Optimal Performance Targets v5.7*

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Abstract

We study a class of contracts that is becoming ever more common among executives, in which the manager earns a discrete bonus if his performance clears an explicit threshold. These performance targets provide the firm with an additional instrument to resolve its moral hazard problem with its manager. The performance target can achieve first-best under risk neutrality, with a target precisely equal to the desired effort that the firm seeks to induce. The optimal bonus increases in risk. If the manager is sufficiently risk averse, the firm will shade the optimal target below equilibrium effort to provide a form of insurance to the manager, outside of the standard reduction in the bonus.

1 Introduction

Ever since the Securities and Exchange Commission required companies to disclose details of their executive pay plans in 2006, we have known that many, if not most, companies use some kind of a performance target, in which the manager earns a bonus if his performance clears an explicit threshold. However, the theoretical literature has either examined the class of linear contracts under normally distributed errors and exponential utility (the so-called LEN model), or has articulated optimal nonlinear contracts in full generality that bear little resemblance to contracts used in practice. What is missing is a theoretical exploration of performance targets, motivated by their actual use, to provide both positive prediction and normative guidance.

Until now. We examine the class of performance target contracts under a variety of settings. To fix ideas, we begin with a risk-neutral agent, and show that performance targets achieve first-best, with the target optimally set to efficient effort in an optimal contract. This result bears similarity to the efficiency of rank-order tournaments. And for good reason, since a performance target is like a tournament, except that the target is not a strategic choice by a separate agent, but rather an optimal choice of the firm. The target provides an extra contract parameter, so that the firm can keep one of the other compensation parameters (the bonus) fixed. Thus, the target offers the firm an additional instrument to resolve the manager's effort problem, a theme that will permeate the analysis. Contrast this with the linear contract, in which salary and bonus both change when the environment changes.

We then examine risk. The canonical model predicts that increases in risk will result in smaller bonuses, as the bonus loads risk onto the manager. However, the empirical evidence on the risk-incentives trade-off has been mixed (Prendergast (2002)). Under a performance target, the optimal bonus increases in variation in the manager's performance measure when he is risk neutral. When output variance increases, this dampens incentives to work, as it is less likely that output from a given unit of effort will clear the target (because of the increased noise in the system). To compensate for this, the firm increases the bonus in order to extract effort from the manager. That the bonus increases in noise may help explain the mixed empirical tests of the risk incentives trade-off. These tests largely regress pay-performance sensitivity (PPS) on stock return volatility, and measure PPS through changes in total direct compensation. Here, an increase in risk directly increases the bonus, which will increase direct compensation and therefore PPS, providing a partial explanation for why empirical tests of the risk incentives trade-off have been mixed.

Next, we solve the model under general risk aversion. While it would be efficient to pay the manager a flat wage in order to provide full insurance, this would ruin incentives to work. We show that the firm will optimally select a target below the second-best equilibrium effort level. Just as the second-best program involves a smaller bonus to reduce the manager's exposure to risk, so does the lower target provide this insurance effect to the manager. Once again, the target serves as a substitute instrument for the bonus, as they alternatively resolve the moral hazard problem with the manager.

There will always be two solutions that induce the same effort, given by a low target and a high target. However, even though both targets implement the same effort, the firm is not indifferent. The low target is easier to achieve, and therefore the manager is more likely to receive his bonus, so his expected bonus compensation is higher. Because of this, he will accept a smaller salary to participate. Because of risk aversion, the firm can lower the bonus also to match incentives at the high target. As such, the firm prefers the low-target contract because it can induce identical effort at lower cost.

We focus attention on finding the optimal contract within the class of performance target contracts; We do not solve for the optimal contract under all possible contracts to show that performance targets are globally optimal. Our paper is in the spirit of the LEN literature, which seeks to discover optimal linear contracts within the smaller class of LEN contracts. This focus on a restricted subset of the full contract space has provided much of our core base of knowledge on various incentive schemes and their optimal attributes, such as the risk-incentives trade-off. We depart from the LEN model in our focus on targets (not linear), general forms of risk aversion (not only exponential) and general distributions that are symmetric and single-peaked (not only normal). We write in the spirit of Ross (2004), who urges research to consider properties of contracts that are used in practice, rather than focusing attention exclusively on fully general contracts that are mathematically complex but lack realism.

There's a small empirical literature on performance targets and an even smaller theoretical one. Murphy (2000) is an early empirical analysis of performance targets that finds that internally determined performance standards are more likely to have discontinuous features that lead to income smoothing. Murphy (2000) considers compensation in the form of $s + b(X - \bar{X})$, where X is the manager's performance measure, and $\bar{X}$ is the standard that the manager faces. While this does capture the flavor of a performance that must exceed a standard, it is nonetheless a linear contract in $X - \bar{X}$. Indeed, much of the prior literature assumes targets of this form, and does not model the discontinuous nature of the target explicitly. This paper aims to use the description of performance targets and standards from Murphy (2000), but to model the manager's optimization problem more explicitly.

Murphy (2000) further documents the presence of an "incentive zone" in which the manager's pay is linear within the incentive zone and flat outside of it. We do not consider the optimal incentive zone, as the model itself has enough complexity as is, without examining a linear region in between two different targets. Matějka and Ray (2014) examine the incentive zone in a model of multiple performance targets and differential incentive weights. Gutiérrez Arnaiz and Salas-Fumás (2008) show that the incentive zone collapses to a "dichotomous bonus" (the kind we consider here) when the performance horizon collapses, say from an annual basis to a quarterly basis.

Gutiérrez Arnaiz and Salas-Fumás (2008) solve for the optimal contract in a specific setting.[1] They do not answer the more general question of optimal targets under any symmetric distribution. Their contract is curvilinear in the incentive zone, as it is a function of the likelihood ratio, a standard feature of optimal contracts under risk aversion. This function is convex and then becomes concave after it hits an inflection point, which the authors argue is effectively the performance target. However, because they solve their model in a general continuous framework, they do not have a precise characterization of the optimal bonus and target.

Other theoretical work on targets examines stage financing in venture capital (Dahiya and Ray (2012)) and performance evaluation over multiple periods (Ray (2007)). Indjejikian et al. (2014a) and Gerakos and Kovrijnykh (2013) both consider earnings targets, and the latter paper indeed contains a formal model. However, none of these papers solve for the optimal target. There is of course a large literature on the ratchet effect (Weitzman (1980); Indjejikian et al. (2014b); Aranda et al. (2014); Arnold and Artz (2015); and Bouwens and Kroos (2011)), which primarily concerns dynamic changes in targets over time. These papers often ask whether the ratchet effect exists at all, and generally take the first period target as given, rather than solving for it optimally.

[1] They assume a Symmetric Variance Gamma (SVG) distribution, and the agent makes a one-shot change to the mean of a stochastic process. Madan and Seneta (1987) and Carr and Geman (2002) show that the SVG process fits data from share prices, but as of yet, there is no evidence that SVG fits data from accounting numbers, on which most performance targets are based.

The existing empirical literature on performance targets largely focuses on testing the ratchet effect, i.e., whether increases in targets are positively associated with prior-year performance relative to the target. The evidence of this is mixed, as some papers find a positive association (e.g., Leone and Rock (2002); Holthausen et al. (1995); Murphy (2000); Anderson et al. (2010); Bouwens and Kroos (2011); Kim and Yang (2014)), while others do not (e.g., Indjejikian and Nanda (2002); Indjejikian and Matějka (2006); Choi et al. (2012)). Some of the more recent papers on performance targets that take advantage of the executive compensation disclosure requirements of 2006 could be ripe environments to test some of the predictions of this paper. In particular, after 2006, the Securities Exchange Commission, the U.S. equities regulator, required any U.S. company that trades on U.S. exchanges to disclose details of their compensation plans, such as performance targets, in setting executive pay. Because such performance targets rely on observable performance measures like earnings or stock price, it would be possible to calculate the variance of those performance measures to test Proposition 2. Specifically, a stock price target lends itself to calculating volatility over some fixed horizon, such as 30-, 60-, or 90-day lag, which could be an empirical proxy for variance in the performance measure. For earnings targets, a reasonable empirical proxy for variance would be the volatility of the quarterly EPS targets over an annual or multiyear horizon. Thus, executive performance targets based on stock prices are particularly well suited for testing the empirical predictions of this paper, and more and more research today utilizes this post-2006 disclosure, so such a test would naturally fit into that emerging literature.

All these papers provide a modicum of evidence supporting the ratchet effect, which is fundamentally a dynamic incentive story; the model and application here is to a static contract. Perhaps the greatest contribution of our theory is simply establishing that the optimal target will lie beneath the equilibrium effort that the principal seeks to induce. This is a statement fundamentally about target difficulty. There is an emerging literature on target difficulty, which offers conditions on how the target difficulty changes with respect to the exogenous parameters. Ultimately, our contribution is to prove that firms will set achievable targets in equilibrium, which provides some of the foundational assumptions for the entire empirical literature on targeting. Our objective is to provide a theoretical justification for much of the empirical work in this area.

Of course, it is somewhat odd that the existing theoretical and empirical literature on performance targets remains preoccupied with target ratcheting, an inherently dynamic concept, when there is an absence of theory (or empirics) on the more elemental problem of how to set static performance targets. In that sense, the cart has come before the horse. This is unusual, since historically, agency theory first solved the static single-agent problem before it moved on to solve multi-agent and/or dynamic incentive problems. This paper seeks an answer to that more basic question of how the firm sets a performance target in a static setting. The main theoretical prediction is on the relation between the optimal target and the equilibrium effort chosen by the manager. Even though managerial effort is unobservable, one proxy for equilibrium effort may be prior performance. In that case, this paper is more consistent with the contrarian literature that shows that managers with high performance are more likely to clear future targets (e.g. Indjejikian and Nanda (2002); Indjejikian and Matějka (2006); Choi et al. (2012)).

The paper proceeds as follows. Section 2 considers the base model under risk neutrality and discusses the risk-incentives trade-off. Section 3 introduces managerial risk aversion. Section 4 compares performance targets to linear contracts. Section 5 concludes.

2 The Model

To motivate the model, consider some sample executive pay contracts curated from proxy statements of corporate filings. In 2010, McDonald's set a target for operating income at $7.24 billion. If the CEO hit this target, his payout was $2,160,000. This target was discrete, in that it offered a fixed cash payment if the performance cleared the target and nothing otherwise. Other companies have imposed similar discrete targets. For example, Bank of America in 2011 set a 3-year average ROA target, awarding 33% of the total bonus if the executive's actual ROA exceeded 50 bps, and nothing otherwise. Barnes & Noble enforced an adjusted EBIT target of -$178.27 million, a low bar given the digital business was expected to have significant cash flow requirements in Fiscal 2014. Chevron set a target based on invested capital with no performance shares awarded if ROIC fell below 18%, 8% awarded if it exceeded 18%, 40% awarded if it exceeded 20%, and 80% awarded if it exceeded 22% or higher. Roughly 38% of the CEO's compensation was paid in performance shares, delivered in cash.

These are all examples of sample executive contracts that contain some kind of discrete performance target, in which performance must clear an explicit threshold. Often, such targets operate at the low end of performance, in which the executive must obtain a minimal level of performance in order to receive any kind of payout at all. Sometimes the payout rises linearly with performance, in which case the board interpolates a bonus number for performance in between two discrete targets.[2] Nonetheless, even absent interpolation, many executive pay contracts contain some kind of discrete target. The Incentive Lab database covers the top 750 firms (measured by market capitalization) over 1998 to 2012, which encompasses 4,673 unique CEO IDs. Of the 2,424 CEOs that use absolute metrics, 1,666 (or 69%) have some discrete performance target. Of the 2,427 that use relative metrics, 1,088 or (45%) have some discrete performance target. To keep the analysis focused, we will examine only a single performance target with no interpolation. Of course, multiple performance targets would be a straightforward generalization of the theory developed here.

The performance target contract is really a class of contracts and, like all theoretical models, is a necessary simplification of reality. There is wide variation among executive contracts on the kinds of performance targets they deploy. Some have only single jumps, some have multiple jumps. Some have regions that are flat after the jump, while others grant options as a bonus, which itself increases in value with firm performance. It would be impossible to document all the different variations, since that would result in a multitude of models and defeat the purpose of theoretical analysis. Here, our purpose is simply to isolate the properties of a model that all performance targets share, namely, the base structure of a single target and a single bonus.

