Memo

ShTo: Professor Vanderbeek
From: Michael Byckovski, Brandon Porath, Max Porter, Christian Tello, Remha Yohannes
Date: November 12, 2019
Re: Shaft Design Project

Summary:

The project aims to design a shaft layout that will be able to run maintenance free for a period of 3 years, being operated 5 days/week, 8 hours/day. The shaft will spin at a constant RPM of 1725, transmitting 2 HP to the shaft, and powering a 6 inch, 20° spur gear with its mating gear, and a cooling fan. Quantitative methods will be used to gain in-depth insight into each individual component. In addition, design considerations will adhere to effective stresses associated with a minimum factor of safety (2.5). This data will be contextualized and culminate in a final shaft design.

The primary analysis and verification tool in this design study was MathCAD. The program was utilized to calculate bearing life and critical stress concentration values and locations along the shaft. With these considerations, we chose appropriate bearing types and size, keyway dimensions, and shoulder locations and fillets. Our current shaft design meets all these analysis requirements while maintaining a critical factor of safety at 6.765. Shaft material was 1020 cold drawn steel. This material is common for shafts due to its high strength and low cost.

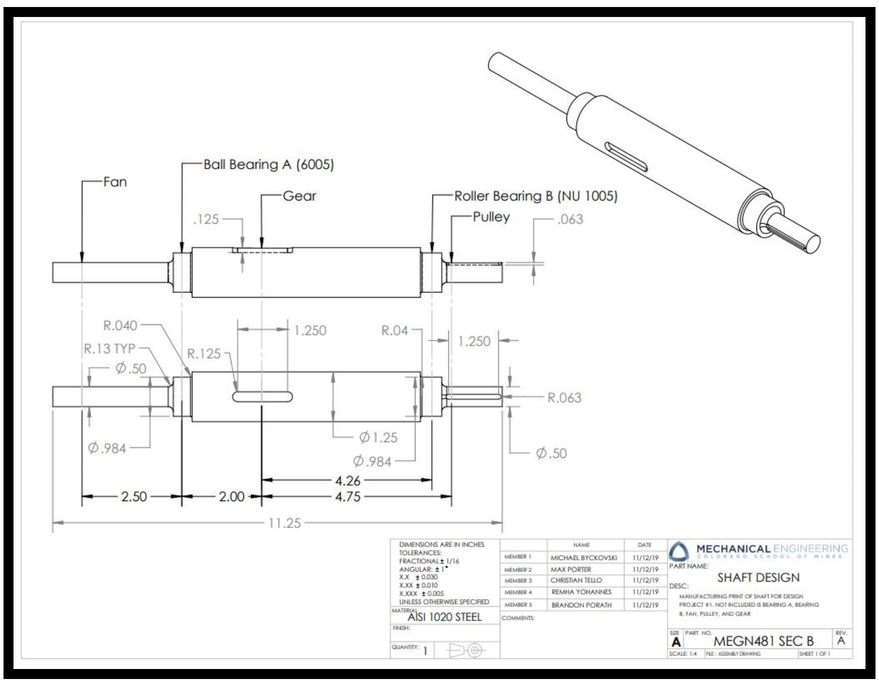


Diagram 1 of Shaft Print: Full view in Appendix [1]

Methods:

To simplify our analysis, we took into account common material assumptions. We assume component and shaft material will act in accordance to its standard yield, tensile and compressive strengths as described in the textbook and thermal contraction and expansion of the shaft and components is negligible. We also treated the bearings as pin supports in stress analysis and based component calculations off their centers of mass.

The analysis was primarily performed in MathCAD. Firstly, the reaction forces and resultant torque on the shaft was calculated using known speeds, horsepower, pressure angle and pitch diameter.We determined critical locations from resultant loading, shear, and moment diagrams of the shaft. After identifying the critical locations, we calculated stress concentration factors and failure limits. Using the conservative DE-Gerber criteria, calculated endurance strength, material properties, and a minimum factor of safety of 2.5, we determined critical diameters along the shaft at component interfaces. Verification analysis was performed to confirm stress locations and limits. Because we identified critical stress locations, FEA was not required and therefore saved us time and money. The MathCAD calculations are illustrated in the Appendix [2].

