torsion Effect

$d := 0.625$ Restating Shoulder Diameter

$k_b := \left(\frac{d}{0.3}\right)^{-0.107}$ Actual Size Factor at Shoulder

$S_e := k_a \cdot k_b \cdot S'_e = 27.741\ ksi$ Actual Endurance Limit at Shoulder

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = 304.15\ lbf \cdot in$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 163.347\ lbf \cdot in$$

$d := 0.625 \cdot in$ Restating Shoulder Diameter

$n := \left(\frac{(\pi \cdot d^3)}{16}\right) \cdot \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 3.586$ Factor of Safety at Shaft #1 Shoulder

$D = 0.75\ in$ Restating Major Diameter

$r = 0.063\ in$ Restating Shoulder Fillet

Critical Location: Yielding

Effective Material Stress

$$\sigma_{vm} := \left(\left(\frac{(32 \cdot K_f \cdot (M_m + M_a))}{\pi \cdot d^3} \right)^2 + 3 \cdot \left(\frac{(16 \cdot K_{fs} \cdot (T_m + T_a))}{\pi \cdot d^3} \right)^2 \right)^{\frac{1}{2}} = 9.742 \ ksi$$

$n_y := \dfrac{S_y}{\sigma_{vm}} = 5.851$ Material Yielding Factor of Safety

Critical Location: Keyway

$r := d \cdot (0.02) = 0.013 \ in$ Typical Keyway r-d Relationship

$\dfrac{D}{d} = 1.2$ $\dfrac{r}{d} = 0.02$

$K_t := 2.14$ Keyway First Iteration Estimate (Table 7-1)

$q := 0.47$ Notch-Sensitivity to Bending (Figure 6-26)

$K_{ts} := 3.0$ Keyway First Iteration Estimate (Table 7-1)

$q_s := 0.47$ Notch-Sensitivity to Torsion (Figure 6-27)

$K_f := 1 + q \cdot (K_t - 1) = 1.536$ Combined Bending Effect

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 1.94$ Combined Torsion Effect

$M_a := 18.34 \cdot lbf \cdot in$ Alternating Moment at Keyway Edge (Found via Moment Diagrams)

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = 513.844 \ lbf \cdot in$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 275.535 \ lbf \cdot in$$

$d := 0.625 \cdot in$ Restate Shaft Diameter at Keyway

$$n := \left(\frac{(\pi \cdot d^3)}{16}\right) \cdot \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 2.123$$

Factor of Safety at Shaft #1 Keyway

$$w := \frac{3}{16} \cdot in \qquad h := \frac{1}{8} \cdot in$$

Keyway Dimensions (Table 7-6)

$$F := \frac{2 \cdot T_i}{d} = 748.8\ lbf$$

Force at Surface of Shaft #1

$$S_{sy} := S_y \cdot .577 = 32.889\ ksi$$

Material Shear Strength

$$l_{key\_s} := F \cdot \frac{n}{w \cdot S_{sy}} = 0.258\ in$$

Length Required to Resist Shear

$$l_{key\_c} := 2 \cdot F \cdot \frac{n}{w \cdot S_y} = 0.298\ in$$

Length Required to Resist Crushing

Keyway must be at least 0.298 in. long or it will fail in Shear (Failure Mode)

Shaft 2 Design

Material: 1020 CD Steel

$S_y := 57\ ksi$	Yielding Strength
$S_{ut} := 68$	Ultimate Tensile Strength
$S'_e := 0.5 \cdot S_{ut}$	Test Specimen Endurance Limit

Critical Location: Shoulder

$n := 2$	Assume a Factor of Safety of 2
$K_t := 1.7 \qquad K_{ts} := 1.5 \qquad d := 1.0$	Conservative First Guess (Table 7-1)
$K_f := K_t \qquad K_{fs} := K_{ts}$	Assume Kf=Kt, Kfs=Kts
$k_a := 2.7 \cdot S_{ut}^{-0.265}$	Surface Factor
$k_b := \left(\dfrac{d}{0.3}\right)^{-0.107}$	Size Factor for Assumed Diameter
$S'_e := 34 \cdot ksi \qquad S_{ut} := 68 \cdot ksi$	Restated Test Specimen Endurance Limit
$S_e := k_a \cdot k_b \cdot S'_e = 26.38\ ksi$	Endurance Limit at Shoulder
$M_a := 9.31 \cdot lbf \cdot in$	Alternating Moment at Shoulder (Found via Moment Diagram)
$M_m := 0 \cdot lbf \cdot in$	Mean Moment
$T_a := \dfrac{(585 + 175)}{2} \cdot lbf \cdot in = 380\ lbf \cdot in$	Alternating Torque
$T_m := \dfrac{(585 - 175)}{2} \cdot lbf \cdot in = 205\ lbf \cdot in$	Mean Torque