2.1 Risk-neutral Benchmark

A risk-neutral principal (the firm) contracts with a risk-neutral and effort-averse manager (the agent). The manager exerts unobservable effort $e \geq 0$ at a cost of effort $C(e) = 0.5ce^2$, so C is strictly increasing and convex. The manager's performance measure is given by

$$q = e + \varepsilon, \tag{1}$$

where ε conditional on each effort follows a continuous distribution G with mean 0 and

[2] In this case, the company simply takes the weighted average of performance and the weighted average of the payout that matches the weight on the performance. For example, suppose the contract offers the executive b_1 if performance clears t_1 and b_2 if performance clears t_2. If actual performance is $\lambda t_1 + (1-\lambda)t_2$, then the interpolated payout is $\lambda b_1 + (1-\lambda)b_2$.

variance σ^2 that has unimodal and symmetric density function g.[3] [4] Call e^* the first-best effort, given as the solution to $C'(e^*) = 1$, so $e^* = 1/c$. Observe that total surplus is $e^* - C(e^*) = \frac{1}{2c} > 0$. The firm offers the manager a contract (t, s, b), where t is the performance target, s is the salary, and b is the bonus, contingent on performance. The manager earns the bonus if performance exceeds the target:

$$Pay = \begin{cases} s & \text{if } q < t, \\ s + b & \text{if } q \geq t. \end{cases} \tag{2}$$

This fits the simplest description of a performance target, where performance must exceed a threshold before the manager earns a payment. There is a discontinuity in the manager's payoff, jumping from s to $s + b$, when performance exceeds the target.[5]

The probability that the manager receives his bonus is

$$P \equiv Prob(q \geq t) = Prob(\varepsilon \geq t - e) = G(e - t), \tag{3}$$

since by symmetry of g, $G(x) = 1 - G(-x)$. Observe that the probability increases in effort and decreases in the target:

$$\frac{\partial P}{\partial e} = g(e - t) = -\frac{\partial P}{\partial t}. \tag{4}$$

Higher targets directly reduce the manager's probability of achieving his bonus. Target and effort work in exactly opposite directions on this probability. The expected utility of the manager is

$$EU = s + bG(e - t) - C(e). \tag{5}$$

[3]Two additional conditions are placed on the distribution ϵ: $f_e(q, e)/f(q, e)$ is nondecreasing in q (Monotone likelihood ratio condition) (As f is continuous, $f(q, e)$ is uniformly bounded from zero and so $f_e(q, e)/f(q, e)$ is also bounded.) and $\int_{-\infty}^{q} f(x, e)dx$ is concave in e (Convexity of the distribution function hypothesis), following Rogerson (1985).

[4]The symmetry of the error distribution is not necessary, but does dramatically ease calculation. The most common distributions, such as the normal, are symmetric.

[5]Recall from real analysis that step functions can arbitrarily approximate continuous functions, as the number of steps tends to infinity (Lebesgue's Monotone Convergence Theorem). Thus, if we expand the class of contracts to include multiple performance targets with multiple bonuses, then in the limit, these contracts will arbitrarily approximate the optimal (nonlinear) contract for the same reason.

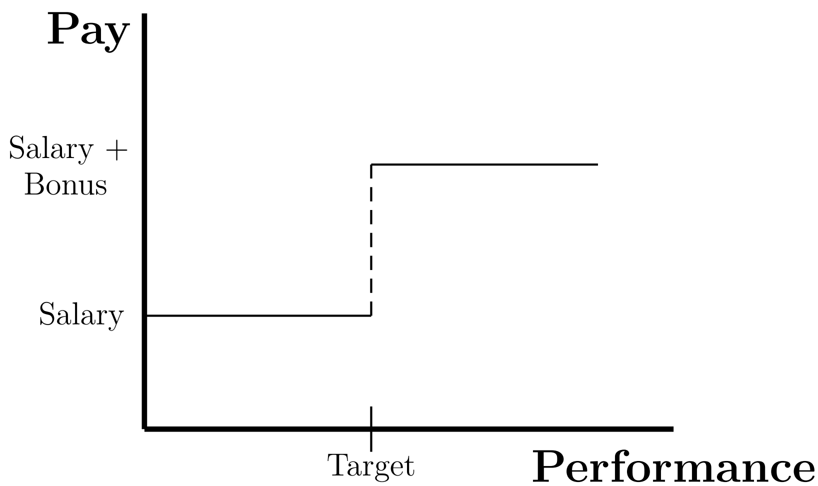


Figure 1: Discontinuity in Manager's Pay

The manager can select effort at cost $C(e)$ to maximize his expected payoff. The solution to this problem generates the incentive constraint for the manager:

$$bg(e - t) = C'(e). \tag{IC}$$

The manager equates the marginal cost of effort to the expected marginal benefit, which is the change in the probability of achieving the bonus, times the unconditional bonus itself. This marginal effect on the change in probability is represented by the term g, and will permeate the analysis. While higher targets unilaterally decrease the probability of clearing the target, the effect on the change in probability is ultimately what matters. Indeed, the firm picks a contract that induces the manager's effort, and the difference between the target and effort will ultimately drive the manager's incentives. As is common, assume the manager faces an outside opportunity $\bar{u}$ in order to induce participation. The manager's expected payoff must exceed this opportunity, and therefore impose the standard participation constraint (PC) that $EU \geq \bar{u}$.

The contract has three instruments—the salary, the bonus, and the target—to control a unidimensional effort-choice subject to two constraints: the participation constraint and the incentive constraint. As such, the target provides an extra degree of freedom and this will, in general, lead to a continuum of equilibria. Specifically, observe that from the incentive constraint (IC), any pair of bonuses and targets that satisfy IC will induce the same effort level. To reduce the number of equilibria, we make the additional assumption that the firm will select the smallest bonus when multiple equilibria exist. This can be justified on the grounds that incentive compensation at firms can be controversial, as it may lead to political outrage. For example, recall the backlash that banks faced after the U.S. financial crisis for honoring the bonus payments written into their executive contracts when the banks received government financial assistance. While we do not model political outrage explicitly, we simply represent this as the additional constraint by which the firm will choose the lowest bonus among a set of equilibria that generate the same profits.

The firm maximizes expected profits, subject to the incentive and participation constraints. The solution to this problem generates the optimal efficient contract. Before we solve the optimal contract, we impose following assumption on the distribution G and the cost parameter c.

Assumption 1 *Assume that the distribution G and the cost parameter c satisfy*

$$\max_{\epsilon \in [-1/c, 0]} g'(\epsilon) < cg(0).$$

This assumption is imposed to guarantee the globally satisfied second order condition. The condition $\max_{\epsilon \in [-1/c, 0]} g'(\epsilon) < cg(0)$ holds under many single-peaked distributions, such as the normal distribution and the logistic distribution, when the cost function is sufficiently convex. Intuitively, when the second derivative of the cost function c increases, the left-hand side $\max\limits_{\epsilon \in [-1/c, 0]} g'(\epsilon)$ is weakly decreasing and the right-hand side $cg(0)$ is strictly increasing. As long as $g'(\cdot)$ is bounded above, we can always find some c such that the condition above is satisfied. The following lemma gives a sufficient condition for the normal distribution.

Lemma 1 *If $c > \frac{e^{-1/2}}{\sigma}$, then $\max_{\epsilon \in [-1/c, 0]} g'(\epsilon) < cg(0)$ under the normal distribution.*

The details are provided in the appendix. We next state our first proposition given that Assumption 1 holds.

Proposition 1 *The optimal contract that implements first-best effort e^* is (t^*, s^*, b^*)*

where

$$t^* = e^*, \tag{6}$$

$$b^* = \frac{1}{g(0)}, \tag{7}$$

$$s^* = \bar{u} + C(e^*) - \frac{1}{2g(0)}. \tag{8}$$

The proposition provides the optimal contract, which in this case is efficient. This should come as no surprise, as the manager is risk neutral, and can fully "internalize" the value of his effort under the optimal contract. Compare this to the usual efficient linear contract that makes the agent the full residual claimant on firm output, where the firm extracts rents from the manager through a (possibly) negative salary. Here, the efficient contract is nothing like the "sell the firm" contract. The firm will set the target to the efficient effort level, and then select the salary and bonus to solve the participation and incentive constraints, respectively. Proposition 1 formally proves that the firm can select the target equal to efficient effort under risk neutrality. This conforms to the common intuition that the target can equal the effort that the principal seeks to induce, which in this case is first-best effort.[6]

The performance target offers a discrete jump in payoff if performance clears the target. Consider this a "prize" of b, the difference in payoff from clearing the target versus not. In equilibrium, $t^* = e^*$, so (IC) in equilibrium becomes

$$b^* = \frac{1}{g(0)}. \tag{9}$$

The relationship between target and effort is nontrivial, since a shift in the target t will immediately shift equilibrium effort $e(s, b, t)$. However, the proof of Proposition 1 shows that because the manager's participation constraint will bind, firm profits equal total surplus, and therefore the firm can afford to achieve efficiency. Given that the firm seeks to implement e^*, the optimal target will pin down equilibrium efficient effort exactly. This occurs precisely when the returns to managerial effort are highest, the point when a marginal increase in effort leads to the greatest <u>change</u> in probability.

[6]Gutiérrez Arnaiz and Salas-Fumás (2008) offer a numerical example in which the performance standard is set equal to the mode of the performance distribution. But without formally solving for the optimal contract performance target, it is impossible to say for sure whether the target lies above or below equilibrium effort.

This is exactly when the density g hits its maximum at $t^* - e^* = 0$.[7] Moreover, the optimal salary compensates the manager for his outside opportunity and cost of effort, but then deducts half of his bonus from his salary upfront. Indeed, this is necessary in order to provide the manager with strong effort incentives. Comparative statics on the proposition immediately generate the following corollaries. First consider the effect from the changes in the outside options.

Corollary 1 *The optimal bonus is unchanged in the manager's outside options* $\left(\frac{\partial b^*}{\partial \bar{u}} = 0\right)$*, while the optimal salary increases in the manager's outside options* $\left(\frac{\partial s^*}{\partial \bar{u}} > 0\right)$*.*

The participation constraint ensures that the manager meets his outside options. As with the canonical model, increasing outside opportunities forces the firm to increase the salary in order to retain the manager. Now consider changes in the cost of effort, which tracks the quality or productivity of the manager.

Corollary 2 *As the manager's marginal cost of effort increases, the optimal target decreases, the optimal salary decreases, and the optimal bonus is unchanged.*

Because our optimal target is $t^* = e^* = \frac{1}{c}$, it is immediate that the firm will decrease the target as the manager's cost of effort rises. In fact, this is the only comparative static in which the target changes. As effort becomes costly, first-best effort becomes lower to save on the managers' disutility, and therefore social welfare. The corollary shows that when marginal cost of effort increases, firm decreases the target, which decreases effort. These two countervailing effects, higher marginal cost of effort and lower level of effort, lead to lower total cost of effort at optimum. As a result, the firm does not have to pay as much to the manager to compensate for the cost of effort so it will simultaneously decrease salary. The countervailing effect comes from the fall in the target. Indeed, the target is a powerful instrument and has a direct effect on effort. The higher marginal cost of effort counteracts the salary-reducing effect from lower effort. However, the salary still fall in equilibrium.

There is a close theoretical analogy between rank-order tournaments (Lazear and Rosen (1981)) and performance targets. Both rely on a relative comparison of output in order to secure an external prize. In performance targets, that comparison is against

[7] The unimodal condition and symmetry imply that the mean of the distribution is its maximal point, which is why the firm will set the difference between target and effort to be equal to this mean of 0.

an standard set by the firm, whereas in tournaments that comparison is made against the output of another strategic player in the game. In both models, an increase in risk dampens incentives to provide effort, and both models can implement first-best under risk neutrality. Here, the bonus reduces to $g(0)^{-1}$, which in the case of a normal distribution is simply $\sigma\sqrt{2\pi}$, so the bonus increases in risk unambiguously. This feature of how the bonus reacts to a change of risk is quite general, as we show next.

2.2 Increases in Risk

There's hardly a more celebrated result in agency theory than the risk-incentives trade-off. The standard LEN model of linear contracts, exponential utility, and normal errors deviates from efficiency because of the risk premium that the firm must pay the manager to bear risk, through mean-variance preferences that include a disutility for risk. This workhorse model of contract theory, nicely summarized in Prendergast (1999), posits that as risk (measured through the variance of the error distribution) increases, optimal incentives should decrease, since the optimal bonus from that model is $(1 + rc\sigma^2)^{-1}$. Because of this, the firm reduces the optimal bonus away from that which would guarantee efficiency.

The existing literature on the risk-incentives trade-off has been mixed. Some papers find a positive relationship (Core and Guay (1999), Oyer and Schaefer (2001), Core and Guay (2001), Nam et al. (2003), and Coles et al. (2006)), some find a negative relationship (Lambert and Larcker (1987), Aggarwal and Samwick (1999), and Jin (2002)), and some find no relationship at all (Garen (1994), Yermack (1995), Bushman et al. (1996), Ittner et al. (1997), and Conyon and Murphy (2000)). Most of these papers measure risk as volatility of stock returns and measure incentives as pay-performance sensitivity, measured as changes in direct compensation for a given change in performance. The existing literature has not made a conclusive statement on whether incentives optimally increase or decrease with risk. This calls into question whether the theory is even valid, if it holds under such special circumstances. Indeed, a raft of papers have offered conditions under which the trade-off reverses, giving a positive relationship between risk and incentives (e.g., Dutta (2008) and Prendergast (2002)).

