JOURNAL OF MANAGEMENT ACCOUNTING RESEARCH
Vol. 28, No. 1
Spring 2016
pp. 27–37

American Accounting Association
DOI: 10.2308/jmar-51285

Reconciling Full-Cost and Marginal-Cost Pricing

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ABSTRACT: Despite the clear prescription from economic theory that a firm should set price based only on variable costs, firms routinely factor fixed costs into pricing decisions. We show that full-cost pricing (FCP) can achieve the optimal price. FCP marks up variable cost with the contribution margin per unit, which, in equilibrium, includes the fixed cost. FCP converges to the optimal price when the firm can estimate its equilibrium income. We compare FCP to alternative pricing algorithms that require less information, but converge to optimal price under more narrow conditions than FCP.

Keywords: full-cost pricing; marginal costs; efficiency.

INTRODUCTION

Economic theory provides an unambiguous prediction on how a profit-maximizing firm should set its price for a product: the firm includes only marginal (or variable) costs. It should not factor fixed costs into the pricing decision. And yet, a wealth of survey evidence (Shim and Sudit 1995; Noreen and Burgstahler 1997; Shim 1993; Govindarajan and Anthony 1983) shows that more than 60 percent of American manufacturing companies do include fixed costs into prices, called full-cost pricing (FCP). We seek to resolve this puzzle between economic theory and real accounting practices with the following observation: optimal pricing by a monopolist requires accurate knowledge of the demand curve, even though firms rarely have such knowledge, and certainly not to the extent and precision needed for accurate pricing. Instead, FCP can, in fact, lead to the optimal price. We provide a specific algorithm by which the firm can formulate its full-cost price and achieve the optimal price. This may be achieved even when knowledge of equilibrium income is imperfect. Thus, full-cost pricing is simply an implementation of, rather than a contradiction to, optimal pricing.

We conduct the analysis in a setting where a monopolist faces non-trivial (downward-sloping) demand and has a linear cost curve. In the classical textbook solution, the firm maximizes profits by equalizing marginal revenue and marginal cost, thereby marking up variable cost in a manner proportional to the elasticity of demand. Fixed costs vanish from the firm's optimization problem and, therefore, do not factor into the optimal price. Instead, they only affect the firm's decision on whether to enter the market. For the firm to pick the price correctly, it must know the parameters of the demand curve, since the optimal price (via marginal revenue) is a function of the demand parameters. However, if these strong informational requirements are not met, pricing resulting from erroneous estimates of demand will be suboptimal, which may explain why firms do not actually price according to economic theory.

In contrast, the firm can alternatively set a full-cost price that includes its fixed costs. Rather than maximizing profits explicitly and arriving at optimal price through first order conditions, the full-cost price emerges from an equilibrium condition. Income of the firm at the equilibrium price equals, by definition, equilibrium income. Rearranging this identity, equilibrium price equals variable cost plus equilibrium contribution margin per unit (fixed cost plus equilibrium income, all divided by quantity). Therefore, if the firm can measure its equilibrium income, it can "back out" an estimate of equilibrium price. This

We thank John Cannavo and Kyle Choy for outstanding research assistance.

The views expressed in this paper are those of the authors and do not necessarily reflect the view of the Board of Governors of the Federal Reserve System or its staff.

Editor's note: Accepted by Ranjani Krishnan.

Submitted: November 2014
Accepted: August 2015
Published Online: September 2015

yields a candidate full-cost price, which can then be used to estimate the optimal price. In order to achieve the optimal price, the firm must have some knowledge or estimate of optimal (equilibrium) income.

We consider full-cost pricing in both a static and dynamic sense. In the static model, the firm sets a single price and FCP achieves the optimal price if and only if the firm knows equilibrium income and quantity. However, full-cost pricing also achieves optimal pricing in a dynamic setting when the firm has no knowledge of its equilibrium quantity (and only has an estimate of equilibrium income). We provide an algorithm by which the firm prices over time, setting the initial price equal to its variable cost and revising the price upward by the equilibrium contribution margin per unit (where the number of units is the quantity the market purchases based on the price in the prior stage). The full-cost price then converges to the optimal price. Variable cost is a natural starting value, since a competitive benchmark sets price equal to the marginal (variable) cost. This convergence works both when the firm knows equilibrium income exactly, and when the firm does not know income exactly, but has a (consistent) estimate of it. This form of price experimentation, documented in surveys of company practice, actually has a theoretical foundation.

Finally, we compare full-cost pricing against an alternative algorithm based on the firm's first derivative of its profit function, which we call the first order approach. This requires even less information than FCP, since the firm simply picks a candidate and increases or decreases it based on whether marginal profit is increasing or decreasing. This works because the first order condition guarantees that marginal profits will be zero at the optimum. Eventually, this algorithm converges to the optimal price as the marginal increments to profits shrink, signaling convergence. We give conditions under which the first order approach converges to optimal price. These conditions restrict the parameters of the demand curve. As such, even though full-cost pricing requires higher informational requirements, it works under a much broader class of settings than the first order algorithm. Ultimately, the firm will choose the algorithm based on whether it has better information on income or demand.