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = 987.776\ lbf \cdot in$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 532.606\ lbf \cdot in$$

$$d := \left(\frac{(16 \cdot n)}{\pi} \left(\frac{A}{S_e} + \frac{B}{S_{ut}} \right) \right)^{\frac{1}{3}} = 0.773\ in$$ Conservative DE-Goodman

$d := 0.8125 \cdot in$ Resize Shoulder Diameter to Next Size

$D := (1.2) \cdot d = 0.975\ in$ Typical Shoulder D-d Relationship

$D := 1.0 \cdot in$ Resize Major Diameter to Closest Next Size

$r := d \cdot (0.1) = 0.081\ in$ Typical Shoulder Fillet Radius

$\frac{D}{d} = 1.231 \qquad \frac{r}{d} = 0.1$

$K_t := 1.63$ Stress Concentration due to Bending (Figure A-15-9)

$q := 0.73$ Notch-Sensitivity to Bending (Figure 6-26)

$K_{ts} := 1.37$ Stress Concentration due to Torsion (Figure A-15-8)

$q_s := 0.76$ Notch-Sensitivity to Torsion (Figure 6-27)

$K_f := 1 + q \cdot (K_t - 1) = 1.46$ Combined Bending Effect

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 1.281$ Combined Torsion Effect

$d := 0.8125$ Restating Shoulder Diameter

$k_b := \left(\frac{d}{0.3} \right)^{-0.107}$ Actual Size Factor at Shoulder

$S_e := k_a \cdot k_b \cdot S'_e = 26.973\ ksi$ Actual Endurance Limit at Shoulder

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = 843.697\ lbf \cdot in$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 454.916\ lbf \cdot in$$

$d := 0.8125 \cdot in$ Restating Shoulder Diameter

$$n := \left(\frac{(\pi \cdot d^3)}{16}\right) \cdot \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 2.774$$

Factor of Safety at Shaft #2 Shoulder

$D = 1\ in$ Restating Major Diameter

$r = 0.081\ in$ Restating Shoulder Fillet

Critical Location: Yielding

Effective Material Stress

$$\sigma_{vm} := \left(\left(\frac{(32 \cdot K_f \cdot (M_m + M_a))}{\pi \cdot d^3}\right)^2 + 3 \cdot \left(\frac{(16 \cdot K_{fs} \cdot (T_m + T_a))}{\pi \cdot d^3}\right)^2\right)^{\frac{1}{2}} = 12.329\ ksi$$

$$n_y := \frac{S_y}{\sigma_{vm}} = 4.623$$

Material Yielding Factor of Safety

Critical Location: Keyway

$r := d \cdot (0.02) = 0.016\ in$ Typical Keyway r-d Relationship

$$\frac{D}{d} = 1.231 \qquad \frac{r}{d} = 0.02$$

$K_t := 2.14$ Keyway First Iteration Estimate (Table 7-1)

$q := 0.55$ Notch-Sensitivity to Bending (Figure 6-26)

$K_{ts} := 3.0$ Keyway First Iteration Estimate (Table 7-1)

$q_s := 0.55$ Notch-Sensitivity to Torsion (Figure 6-27)

$K_f := 1 + q \cdot (K_t - 1) = 1.627$ Combined Bending Effect

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.1$ Combined Torsion Effect

$M_a := 17.93 \cdot lbf \cdot in$ Alternating Moment at Keyway Edge (Found via Moment Diagrams)

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = (1.383 \cdot 10^3)\ lbf \cdot in$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 745.648\ lbf \cdot in$$

$d := 0.8125 \cdot in$ Restate Shaft Diameter at Keyway

$$n := \left(\frac{(\pi \cdot d^3)}{16}\right) \cdot \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 1.692$$

Factor of Safety at Shaft #2 Keyway

Not Acceptable Factor of Safety So Pick New Material

Repick Shaft #2 Material (1050 Steel)

$S_{ut} := 100$ 1050 Steel Ultimate Tensile Strength

$S_y := 84 \cdot ksi$ 1050 Steel Yielding Strength

$S'_e := 50 \cdot ksi$ Test Specimen Endurance Limit

$k_a := 2.7 \cdot S_{ut}^{-0.265}$ Recalculated Surface Factor

$S_e := k_a \cdot k_b \cdot S'_e = 35.813\ ksi$ 1050 Steel Endurance Limit at Keyway

$q := 0.65$ Notch-Sensitivity to Bending (Figure 6-26)

$q_s := 0.65$ — Notch-Sensitivity to Torsion (Figure 6-27)

$K_f := 1 + q \cdot (K_t - 1) = 1.741$ — Recalculated Combined Bending Effect

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1) = 2.3$ — Recalculated Combined Torsion Effect

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_a)^2 + 3 \cdot (K_{fs} \cdot T_a)^2} = (1.515 \cdot 10^3)\ \mathbf{lbf \cdot in}$$

$$B := \sqrt[2]{4 \cdot (K_f \cdot M_m)^2 + 3 \cdot (K_{fs} \cdot T_m)^2} = 816.662\ \mathbf{lbf \cdot in}$$