Results:

Through calculation, we determined failure was most likely at the gear. Using DE-Gerber criteria, and a factor of safety of 2.5, the minimum diameter of the shaft at the location of the gear was deemed to be .382". Since the bore size of the gear increased in increments of eighths of an inch, the shaft's diameter was increased to 0.5". Once the shaft's diameter was rounded up to 0.5", our factor of safety increased to 6.765. At this point, 0.5" was set as our minimum shaft diameter, and the gear diameter was increased to accomodate for the mounted bearing. The final shaft diameter at the location of the gear finished out at 1.25". With the shaft diameter verified, bearing A was chosen to be deep groove ball bearing [6005] to handle radial and axial loads. A 25mm bore was chosen because smaller sizes had rated loads that were less than the calculated load. Bearing B was selected to be a Cylindrical Roller Bearing [NU 1005] with a 25mm bore in order to promote uniformity in the shaft. The bearing choice iteration is documented in MathCAD. The calculated load was less than the rated loads of the bearings at this dimension. The size of the keyways used to secure the pulley and gear were determined using the shaft diameter, and Table 7-6 in the book. For the gear key, the width and height for the gear key were each ¼ inch and the thickness was ⅛ inch. The shaft diameter was 1.25 inch. The minimum required shear length was 0.007 feet and the crush length was 0.008 feet. The width and height for the pulley key were each ⅛ in and the thickness was 1/16 in. The shaft diameter at this location is 0.5". The shear length was 0.033 feet and the crush length was 0.038 feet. In the design, the length of the keyways was overshot to allow for accommodation, since the mounting could be done using #3 set screws located in Table 7-4.

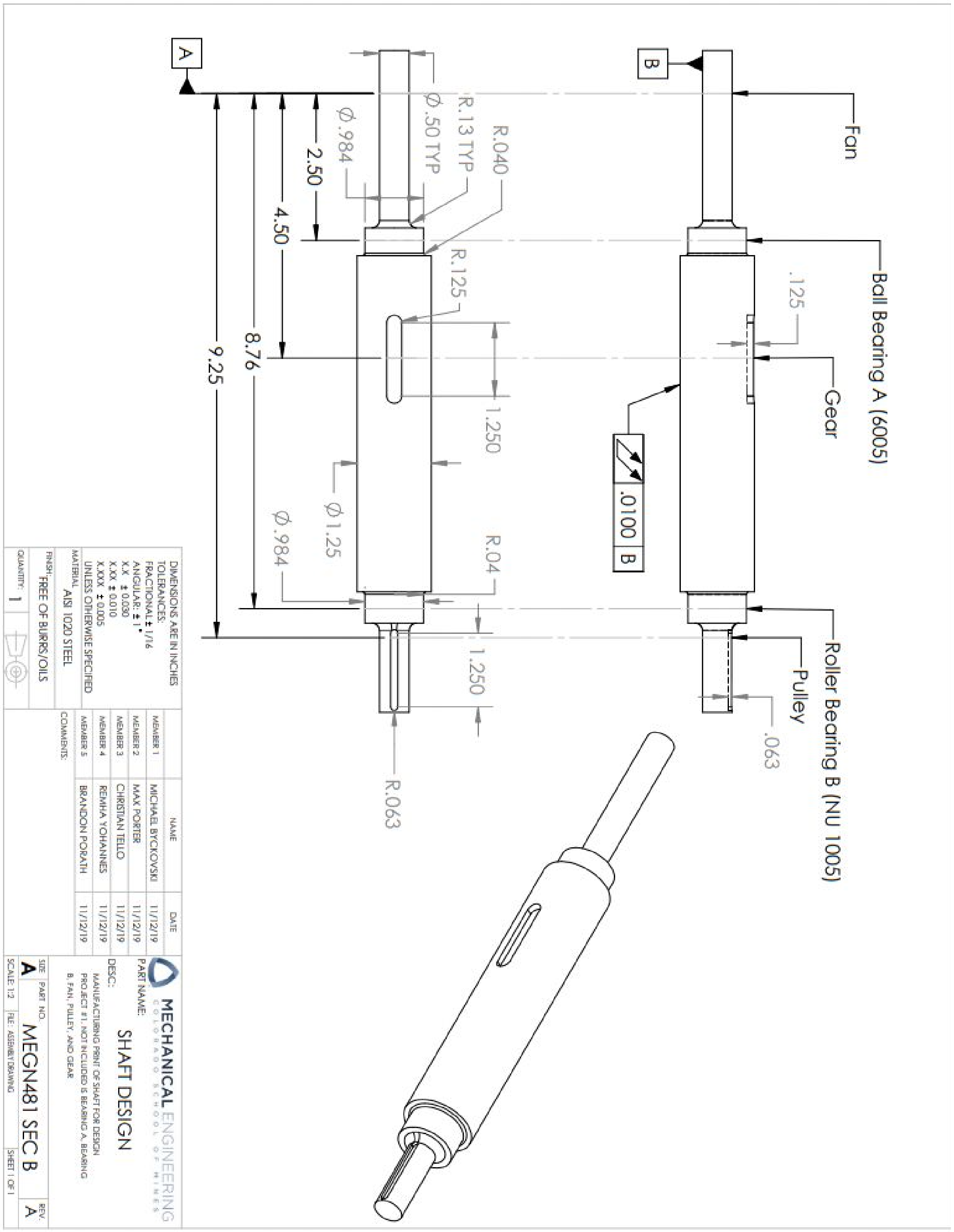
All of these specifications were met or exceeded in the design of the shaft. The key length was exceeded, as the keyways will be machined by a CNC mill and the extra length will add negligible machine time. The keyway length will be the full width of the gear and pulley as it will be broached and the extra length will be utilized to add strength and decrease the likelihood of premature failure. The fillet at the shoulders exceeds the minimum in order to minimize stress concentrations and fatigue. The fillet at the shoulder that locates the deep groove ball bearing and transmits axial load matches the fillet on the bearing itself. This helps minimize the stress concentration while still being able to locate the bearing and transmits load. The shaft was designed to be easy to assemble with all components sliding on from either end up against a shoulder as shown in appendix 4. The gear is an exception with no shoulder as it decreases stock size and material removed.