To fix ideas, consider Apple Inc., currently the world's most valuable company by market capitalization. Apple is virtually a monopoly in the high-end mobile device market and its innovative products make estimation of demand difficult. However, Apple has a history of high-profit products and it is conceivable that they seek to maintain those profit margins over time. There exists an enormous cottage industry on Wall Street of equity analysts that seek to predict Apple's earnings/profits for the next quarter, and also for estimating the true marginal cost of its flagship product (the iPhone). Many of these analysts frequently comment that estimating Apple's demand is notoriously difficult, since its products are new. Historical data are unlikely to be a useful guide, but Apple's prior historical earnings are available. Indeed, the former CEO of Apple, Steve Jobs, often remarked that he aimed to create products that consumers do not yet know that they need and, at the same time, maintain high standards as the luxury technology player in the technology space (Isaacson 2011). It is hard for an innovative firm like Apple to apply the marginal-cost approach because it sells new products and has no information on what that demand function looks like, and can apply full-cost pricing using the market's estimate for equilibrium income.

Accounting research has developed a large volume of survey evidence to show that full-cost pricing dominates managerial practices (Shim and Sudit 1995; Emore and Ness 1991; Cooper 1990; Govindarajan and Anthony 1983; Gordon, Cooper, Falk, and Miller 1981; Drury and Tayles 2000; Bright, Davies, Downes, and Sweeting 1992; Scapens 1983). This debate between real accounting practices and economic theory has persisted for quite some time (as early as Friedman [1953], and as recently as Lucas [2003]). Yet formal theoretical models of this debate have appeared only recently. Al-Najjar, Baliga, and Besanko (2005) consider a case of a Bertrand oligopoly and find that firms that follow a naive adaptive learning process to adjust prices will eventually include fixed costs into their pricing (which they call the sunk-cost bias). Their adaptive learning algorithm is similar in spirit to ours in that the firm prices dynamically over time, although their emphasis is slightly different, focusing on strategic interaction between firms. The sunk-cost bias persists because firms do not optimize in their models, but instead adopt a behavioral strategy of adaptive learning. In contrast, we do not assume at the outset that firms do not optimize. Nonetheless, Al-Najjar et al. (2005) is a recent paper in the economic literature that squarely attacks the same puzzle we do, even though they arrive at different conclusions and justifications for full-cost pricing.

Netzer and Thepot (2008) also seek to understand the paradox, but work within a leader-follower game of a two-tier organization, namely, an upstream unit that produces capacity and a downstream unit that chooses output level. They find that full-cost pricing dominates other pricing rules in a Cournot oligopoly case, stemming from the two-tiered structure of their organization. Finally, Noreen and Burgstahler (1997) arrive at a negative result of full-cost pricing, namely, that in a multi-product firm with fixed costs, full-cost pricing does not necessarily lead to a satisfactory profit. Other papers seek to explain absorption costing practices (such as full-cost pricing) in a strategic environment, such as Alles and Datar (1998), Narayanan and Smith (2000), and Hughes and Kao (1998). In contrast, we seek to focus our analysis on the noncompetitive monopoly case.

There is a long literature in accounting on the relationship between the fixed capacity of investments to justify full-cost pricing. Surveys of this voluminous literature can be found in Balakrishnan and Sivaramakrishnan (2002). While we do not include a model of capacity in this paper (to focus on the pricing problem in isolation), the prior accounting models provide a wealth of detail that we take as given. The focus of this paper is on the convergence properties of full-cost pricing. There are other reasons the firm may use a full-cost approach, even if we do not model the capacity decision explicitly.

MARGINAL-COST PRICING

Consider a monopolist who sells a single product in a market. The firm sets the price $p > 0$, and market demand is $q(p) > 0$. Assume throughout that demand is decreasing, so $q'(p) < 0$ for all p. The firm's cost of producing q units of the product is $C(q) = F + \upsilon q$. Observe that the marginal cost $C'(q) = v$ is the constant variable cost of production. The fixed cost F includes fixed manufacturing overhead; fixed selling, general, and administrative (SG&A); and any other fixed costs of the firm.

The classical textbook analysis solves the inverse problem of a firm choosing quantity against inverse demand $p(q)$. Because our focus is on pricing, we will consider the more intuitive (although slightly more algebraically involved) formulation of the firm choosing price directly. The firm chooses price p to maximize income (profit) $y(p)$, where:

$$y(p) = pq(p) - C\Big(q(p)\Big) = (p - \upsilon)q(p) - F$$

We further assume the regularity condition that the firm's income function is strictly concave, so $y''(p) < 0$ for all p. This is equivalent to assuming that the firm's demand is not "too convex." For example, linear demand is strictly decreasing, but weakly concave. This second order sufficient condition guarantees a unique solution p^* to the firm's problem. The first order condition for the firm's problem gives the optimal price:

$$p^* = \upsilon - \frac{q(p^*)}{q'(p^*)} \tag{1}$$

This condition implicitly delivers the optimal price as a function of the demand curve. Optimal quantity is simply $q^* = q(p^*)$. Since demand slopes down, $q'(p) < 0$, so the optimal price is a positive markup from the firm's variable cost. In particular, this markup is a function of the elasticity of demand $\in = pq'(p)/q(p)$, and so $p^* = \upsilon/(1 + 1/\in)$, a rearrangement of the Lerner equation. The firm bases its markup purely on conditions of demand, charging higher markups when consumers are willing to pay more (less elastic demand). Importantly, the optimal price is *not* a function of F.

