Memo

To: Professor Vanderbeek
From: Michael Byckovski, Brandon Porath, Max Porter, Christian Tello, Remha Yohannes
Date: December 3, 2019
Re: Compound Reduction Gear Train

Summary:

The project aims to design a speed reduction transmission gearbox. This would effectively reduce rotational speed and increase torque. With an input power and rotational speed of 2.5 HP and 800 RPM respectively, the gearbox configuration must output a torque of 7750±500 lbf-in. and a rotational speed of 20±10 RPM. Final design considerations must be overall dimension and spacing of the gearbox. To minimize the gearbox size, shaft center to center distance should not be greater than 6”. Quantitative methods will be used to gain in-depth insight into characteristic gear dimensions and forces. This data will be contextualized and culminate in final gear selections and gearbox configuration.

The primary analysis and verification tool in this design study was MathCAD. The program calculated characteristic gear dimensions and forces that can be utilized in AGMA bending and surface failure approaches, the two most common gear failure modes. With appropriate specifications and ratings, a pinion and gear were selected from the Boston Gear catalog and meshed and assembled in the dimensioned gearbox using Solidworks solid modeling.

Methods:

To simplify our analysis, we made both gears and both pinions in the gear train identical. With this, only two sets of calculations were required. Due to the open ended nature of the design, we also assumed many of the AGMA parameters in the analysis. These include a uniform power source and driven machine, a gearbox operating with oil below 250 °F, teeth with full depth contact, and a system rated for 10,000,000 cycles. Other estimates were based on industry standards. These include stock sized, steel gears and pinions, desired diametral pitch and pressure angle, and standard gear and pinion surface hardness.

The analysis was primarily performed in MathCAD. Firstly, the gear teeth number and diameters were calculated given input and output speeds and power limitations. An AGMA approach of finding bending and contact stress failure allowed us to identify the critical gearbox components. Bending and pitting failure modes were analyzed for both pinion and gear. Now with gear size, teeth number, and allowable stress rating, we selected appropriate catalog gears from Boston gear. These were assembled into a final gear box design and interference analysis in Solidworks was performed to verify the sizing calculations. Because we identified critical failure mode of bending stress on the larger gear through MathCAD failure analysis, FEA was not required and therefore saved us time and money. The MathCAD calculations are illustrated in the Appendix A.

Results:

Based on our calculations, we determined that the critical component of the gearbox is the pinion. Specifically, the pinion has the largest factor of safety failing in bending. Conducting an AGMA failure analysis, the factors of safety associated with gear tooth bending and pinion tooth bending are greater than contact stress factors of safety. The most critical factor of safety was 4.964, associated with bending failure of the pinion. Regarding design parameters, final shaft speed is 25.92 rpm and center to center shaft distance is 5.9 inches. Both are within the specified design range. In addition, the diametral pitch and pressure angle are 10 teeth per inch and 20° respectively with a gear ratio of 5.6. Because the pinions experience faster speeds, we chose a Grade 2 steel while the gears are made cheaper with Cast Iron. After interference analysis, we specified the pinions to each have 18 teeth and the gears to have 100 teeth. All four gear sizes are standard which cuts down on manufacturing cost.

The combined load from the gears on the primary gearbox shaft is 465.8 pound-force. This is calculated by summing the reaction forces applied to the shaft from gear 3 and pinion 4. The forces transmitted from the gears are relatively small so any low carbon steel shaft is acceptable for this gearbox application. The contact stress experienced by both gears is 61.5 ksi and 61.1 ksi for the pinions. Unfortunately, this is well above the Boston Gear standard for safe static stresses. While the gear mesh parameters and interference study are sound, we can revisit AGMA failure analysis assumptions that have large impacts on final safety factors and stress.

Upon final assembly, we ensured the gears have appropriate clearance for oil circulation and that a minimum gearbox wall thickness is achieved. We also introduced ball bearings on the ends of shafts for additional support. All bearings have a dynamic rating of 2150 lb capacity which is well within our operating range. The final gearbox dimensions are 11 x 8 x 19 inches.