Here, the bonus is a reward to the manager for clearing the target, and as the incentive constraint shows, it will equilibrate the marginal cost of effort against the change in probability of clearing the target $g(e - t)$, times the unconditional "prize" of

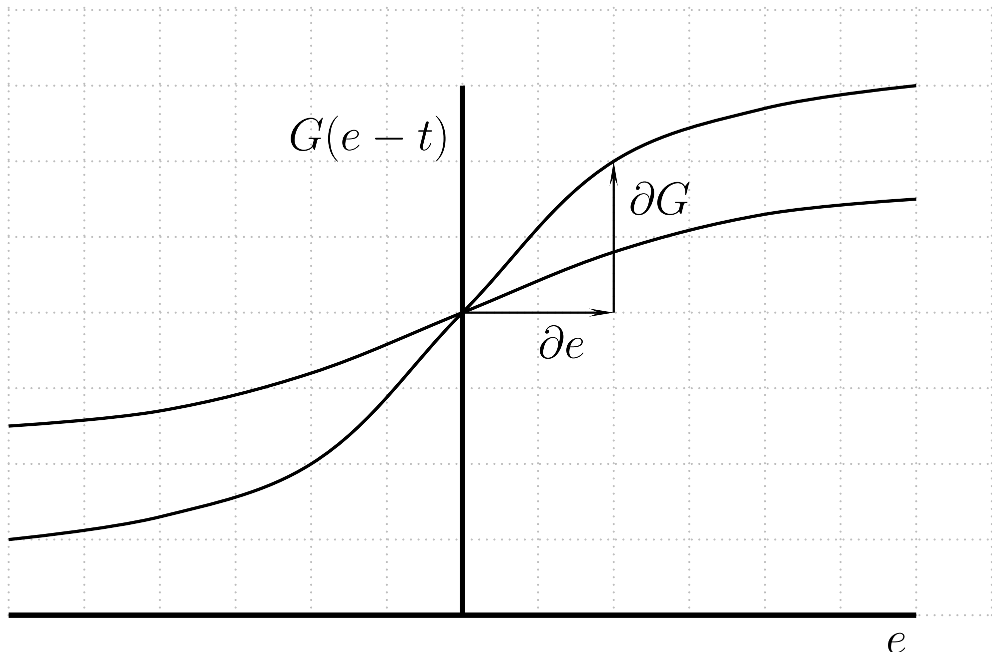


Figure 2: An increase in risk. The steeper CDF second-order stochastically dominates the flatter CDF.

b. Unlike the canonical model, there is no disutility for risk that holds over the entire domain of the manager's utility function. Rather, only incentives at the target matter. The assumption of risk neutrality here is to focus on a competing effect, namely the effect of noise on the probability of clearing the target.[8] This effect will still permeate a model of risk aversion, though it may be muted because of the need for insurance. Of course, linear contracts allow no positive relationship for risk and incentives under any conditions.

Proposition 2 *Let g_i be a mean-zero, unimodal distribution for $i = 1, 2$. Let b_i^* be the optimal bonus from Proposition 1. If G_1 second-order stochastically dominates G_2, then $b_1^* < b_2^*$.*

Said differently, as risk increases, this dampens the agent's incentives to produce effort. To compensate for this reduction in incentives, the firm must increase the size of

[8]Prendergast (2002) also assumes risk-neutral agents in order to avoid the standard trade-off. It is possible that the trade-off may emerge under risk aversion, though the model does not permit closed-form solutions of this. Instead, numerical simulations do show that the risk incentives trade-off vanishes under risk aversion.

the prize, for the same logic as occurs in tournaments. Thus the optimal bonus exactly balances the increase in variance. This fits exactly Proposition 2(b) of Gutiérrez Arnaiz and Salas-Fumás (2008), who find the same unambiguous result that the bonus size increases in volatility. Even though Gutiérrez Arnaiz and Salas-Fumás (2008) use a more specific model (SVG process), they also find the same reversal of the risk-incentives trade-off.

The term $g(0)^{-1}$ is a proxy for the variance: As the variance on output rises, the tails of the density g will increase and its maximal point $g(0)$ will sink. Recall that G represents the probability of clearing the target, and g is the change in this probability. So, under a higher variance, a marginal change in effort will lead to a smaller change in probability. It is the excess noise that forces the manager to reduce effort. Figure 2 shows two distribution functions, one that is second-order stochastically dominant over the other. Remember that a marginal increase in effort changes the probability of achieving the target, and so it is the change in probability (the slope of the distribution function) that matters. In the low variance case, the slope of the distribution is steeper around the mean of 0, so a marginal increase in effort leads to a higher probability of hitting the target than under a high variance distribution. Proposition 2 proves this rigorously under two distributions.

There is a natural question of whether the bonus in this model can compare to the bonus in the linear model. Recall that the bonus in the linear model is the slope of the contract, and therefore directly maps into a conceptual definition of pay for performance. Because of the discrete nature of the target, there is no natural analogue to this slope, which is the marginal change in pay for a marginal unit of effort. Nonetheless, the matter is largely immaterial, because empirical estimates of PPS will almost always increase in the discrete bonus of this model.

For example, Aggarwal and Samwick (1999), Jin (2002), and Guay (1999) seek to estimate the risk-incentives trade-off by regressing pay for performance sensitivity (PPS) on risk, usually measured through the volatility of stock returns. Jin (2002) defines PPS as changes in total direct compensation, as well as changes in the re-evaluation of stock and stock options. An increase in the bonus of Figure 1 will increase total direct compensation, and therefore will have an upward effect on the empirical measure of PPS. This will confound the risk-incentives trade-off.

3 Risk Aversion

Now consider that the manager is risk averse and has a utility function u that is strictly increasing, concave, and bounded below.[9] The firm still writes a contract (s, t, b) as before, with a similar bonus and target structure:

$$Utility = \begin{cases} u(s) & \text{if } q < t, \\ u(s+b) & \text{if } q \geq t. \end{cases} \tag{10}$$

The discontinuity in the manager's payoff now jumps from $u(s)$ to $u(s+b)$ when output exceeds the target. The expected utility of the manager is

$$EU = \int_{-\infty}^{t-e} u(s)g(\varepsilon)d\varepsilon + \int_{t-e}^{\infty} u(s+b)g(\varepsilon)d\varepsilon - C(e). \tag{11}$$

The integral splits at $t-e$ because that is exactly the point for ε such that the manager earns the bonus ($q \geq t$, or $\varepsilon \geq t-e$). Impose the standard participation constraint (PC) that $EU \geq \bar{u}$. Observe that because the payoff to the manager takes only two discrete values, $u(s)$ and $u(s+b)$, the firm can completely control the manager's behavior through the choice of these two payoff levels. As such, the compensation terms pass out of the integral and we can re-write EU as:

$$EU = u(s)G(t-e) + u(s+b)G(e-t) - C(e). \tag{12}$$

Thus, the expectation makes the discrete payoff structure continuous, and so the manager can select effort to maximize his expected payoff. The solution to this problem, given by the first-order condition, generates the incentive constraint for the manager:

$$g(e-t)\Big(u(s+b) - u(s)\Big) = C'(e). \tag{IC}$$

As before, the benefit of effort includes its effect on changing the probability of clearing the target, expressed in the term $g(e-t)$. Observe that under risk neutrality,

[9]To preclude Mirrlees' "unpleasant theorem" when the agent is risk-averse, we assume that the utility function is bounded below, particularly not arbitrarily close to negative infinity when the wealth approaches zero. This rules out some utility specification, such as $\log(W)$. Moroni and Swinkels (2014) show that existence is assured when there is a finite lower bound on utility. Hence the assumption necessary to get results in Mirrlees (1999) does not hold and the principal cannot approximate the first-best outcome by introducing arbitrarily severe punishments.

the utility spread collapses to the bonus as a special case. The incentive constraint now contains the term $u(s+b) - u(s)$, which we call the utility spread. This is the gain in utility from achieving the bonus. Since utility is increasing, the spread rises in the bonus ($u'(s+b) > 0$).

The concavity of the utility function prevents first-best contract being the optimal choice of the firm, which is shown below to lead to an effort distortion. To see this, imagine that the firm attempted to implement first-best effort using a contract. Plugging this into the incentive constraint generates $g(e-t)\Delta = 1$, where Δ is the utility spread. Rearranging terms gives

$$g(e-t) = \frac{1}{\Delta} > \frac{1}{b^*} = g(0). \tag{13}$$

where the bonus on the right-hand side is set at the first-best level, and $\Delta < b^*$ by risk aversion. But of course, this is impossible since the distribution peaks at zero. So, in fact, (IC) will hold at an effort level distorted away from first-best. This occurs precisely when the utility spread is smaller than the optimal bonus, which must occur since the manager is risk averse and the optimal target lies away from the efficient effort ($t < \hat{e}^*$). Indeed, both the utility spread falls short of the bonus, and the change in probability lies beneath its maximal point. And thus the marginal benefit is less than the first-best marginal cost of one, yielding the effort distortion.

Now let's consider the firm's problem under risk aversion. The firm maximizes expected profits, subject to the incentive and participation constraints. The full program involves expected profits less a multiplier for both constraints:

$$\max_{(s,b,t)} e - \left(s + bG(e-t)\right). \tag{14}$$

subject to

$$u(s) + \Big(u(s+b) - u(s)\Big)G(e-t) - C(e) \geq \bar{u}, \tag{PC}$$

$$g(e-t)\Big(u(s+b) - u(s)\Big) = C'(e). \tag{IC}$$

As in the case of risk neutrality, we assume the firm will choose the smallest bonus among the contracts that generate the same profits, if there are multiple such contracts. This will allow us to focus on a unique contract that emerges from the program above.

Lemmas 2, 3 and Proposition 3 solve for this program, and we discover that the target plays an important role in balancing the risk and incentives problem:

Lemma 2 *For any effort,* (PC) *binds for the optimal contract that induces this effort.*

If the participation constraint does not bind, the firm can save some costs but induce the same effort through lower salary and bonus.

Lemma 3 (IC) *binds for any contract.*

If the incentive constraint does not bind, the manager can always improve his expected payoff by marginally changing the effort level.

Proposition 3 *Under risk aversion, both* (IC) *and* (PC) *bind for the optimal performance target contract. If the agent is sufficiently risk averse, the optimal target lies below equilibrium effort* ($\hat{t} < \hat{e}$)*.*[10]

We measure the level of risk aversion in terms of absolute risk aversion and the details are provided in the Appendix. The primary result is that the firm will shade the target downward to handle the manager's risk aversion. Proposition 3 shows that the target, in addition to the bonus, offers insurance. This removes the insurance burden from the bonus and puts it on the target, as often occurs when the firm has multiple instruments to design optimal compensation. Recall under the benchmark model that the incentive constraint equalizes the marginal cost of effort against its marginal return. In the structure of this model, that is equivalent to a horizontal line passing through the distribution of effort, as shown in Figure 3.

This occurs because of the symmetry of the error distribution. From (IC), the marginal cost of effort must equal the marginal return, which is the marginal change in the probability of clearing the target times the size of the prize, the utility spread. Because g is symmetric, there will always be two targets symmetrically distributed around equilibrium effort that solves (IC). To see this visually, imagine a horizontal line passing through the density g. The coordinates of the x-axes of the intersection

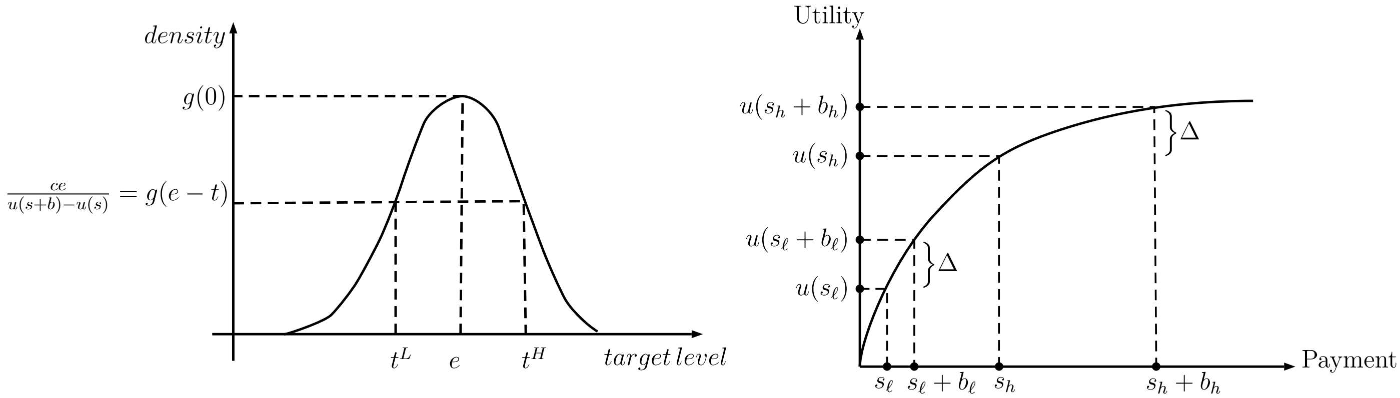
[10] We derived the property that must be satisfied under the optimal contract (assuming that an *optimal performance target* contract exists). When an optimal contract exists, we show that the equilibrium effort must be greater than the target.

points are the optimum targets that satisfy (IC). There will always be two solutions to this problem, as Figure 3 illustrates.