The benchmark model does not consider fixed costs in the pricing decision because it falls out of the firm's optimization problem. The firm chooses its price on the margin, balancing the marginal variable cost against marginal revenue, which the demand curve determines. Only firms with sufficiently low fixed costs will enter the market, namely, those with fixed costs less than the equilibrium contribution margin ($F < (p^* - \upsilon)q^*$). Firms that do enter the market will make non-negative profits in equilibrium. Thus, the fixed cost only affects the decision to enter, not the pricing decision.

STATIC FULL-COST PRICING

The survey evidence reviewed in the introduction suggests that firms routinely include fixed costs into their pricing decisions. This method of full-cost pricing directly conflicts with the benchmark model above, where the optimal price relies only on the variable cost and the elasticity of demand. In addition, there are informational requirements (discussed below) for FCP to achieve optimal pricing. However, FCP does have an advantage: it does not require knowledge of the demand curve. It only requires knowledge (or an estimate) of equilibrium returns. And, as we will show, it can lead to optimal price (despite doing so in a different way than the textbook model does).

First, we show a limited case: if the firm knows (or can estimate) its equilibrium income, then FCP immediately achieves (or estimates) optimal pricing if and only if the firm knows its equilibrium quantity. This necessary and sufficient condition entirely characterizes when static full-price costing can immediately (without experimentation) achieve (or estimate) optimal pricing. In the next sections, we will discuss dynamic FCP, which requires firms to *only* know returns (not quantities).

To fix ideas, recall that the firm faces demand $q(p)$ and optimizes income $y(p)$ over price, delivering an optimal price p^*, an optimal quantity $q^* = q(p^*)$, and optimal (equilibrium) income $y^* = y(p^*)$. The optimal price, quantity, and income all follow from the demand curve, which the firm takes as given. Solving for p^* at this y^* delivers:

$$p^* = \upsilon + \frac{F + y^*}{q^*}.$$

This is simply the price that maximizes income. It is also a rewriting of the identity that price equals variable cost plus contribution margin per unit, since contribution margin per unit is precisely price minus variable cost. Thinking of the pricing function in this formula, as the sum of variable cost and contribution margin per unit, will be a constant theme throughout the paper. Since the income function is strictly concave, this price p^* is unique. Of course, the firm cannot simply pick this p^* since it does not know $q^* = q(p^*)$. But it does have estimates of income and quantity, with unbiased estimators $\hat{y}$ and $\hat{q}$, respectively (so $E\hat{y} = y^*$ and $E\hat{q} = q^*$). The signal $\hat{q}$ is a forecast of quantity, which the firm uses to plan production and sales, while the signal $\hat{y}$ is the firm's estimate of its equilibrium income, based on historical market data and subjective priors. Therefore, consider the candidate estimator of optimal price:

$$\hat{p} = \upsilon + \frac{F + \hat{y}}{\hat{q}}, \tag{2}$$

where the estimators $\hat{q}$ and $\hat{y}$ replace their equilibrium counterparts.

Observe that $\hat{p}$ is a markup on variable cost, but unlike the markup from the benchmark model, which is based on the elasticity of demand, this markup contains the fixed cost plus equilibrium income. This is precisely FCP, since it includes the fixed cost in the price. More specifically, the price is a function of the unit cost under absorption costing, $\upsilon + \frac{F}{\hat{q}}$. Absorption costing includes fixed manufacturing overhead per unit in the unit cost, and is both required by U.S. generally accepted accounting principles (GAAP) and taught extensively in managerial accounting textbooks. This full-cost price $\hat{p}$ immediately achieves the optimal price p^* under the following condition (all proofs in Appendix A):

Proposition 1: Suppose the firm has an unbiased estimator of equilibrium income ($E\hat{y} = y^*$). The static full-cost price is an unbiased estimator of the optimal price ($E\hat{p} = p^*$) if and only if the firm knows equilibrium quantity.

This proposition shows that the firm's knowledge of its equilibrium quantity completely characterizes whether FCP can achieve (or unbiasedly estimate) optimal pricing. Of course, if the firm knows its equilibrium quantity q^*, it is clear from the equations for $\hat{p}$ and p^* above that the two will coincide, on average, since $\hat{y}$ is unbiased. More surprising is that *anytime* FCP achieves optimal pricing $\left(E(\hat{p}) = p^*\right)$, it must be the case that the firm knows its equilibrium quantity. The proof relies on a simple application of Jensen's inequality.