$d := 0.8125 \cdot \mathbf{in}$ $\quad S_{ut} := 100 \cdot \mathbf{ksi}$ — Restate Shaft Diameter/Material at Keyway

$$n := \left(\frac{(\pi \cdot d^3)}{16}\right) \cdot \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 2.087$$

Factor of Safety at Shaft #2 Keyway

$d := 0.875\ \mathbf{in}$

$$n := \left(\frac{(\pi \cdot d^3)}{16}\right) \cdot \left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 2.606$$

Acceptable Factor of Safety

$w := \frac{3}{16} \cdot \mathbf{in}$ $\quad h := \frac{1}{8} \cdot \mathbf{in}$ — Keyway Dimensions (Table 7-6)

$F := \frac{2 \cdot T_f}{d} = (1.337 \cdot 10^3)\ \mathbf{lbf}$ — Force at Surface of Shaft #2

$S_{sy} := S_y \cdot .577 = 48.468\ \mathbf{ksi}$ — Material Shear Strength

$l_{key\_s} := F \cdot \frac{n}{w \cdot S_{sy}} = 0.383\ \mathbf{in}$ — Length Required to Resist Shear

$l_{key\_c} := 2 \cdot F \cdot \frac{n}{w \cdot S_y} = 0.443\ \mathbf{in}$ — Length Required to Resist Crushing

Keyway must be at least 0.382 in. long
or it will fail in Shear (Failure Mode)

Bearing Sizing and Analysis

Bearing (x2) Shaft #1 : Deep Groove Ball Bearing

$a := 3 \qquad V := 1$	Ball Bearing (Inner Ring Rotates)
$L_{10} := 10^6$	Rated Life
$n_i := 750$	Shaft #1 Speed
$\zeta_D := 438000$	Shaft #1 Desired Life (hours)
$L_D := \zeta_D \cdot n_i \cdot 60 = 1.971 \cdot 10^{10}$	Desired Life
$R_{A.x} := 3.39 \cdot lbf$	Reaction Force x-dir
$R_{A.y} := 9.31 \cdot lbf$	Reaction Force y-dir
$F_a := \sqrt[2]{(R_{A.x})^2 + (R_{A.y})^2} = 9.908\ lbf$	Reaction Force at Bearing
$F_D := F_a = 9.908\ lbf$	Desired Radial Load
$x_D := \dfrac{L_D}{L_{10}} = 1.971 \cdot 10^4$	Multiple of Rating Life
$\theta := 4.459$	Manufacturer #2 Characteristic Parameter
$x_0 := 0.02$	Manufacturer #2 Characteristic Parameter
$b := 1.483$	Manufacturer #2 Shape Parameter
$R_D := 0.95$	System Reliability
$a_f := 2$	Bearing System Factor of Safety

Calculated Load Rating

$$C_{10} := a_f \cdot F_D \cdot \left[\frac{x_D}{x_0 + (\theta - x_0) \cdot (1 - R_D)^{\frac{1}{b}}} \right]^{\frac{1}{a}} = [631.557]\ lbf$$

Bearing (x2) Shaft #2 : Deep Groove Ball Bearing

$a := 3 \qquad V := 1$	Ball Bearing (Inner Ring Rotates)
$L_{10} := 10^6$	Rated Life
$n_f := 300$	Shaft #1 Speed
$\zeta_D := 438000$	Shaft #1 Desired Life (hours)
$L_D := \zeta_D \cdot n_f \cdot 60 = 7.884 \cdot 10^9$	Desired Life
$R_{A.x} := 3.39 \cdot lbf$	Reaction Force x-dir
$R_{A.y} := 9.31 \cdot lbf$	Reaction Force y-dir
$F_a := \sqrt[2]{(R_{A.x})^2 + (R_{A.y})^2} = 9.908\ lbf$	Reaction Force at Bearing
$F_D := F_a = 9.908\ lbf$	Desired Radial Load
$x_D := \frac{L_D}{L_{10}} = 7.884 \cdot 10^3$	Multiple of Rating Life
$\theta := 4.459$	Manufacturer #2 Characteristic Parameter
$x_0 := 0.02$	Manufacturer #2 Characteristic Parameter
$b := 1.483$	Manufacturer #2 Shape Parameter
$R_D := 0.95$	System Reliability
$a_f := 2$	Bearing System Factor of Safety

Calculated Load Rating

$$C_{10} := a_f \cdot F_D \cdot \left[\frac{x_D}{x_0 + (\theta - x_0) \cdot (1 - R_D)^{\frac{1}{b}}} \right]^{\frac{1}{a}} = [465.335]\ lbf$$

Appendix D:

Citations:

[1] Budynas, Richard G., and J. Keith Nisbett. *Shigley's Mechanical Engineering Design*. McGraw-Hill Education, 2020.
[2] SKF Bearings Catalog
[3] Boston Gears Catalog
[4] McMaster Carr Catalog