The low and high targets will equivalently induce the same equilibrium effort. Recall that the probability of clearing the target, P, decreases in the level of the target; as such, the manager has a lower chance of receiving the bonus with high targets. Therefore, the manager receives a higher expected bonus from a low target than a high target, so he requires less salary in order to participate. Said differently, the principal must pay a premium to the manager in order to induce participation under a high target. Since both targets generate the same equilibrium effects, the high target has no benefit for output, only a higher cost to induce participation.

This result follows fundamentally from risk aversion. Recall that the utility spread is the difference in utility from receiving the bonus versus receiving the salary alone. For any fixed bonus, this spread falls in the salary level because of diminishing marginal utility (driven by the risk aversion, illustrated in Figure 3b). Therefore, when the firm offers a high target with a low probability of payout, it must offer a corresponding high salary to guarantee participation. That high salary, call it s_H, paired with a given bonus, call it b_H, determines the utility spread and therefore effort incentives. A low target raises the probability of payout, and the firm can afford to pay a lower salary to guarantee participation. Because of diminishing marginal utility for a fixed bonus b_H, the utility spread at the low salary will exceed the utility spread at the high salary, since the utility curve is steeper at the lower salary level. To keep incentives unchanged, the firm can therefore lower the bonus to some $b_L < b_H$, which will match exactly the utility spread and therefore the incentives at the prior contract. To see this visually, observe in Figure 3b that the diminishing marginal utility (risk aversion) forces $b_L < b_H$ in order for incentives (Δ) to be identical at both contracts. Thus, the low target pairs with a low salary and low bonus, and offers the same incentives as the high target with a high salary and high bonus. When the agent is sufficiently risk averse, the firm can induce the same level of effort at lower cost using low target.

As such, in every equilibrium we always have $\hat{t} < \hat{e}$ if the manager is sufficiently risk averse. The optimal performance target contract under risk aversion is inherently complex, as salary, bonus, and target jointly and simultaneously determine effort. It is impossible to change one variable alone without changing others as well. In particular, a change in salary will affect the incentives of a risk-averse manager. Even with this complexity, we show that the participation constraint will always bind. The firm will



(a) Dual Performance Targets.

(b) Optimality of lower target.

Figure 3

always be able to select a target such that it extracts the full rent from the manager.

To see why, observe that the firm has at its disposal a target that can always serve as an extra instrument to modulate the manager's expected utility. If the target is strictly below equilibrium effort, so $\hat{t} < \hat{e}$, then a slack participation constraint leaves rents for the manager. But the firm can always simultaneously lower the salary and raise the target, keeping equilibrium effort constant. Lowering the salary will tighten the participation constraint, since the manager's expected payoff will fall. Raising the target will further tighten the participation constraint, as the manager certainly prefers low targets to high targets. But both these actions will raise effort and, therefore, profits for the firm, and thus allow the firm to implement the same effort at a lower cost (a lower salary and lower expected compensation). The firm will do this until the participation constraint binds.

The first-order condition of the manager's problem may have multiple solutions. In particular, because the distribution is unimodal, there may be multiple solutions. However, the full proof of proposition 3 shows that when the manager is sufficiently risk averse, the firm will implement a particular effort at lower cost by choosing the low-target contract. Furthermore, the manager will select the equilibrium effort even if there are other solutions to the (IC) constraint. This helps to eliminate most of the other effort levels that satisfy the incentive constraint.

Given that, in equilibrium, the optimal effort always exceeds the optimal target $(\hat{e} > \hat{t})$, we can generate following comparative statics on the incentive constraint.

Lemma 4 *The effort chosen by the manager rises in the bonus, falls in the salary, and rises in the target if effort exceeds the target* ($\hat{e} > \hat{t}$)*.*

Corollary 3 follows directly from Lemma 4.

Corollary 3 *Equilibrium effort rises in the bonus, falls in the salary, and rises in the target.*

The first effect from the bonus is the same as the canonical model: higher pay-forperformance sensitivity (PPS) induces the manager to work more. The second result is more surprising. Given the specific formulation of the LEN model, salary has no effect on effort incentives, because of constant absolute risk aversion that fails to capture the wealth effect (Makarov and Schornick, 2010; Peress, 2003). It is then straightforward in the LEN model that the firm is then able to hold the manager to his participation constraint, since it can lower salary without affecting (IC). But in our model, salary affects the utility spread and therefore incentives. This is because the utility of wealth and the disutility of effort enter the utility function separately. The utility function we considered is in line with a large class of preferences assumed in the literature (see Holmstrom et al. (1979)) and does not exclude the wealth effect. Under a general form of risk aversion, risk preferences at any point depend on wealth at that point (Cohn et al., 1975; Friend and Blume, 1975). In our model, specifically, because the manager has concave utility and therefore diminishing marginal utility, for a fixed bonus, a higher salary will cause the utility spread to shrink (since $u'(s+b) < u'(s)$). This will decrease effort incentives. This is consistent with the fact that the LEN model does not incorporate the wealth effect, but the impact on incentives does exist for a general class of utility functions.

3.1 Simulation of the Optimal Contract

Observe that the optimal contract from (14) does not have a closed-form solution, so simulation is necessary. However, the incentive constraint does not have a closed-form solution either, so we cannot first derive the effort, e as a function of (s, b, t). Therefore we use the “fmincon” function in MATLAB to solve the optimization problem. To generate the simulation results, we take the following steps.

1. We assume $C(e) = 0.5ce^2$, $u(x) = \sqrt[3]{x}$, $g(\epsilon) = N(0, \sigma^2)$. Further, we let $c = 0.1$, $\bar{u} = 1$.

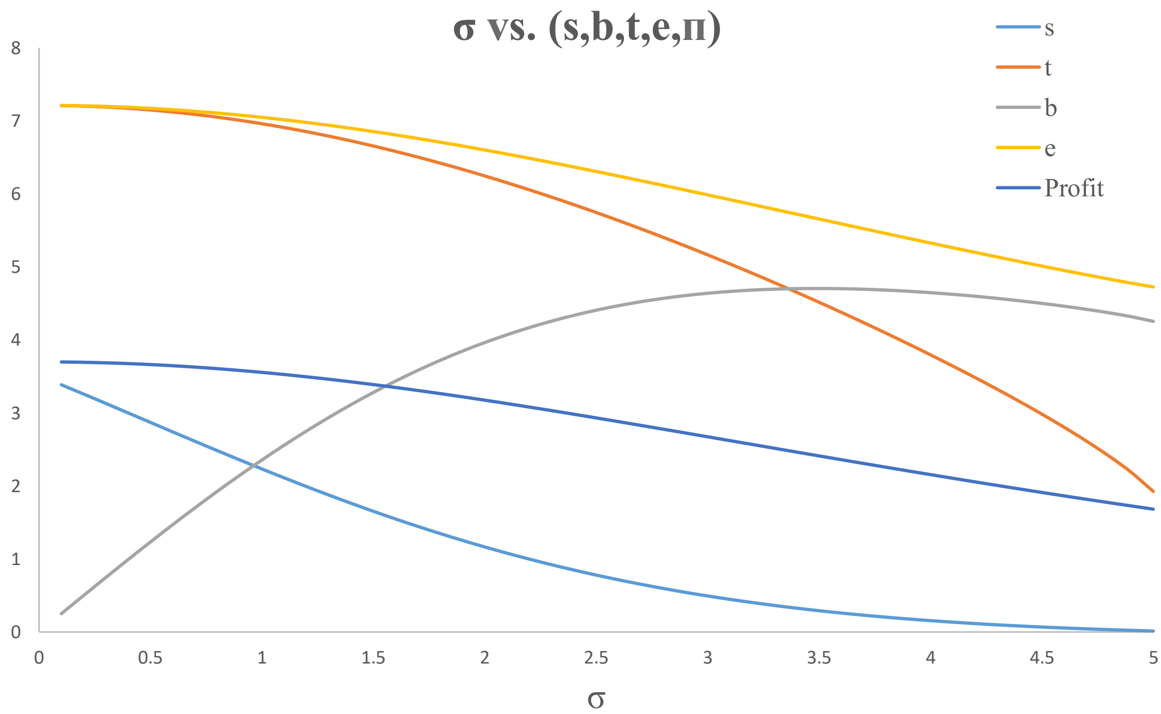


Figure 4: Numerical simulations under cubic utility and mean-zero normal errors. The graph shows the optimal contract as a function of sigma, when the outside option is fixed at zero.

2. We define a function named "nlcon(x, sigma)" in MATLAB to capture the constraints in the optimization problem. Here, x is a vector with four elements, (s, t, b, e). We include both binding (IC) and (PC) constraints in this function. Further, we also require (s, t, b, e) to be non-negative.

3. We multiply profit function (14) by negative one as our objective function.

4. We let σ vary from 0.1 to 5 and calculate optimal x vector (s^*, t^*, b^*, e^*) that minimizes the objective function under specified constraints and we obtain the corresponding maximum profit derived from the minimized objective function.

Figure 4 shows numerical simulation from the example with cubic utility and normal errors. MATLAB code is available on request. In the graph, we show the optimal contract as a function of σ. Notice that the optimal target always lies below equilibrium effort. In addition, bonus increases with risk first, and when risk is large enough, the bonus starts to fall in risk (in this case, approximately when $\sigma^2 > 3.5^2 = 12.25$). Thus, in this case with cubic utility, there is not always a risk-incentives trade-off as predicted in linear contract. Under performance target contract with risk-averse manager, there are two forces that affect the relation between bonus and risk. The first force comes from the noise on the probability of clearing the target. In order to economize on the costs associated with transferring risk to the agent, the firm reduces the target. The second force comes from the risk-incentives trade-off, where the firm reduces the bonus when risk increases, because the bonus loads risk onto the manger. The simulation result suggests that both forces potentially contribute to the relation between bonus and risk. When risk is low, the first force dominates. The risk-incentives trade-off dominates when risk gets too high. Of course, even in this simple example, the implicit equations that define both the manager's and firm's optimal solutions are sufficiently complex that the optimal bonus is non-monotonic in risk.

4 Comparison with Linear Contracts

To put our results in perspective, it may help to consider a simple example that illustrates how performance target contracts can perform better than linear contracts. In this section, we consider a liner contract where the wage paid to the manager is $w(q) = s + bq$ and agent's cost of effort $C(e) = \frac{1}{2}ce^2$.

4.1 Risk-neutral Manager

Suppose that the manager is risk-neutral and has a quadratic cost of effort $C(e) = \frac{1}{2}ce^2$. Output is $q = e + \varepsilon$ as before, with the error term normally distributed, i.e., $\varepsilon \sim N(0, \sigma^2)$. Under this specific model, the manager will pick effort $\hat{e}$ that maximizes expected utility:

$$\begin{aligned} EU &= s + bE(q|e) - \frac{1}{2}ce^2, \\ &= s + be - \frac{1}{2}ce^2. \end{aligned} \tag{15}$$

Solving manager's problem we get $\hat{e} = b/c$. In order to implement the first-best effort $e^* = 1/c$, the principal needs to pay the agent a bonus $\hat{b} = 1$. As is common, assume the manager faces an outside option $\bar{u}$. The manager's expected payoff must be at least

$\bar{u}$ to induce participation, $EU \geq \bar{u}$. We will have salary $\hat{s} = \bar{u} - 1/(2c)$. The cost of implementing e^* is the manager's expected wage of $Ew = \hat{s} + \hat{b}e^* = (\bar{u} - \frac{1}{2c}) + \frac{1}{c} = \bar{u} + \frac{1}{2c}$. Indeed, when agent is risk neutral, the principal will "sell the firm" to the manager and get the full surplus. Meanwhile, the agent will get "dollar-for-dollar" payoff for his own effort.