Conclusions:

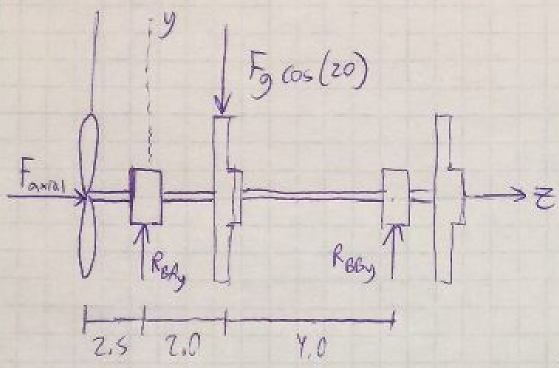
Our final results make sense, since our actual factor of safety will exceed the initial calculated value of 6.765, which is above the minimum factor of safety of 2.5. While our shaft isn't the most cost efficient, it has far exceeded the requirements outlined in the prompt, and reduced the risk at critical point A, on the gear. The dominant failure mode is failure by crushing, which means that the gear and pulley key lengths must be 0.008 ft and 0.038 ft, respectively. For these reasons, we are very confident in our results.

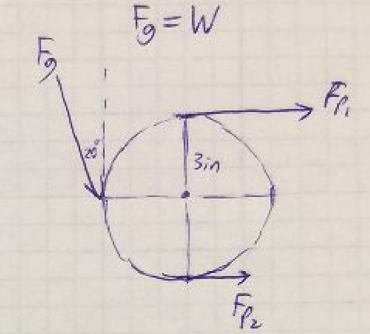
Appendix:

[1] Shaft Drawing



[2]. Reaction Force Calculations:





$T = -F_{P_1}(3) + F_{P_2}(3) + F_g \cos(20)(3)$

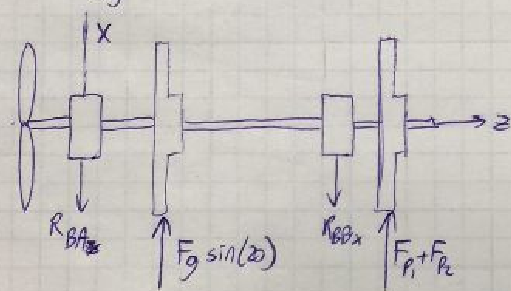
$F_{P_2} = 5.59 \text{ lbf} \quad F_{P_1} = 27.93 \text{ lbf}$