DYNAMIC FULL-COST PRICING

While static FCP requires strong informational assumptions to achieve optimal pricing, this is not the case for dynamic pricing. Dynamic pricing refers to the process of experimentation that firms routinely engage in so they are able to "search" for the optimal price. Indeed, firms rarely pick a single price and hold it forever, but instead may change these prices over time. Some of the initial changes may occur in the lab based on focus groups or in sample populations, but many of the changes occur in real markets over time. Here, we show that if the firm sets full-cost prices over time, then these prices will converge toward the optimal price. This is the best defense of the practice of FCP, namely, that it allows firms to eventually reach their optimal price. And the firm can do this without any knowledge of its equilibrium quantity q^*, using only knowledge of y^*.

To gain traction on the dynamic pricing problem, we first suppose that the firm knows its equilibrium income. This will lay bare the intuition behind the convergence algorithm. After that, we consider the more general case, when the firm does not know its equilibrium income precisely, but has an estimate of it. We view this later case as a realistic and plausible scenario.

Convergence When Income is Known

Suppose the firm knows its equilibrium income y^*. Recall that at the optimal price $y(p^*) = y^*$, or $(p^* - \upsilon^*)q^* - F = y^*$. This is simply the identity for equilibrium income. In other words, equilibrium income is simply income evaluated at the optimal price p^*. Solving this equation for price shows that:

$$p^* = \upsilon + \frac{F + y^*}{q^*} \tag{3}$$

Consider a candidate convergence algorithm that simply replaces q^* with $q(p_t)$ and p^* with p_{t+1}. It is natural to suppose this price might converge to the optimal price, since the optimal price is the unique price that satisfies the identity in Equation (3), above. Below, we define this algorithm and explore its convergence properties.

First, let $p_0 = \upsilon$ be the starting price. Then, set:

$$p_{t+1} = \upsilon + \frac{F + y^*}{q(p_t)} \tag{4}$$

This defines an algorithm that delivers a price path. The initial value is the firm's marginal cost (variable cost), which the firm knows. Also, since demand slopes down, the price path is an increasing sequence. Starting below optimal price ($p_0 = \upsilon < p^*$) assumes convergence from below. We can rewrite the equation above as:

$$Price = Marginal\,Cost + Equilibrium\,Contribution\,Margin\,per\,Unit \tag{5}$$

where the "per unit" is evaluated at $q(p_t)$, demand based on the price in the prior stage.

Observe that each price in the price path in Equation (4) is a full-cost price. The "new" price p_{t+1} is a function of the "old" price, p_t. In particular, it is a markup on variable cost, but the markup itself, unlike the classical model, relies not on the elasticity of demand, but instead on the fixed cost and equilibrium income. Furthermore, $\upsilon + F/q(p_t)$ is the unit cost under absorption costing. The pricing algorithm sets the new price equal to the unit cost plus unit profit, where the number of units is determined by demand at the old price, p_t. The market then demands $q(p_t)$ and the firm sets its new price, p_{t+1}, based on the formula for p_{t+1} above. The firm then iterates in this manner in every subsequent stage. The next proposition shows that this converges to the optimal price:

Proposition 2: Suppose the firm knows its equilibrium income y^*. The full-cost price p_t converges to the optimal price p^*.

The pricing algorithm, thus, has the attractive property that it converges to p^*. It is also based on real accounting practice: GAAP requires absorption costing, and the proposition shows that pricing based on absorption costing leads to optimal outcomes. The proof of the proposition gives conditions under which the full-cost price p_t converges to the optimal price p^*. The price path is a nonlinear first order difference equation. It is nonlinear because of the nonlinear demand curve $q(p_t)$, and it is first order because it connects p_{t+1} to p_t. And since $p_0 = \upsilon < p^*$, FCP converges to p^*, as desired.

Convergence When Income is Unknown

Suppose the firm does not know its equilibrium income precisely, but must estimate it based on historical market data and internal cost information. Assume the firm has an estimate $\hat{y}$ of its equilibrium quantity y^*. We consider the case where this estimator is consistent for y^*. We will treat the case of unbiasedness next (see Figure 1).

The convergence algorithm will be similar, although not identical, to before. Rather than using equilibrium income y^*, the firm will use its estimate $\hat{y}$ in its place. The convergence algorithm works as follows. First, the firm picks price $p_0 = \upsilon$. Then, for each $t \geq 0$, the firm selects the new price as:

$$p_{t+1} = \upsilon + \frac{F + \hat{y}}{q(p_t)}$$

Just as before, this is FCP because it includes the fixed cost in the pricing algorithm. Also, as before, the price is simply the unit price under absorption costing plus the estimated profit per unit. The firm need not know its equilibrium quantity q^*, since it endogenously generates demand through the prices p_t it offers in the market. This price path now has additional uncertainty because it relies on an estimate of income, rather than its true value. Nonetheless, provided that this estimate is consistent, we still arrive at convergence.

Proposition 3: If $\hat{y}$ is a consistent estimator of y^*, then the full-cost price p_t converges to the optimal price p^*.