Conclusions:

In conclusion, the pinion has the highest factor of safety. This ensures the pinion will fail last. This is realistic because the pinions are made from steel, while the gears are of cast iron. Since steel has better mechanical properties than cast iron, we expect the cast iron to fail first. This is ideal, because pinions are typically the most expensive components of the gearbox and it is cost effective to have gear fail in bending first.

Appendix A:

MathCAD calculations

Machine Design Project #2
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12/3/19

Given:

$H_i := 2.5 \cdot hp$ — Input shaft horsepower

$n_i := 800 \cdot \left(\frac{1}{min}\right) \quad n_f := 25.92 \cdot \left(\frac{1}{min}\right)$ — Input and output shaft speed

$T_{final} := 7750 \cdot lbf \cdot in$ — Output torque

$T_i := \frac{H_i}{n_i} = 103.125 \; lbf \cdot ft$ — Input torque

$Phi := 20 \cdot deg$ — Desired pressure angle

$P_d := 10 \cdot \left(\frac{1}{in}\right)$ — Desired diametral pitch

$w_i := n_i = 800 \left(\frac{1}{min}\right)$ — Input shaft speed

$w_f := n_f = 25.92 \; \frac{1}{min}$ — Output shaft speed

$m_v := \frac{n_i}{n_f} = 30.864$ — Angular velocity ratio

Gear Tooth Number Analysis:

mv=N2/N3*N4/N5
assume N2/N3=N4/N5
mv=(N2/N3)^2
N2/N3=(3/80)^1/2

System of equations

$k := 1$

Full-depth teeth

$m := \sqrt{m_v} = 5.556$

Gear ratio (Guess/check)

Minimum number of teeth

$$N_p := \frac{(2 \cdot k)}{(1 + 2 \cdot m) \cdot (\sin(20 \cdot deg))^2} \cdot \left(m + \sqrt[2]{m^2 + (1 + 2 \cdot m) \cdot (\sin(20 \cdot deg))^2}\right) = 15.864$$

$N_2 := 18$ $\quad N_4 := 18$

Pinion teeth number

$N_3 := N_2 \cdot m = 100$

System of equations

$N_3 = 100$ $\quad N_5 := N_3 = 100$

Gear teeth number

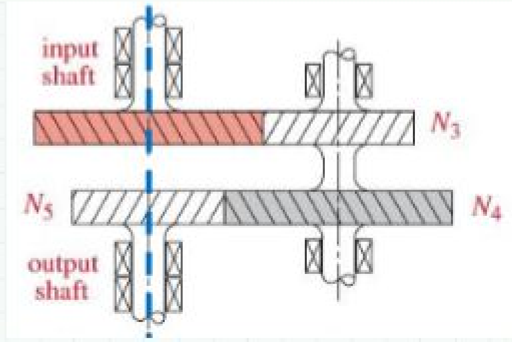
Output shaft speed verification

$w_{final} := 800 \cdot \left(\frac{N_2}{N_3}\right)^2 = 25.92$ rpm which is below 30 rpm so acceptable parameters

$m_G := \frac{N_3}{N_2} = 5.556$

Gear ratio

Pitch Circle Diameters & Pitch Circle Radius



$$d_{24} := \frac{N_2}{P_d} = 1.8\ in \qquad r_{24} := \frac{d_{24}}{2} = 0.9\ in$$

$$d_{35} := \frac{N_3}{P_d} = 10\ in \qquad r_{35} := \frac{d_{35}}{2} = 5\ in$$

Nominal center to center distance

$$C := r_{24} + r_{35} = 5.9\ in$$

$$a_p := \frac{1}{P_d} = 0.1\ in$$

$$Z := \sqrt{\left(r_{24}+a_p\right)^2 - \left(r_{24}\cdot\cos(20\cdot deg)\right)^2} + \sqrt{\left(r_{35}+a_p\right)^2 - \left(r_{35}\cdot\cos(20\cdot deg)\right)^2} - C\cdot(\sin(20\cdot deg))$$