$\Sigma M_{A_x} = 0 = -F_g \cos(20)(2) + R_{BB_y}(6)$

$R_{BB_y} = 8.12 \text{ lbf}$

$\Sigma F_y = 0 = R_{BA_y} - F_g \cos(20) + R_{BB_y}$

$R_{BA_y} = 16.24 \text{ lbf}$



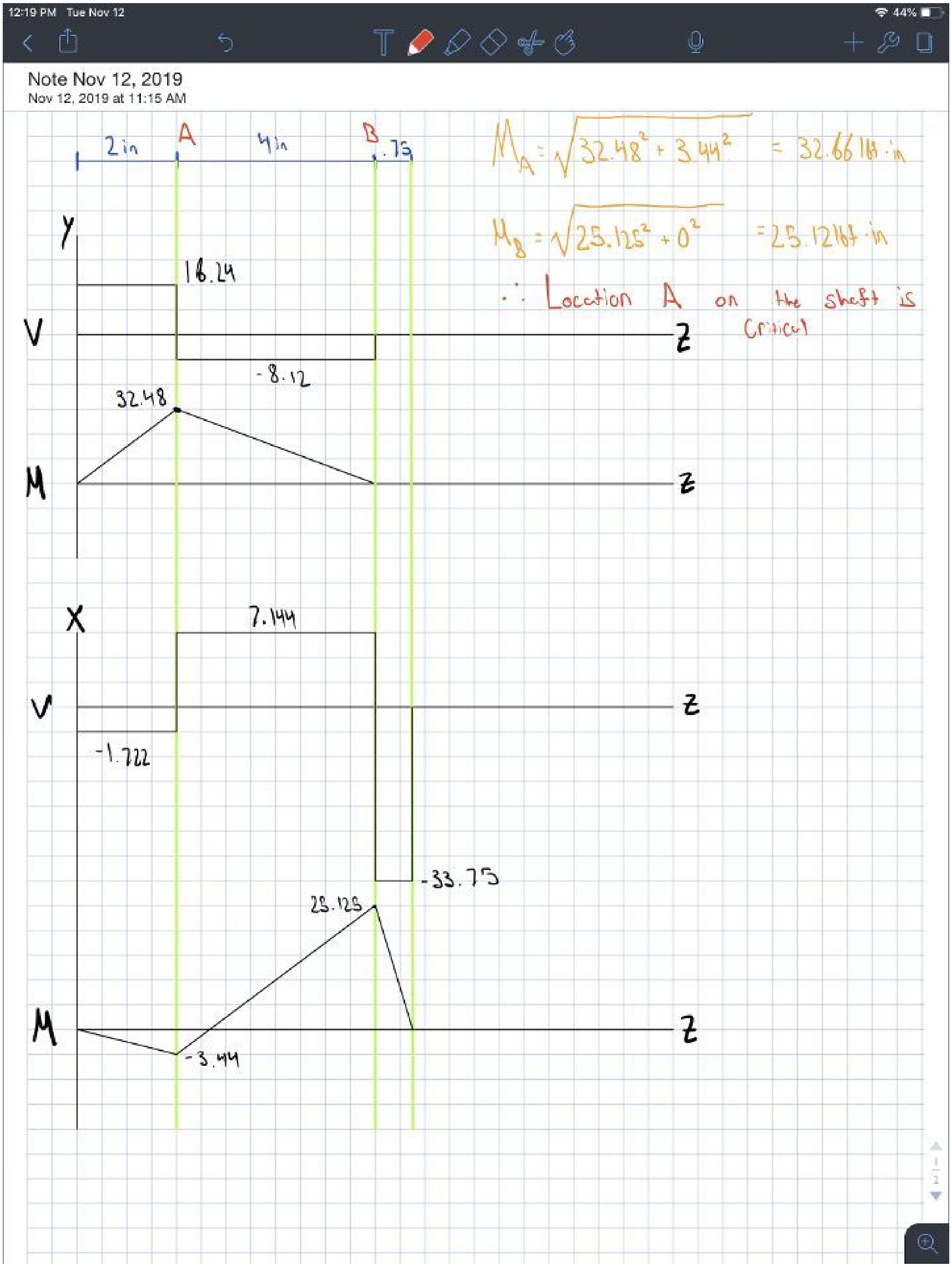
$\Sigma M_{A_y} = 0 = F_g \sin(20)(2) \cancel{+ R_{BB_x}(6)} - R_{BB_x}(6) + (F_{P_1} + F_{P_2})(6.75)$