From section 2.1, we know that the first best can be implemented by a performance target contract with salary $s = \bar{u} + C(e^*) - 1/(2g(0))$, bonus $b = b^* = 1/g(0)$, and target $t = e^* = 1/c$. Under this contract, the manager's wage function is

$$w(q) = \begin{cases} s^* + b^* & \text{if } q \geq e^*, \\ s^* & \text{otherwise.} \end{cases} \tag{16}$$

The firm will get the (maximized) total surplus. The probability of success of reaching the target is $P = Prob(q \geq e^*) = 1 - G(e^* - e^*) = 1 - G(0) = \frac{1}{2}$.

The cost of implementing e^* is the same as the linear contract: $Ew = P(s^* + b^*) + (1 - P)(s^*) = s^* + \frac{b^*}{2} = \bar{u} + C(e^*) = \bar{u} + \frac{1}{2c}$. Therefore, when the agent is risk neutral, both linear contract and performance target contract can achieve first-best effort and the principal gets the total surplus under both contracts.

4.2 Risk-averse Manager with Mean-variance Preferences

Now consider a risk-averse manager with mean-variance preferences. First consider a linear contract, where wage $w = s + bq$. Manager's expected utility then becomes

$$EU_L = E(w) - \frac{r}{2}V(w) - C(e) = s + be - \frac{r}{2}b^2\sigma^2 - C(e). \tag{17}$$

leading to $e = b/c$ for *(IC)*. The subscript L denotes "linear." The firm's problem allows substituting in *(IC)* and *(PC)*, so expected profits are

$$\Pi_L(b) = e - \frac{r}{2}b^2\sigma^2 - C(e) - \bar{u}. \tag{18}$$

Maximizing profit over b gives $b_L = (1 + rc\sigma^2)^{-1}$. So equilibrium profit is now

$$\Pi_L = \frac{b_L}{c} - \frac{{b_L}^2}{2}\left(r\sigma^2 + \frac{1}{c}\right) - \bar{u}, \tag{19}$$

$$= \frac{1}{c + rc^2\sigma^2} - \frac{1}{2(1 + rc\sigma^2)^2}\left(r\sigma^2 + \frac{1}{c}\right) - \bar{u}. \tag{20}$$

Under the performance target contract, the manager's wage function is given by:

$$w = \begin{cases} s+b & \text{if} \quad q \geq t, \\ s & \text{if} \quad q < t. \end{cases} \tag{21}$$

The probability of clearing the target is $P \equiv G(e-t)$. Observe that:

$$w^2 = \begin{cases} (s+b)^2 & \text{with probability} \quad P, \\ s^2 & \text{with probability} \quad 1-P. \end{cases} \tag{22}$$

Therefore, we can calculate:

$$E(w) = s + bP, \tag{23}$$

$$E(w^2) = (s+b)^2 P + (s^2)(1-P), \tag{24}$$

$$= s^2 + P(b^2 + 2sb). \tag{25}$$

$$\Rightarrow V(w) = E(w^2) - (Ew)^2 = Pb^2(1-P). \tag{26}$$

The variance of the manager's wage is a more complex function of the probability of clearing the target. If this probability is extreme (either 0 or 1) then the variance is 0, and the manager will simply maximize expected wage, as in the linear model with risk neutrality. This should be intuitive because, if the manager is certain to succeed or fail, there is no uncertainty in his wage contract.

Since the wage depends on whether the manager clears the target (which itself depends on the realization of ε), the wage is therefore a random variable. The manager's payoff is a function of the moments of the wage function. Represent the manager's mean-variance preferences as:

$$EU = E(w) - \frac{r}{2}V(w) - C(e).$$

where r is a constant absolute risk-aversion parameter. Inserting $E(w)$ and $V(w)$ into EU:

$$EU = (s + bP) - \frac{r}{2}Pb^2(1-P) - C(e).$$

Maximizing expected utility with respect to effort and rearranging terms gives the incentive constraint:

$$\left(b - \frac{r}{2}b^2(1-2P)\right) g(e-t) = ce. \tag{IC}$$

Because (PC) binds by Lemma 2, the firm's expected profit given effort e is

$$e - (s + bP) = e - \left(\bar{u} + \frac{r}{2}Pb^2(1-P) + C(e)\right). \tag{27}$$

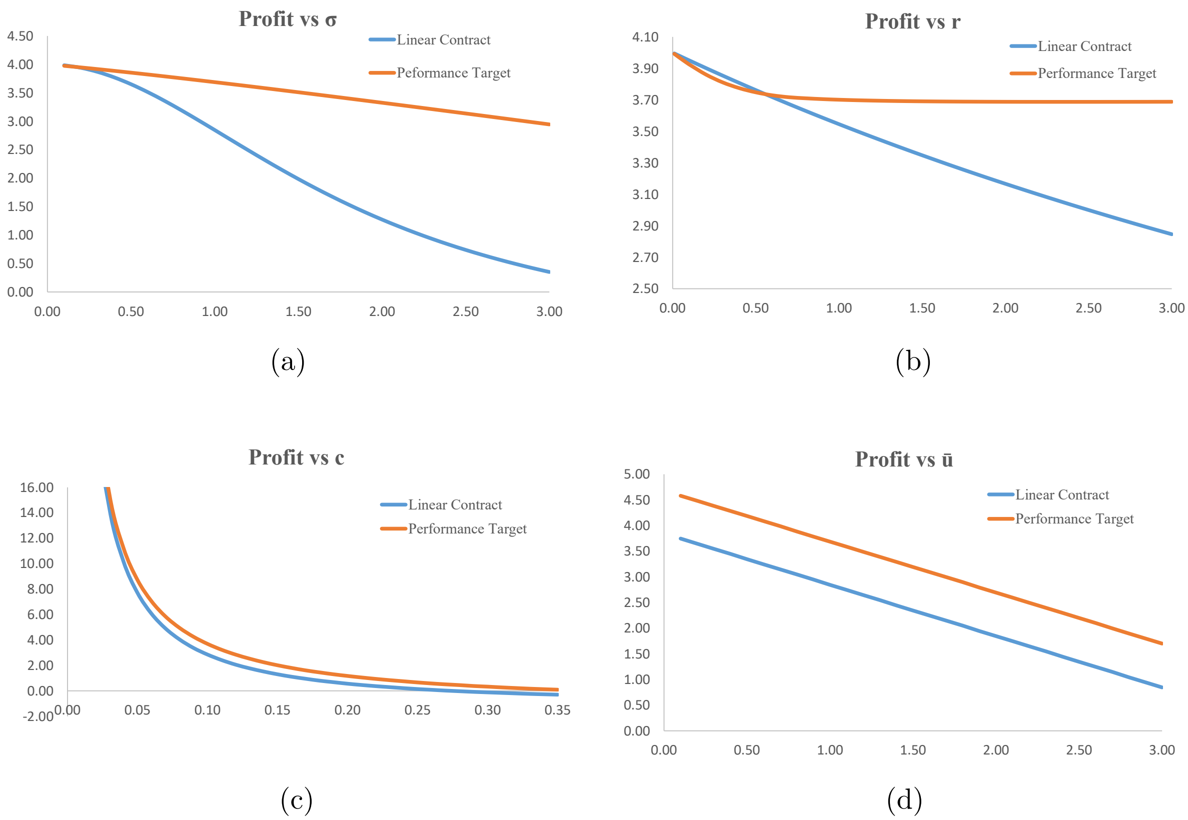


Figure 5: Profit of mean-variance preferences under performance target contract and the linear contract.

Proposition 4 *The optimal performance target contract under mean-variance preference is characterized by the following system of equations:*

$$\left(b - \frac{r}{2}b^2(1-2P)\right) g(e-t) = ce; \qquad (IC)$$

$$\frac{\partial e}{\partial t} = \frac{rb^2 g(e-t)^2 + (b-(r/2)b^2(1-2P))g'(e-t)}{rb^2 g(e-t)^2 + (b-(r/2)b^2(1-2P))g'(e-t) - c}; \qquad (28)$$

$$\frac{\partial e}{\partial b} = \frac{-(1-rb(1-2P))g(e-t)}{rb^2 g(e-t)^2 + (b-(r/2)b^2(1-2P))g'(e-t) - c}; \qquad (29)$$

$$[1 - bg(e-t)]\frac{\partial e}{\partial t} + [bg(e-t) - ce] = 0; \qquad (30)$$

$$[1 - bg(e-t)]\frac{\partial e}{\partial b} = rbP(1-P). \qquad (31)$$

The proofs are in the Appendix. When $r = 0$, $bg(e-t) = 1$ by (31) and hence $1 = bg(e-t) = ce$ by (30). So the optimal contract implements the first-best effort

$e^* = 1/c$. For general $r > 0$, it is clear that there is no closed-form solution to the optimal contract and the effort induced. Nonetheless, we were able to simulate some insight into the question of the optimality of performance targets versus linear contracts in this model of mean-variance preferences. This allows more of an apples-to-apples comparison of the firm's profit under the two contract types. Figure 5 shows plots for the two different contracts. Overall, the performance target contracts strictly dominate the linear ones, except when the agent has a low level of risk aversion or the risk is very low (where the two profits differ by minimal amount).[11] In most other regions, performance target contracts fare better than linear contracts. In reality, executive contracts are a mix of both linear and performance target contracts, with discrete jumps and linear regions in between them. This analysis studies the two extremes and the simulation results lend support to the wide use of performance target contracts.

5 Conclusion

Until very recently, academic researchers have largely guessed what executive contracts actually look like. In the face of such lack of knowledge about these specific contracts, linear models are good first approximations, given their simplicity and robustness. This has generated the large LEN literature in accounting and finance (on the theory side), coupled with linear tests of the risk incentives trade-off (on the empirical side). Yet, empirical tests of the risk incentives trade-off remain weak, and a comprehensive test that combines models of actual contracts, with a precise fit to empirical data, remains elusive.

Since the SEC required more disclosure of executive contracts in 2006, we now have a better sense of what form CEO pay actually takes. A noteworthy feature of these contracts is the reliance on a performance target of some kind, which involves an explicit payout according to a pre-specified target. This paper models such contracts explicitly. We show that under, risk neutrality, the performance target contract can achieve first-best with a target precisely equal to the desired effort that the firm seeks to induce. When the manager is sufficiently risk averse, the firm will shade the optimal target

[11]In our simulition results, we set vary σ from 0.1 to 3 with a 0.1 interval. Linear contract outperforms performance target contract when $\sigma = 0.1$ and performance target contract starts to outperform linear contract from $\sigma = 0.2$. This indicates performance target contract dominates linear contract most of the time.

below equilibrium effort to provide a form of insurance to the manager. Our paper indicates that the performance target is indeed a powerful tool that provides the firm with an additional instrument to resolve moral hazard problems with the manager.

Nevertheless, this paper generates a host of new intuitions and insights that can be tested against executive pay data: (1) the optimal bonus increases in risk, (2) the target provides insurance (in addition to the bonus) to help resolve the manager's moral hazard problem, and (3) how the optimal bonus and target fare against a linear contract. This result is in contrast to the incentive-risk trade-off in traditional LEN model.[12]

The trend toward more disclosure makes executive contracts available to the analyst, who can then tailor the theory and generate more precise empirical predictions than were possible before. This new research agenda mixes theory with empirics at a more intimate level, since the contract itself emerges from practice. We remain optimistic about how future work can explore dynamic effects, earnings management, the informativeness principle, team incentives, and many other theoretical questions regarding contracts used in practice, of which performance targets are just one example.[13] We hope that future research can build upon our model and generate empirical implications that can inspire future researchers to test those predictions against data.

6 Appendix

Proof of Lemma 1: The probability density function of the normal distribution with mean 0 and variance σ^2 is

$$g(\epsilon) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\epsilon^2/(2\sigma^2)}. \tag{32}$$

It is clear that $g(0) = \frac{1}{\sqrt{2\pi\sigma^2}}$ and $g'(\epsilon) = -(\frac{1}{\sqrt{2\pi\sigma^2}})(\frac{\epsilon}{\sigma^2})\, e^{-\epsilon^2/(2\sigma^2)}$. So $g'(\epsilon) > 0$ when $\epsilon < 0$, $g'(\epsilon) = 0$ when $\epsilon = 0$, and $g'(\epsilon) < 0$ when $\epsilon > 0$. Further note that

$$g''(\epsilon) = (\frac{1}{\sqrt{2\pi\sigma^2}})(\frac{1}{\sigma^2})e^{-\epsilon^2/(2\sigma^2)}(\frac{\epsilon^2}{\sigma^2} - 1). \tag{33}$$

[12]When the agent is risk averse, optimal bonus first increases in risk and then decreases in risk. Still this result suggest the relation between bonus and risk is contingent on the level of risk. Indeed, empirical studies find mixed evidence on the relation between incentive bonus and risk.

[13]For example, an open question related with earnings management is the firm's desire to discourage earnings management. A full exploration of this phenomenon is outside the scope of this paper, but worthy of future research. Bizjak et al. (2014) finds empirically that performance targets induce real earnings management rather than accruals management, but to date there is no theoretical investigation of this question.