$Z = 0.499\ in$ Length of action

$p_{c24} := \pi\cdot\frac{d_{24}}{16} = 0.353\ in$ Circular pitch- Mesh#1

$p_{c35} := \pi\cdot\frac{d_{35}}{83} = 0.379\ in$ Circular pitch- Mesh#2

$p_b := p_{c24}\cdot\cos(20\cdot deg) = 0.332\ in$ Base pitch

$m_p := \frac{Z}{p_b} = 1.503$ Contact ratio

$m_G := \frac{N_3}{N_2} = 5.556$ Gear ratio

AGMA Bending Analysis- Pinion

$d_{24}=1.8\ in$	Pinion pitch diameter
$V:=(\pi\cdot d_{24}\cdot n_i)=376.991\ \frac{ft}{min}$	Pitch- line velocity
$W_t:=\frac{H_i}{V}=218.838\ lbf$	Tangential transmitted load
$K_o:=1$	Overload factor
$K_v:=\frac{\left(50+\sqrt[2]{335.103}\right)}{50}=1.366$	Dynamic factor
$F:=1.5\left(\frac{1}{in}\right)$	Face width
$Y_p:=0.309$	Lewis form factor @ 18 teeth
$K_{s.p}:=1.192\left(\frac{\left(F\cdot\sqrt[2]{Y_p}\right)}{P_d}\right)^{0.0535}=1.044$	Size factor
$K_m:=1$	Load distribution factor
$K_b:=1$	Rim- thickness factor
$J_p:=0.33$	Pinion geometry factor
$F_2:=1.5\ in$	Face width
$\sigma_p:=\frac{W_t\cdot K_o\cdot K_v\cdot K_{s.p}\cdot P_d\cdot K_m\cdot K_b}{F_2\cdot J_p}=6.303\ ksi$	Pinion bending stress
$H_{B.p}:=240\cdot psi$	Pinion Brinell hardness
$S_{t.p}:=77.3\cdot H_{B.p}+12800\ psi=31.352\ ksi$	Endurance strength
$K_T:=1$	Temperature correction factor
$R:=0.99$	Reliability
$K_R:=0.50-0.109\ \ln(1-R)=1.002$	Reliability factor
$Y_N:=1$	Stress cycle factor
$S_{F.p}:=\frac{\left(\frac{S_{t.p}\cdot Y_N}{K_T\cdot K_R}\right)}{\sigma_p}=4.964$	Factor of safety (Bending)

AGMA Bending Analysis- Gear

$Y_G := 0.447$ Lewis form factor @ 100 teeth

$K_{s.G} := 1.192 \left(\frac{\left(F \cdot \sqrt[2]{Y_G} \right)}{P_d} \right)^{0.0535} = 1.054$ Size factor

$J_G := 0.43$ Gear geometry factor

$\sigma_G := \frac{W_t \cdot K_o \cdot K_v \cdot K_{s.G} \cdot P_d \cdot K_m \cdot K_b}{F_2 \cdot J_G} = 4.885\ ksi$ Gear bending stress

$H_{B.G} := 201 \cdot psi$ Gear Brinell hardness

$S_{t.G} := 13000 \cdot psi$ Endurance strength for Class 40 gray cast iron

$S_{F.G} := \frac{\left(\frac{S_{t.G} \cdot Y_N}{K_T \cdot K_R} \right)}{\sigma_G} = 2.656$ Factor of safety (Bending)

AGMA Pitting Analysis- Pinion

$C_p := 2100 \cdot \sqrt[2]{psi}$ Elastic coefficient

$C_f := 1$ Surface condition factor

$I := \frac{\cos(Phi) \cdot \sin(Phi) \cdot m_G}{2 \cdot (m_G + 1)} = 0.136$ Geometry factor for pitting resistance