$R_{BB_x} = 40.65 \text{ lbf}$

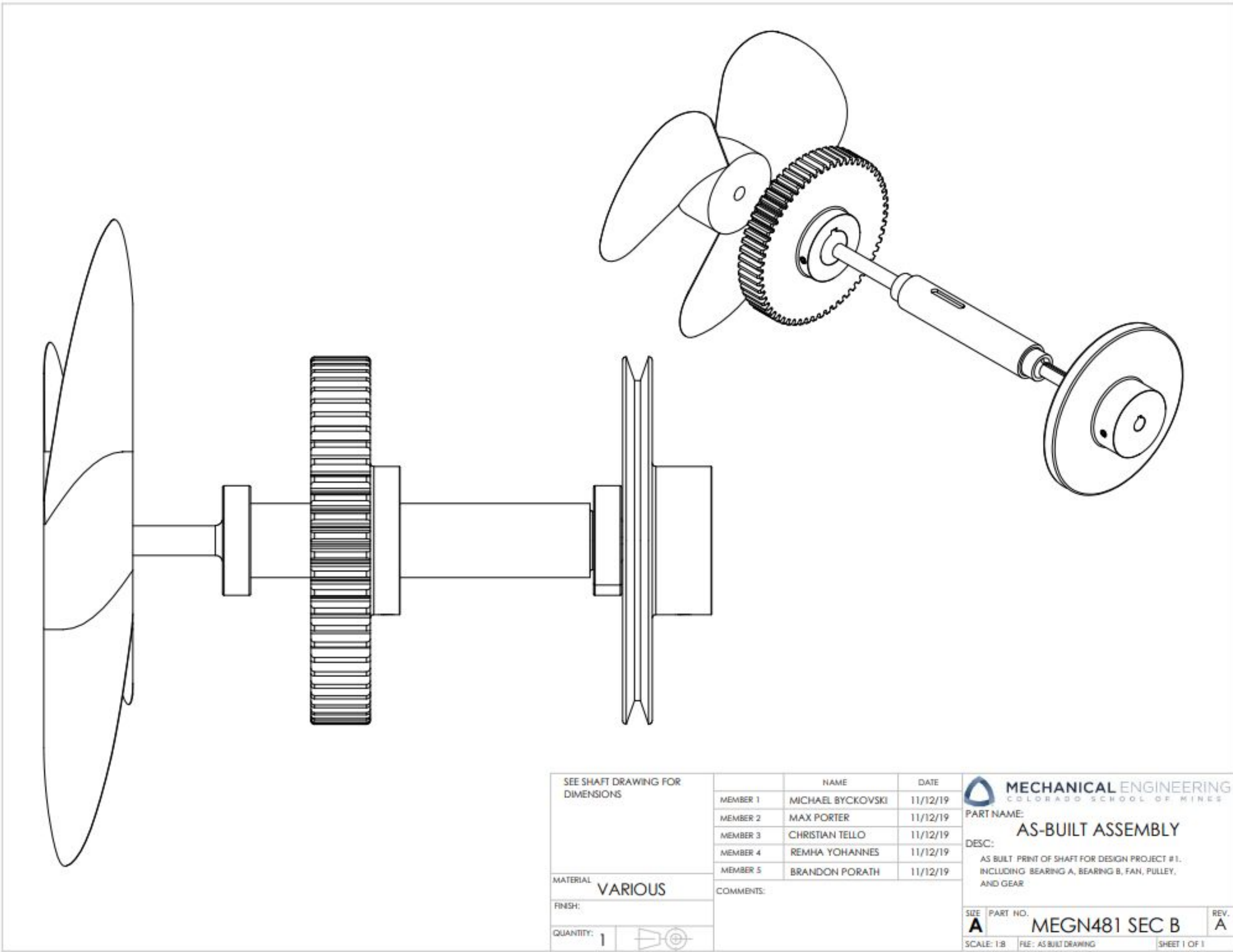
$\Sigma F_x = 0 = -R_{BA_x} + F_g \sin(20) - R_{BB_x} + F_{P_1} + F_{P_2}$