So $g'(\epsilon)$ is strictly increasing in ϵ for $\epsilon < -\sigma$, reaching the (global) maximum at $\epsilon = -\sigma$, and strictly decreasing in ϵ for $-\sigma < \epsilon \leq 0$.

It follows that a sufficient condition for $\max_{\epsilon \in [-1/c, 0]} g'(\epsilon) < cg(0)$ under the normal distribution is that

$$c > \frac{e^{-1/2}}{\sigma}, \tag{34}$$

i.e., the cost function is convex enough or the performance measure spreads out. This is because the maximum value $g'(-\sigma) = (\frac{1}{\sqrt{2\pi\sigma^2}})(\frac{1}{\sigma})\, e^{-1/2}$ is greater than or equal to $g'(\epsilon)$ for any $\epsilon \in [-1/c, 0]$. Further, $cg(0) = \frac{c}{\sqrt{2\pi\sigma^2}}$. If $g'(-\sigma) < cg(0)$, which simplifies to $\frac{e^{-1/2}}{\sigma} < c$, it must be that $g'(\epsilon) \leq g'(-\sigma) < cg(0)$ for all $\epsilon \in [-1/c, 0]$, since $g'(\cdot)$ hits the maximum at $-\sigma$. Therefore, $\max\limits_{\epsilon \in [-1/c,0]} g'(\epsilon) < cg(0)$ when $\frac{e^{-1/2}}{\sigma} < c$. ■

Hence Assumption 1 can be satisfied when the cost function is sufficiently convex or the distribution sufficiently spreads out.

In Proposition 1, we claim that the contract $t^* = e^*$, $b^* = \frac{1}{g(0)}$, and $s^* = \bar{u} + C(e^*) - \frac{1}{2g(0)}$ is the optimal contract (with the smallest bonus). Claims 1 and 2 prove this proposition. We first show that this contract uniquely implements the first-best effort e^*.

Claim 1 *The contract* $(s^*, t^*, b^*) = (\bar{u} + \frac{1}{2c} - \frac{1}{2g(0)}, \frac{1}{c}, \frac{1}{g(0)}$ *uniquely implements* $e^* = \frac{1}{c}$.

Proof of Claim 1: Note that $e^* = \frac{1}{c}$ and $C(e^*) = \frac{1}{2c}$. First, we show that (IC) holds uniquely when $e = \frac{1}{c} = e^*$.

Observe that the FOC holds when $e = e^*$, i.e.,

$$b^* g(e - t^*) - C'(e) = [1/g(0)]g(e - 1/c) - ce, \tag{35}$$

$$= [1/g(0)]g(0) - 1, \tag{36}$$

$$= 0, \tag{37}$$

where the second equality is satisfied when we replace e by $e^* = 1/c$. Furthermore, the *SOC* holds globally under our assumption, i.e.,

$$b^* g'(e - t^*) - C''(e) = [1/g(0)]g'(e - 1/c) - c < 0 \text{ for all } e \geq 0. \tag{38}$$

This is equivalent to

$$g'(\epsilon) < cg(0) \text{ for all } \epsilon \geq -1/c. \tag{39}$$

By the assumption of unimodal probability distribution, $g'(\epsilon) < 0$ for all $\epsilon > 0$. So $g'(\epsilon) < 0 < cg(0)$ for all $\epsilon > 0$. Because the continuous function g' satisfies

$\max_{[-1/c,0]} g'(\epsilon) < cg(0)$, $g'(\epsilon) < cg(0)$ for all $\epsilon \in [-1/c, 0]$. Hence $g'(\epsilon) < cg(0)$ for all $\epsilon \geq -1/c$ and equivalently $[1/g(0)]g'(e - 1/c) - c < 0$ for all $e \geq 0$. So $e^* = 1/c$ is the unique maximizer for the manager's expected payoff given this contract.

Then we check (PC) is satisfied under this contract which implements $e = e^* = 1/c$:

$$\begin{aligned} & u(s)G(t-e) + u(s+b)G(e-t) - C(e), && (40)\\ &= s + bG(e-t) - C(e), && (41)\\ &= \bar{u} + 1/(2c) - 1/[2g(0)] + [1/g(0)]G(e - 1/c) - (1/2)ce^2, && (42)\\ &= \bar{u} + 1/(2c) - 1/[2g(0)] + [1/g(0)]G(0) - (1/2)c(1/c)^2, && (43)\\ &= \bar{u}, && (44) \end{aligned}$$

where the first equality follows from risk neutrality and the third equality is satisfied when we replace e by $e^* = 1/c$. ■

Next, we show that (s^*, t^*, b^*) is the optimal contract that has the lowest bonus.

Claim 2 *The contract* $(s^*, t^*, b^*) = (\bar{u} + 1/(2c) - 1/[2g(0)], 1/c, 1/g(0))$ *is optimal and the bonus is the smallest among all optimal contracts.*

Proof of Claim 2: (PC) binds under (s^*, t^*, b^*) by Claim 1. Because the firm maximizes her payoff at the first-best effort e^*, (s^*, t^*, b^*) is an optimal contract. We show that the bonus in an optimal contract cannot be lower than $1/g(0)$. Because the effort implemented is e^* under an optimal contract, FOC must hold at e^* under such contract. Otherwise, the manager will make an effort other than e^* by continuity and the firm is strictly worse off. If the bonus is strictly lower than $1/g(0)$, the first derivative of the manager's expected payoff at e^* is then

$$\begin{aligned} bg(e^* - t) - C'(e^*) &= bg(e^* - t) - ce^*, && (45)\\ &= bg(e^* - t) - 1, && (46)\\ &\leq bg(0) - 1, && (47)\\ &< 1 - 1 = 0, && (48) \end{aligned}$$

which implies a contradiction. Hence, the contract $(s^*, t^*, b^*) = (\bar{u}+1/(2c)-1/[2g(0)], 1/c, 1/g(0))$ is the optimal contract with the smallest bonus. ■

Proposition 1 then follows from Claims 1 and 2.

Proof of Proposition 2: We use second-order stochastic dominance to measure an increase in the dispersion of the distribution. Assume g_i for $i = 1, 2$ are two probability densities over the real line with mean 0 and finite variance that both satisfy the single-peaked condition. Suppose G_1 is second-order stochastically dominant over G_2. By definition, for all $w \in (-\infty, \infty)$, we have $S_1(w) < S_2(w)$ and $S_1(\infty) = S_2(\infty)$ where

$$S_i(w) = \int_{-\infty}^{w} G_i(w)\mathrm{d}w. \tag{49}$$

Suppose $g_2(0) > g_1(0)$. By the single-peaked condition, g is increasing over its negative domain, so $g_i(0) > g_i(x)$ for each $x < 0$. Now $g_2(0) > g_1(0)$, both densities are strictly increasing, and they both integrate to the same value, $G_1(0) = G_2(0) = \frac{1}{2}$, at the end point of the interval $(-\infty, 0)$. Then there exists a $z \in (-\infty, \infty)$ such that

$$g_2(x) < g_1(x),\ \forall x < z. \tag{50}$$

Integrate both sides of this inequality over $(-\infty, x)$ for each $x < z$ to generate

$$G_2(x) < G_1(x),\ \forall x < z. \tag{51}$$

Integrate over $(-\infty, z)$ to arrive at

$$S_2(z) < S_1(z). \tag{52}$$

This contradicts the definition of SOSD. Therefore $g_2(0) < g_1(0)$, and so the optimal bonus from Proposition 1 is

$$b_1^* = \frac{1}{g_1(0)} < \frac{1}{g_2(0)} = b_2^*. \tag{53}$$

■

For shorthand, denote

$$\Delta := u(s+b) - u(s) > 0 \text{ and } \Delta' := u'(s+b) - u'(s) < 0. \tag{54}$$

because utility is increasing and concave.

Proof of Lemma 2: Suppose that for the cheapest contract (s, t, b) that induces an effort e, $u(s)G(t-e) + u(s+b)G(e-t) - C(e) > \bar{u}$. Let $\alpha = u(s)G(t-e) + u(s+b)G(e-t) - C(e) - \bar{u} > 0$. Consider an alternative contract (s', t', b') such that $t' = t$, $u(s') = u(s) - \alpha$, and $u(s'+b') = u(s+b) - \alpha$. Because

$$
\begin{aligned}
& u(s')G(t-e) + u(s'+b')G(e-t) - C(e), && (55)\\
& \quad = [u(s) - \alpha]G(t-e) + [u(s+b) - \alpha]G(e-t) - C(e), && (56)\\
& \quad = u(s)G(t-e) + u(s+b)G(e-t) - C(e) - \alpha, && (57)\\
& \quad \geq u(s)G(t-\tilde{e}) + u(s+b)G(\tilde{e}-t) - C(\tilde{e}) - \alpha, && (58)\\
& \quad = [u(s) - \alpha]G(t-\tilde{e}) + [u(s+b) - \alpha]G(\tilde{e}-t) - C(\tilde{e}), && (59)\\
& \quad = u(s')G(t-\tilde{e}) + u(s'+b')G(\tilde{e}-t) - C(\tilde{e}) \text{ for all } \tilde{e}, && (60)
\end{aligned}
$$

(IC) still holds and the agent will still choose e. Moreover, $u(s')G(t-e) + u(s'+b')G(e-t) - C(e) = [u(s)-\alpha]G(t-e) + [u(s+b)-\alpha]G(e-t) - C(e) = u(s)G(t-e) + u(s+b)G(e-t) - C(e) - \alpha = \bar{u}$. So (PC) also holds. Then the contract (s', t', b') implements effort e but gives the firm a strictly higher payoff (or a strictly lower payout) than (s, t, b), which contradicts the assumption that (s, t, b) is the cheapest contract that induces e. ■

Proof of Lemma 3: The incentive constraint is

$$g(e-t)[u(s+b) - u(s)] = C'(e) = ce. \tag{61}$$

Let e^* be the manager's effort induced in equilibrium. Suppose that $g(e^*-t)[u(s+b) - u(s)] > C'(e^*) = ce^*$. Because the distribution function and the marginal cost of effort are continuous in effort, there is $e' > e^*$ such that $g(e-t)[u(s+b) - u(s)] = \Delta g(e-t) > C'(e) = ce$ for all $e \in [e^*, e']$. The manager's expected payoff from selecting e^* is

$$u(s) + \Delta G(e^*-t) - C(e^*) = u(s) + \Delta G(-t) + \int_0^{e^*} (\Delta g(e-t) - ce)de. \tag{62}$$

On the other hand, the manager's expected payoff from selecting e' is

$$
\begin{aligned}
&u(s) + \Delta G(e' - t) - C(e') && (63)\\
&\quad = u(s) + \Delta G(-t) + \int_0^{e'} (\Delta g(e-t) - ce)de && (64)\\
&\quad = u(s) + \Delta G(-t) + \int_0^{e^*} (\Delta g(e-t) - ce)de + \int_{e^*}^{e'} (\Delta g(e-t) - ce)de && (65)\\
&\quad = [u(s) + \Delta G(e^* - t) - C(e^*)] + \int_{e^*}^{e'} (\Delta g(e-t) - ce)de && (66)\\
&\quad > u(s) + \Delta G(e^* - t) - C(e^*), && (67)
\end{aligned}
$$

where the inequality follows from $\Delta g(e-t) > ce$ for $e \in [e^*, e']$. This implies that e^* is not the equilibrium effort of the manager, which contradicts the hypothesis. Similarly, we can show that the manager can improve his payoff by selecting a marginally lower effort if $g(e^* - t)[u(s+b) - u(s)] < C'(e^*) = ce^*$. This completes the proof. ■

In Proposition 3, we claim that the effort that is actually implemented can never be less than the target under the optimal contract. This is shown by demonstrating that a contract with a lower target would implement the same effort at a lower cost for the firm. We include the technical discussion in the proof. In particular, we show that under the contract with lower target, the manager will select the equilibrium effort under the contract with higher target even if there are other solutions to the (IC) constraint for the low-target contract. The argument applies to any (strongly) unimodal distribution. We prove Proposition 3 in a series of steps, which we state and prove as claims.

It is clear that $\hat{e} \neq \hat{t}$ when the manager is risk averse. We first prove that if an effort e is implemented by a contract with target $t > e$ and spread Δ, it is implemented by another contract with target $2e - t$ that is less than e and the same spread Δ as well.

Claim 3 *If a contract with $t = \hat{e} + z$ implements effort $\hat{e}$ for some $z > 0$, then so does a contract with $t = \hat{e} - z$ and the same spread.*

Proof of Claim 3: Let (s, b, t_H) be a contract with salary s, bonus b, and target t_H that generates effort $\hat{e}$ such that $\hat{e} < t_H$. Let $\Delta = u(s+b) - u(s)$ be the spread. Let $z = t_H - \hat{e} > 0$. We show that the effort $\hat{e}$ is also implemented by a contract with a target $t_L = \hat{e} - z$ and the same spread Δ. It is clear that $t_L < \hat{e} < t_H$.