$\sigma_{c.p} := C_p \cdot \sqrt[2]{\left(\frac{W_t \cdot K_o \cdot K_v \cdot K_{s.p} \cdot K_m \cdot C_f}{d_{24} \cdot F_2 \cdot I} \right)} = 61.173\ ksi$ Pinion contact stress

$S_{c.p} := 322 \cdot (H_{B.p}) + 29100 \cdot psi = 106.38\ ksi$ Allowable contact stress

$Z_N := 1$ Stress life cycle factor

$C_{H.p} := 1$ Hardness ratio factor for pitting resistance

$S_{H.p} := \frac{\left(\frac{S_{c.p} \cdot Z_N \cdot C_{H.p}}{K_T \cdot K_R} \right)}{\sigma_{c.p}} = 1.736$ Factor of safety (Pitting)

AGMA Pitting Analysis- Gear

$$\sigma_{c.G} := C_p \cdot \sqrt[2]{\left(\frac{W_t \cdot K_o \cdot K_v \cdot K_{s.G} \cdot K_m \cdot C_f}{d_{24} \cdot F_2 \cdot I}\right)} = 61.475\ ksi$$

Gear contact stress

$$S_{c.G} := 80000 \cdot psi$$

Allowable contact stress

$$A' := 8.98 \cdot \left(10^{-3}\right) \cdot \left(\frac{H_{B.p}}{H_{B.G}}\right) - 8.29\ \left(10^{-3}\right) = 0.002$$

$$C_{H.G} := 1 + A' \cdot \left(m_G - 1\right) = 1.011$$

Hardness ratio factor for pitting resistance

$$S_{H.G} := \frac{\left(\frac{S_{c.G} \cdot Z_N \cdot C_{H.G}}{K_T \cdot K_R}\right)}{\sigma_{c.G}} = 1.313$$

Factor of safety (Pitting)