$R_{BA_x} = 1.722 \text{ lbf}$

[3]. Shear and Moment Diagram Calculations:



[4]. Assembly Exploded View:



[5] MathCAD calculations

Machine Design
Michael Byckovski, Brandon Porath, Max Porter, Christian Tello, Remha Yohannes

Givens:

Pulley

$n := 28.75 \cdot \frac{1}{s}$

$H := 2\ ft \cdot \frac{lb}{s}$

Fan

$F_{fan\_axial} := 45\ lbf$

Gear

$D_p := 0.5 \cdot ft$

$Phi := 20 \cdot deg$

General

$n_f := 2.5$

$R_D := .95$

Weibull Parameters

$x_0 := .02$

$\theta := 4.459$

$b := 1.483$

Material: 1020 CD Steel

Life values

$S_y := 50.8\ ksi \qquad S_{ut} := 60.9 \qquad S'_e := 0.5 \cdot S_{ut}$

$L_R := 10^6$

$L_{10} := 10^6$

$L_D := 6.4584 \cdot 10^8$

Gear forces on shaft

$T := 6.09 \cdot lbf \cdot ft \qquad H = \frac{T \cdot n}{63025}$

$W_{gear\_tangential} := \frac{2 \cdot T}{D_p} = 24.36\ lbf$

$W_{gear\_radial} := W_{gear\_tangential} \cdot \tan(Phi) = 8.866\ lbf$

$W := \frac{W_{gear\_tangential}}{\cos(Phi)} = 25.923\ lbf$

Forces At Pulley

$F_{p1} := 27.925 \cdot lbf \qquad F_{p2} := 5.585 \cdot lbf$

Forces At Pulley

$F_{p1} := 27.925 \cdot lbf \quad F_{p2} := 5.585 \cdot lbf$

$F_{p1} := 5 \cdot F_{p2}$

$T := 3 \left(-F_{p1} + F_{p2} + W_{gear\_tangential}\right) \cdot in$

Reaction Force of Bearings

$R_{BBy} := W_{gear\_tangential} \cdot \frac{2}{6}$

$R_{BBy} = 8.12\ lbf$

Reaction Force of Bearings

$$R_{BBy} := W_{gear\_tangential} \cdot \frac{2}{6}$$

$$R_{BBy} = 8.12\ \boldsymbol{lbf}$$

$$R_{BAy} := W_{gear\_tangential} - R_{BBy}$$

$$R_{BAy} = 16.24\ \boldsymbol{lbf}$$

$$R_{BBx} := \frac{\left(W \cdot \sin\left(Phi\right) \cdot 2 + 6.75\ \left(F_{p1} + F_{p2}\right)\right)}{6}$$

$$R_{BBx} = 40.654\ \boldsymbol{lbf}$$

$$R_{BAx} := W \cdot \sin\left(Phi\right) + F_{p1} + F_{p2} - R_{BBx}$$

$$R_{BAx} = 1.722\ \boldsymbol{lbf}$$

Combine Bending Moments at Bearings A and B

$$M_A := 32.66 \cdot \boldsymbol{lbf} \cdot \boldsymbol{in}$$

$$M_B := 25.12 \cdot \boldsymbol{lbf} \cdot \boldsymbol{in}$$

Bearing B located 4 inches right of gear center and 0.75 inches left of pulley center

$$T = 6.06\ \boldsymbol{lbf} \cdot \boldsymbol{in}$$

Location Gear Fatigue Stress Concentration

$K_t := 2.114$ $K_{ts} := 3.0$ $d := 1.0 \cdot \boldsymbol{in}$ $r := .02 \cdot d$

$r = 0.02\ \boldsymbol{in}$

Location Gear Fatigue Stress Concentration

$K_t := 2.114$ $K_{ts} := 3.0$ $d := 1.0 \cdot in$ $r := .02 \cdot d$

$r = 0.02\ in$

Assume Kf=Kt, Kfs=Kts

$K_f := K_t$

$K_{fs} := K_{ts}$

$k_a := 2.7 \cdot S_{ut}^{-0.265} = 0.909$

$k_b := \left(\frac{1}{0.3}\right)^{-0.107} = 0.879$

$S_e := k_a \cdot k_b \cdot S'_e = 24.326\,\text{kps}$

Mm=Ta = 0, there's no alternating torque

$A := \sqrt[2]{4 \cdot \left(K_f \cdot M_A\right)^2} = 0.138\ kip \cdot in$

Mm=Ta = 0, there's no alternating torque

$$A := \sqrt[2]{4 \cdot \left(K_f \cdot M_A\right)^2} = 0.138 \ \mathbf{kip \cdot in}$$