First, we claim that the (IC) condition holds at the effort $\hat{e}$ for the contract with target t_L. Note that the spread Δ is the same for the two contracts. Then we have

$$g(\hat{e} - t_L)\Delta - c\hat{e} = g(z)\Delta - c\hat{e} = g(-z)\Delta - c\hat{e} = g(\hat{e} - t_H)\Delta - c\hat{e} = 0, \tag{68}$$

where the first equality follows from the definition of t_L, the second equality follows from the symmetry of g, the third equality follows from the definition of z, and the fourth equality follows from the (IC) constraint under the contract with target t_H. Hence $\hat{e}$ is *one* solution to the (IC) condition for the contract with target t_L by Equation (68).

Next, we show that $\hat{e}$ is the *unique* optimal choice of the manager under the contract with target t_L, even if there could be multiple solutions to (IC). Suppose that the manager's payoff can be maximized by another effort $e' \neq \hat{e}$ under the contract with target t_L. Since ce is strictly increasing in e and $g(e - t_L)\Delta$ is strictly decreasing for $e > t_L$, there is at most one solution greater than t_L to (IC) under the contract with target t_L. The argument above shows that $\hat{e}$ is one solution to (IC). Because $\hat{e} > t_L$, $\hat{e}$ is the *unique solution greater than* t_L to (IC) under the contract with target t_L. Then e' must be less than t_L and $\hat{e}$, since we assumed that $e' \neq \hat{e}$.

It follows from the manager's payoff being maximized at e' that he would get a weakly higher payoff from e' than $\hat{e}$ under the contract with target t_L. This implies that

$$\int_{e'}^{\hat{e}} (\Delta g(e - t_L) - ce)de, \tag{69}$$

$$=[u(s) + \Delta G(-t_L) + \int_0^{\hat{e}} (\Delta g(e - t_L) - ce)de] - [u(s) + \Delta G(-t_L) + \int_0^{e'} (\Delta g(e - t_L) - ce)de], \tag{70}$$

$$=[u(s) + \Delta G(\hat{e} - t_L) - C(\hat{e})] - [u(s) + \Delta G(e' - t_L) - C(e')] \leq 0, \tag{71}$$

where the inequality follows from the assumption. Further, note that $e - t_H = e - \hat{e} - z$ and $e - t_L = e - \hat{e} + z$. When $e < \hat{e}$, it is clear that

$$|e - t_H|^2 = |e - \hat{e} - z|^2 = (\hat{e} - e)^2 + 2(\hat{e} - e)z + z^2 > (e - \hat{e})^2 + 2(e - \hat{e})z + z^2 = |e - \hat{e} + z|^2 = |e - t_L|^2, \tag{72}$$

because $e - \hat{e} < 0$ and $z > 0$. So $g(e - t_H) = g(|e - t_H|) < g(|e - t_L|) = g(e - t_L)$ by symmetry and single-peakedness of the unimodal distribution. It hence follows that $g(e - t_H)\Delta < g(e - t_L)\Delta$ for any $e < \hat{e}$, including e'. Then we have

$$[u(s) + \Delta G(\hat{e} - t_H) - C(\hat{e})] - [u(s) + \Delta G(e' - t_H) - C(e')], \tag{73}$$

$$= [u(s) + \Delta G(-t_H) + \int_0^{\hat{e}} (\Delta g(e - t_H) - ce)de] - [u(s) + \Delta G(-t_H) + \int_0^{e'} (\Delta g(e - t_H) - ce)de], \tag{74}$$

$$= \int_{e'}^{\hat{e}} (\Delta g(e - t_H) - ce)de, \tag{75}$$

$$< \int_{e'}^{\hat{e}} (\Delta g(e - t_L) - ce)de \leq 0, \tag{76}$$

where the strict inequality follows from $g(e - t_H)\Delta < g(e - t_L)\Delta$ for $e < \hat{e}$ (and $e' < \hat{e}$), and the last inequality follows from (69). Hence the manager's expected payoff $u(s) + \Delta G(\hat{e} - t_H) - C(\hat{e})$ from selecting $\hat{e}$ under the contract with target t_H is strictly lower than the payoff $u(s) + \Delta G(e' - t_H) - C(e')$ from selecting e'. This contradicts the assumption that the contract with target t_H implements the effort $\hat{e}$. We therefore conclude that the contract with target t_L implements the effort $\hat{e}$ as well and that this is the *unique* optimal effort for the manager. ■

Next we show that the low-target contract requires less salary and bonus to implement if it gives the same expected payoff to the manager.

Claim 4 *Let (s, b, t) be a contract that implements effort $\hat{e} < t$. Then the salary and bonus would be lower for another contract $(s', b', 2\hat{e} - t)$ with the same spread if it gives the manager equal expected payoff, i.e., $s' < s$ and $b' < b$.*

Proof of Claim 4: It is clear that $2\hat{e} - t < \hat{e}$. We so call the contract with target t high-target contract, denoted by (s_H, b_H, t_H), and the contract with target $2\hat{e} - t$ low-target contract, denoted by (s_L, b_L, t_L). Claim 3 shows that the manager will indeed select effort $\hat{e}$ under the contract with target t_L (and the same spread). We next show that the salary and the bonus to induce the same payoff for the manager would be lower under the contract with target t_L.

Let $\hat{u}$ be the manager's expected payoff under the contract (s_H, b_H, t_H). Then

$$\hat{u} = u(s_H) + \Delta G(\hat{e} - t_H) - C(\hat{e}) = u(s_H) + \Delta G(-t_H) + \int_0^{\hat{e}} (\Delta g(e - t_H) - ce)de. \tag{77}$$

By the assumption of unimodal distribution, $g(e-t_H)\Delta < g(e-t_L)\Delta$ for $e < \hat{e}$. So

$$\int_0^{\hat{e}} (\Delta g(e-t_H) - ce)de < \int_0^{\hat{e}} (\Delta g(e-t_L) - ce)de. \tag{78}$$

Furthermore, $-t_H < -t_L < 0$ and $\Delta > 0$. This implies that $\Delta G(-t_H) < \Delta G(-t_L)$. Hence, if the same payoff $\hat{u}$ is induced by the contract with target t_L, the manager's salary s_L satisfies

$$\begin{aligned} u(s_L) =& \hat{u} - [\Delta G(-t_L) + \int_0^{\hat{e}} (\Delta g(e-t_L) - ce)de], && (79)\\ <& \hat{u} - [\Delta G(-t_H) + \int_0^{\hat{e}} (\Delta g(e-t_H) - ce)de], && (80)\\ =& u(s_H), && (81) \end{aligned}$$

because the spread Δ is the same under these two contracts. The strictly increasing payoff function $u(\cdot)$ implies that $s_L < s_H$.

Moreover, we have

$$u(s_H + b_H) - u(s_H) = \Delta = u(s_L + b_L) - u(s_L). \tag{82}$$

The marginal utility $u'(\cdot)$ is strictly decreasing because of risk aversion, so

$$u(s_H + b_L) - u(s_H) < u(s_L + b_L) - u(s_L). \tag{83}$$

It follows that

$$u(s_H + b_L) - u(s_H) < u(s_H + b_H) - u(s_H), \tag{84}$$

which implies $b_L < b_H$ by strictly increasing utility function. ■

The next three claims will be useful in proving Lemma 5: a low-target contract is cheaper to implement for a sufficiently risk averse manager.

Claim 5 *If a twice continuously differentiable function f satisfies $f''(\cdot) > (<)(=)0$, then*

$$\frac{f(a)+f(b)}{2} > (<)(=)\frac{1}{b-a}\int_a^b f(t)dt \text{ for } a < b. \tag{85}$$

Proof of Claim 5: For any x such that $0 < x < b-a$, $a < a+x < b$ and $a < b-x < b$. It is clear that

$$
\begin{aligned}
&(f(b) - f(b-x)) - (f(a+x) - f(a)), && (86)\\
&= \int_{b-x}^{b} f'(t)dt - \int_{a}^{a+x} f'(t)dt, && (87)\\
&= \int_{a}^{a+x} f'(t+b-x-a)dt - \int_{a}^{a+x} f'(t)dt, && (88)\\
&= \int_{a}^{a+x} [f'(t+b-x-a) - f'(t)]dt, && (89)\\
&= \int_{a}^{a+x} \int_{t}^{t+b-x-a} f''(s)dsdt > (<)(=)0. && (90)
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&f(b)(b-a) - \int_{a}^{b} f(y)dy, && (91)\\
&= f(b)(b-a) - \int_{0}^{b-a} f(b-x)dx, && (92)\\
&= \int_{0}^{b-a} (f(b) - f(b-x))dx, && (93)\\
&> (<)(=) \int_{0}^{b-a} (f(a+x) - f(a))dx, && (94)\\
&= \int_{0}^{b-a} f(a+x)dx - f(a)(b-a), && (95)\\
&= \int_{a}^{b} f(y)dy - f(a)(b-a). && (96)
\end{aligned}
$$

The Inequality (85) then follows, because

$$[f(a) + f(b)](b-a) > (<)(=) 2\int_{a}^{b} f(y)dy. \tag{97}$$

■

Claim 6 *If a three-times continuously differentiable function h satisfies $h'''(\cdot) > (<)(=)0$, then $\frac{h(a+x)-h(a-x)}{2x}$ is strictly increasing (strictly decreasing) (constant) in $x > 0$ for any given a.*

Proof of Claim 6: We show that the first derivative of $\frac{h(a+x)-h(a-x)}{2x}$ with respect to x is positive (negative) (zero) if $h'''(\cdot) > (<)(=)0$.

The first derivative is

$$\frac{d}{dx}\left(\frac{h(a+x)-h(a-x)}{2x}\right) = \frac{[h'(a+x)+h'(a-x)](2x) - 2[h(a+x)-h(a-x)]}{4x^2}, \tag{98}$$

$$= \frac{[h'(a+x)+h'(a-x)]/2 - [h(a+x)-h(a-x)]/(2x)}{x}, \tag{99}$$

$$= \frac{[h'(a+x)+h'(a-x)]/2 - \int_{a-x}^{a+x} h'(t)dt/(2x)}{x} > (<)(=)0. \tag{100}$$

if $h'''(\cdot) = (h')'' > (<)(=)0$ by Claim 5. Hence for any a, $\frac{h(a+x)-h(a-x)}{2x}$ is strictly increasing (strictly decreasing) (constant) in $x > 0$ if $h'''(\cdot) > (<)(=)0$. ■

Claim 7 *If two contracts with the same spread (s_H, b_H, t_H) and $(s_L, b_L, 2\hat{e} - t_H)$ give manager the same expected payoff, the low-target contract is less costly to implement if and only if*

$$\frac{u^{-1}(u(s_L)+\Delta) - u^{-1}(u(s_H))}{(u(s_L)+\Delta) - u(s_H)} < \frac{u^{-1}(u(s_H)+\Delta) - u^{-1}(u(s_L))}{(u(s_H)+\Delta) - u(s_L)}, \tag{101}$$

where Δ is the common spread.