Failure Mode- Pinion

$$S_{H.p}{}^2 = 3.012$$

$$S_{F.p} = 4.964$$

$$S_{F.p} > S_{H.p}{}^2$$

Threat in the pinion is from bending

Failure Mode- Gear

$$S_{H.G}{}^2 = 1.724$$

$$S_{F.G} = 2.656$$

$$S_{F.G} > S_{H.G}{}^2$$

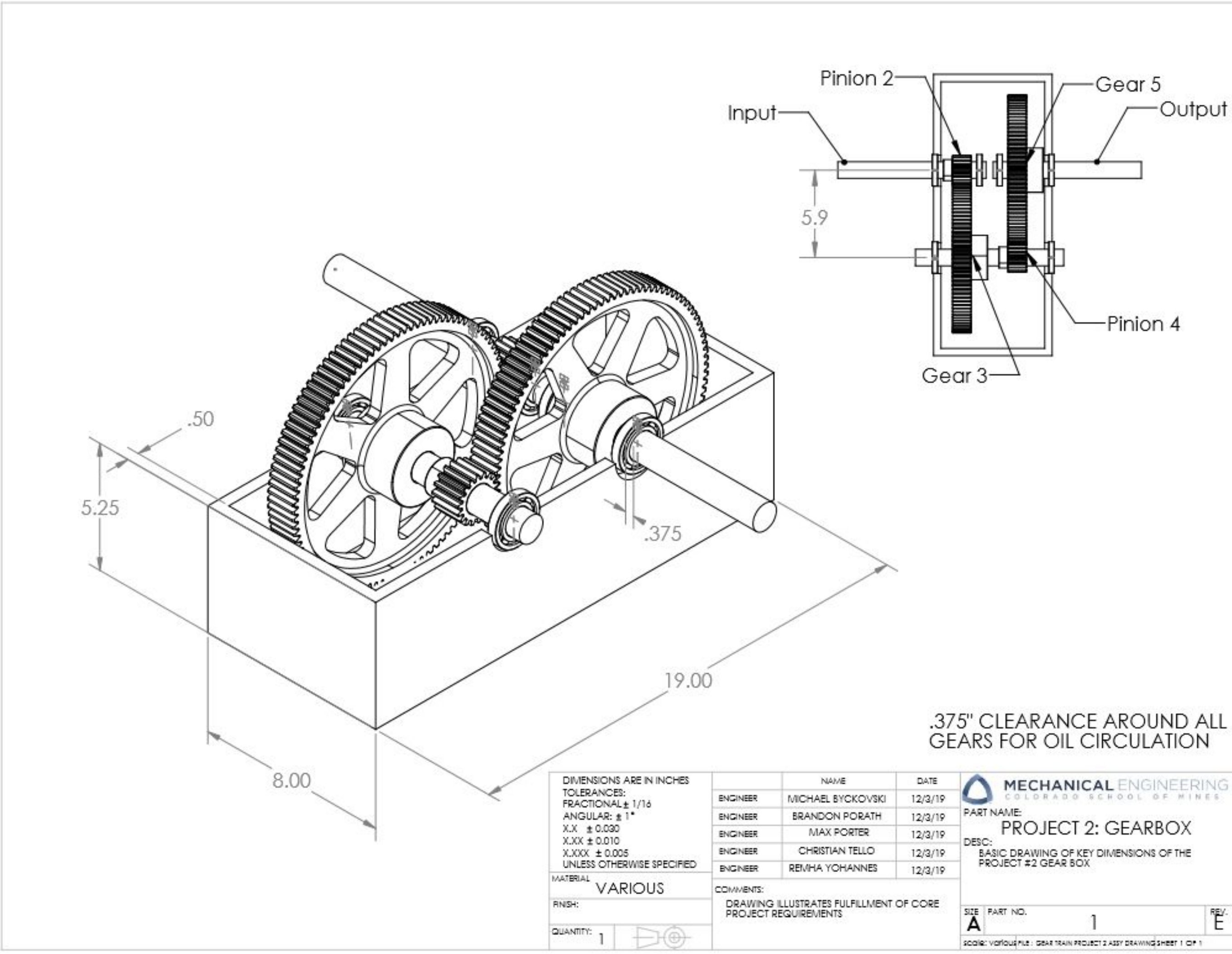
Threat in the gear is from bending

Geartrain Shaft Analysis

$W_t = 218.838\ \mathbf{lbf}$	Tangential transmitted load
$F_{23.t} := W_t = 218.838\ \mathbf{lbf}$	Pinion(2) to gear(3) tangential load
$F_{23.r} := F_{23.t} \cdot \tan(Phi) = 79.651\ \mathbf{lbf}$	Pinion(2) to gear(3) radial load
$F_{a3.x} := -F_{23.t} = -218.838\ \mathbf{lbf}$	Shaft x-dir. reaction to gear(3)
$F_{a3.y} := -F_{23.r} = -79.651\ \mathbf{lbf}$	Shaft y-dir. reaction to gear(3)
$F_{a3} := \sqrt[2]{\left(F_{a3.y}\right)^2 + \left(F_{a3.x}\right)^2} = 232.883\ \mathbf{lbf}$	Shaft reaction to gear(3)
$F_{a4} := F_{a3} = 232.883\ \mathbf{lbf}$	Shaft reaction to pinion(4)
$F_a := F_{a3} + F_{a4} = 465.765\ \mathbf{lbf}$	Total load on gear train shaft
$l := 3.75 \cdot \mathbf{in}$	Length of shaft from support to point load
$y := 0.56\ \mathbf{in}$	Shaft radius
$mass := 10\ \mathbf{kg}$	Shaft mass
$M_a := l \cdot F_a = 145.552\ \mathbf{lbf} \cdot \mathbf{ft}$	Moment induced through gear force
$I := \dfrac{mass \cdot l^2}{3} = 0.326\ \mathbf{kg} \cdot \mathbf{ft}^2$	Area moment of inertia
$\sigma_b := \dfrac{M_a \cdot y}{I} = 304.52\ \dfrac{\mathbf{ft}}{\mathbf{s}^2}$	Bending stress on shaft