$$B := \sqrt[2]{3 \ \left(K_{fs} \cdot T\right)^2} = 0.031 \ \mathbf{kip \cdot in}$$

$$d := \left(\left(\frac{\left(8 \cdot n_f \cdot 0.138\right)}{\pi \cdot S_e}\right)\left(1 + \left(1 + \frac{\left(2 \cdot 0.031 \cdot S_e\right)}{0.138 \cdot S_{ut}}\right)^2\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = 0.382$$

DIAMETER @Gear needs to
be 0.5inches due to rounding

$d := .5$ $\quad D := .625$ $\quad r := .1 \cdot d$ $\quad q := .5$ $\quad q_s := .54$ $\quad \frac{D}{d} = 1.25$

$K_{ts} := 1.3$ $\quad K_t := 1.7$ $\quad \frac{r}{D} = 0.08$

$K_f := 1 + q \cdot \left(K_t - 1\right) = 1.35$ $\qquad K_{fs} := 1 + q_s \cdot \left(K_{ts} - 1\right)$

$$A := \sqrt[2]{4 \cdot \left(K_f \cdot M_A\right)^2} = 0.088 \ \mathbf{kip \cdot in}$$

$$B := \sqrt[2]{3 \ \left(K_{fs} \cdot T\right)^2} = 0.012 \ \mathbf{kip \cdot in}$$

$K_f := 1 + q \cdot (K_t - 1) = 1.35$

$K_{fs} := 1 + q_s \cdot (K_{ts} - 1)$

$$A := \sqrt[2]{4 \cdot (K_f \cdot M_A)^2} = 0.088\ \mathbf{kip \cdot in}$$

$$B := \sqrt[2]{3\ (K_{fs} \cdot T)^2} = 0.012\ \mathbf{kip \cdot in}$$

$$n := \left(\left(\frac{8\ .088}{\pi \cdot d^3 \cdot S_e} \right) \left(1 + \left(1 + \left(\frac{2 \cdot .012 \cdot S_e}{.088 \cdot S_{ut}} \right)^2 \right)^{\frac{1}{2}} \right) \right)^{-1} = 6.765$$

n > 2.5, because the shaft was rounded up from .382 inches to .5 inches, which is a substantial increase in the diameter, but was what we needed in order to accomodate the prompt of increasing by standard sizes of 1/8", we also made the radius of the fillets less sharp.

Bearing A: Deep groove ball bearing

$a := 3 \qquad V := 1$

$F_a := F_{fan\_axial} = 45\ lbf$

$F_r := \sqrt{(R_{BAy})^2 + (R_{BAx})^2} = 16.331\ lbf$

$x_d := \frac{L_D}{L_{10}} = 645.84$

10 mm bore

$X_2 := 0.56 \qquad C_0 := 503.57 \cdot lbf$

$\frac{F_a}{C_0} = 0.089$

$Y_2 := 1.53$

$F_{e1} := X_2 \cdot V \cdot F_r + Y_2 \cdot F_a = 77.995\ lbf$

$$C_{10} := n_f \cdot F_{e1} \cdot \left[\frac{x_d}{x_0 + (\theta - x_0) \cdot (1 - R_D)^{\frac{1}{b}}} \right]^{\frac{1}{a}} = [8.846]\ kN$$

Calculated load is greater than bearing rated load so choose larger bearing bore

20 mm bore

$X_2 := 0.56$ $\quad C_0 := 1393.8 \cdot lbf$

$\dfrac{F_a}{C_0} = 0.032$

$Y_2 := 1.95$

$$F_{e1} := X_2 \cdot V \cdot F_r + Y_2 \cdot F_a = 96.895\ \boldsymbol{lbf}$$

$$C_{10} := n_f \cdot F_{e1} \cdot \left[\frac{x_d}{x_0 + (\theta - x_0) \cdot (1 - R_D)^{\frac{1}{b}}} \right]^{\frac{1}{a}} = [10.989]\ \boldsymbol{kN}$$