The proof mirrors the one from the earlier section, with the only difference in the nature of convergence. Instead of relying on the true value y^*, the proof uses convergence in probability, since the estimator $\hat{y}$ is a consistent estimator of y^*. The logic is similar. The firm forms the sample price path as before, now using the estimator $\hat{y}$ in place of equilibrium income y^*. The steady-state equilibrium of the dynamic model satisfies $y(p) = \hat{y}$, which converges in probability to the optimal income y^*. And because the income function is strictly concave and has a unique solution, the steady-state converges to the optimal price as the number of draws of the estimator tends toward infinity. Note that there are two relevant dimensions here. The first is time, which indexes the price path as the firm experiments with different prices. The second is the number of draws of the estimator $\hat{y}$. The probability limit refers to the behavior of the estimator as the number of draws becomes large. Proposition 2 shows that the price path p_t converges to the optimal price p^* as the number of draws tends toward infinity. This is a theoretical property describing the accuracy of the price path p_t based on the accuracy (consistency) of the estimator $\hat{y}$.

Unbiased Estimator of Income

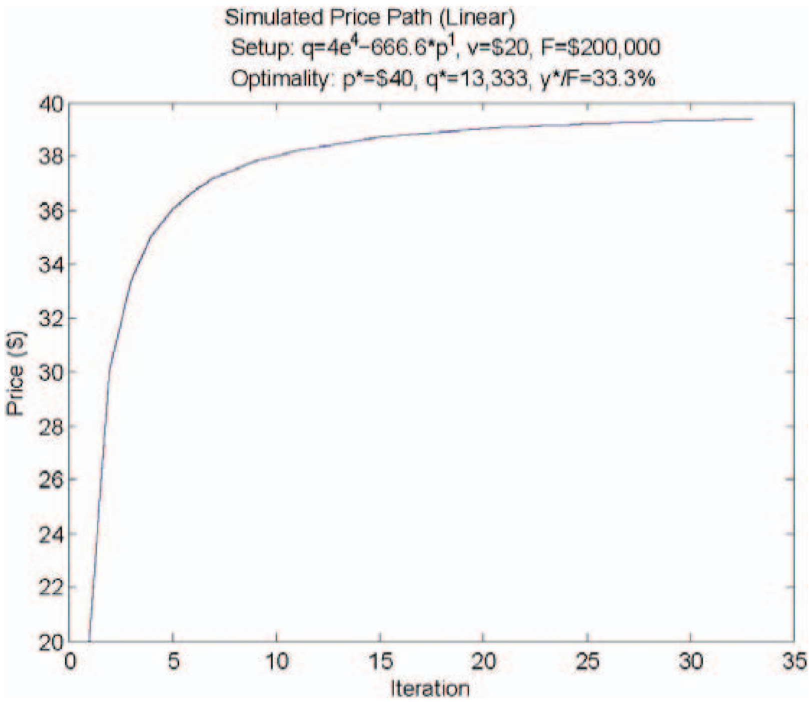
The criterion chosen for the estimator above, consistency, was deliberately strong. A weaker criterion does not lead to convergence. If $\hat{y}$ is unbiased, so $\left(E(\hat{y}) = y^*\right)$, then the full-cost price will not converge to the optimal price:

Proposition 4: If $\hat{y}$ is an unbiased estimator of y^*, the full-cost price p_t does not converge to the optimal price.

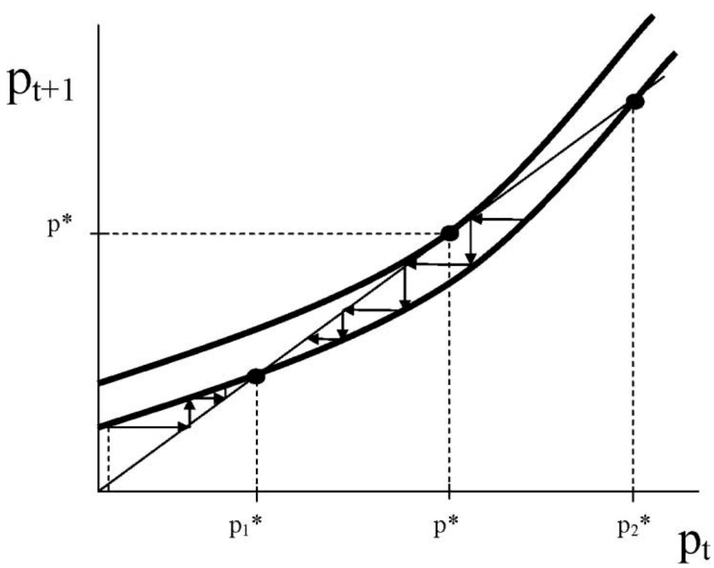
Recall that expectation is a linear operator, whereas the profit function is strictly concave. Because of this, the expectations operator cannot pass inside of the profit function, and this disrupts the convergence proof. The strict concavity of the profit function, combined with Jensen's inequality, delivers a contradiction, which prevents convergence. This was not a problem when the criterion for the estimator was consistency because probability limits can pass seamlessly between nonlinear

FIGURE 1
Convergence under Linear Demand

Panel A: Simulated Price Path



Panel B: Phase Diagram



Panel A plots the simulated price path and Panel B plots the phase diagram. The full-cost price p_t converges to p^* in probability, but to p_1^* in expectation.

functions. Thus, FCP has informational benefits (relative to economic pricing) and it does converge to optimal pricing in a dynamic setting, but only when we use a different criterion for the estimator (consistency rather than lack of bias).