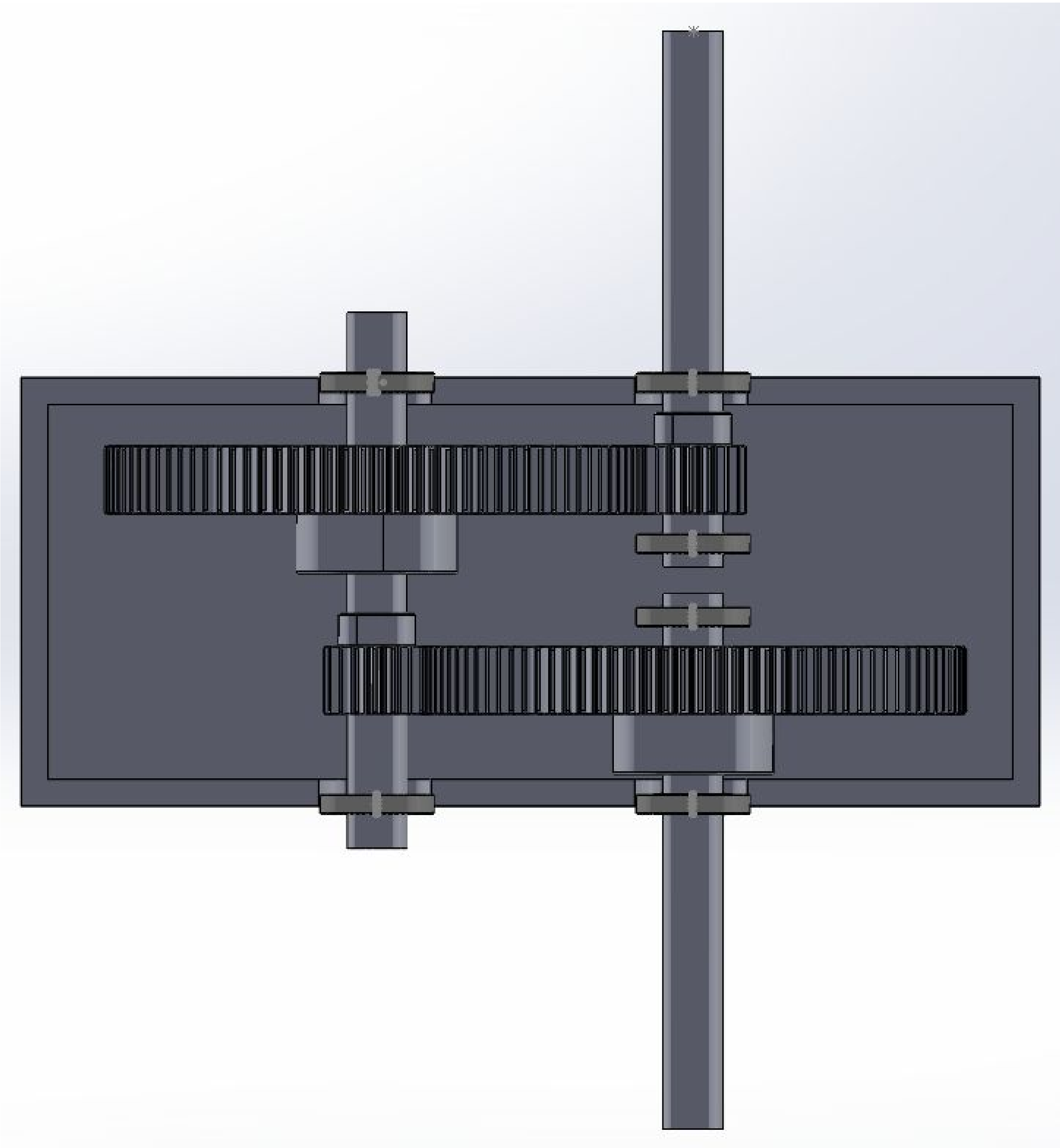
Appendix B:

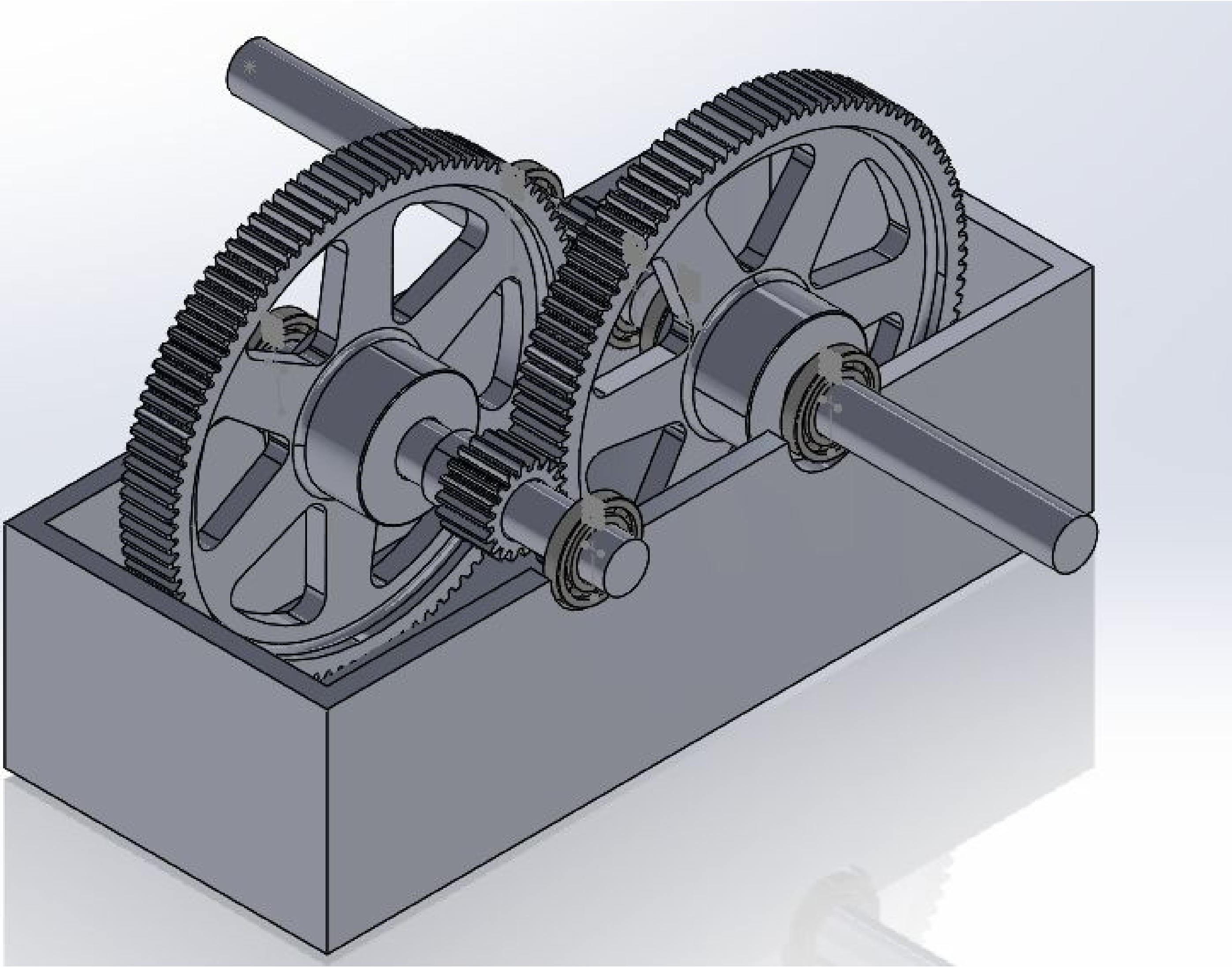
Assembly Drawing

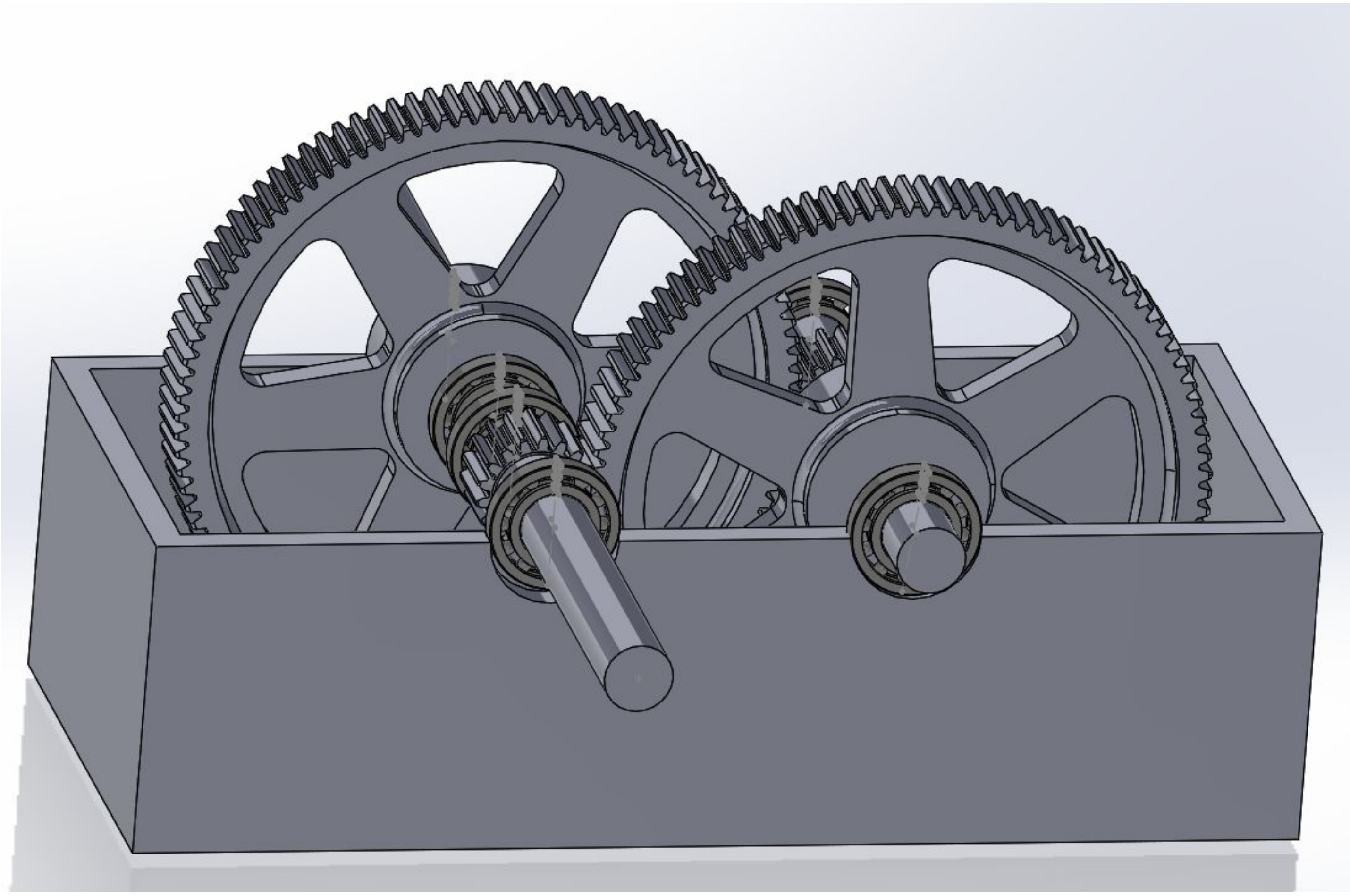


Appendix C:

Assembly Views







Appendix D:

Citations:

Budynas, Richard G., and J. Keith Nisbett. *Shigley's Mechanical Engineering Design*. McGraw-Hill Education, 2020.