Calculated load is greater than bearing
rated load so choose larger bearing bore

25 mm bore

$X_2 := 0.56$ $\quad C_0 := 1562.4 \cdot \boldsymbol{lbf}$

$\dfrac{F_a}{C_0} = 0.029$

$Y_2 := 1.98$

$$F_{e1} := X_2 \cdot V \cdot F_r + Y_2 \cdot F_a = 98.245\ \boldsymbol{lbf}$$

$$F_{e1} := X_2 \cdot V \cdot F_r + Y_2 \cdot F_a = 98.245\ lbf$$

$$C_{10} := n_f \cdot F_{e1} \cdot \left[\frac{x_d}{x_0 + (\theta - x_0) \cdot (1 - R_D)^{\frac{1}{b}}} \right]^{\frac{1}{a}} = [11.142]\ kN$$

Bearing rated load is greater than
calculated load so 25mm bore is acceptable

Bearing B: Cylindrical Roller Bearing

$$a_2 := \frac{10}{3}$$

1

Bearing B: Cylindrical Roller Bearing

$$a_2 := \frac{10}{3}$$

$$C_{10\_2} := n_f \cdot F_r \cdot \left[\frac{x_d}{x_0 + (\theta - x_0) \cdot (1 - R_D)^{\frac{1}{b}}} \right]^{\frac{1}{a_2}} = [1.468] \ kN$$

Calculated C10 is less than all rated loads so use smallest bearing choice

25mm Bore

Set Screw to locate the gear and pulley
Size #3
Seating Torque = 5 lbf*in

Key to secure the gear

$$w_{key@gear} := \frac{1}{4} \cdot in \qquad h_{key@gear} := \frac{1}{4} \cdot in \qquad t_{key@gear} := \frac{1}{8} \ in$$

$$D_{shaft@gear} := 1.25 \cdot in$$

$$T := 6.09 \ ft \cdot lbf$$

$$F_{shaft@gear} := \frac{T}{D_{shaft@gear}} = 116.928 \ lbf$$

Key to secure the gear

$w_{key@gear} := \frac{1}{4} \cdot in$ $h_{key@gear} := \frac{1}{4} \cdot in$ $t_{key@gear} := \frac{1}{8}\ in$

$D_{shaft@gear} := 1.25 \cdot in$

$T := 6.09\ ft \cdot lbf$

$$F_{shaft@gear} := \frac{T}{\frac{D_{shaft@gear}}{2}} = 116.928\ lbf$$

$S_{sy} := .577 \cdot S_y = 29.312\ ksi$

$$l_{shear@gear} := \frac{F_{shaft@gear} \cdot n_f}{S_{sy} \cdot t_{key@gear}} = 0.007\ ft$$ Distortion Energy Thm: shear failure

$$l_{crush@gear} := \frac{2 \cdot F_{shaft@gear} \cdot n_f}{S_y \cdot t_{key@gear}} = 0.008\ ft$$ Failure by crushing

Failure by crushing is dominant failure mode. Therefore gear key length must be at least 0.008 ft long.

Key to secure pulley

$w_{key@pulley} := \frac{1}{8} \cdot in$ $h_{key@pulley} := \frac{1}{8} \cdot in$ $t_{key@pulley} := \frac{1}{16}\ in$

$D_{shaft@pulley} := 0.5 \cdot in$

$T := 6.09\ ft \cdot lbf$

$$F_{shaft@pulley} := \frac{T}{\frac{D_{shaft@pulley}}{2}} = 292.32\ lbf$$

$S_{sy} := .577 \cdot S_y = 29.312\ ksi$

$$l_{shear@pulley} := \frac{F_{shaft@pulley} \cdot n_f}{S_{sy} \cdot t_{key@pulley}} = 0.033\ ft$$ Distortion Energy Thm: shear failure

$$l_{crush@pulley} := \frac{2 \cdot F_{shaft@pulley} \cdot n_f}{S_y \cdot t_{key@pulley}} = 0.038\ ft$$ Failure by crushing

Failure by crushing is dominant failure mode. Therefore pulley key length must be at least 0.038 ft long.

[X]. Citations:

Budynas, Richard G., and J. Keith Nisbett. *Shigley's Mechanical Engineering Design*. McGraw-Hill Education, 2020.