ALTERNATIVE CONVERGENCE METHODS

How does FCP compare against alternative convergence methods? Suppose the firm chooses to find its profit-maximizing price p^* through experimentation of its first order condition. This will provide direct feedback to the firm, since the profit-maximizing price must occur when its first order condition equals zero. To see this, the first derivative of the firm's profit function is:

$$s(p) := q(p) + (p - \upsilon)q'(p) \tag{6}$$

By definition of the optimal price p^*, $s(p^*) = 0$. Consider the following simple algorithm. Begin at some candidate value (initial condition) p_0. Given some price p_t, select a new price:

$$p_{t+1} = p_t + s(p_t). \tag{7}$$

Thus, if the slope at any given price is positive, the firm will increase its price, moving the candidate price closer to the "top of the hill." If the price exceeds the optimal price, the derivative at that point will be negative, instructing the firm to reduce its price. This works since the profit function is globally concave and has a unique optimum—the "top of the hill." Indeed, this algorithm is similar in spirit to numerical methods used in practice to find global maxima using the first order condition as a guide for directional movements (Judd 1998).

Notice that this first order approach uses no information on the fixed cost, nor on equilibrium y^*. It does require knowledge of the first derivative $s(p)$, which, in turn, requires knowledge of the marginal-demand curve $q(p)$, but this can be easily estimated through $\frac{q(p+\varepsilon)-q(p)}{\varepsilon}$, for some ε that is arbitrarily close to 0 and small. This method employs less information than the FCP, but comes at a cost. The first order approach does not converge for a wide class of demand curves. The next proposition shows this under linear demand:

Proposition 5: Under linear demand $q(p) = a - bp$, the first order approach converges to optimal price p^* if and only if $0 < b < 1$.

This is a restriction on the slope of the demand curve. If the parameters on demand fall outside this range, demand is excessively inelastic (b is high), and the price path will diverge. Otherwise, it will converge to the optimal price. The proposition shows convergence only when the coefficient on the first order difference equation has absolute value < 1, a standard criterion for convergence of numerical sequences.

Indeed, the main advantage of this first order approach is its freedom from any knowledge of the firm's fixed cost or equilibrium income that FCP requires. Its disadvantage is that it does not converge under all conditions. Thus, there is a trade-off when selecting convergence algorithms. Either the firm can use FCP, which works under any demand environment, but requires some knowledge of y^*, or it can employ the first order approach, which does not require such knowledge, but has use over a limited class of demand curves. This is a question of where the firm's information is greater. Ultimately, the firm will need to balance knowledge of equilibrium income against knowledge of demand. If the firm seeks to implement its pricing algorithm in as wide a market environment as possible, then FCP can do that, provided the firm has some estimate of equilibrium income.

CONCLUSION

Economics and accounting offer different views on optimal pricing. In the canonical economic model, fixed costs need not be (and should not be) involved in pricing decisions. Notwithstanding this view, it is commonplace in both accounting texts and actual practice to incorporate fixed costs into the pricing decision. We attempt to resolve, in a small way, this tension. Incorporating fixed costs into pricing will, in many cases, lead to optimal prices. In particular, economics-based pricing requires information about underlying demand, and that this information be precise (not estimated). Alternatively, FCP only requires firms to have reliable estimates of equilibrium income. With FCP, firms may still need to undergo some experimentation with prices. But that experimentation is directed by their estimate of income, and will converge to the optimal price.

The model is robust to a number of extensions. In particular, FCP still achieves the optimal price, even in a multi-product firm. Such firms often have complementarities between products, such as an auto manufacturer that makes both cars and trucks. With such complementarities, the price of one product may affect the demand for another. But because the full-cost price is based on equilibrium income and not on any first order condition, these cross-price externalities do not interfere with the convergence properties of the full-cost price. The algorithm is more complex, since the full-cost price at each stage is a vector rather than a scalar, but it converges nonetheless. A second extension is that FCP still converges if the firm decides not to optimize, but to satisfice, i.e., achieve a minimum level of income. We provide conditions under which the full-cost price meets this more relaxed criterion: its forecast of equilibrium quantity must be biased downward. Finally, FCP works under more general non-constant marginal costs, although the formulas become naturally more complex.[1]

We have assumed a monopolist pricing against a demand curve; a natural question is the effect of competition on FCP. Since competitive firms price at marginal cost by definition, this would require specifying more explicitly the production function of the firm. Identifying the universe of production functions under which FCP works would merit further research.[2] While we have considered comparison to one alternative algorithm based on the first order condition, there are, of course, many other possible methods to explore, and the large toolkit of methods in numerical methods in computational economics can serve as a springboard.

The debate on why firms do not actually set prices according to economic theory has persisted for decades. The classic defense of economic theory rested on the "as if" argument: firms behave as if they are solving an optimization problem and setting prices accordingly even though they are not. Our concern here is to model what firms actually do. The extensive survey evidence suggests that they factor fixed costs into their prices. FCP provides an explicit way to do this and still achieves the optimal price from economic theory. We have used no behavioral assumptions, nor have we assumed knowledge of the demand curve, as the benchmark model of economic theory does. This paper is, in fact, a defense of the "as if" argument, since we provide the actual process by which firms can price and, therefore, maximize profits. The longstanding paradox between economic theory and real accounting practices seems less puzzling in light of these observations.