Proof of Claim 7: Claim 3 shows that the same effort $\hat{e}$ will be implemented. Since the manager gets the same expected payoff from two contracts and $G(t_H - \hat{e}) + G(t_L - \hat{e}) = 1$,

$$u(s_H)G(t_H - \hat{e}) + u(s_H + b_H)[1 - G(t_H - \hat{e})] - C(\hat{e}), \tag{102}$$

$$= u(s_L)G(t_L - \hat{e}) + u(s_L + b_L)[1 - G(t_L - \hat{e})] - C(\hat{e}), \tag{103}$$

$$= u(s_L)[1 - G(t_H - \hat{e})] + u(s_L + b_L)G(t_H - \hat{e}) - C(\hat{e}). \tag{104}$$

It follows that $[1 - G(t_H - \hat{e})]/G(t_H - \hat{e}) = [u(s_L + b_L) - u(s_H)]/[u(s_H + b_H) - u(s_L)]$. The conclusion follows from the equivalent statements below: the expected payment is lower for the low-target contract than the high-target contract if and only if

$$s_H G(t_H - \hat{e}) + (s_H + b_H)[1 - G(t_H - \hat{e})] > s_L G(t_L - \hat{e}) + (s_L + b_L)[1 - G(t_L - \hat{e})], \tag{105}$$

$$\Leftrightarrow s_H G(t_H - \hat{e}) + u^{-1}(u(s_H) + \Delta)[1 - G(t_H - \hat{e})] > s_L[1 - G(t_H - \hat{e})] + u^{-1}(u(s_L) + \Delta)G(t_H - \hat{e}), \tag{106}$$

$$\Leftrightarrow [1 - G(t_H - \hat{e})]/G(t_H - \hat{e}) > [u^{-1}(u(s_L) + \Delta) - s_H]/[u^{-1}(u(s_H) + \Delta) - s_L], \tag{107}$$

$$\Leftrightarrow \frac{u(s_L + b_L) - u(s_H)}{u(s_H + b_H) - u(s_L)} > \frac{u^{-1}(u(s_L) + \Delta) - u^{-1}(u(s_H))}{u^{-1}(u(s_H) + \Delta) - u^{-1}(u(s_L))}, \tag{108}$$

$$\Leftrightarrow \frac{(u(s_L) + \Delta) - u(s_H)}{(u(s_H) + \Delta) - u(s_L)} > \frac{u^{-1}(u(s_L) + \Delta) - u^{-1}(u(s_H))}{u^{-1}(u(s_H) + \Delta) - u^{-1}(u(s_L))}. \tag{109}$$

■

Suppose that the manager's payoff is a power function, i.e., $u(w) = w^{\alpha}$ for $0 < \alpha < 1$. So the manager is better off with more wealth and also risk averse. Then $u'(w) = \alpha w^{\alpha-1}$, $u''(w) = \alpha(\alpha - 1)w^{\alpha-2}$, and $u'''(w) = \alpha(\alpha - 1)(\alpha - 2)w^{\alpha-3}$. It follows that $A(w) = -\frac{\alpha(\alpha-1)w^{\alpha-2}}{\alpha w^{\alpha-1}} = (1 - \alpha)w^{-1}$ and $\sqrt{(\frac{1}{3})(\frac{u'''(w)}{u'(w)})} = \sqrt{\frac{1}{3}\frac{\alpha(\alpha-1)(\alpha-2)w^{\alpha-3}}{\alpha w^{\alpha-1}}} = \sqrt{\frac{(1-\alpha)(2-\alpha)}{3}}w^{-1}$. So if $0 < \alpha < 1/2$, it is less costly to induce the low-target contract; if $\alpha = 1/2$, it is equally costly to induce either contract; if $1/2 < \alpha < 1$, it is more costly to induce the low-target contract. The threshold of risk aversion level is then 1/2.

Example 1 *Suppose that $u(\cdot) = \sqrt{\cdot}$, which is strictly concave. We illustrate that the expected compensation is the same under the two contracts through the following numerical example. Suppose that $s_H = 16$, $b_H = 9$, $s_L = 12.25$, $b_L = 8$. Then $u(s_H + b_H) = \sqrt{s_H + b_H} = 5$, $u(s_H) = \sqrt{s_H} = 4$, $u(s_L + b_L) = \sqrt{s_L + b_L} = 4.5$, and $u(s_L) = \sqrt{s_L} = 3.5$. Because the manager's payoff is the same under these two contracts, $G(t_H - \hat{e}) = 0.75$ and $1 - G(t_H - \hat{e}) = 0.25$. We can then verify that the expected payment is 18.25 under both contracts.*

Proof of Proposition 3: By Lemma 5, if the manager is sufficiently risk averse, i.e., $A(\cdot) > \sqrt{(\frac{1}{3})(\frac{u'''(\cdot)}{u'(\cdot)})}$, then the firm will implement the same effort and give the manager the same expected payoff, but make less expected payment by the low-target contract. Hence any contract such that the target is greater than the effort cannot be optimal, because there always exists a lower-target contract that implements the same outcome at a lower cost. This implies that the target is less than effort in equilibrium under the optimal contract. ■

Proof of Lemma 4: Note that (IC) defines the effort level implicitly. Write the second-order condition from the manager's effort problem as

$$g'(e-t)\Delta - c < 0. \tag{SOC}$$

It is clear that the second-order condition for the manager's problem holds at the effort induced.[14] Differentiate (IC) with respect to the bonus:

$$\frac{\partial e}{\partial b} = \frac{g(e-t)u'(s+b)}{c-\Delta g'(e-t)} > 0. \tag{110}$$

This occurs because u is increasing and from the (SOC). Now, differentiate (IC) with respect to salary:

$$\frac{\partial e}{\partial s} = \frac{g(e-t)\Delta'}{c-\Delta g'(e-t)} < 0, \tag{111}$$

from (SOC). Finally, differentiate (IC) with respect to the target:

$$\frac{\partial e}{\partial t} = \frac{g'(e-t)\Delta}{g'(e-t)\Delta - c} \geq 0 \text{ if and only if } e \geq t, \tag{112}$$

because the marginal density is 0 at $e = t$, positive for effort less than t, and negative for effort greater than t. ■

By Proposition 3 we proved above, we know $\hat{t} < \hat{e}$ in equilibrium, and therefore the derivative with respect to target is always positive in equilibrium.

Proof of Proposition 4: Recall that the incentive compatibility constraint is

$$\left(b - \frac{r}{2}b^2(1-2P)\right) g(e-t) = ce. \tag{IC}$$

By the Implicit Function Theorem, we get (28) and (29) from (IC):

$$\frac{\partial e}{\partial t} = \frac{rb^2 g(e-t)^2 + (b-(r/2)b^2(1-2P))g'(e-t)}{rb^2 g(e-t)^2 + (b-(r/2)b^2(1-2P))g'(e-t) - c}, \tag{113}$$

[14] We actually do not need to require that the second-order condition hold for every effort level.

and

$$\frac{\partial e}{\partial b}=\frac{-(1-rb(1-2P))g(e-t)}{rb^2g(e-t)^2+(b-(r/2)b^2(1-2P))g'(e-t)-c}. \tag{114}$$

Further, the firm's problem has been reduced to

$$\max_{(t,b)} \ \{e-\frac{r}{2}Pb^2(1-P)-C(e)-\bar{u}\} \ \text{ subject to } (IC). \tag{115}$$

Taking first-order condition with respect to t, we get

$$[1-\frac{r}{2}b^2g(e-t)(1-2P)-ce]\frac{\partial e}{\partial t}+\frac{r}{2}b^2g(e-t)(1-2P)=0. \tag{116}$$

By (IC), $\frac{r}{2}b^2g(e-t)(1-2P)=bg(e-t)-ce$. Replace $\frac{r}{2}b^2g(e-t)(1-2P)$ with $bg(e-t)-ce$ in the equation above and obtain (30):

$$[1-bg(e-t)]\frac{\partial e}{\partial t}+[bg(e-t)-ce]=0. \tag{117}$$

Taking first-order condition with respect to b, we get

$$[1-\frac{r}{2}b^2g(e-t)(1-2P)-ce]\frac{\partial e}{\partial b}-rbP(1-P)=0. \tag{118}$$

Replace $\frac{r}{2}b^2g(e-t)(1-2P)$ by $bg(e-t)-ce$ and obtain (31):

$$[1-bg(e-t)]\frac{\partial e}{\partial b}=rbP(1-P). \tag{119}$$

Then (IC), (28), (29), (30), and (31) jointly determine the optimal contract. ■

The next Lemma shows that the firm will implement a particular effort at lower cost by low-target contract if the manager is sufficiently risk averse.

Lemma 5 *Let $A(w)=-\frac{u''(w)}{u'(w)}$ be the coefficient of absolute risk aversion, where w is the wealth level.*

(1) If $A(w)>\sqrt{(\frac{1}{3})(\frac{u'''(w)}{u'(w)})}$ for all w, a contract with the same spread and lower target implements the same effort at a lower cost for the firm than the contract with higher target.

(2) If $A(w)<\sqrt{(\frac{1}{3})(\frac{u'''(w)}{u'(w)})}$ for all w, a contract with the same spread and lower target implements the same effort at a higher cost for the firm than the contract with higher target.

(3) If $A(w)=\sqrt{(\frac{1}{3})(\frac{u'''(w)}{u'(w)})}$ for all w, a contract with the same spread and lower target implements the same effort at the same cost for the firm than the contract with higher target.

Proof of Lemma 5: In Claim 4, we have established that $s_L < s_H$ and $b_L < b_H$. So $s_L < s_H < s_L + b_L < s_H + b_H$, because if $s_H \geq s_L + b_L$, the manager gets a strictly higher payoff from the contract with higher target, i.e.,

$$
\begin{aligned}
& u(s_H)G(t_H - \hat{e}) + u(s_H + b_H)[1 - G(t_H - \hat{e})] - C(\hat{e}), && (120)\\
& \quad \geq u(s_L + b_L)G(t_H - \hat{e}) + u(s_H + b_H)[1 - G(t_H - \hat{e})] - C(\hat{e}), && (121)\\
& \quad > u(s_L + b_L)G(t_H - \hat{e}) + u(s_L + b_L)[1 - G(t_H - \hat{e})] - C(\hat{e}), && (122)\\
& \quad = u(s_L + b_L) - C(\hat{e}), && (123)\\
& \quad > u(s_L)G(t_L - \hat{e}) + u(s_L + b_L)[1 - G(t_L - \hat{e})] - C(\hat{e}). && (124)
\end{aligned}
$$

We will show that if the manager is sufficiently risk averse, i.e., $A(\cdot) > \sqrt{(\frac{1}{3})(\frac{u'''(\cdot)}{u'(\cdot)})}$, the expected compensation for the manager is lower under the low-target contract, i.e., $s_H G(t_H - \hat{e}) + (s_H + b_H)[1 - G(t_H - \hat{e})] > s_L G(t_L - \hat{e}) + (s_L + b_L)[1 - G(t_L - \hat{e})]$. Then the firm gets a higher expected payoff to induce $\hat{e}$ under the low-target contract.

Because the utility u is a strictly increasing and continuously differentiable function of the agent's wealth w, the inverse w as a function of u exists. Then we can apply the chain rule for higher derivatives of the inverse function and obtain[15]

$$
\begin{aligned}
\frac{d^3 w}{du^3} &= \left(\frac{dw}{du}\right)^4 \left[3\frac{(u''(w))^2}{u'(w)} - u'''(w)\right], && (125)\\
&= \left(\frac{dw}{du}\right)^4 \left[3\left(-\frac{u''(w)}{u'(w)}\right)^2 u'(w) - u'''(w)\right], && (126)\\
&= \left(\frac{dw}{du}\right)^4 \left[3(A(w))^2 u'(w) - u'''(w)\right]. && (127)
\end{aligned}
$$

It is clear that if $A(w) > (<)\ (=) \sqrt{(\frac{1}{3})(\frac{u'''(w)}{u'(w)})}$, then $\frac{d^3 w}{du^3} > (<)\ (=)\ 0$, i.e., $\frac{dw}{du}$ is a strictly convex (strictly concave) (linear) function of the utility u.

To simplify the notation, let $V = \dfrac{u(s_L) + u(s_H) + \Delta}{2}$. By definition of V, we can write

$$
u(s_L) + \Delta = V + \frac{u(s_L) - u(s_H) + \Delta}{2}, \quad u(s_H) = V - \frac{u(s_L) - u(s_H) + \Delta}{2}; \qquad (128)
$$

$$
u(s_H) + \Delta = V + \frac{u(s_H) - u(s_L) + \Delta}{2}, \quad u(s_L) = V - \frac{u(s_H) - u(s_L) + \Delta}{2}. \qquad (129)
$$

[15] The generalized version is called the "Faà di Bruno's formula".

We have proved that $s_H > s_L$ by Claim 4. It follows that $\frac{u(s_H)-u(s_L)+\Delta}{2} > \frac{u(s_L)-u(s_H)+\Delta}{2}$. Think of the inverse function u^{-1} as the function h in Claim 6: if

$$\frac{d^3w}{du^3} > (<)(=)0, \tag{130}$$

then $\frac{u^{-1}(V+x)-u^{-1}(V-x)}{2x}$ is strictly increasing (strictly decreasing) (constant) in $x > 0$. Because $\frac{u(s_H)-u(s_L)+\Delta}{2} > \frac{u(s_L)-u(s_H)+\Delta}{2}$, we replace x by $\frac{u(s_H)-u(s_L)+\Delta}{2}$ and $\frac{u(s_L)-u(s_H)+\Delta}{2}$ and then obtain

$$\frac{u^{-1}(u(s_H)+\Delta)-u^{-1}(u(s_L))}{(u(s_H)+\Delta)-u(s_L)} > (<)(=)\frac{u^{-1}(u(s_L)+\Delta)-u^{-1}(u(s_H))}{(u(s_L)+\Delta)-u(s_H)}, \tag{131}$$

by Equations (128) when $\frac{d^3w}{du^3} > (<)(=)\ 0$. By Claim 7, the inequalities of (131) imply that the low-target (high-target) contract requires less payment on expectation given that the manager gets the same expected payoff.

Therefore, the arguments of the two paragraphs above show that if $A(w) > (<)\ (=)\sqrt{(\frac{1}{3})(\frac{u'''(w)}{u'(w)})}$ for all w, a contract with the same spread and lower target implements the same effort at a lower (higher) (the same) cost for the firm than the contract with higher target. ∎

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