REFERENCES

Al-Najjar, N., S. Baliga, and D. Besanko. 2005. *The Sunk Cost Bias and Managerial Pricing Practices*. Unpublished manuscript, Northeastern University.

Alles, M., and S. M. Datar. 1998. Strategic transfer pricing. *Management Science* 44: 451–461.

Balakrishnan, R., and K. Sivaramakrishnan. 2002. A critical overview of the use of full-cost data for planning and pricing. *Journal of Management Accounting Research* 14: 3–31.

Bright, J., R. E. Davies, C. A. Downes, and R. C. Sweeting. 1992. The deployment of costing techniques and practices: A UK study. *Management Accounting Research* 3 (3): 201–211.

Cooper, R. 1990. Explicating the logic of ABC. *Management Accounting* 68 (10): 58–60.

Drury, C., and M. Tayles. 2000. *Cost Systems and Profitability Analysis in UK Companies: Selected Survey Findings*. Paper presented to Annual Congress of the European Accounting Association. Available at: http://hdl.handle.net/10068/579928

Emore, J. R., and J. A. Ness. 1991. The slow pace of meaningful change in cost systems. *Journal of Cost Management for the Manufacturing Industry* 4: 36–45.

Friedman, M. 1953. *Essays in Positive Economics*. Chicago, IL: The University of Chicago Press.

Gordon, L., R. Cooper, H. Falk, and D. Miller. 1981. *The Pricing Decision*. A study carried out on behalf of the National Association of Accountants, New York, N.Y., and the Society of Management Accountants of Canada, Hamilton, Ontario, Canada. Montvale, NJ: National Association of Accountants.

Govindarajan, V., and R. N. Anthony. 1983. How firms use cost data in pricing decisions. *Management Accounting* 65: 30–37.

Hughes, J., and J. Kao. 1998. Cross subsidization, cost allocation, and tacit coordination. *Review of Accounting Studies* 2: 265–293.

Isaacson, W. 2011. *Steve Jobs*. New York, NY: Simon & Schuster.

Judd, K. L. 1998. *Numerical Methods in Economics (Scientific and Engineering)*. Cambridge, MA: The MIT Press.

Lucas, M. R. 2003. Pricing decisions and the neoclassical theory of the firm. *Management Accounting* 14: 201–207.

Narayanan, V., and M. Smith. 2000. Impact of competition and taxes on responsibility center organization and transfer price. *Contemporary Accounting Research* 17: 497–529.

Netzer, J.-L., and J. Thepot. 2008. On the optimality of the full-cost pricing. *Economic Behavior and Organization* 68: 282–292.

Noreen, E. W., and D. C. Burgstahler. 1997. Full cost pricing and the illusion of satisficing. *Journal of Management Accounting Research* 7 (1).

Scapens, R. 1983. Closing the gap between theory and practice. *Management Accounting* 61: 34–36.

Shim, E. 1993. *Cost Management in Activity Based Costing in a New Manufacturing Environment*. Unpublished dissertation, Rutgers, The State University of New Jersey.

Shim, E., and E. Sudit. 1995. How manufacturers price products. *Management Accounting* 76 (8): 37–39.

[1] Details on all these extensions are available on request.
[2] For example, the FCP assumptions fail under Cobb-Douglas production under a capacity constraint. Thanks to an anonymous referee for the counterexample.

APPENDIX A

Proof of Proposition 1

The firm knows its equilibrium quantity if the estimator $\hat{q} = q^*$ for all possible realizations of $\hat{q}$. Thus, the proposition states that $\hat{q} = q^*$ is a necessary and sufficient condition for $E(\hat{p}) = p^*$.

Sufficiency

Suppose $\hat{q} = q^*$. Then the static full-cost price is $\hat{p} = \upsilon + \frac{F+\hat{y}}{q^*}$. Taking expectations over both sides:

$$E(\hat{p}) = \upsilon + \frac{F + E\hat{y}}{q^*} = \upsilon + \frac{F + y^*}{q^*} = p^*.$$

Necessity

We prove the contrapositive. Suppose $\hat{q} \neq q^*$ for some realization of the random variable $\hat{q}$. So $\hat{q}$ has a nontrivial distribution. Therefore:

$$E(\hat{p}) = \upsilon + E\left[\frac{F + \hat{y}}{\hat{q}}\right] > \upsilon + \frac{F + y^*}{E(\hat{q})} = p^*,$$

since $\hat{q}$ is unbiased $\left(E(\hat{q}) = q^*\right)$. The inequality follows from Jensen's inequality since the function $f(x) = \frac{1}{x}$ is convex. Thus, $E(\hat{p}) \neq p^*$. ■

Proof of Proposition 2

Here, we prove the more general claim that p_t converges to p^* iff $p_0 < p^*$. This stronger statement implies Proposition 2, since $p_0 = \upsilon < p^*$ by definition of p^*. The convergence algorithm is given by:

$$f(p_t) = p_{t+1} = \upsilon + \frac{F + y^*}{q(p_t)} \textit{ for each } t \geq 0 \tag{8}$$

Observe that:

$$f'(p_t) = \frac{-(F + y^*)q'(p_t)}{q(p_t)^2} > 0$$

for each p_t, since $q'(p_t) < 0$. Furthermore, observe that $f(0) > \upsilon > 0$. Thus, the function $f(p_t) = p_{t+1}$ is the phase diagram for the price path, which starts at a constant above v and increases. In a steady-state equilibrium, $f(p) = p$. Plugging this into (8) gives:

$$f(p) = \upsilon + \frac{y^* + F}{q(p)} = p$$

Rearranging this equation:

$$(p - \upsilon)q(p) - F = y(p) = y^* = y(p^*)$$

Since the income function y is strictly concave, it has a unique maximizer. Therefore, $p = p^*$. So the optimal price p^* is the unique steady-state of the price path. Next, we show $p_0 < p^*$ is necessary and sufficient for convergence.

Sufficiency

Suppose $p_0 < p^*$. Let $g(x) = x$ be the 45-degree line. Observe that $g(p^*) = p^* = f(p^*)$. Because f is increasing, continuous, and because $f(0) > \upsilon > 0 = g(0)$, $f(p) > g(p)$ for each $p < p^*$. Therefore, for each $t \geq 0$:

$$p_{t+1} = f(p_t) > g(p_t) = p_t.$$

So the sequence p_t is increasing. Recall that $f(0) > \upsilon > 0 = g(0)$ and $f(p^*) = p^* = g(p^*)$. Now f and g are both continuous and monotonic, and because f starts at a higher value than g, but ends at the same point, for any p_t and p_{t+1}:

$$f(p_t) - f(p_{t-1}) < g(p_t) - g(p_{t-1}).$$

Substituting in the definitions for f and g, this inequality reduces to:

$p_{t+1} - p_t < p_t - p_{t-1}.$

Thus, the differences in the sequence shrinks in t. Hence, p_t converges to p^*.

Necessity

We prove the contrapositive. Now suppose $p_0 > p^*$. Observe that:

$$f''(p) = \frac{F + y^*}{q(p)^2}\left(\frac{2q'(p)^2}{q(p)} - q''(p)\right)$$

Thus:

$$f''(p) > 0 \textit{ iff } 2q'(p)^2 > q''(p)q(p) \tag{9}$$

since $q'(p) < 0$. The first order condition for p^* is $p^* = \upsilon - \frac{q(p^*)}{q'(p^*)}$. Recall that $y(p) = (p - \upsilon)q(p) - F$. The second derivative is $y''(p) = (p - \upsilon)q''(p) + 2q'(p)$. The second derivative evaluated at p^* is:

$$y''(p^*) = -\frac{q(p^*)}{q'(p^*)}q''(p^*) + 2q''(p^*).$$

Income is globally concave, so $y''(p^*) < 0$. Rewriting, this means $q'(p^*)^2 > q(p^*)q''(p^*)$. Referencing (9), this means $f''(p^*) > 0$. Hence, f is convex at p^*, and p^* is the unique fixed point of f. Therefore, f does not cross g at any other point, and $f(p) > g(p)$ for all $p > p^*$. Therefore:

$p_{t+1} = f(p_t) > g(p_t) = p_t.$

Thus, p_t is an increasing sequence. Now, because f is convex, and $f(p^*) = g(p^*)$, $f'(p) > g'(p)$ for all $p > p^*$. So, for any p_t and p_{t-1}:

$p_{t+1} - p_t = f(p_t) - f(p_{t-1}) > g(p_t) - g(p_{t-1}) = p_t - p_{t-1}.$

Thus, the differences in p_t increase in t. Therefore, this increasing sequence of increasing differences diverges. ■

Proof of Proposition 3

Consider the following convergence algorithm. Pick some initial value p_0. For all $t \geq 0$, let:

$$p_{t+1} = \upsilon + \frac{F + \hat{y}}{q(p_t)}.$$

In a steady-state solution, $p_{t+1} = p_t = p^*$ for all t and, therefore, the equation above becomes:

$$p^* = \upsilon + \frac{F + \hat{y}}{q(p^*)}$$

Rearranging, $y(p^*) = \hat{y}$. Taking the limit as the number of draws of $\hat{y}$ goes to infinity:

$y(\operatorname{plim} p) = \operatorname{plim} y(p) = \operatorname{plim}(\hat{y}) = y^* = y(p^*)$

Because the income function is strictly concave, it has a unique solution. Therefore, plim $p = p^*$. The rest of the argument for convergence is identical to the proof of Proposition 2. ■

Proof of Proposition 4

Take some initial p_0. For each $t \geq 0$, let $p_{t+1} = \upsilon + \frac{F+\hat{y}}{q(p_t)}$.

Suppose the price p_t does converge to some p. In the steady-state equilibrium, $p_{t+1} = p_t = p$. Plugging this into the equation above and rearranging, we have $y(p) = \hat{y}$. Now take expectations over all realizations of $\hat{y}$:

$y\big(E(p)\big) > E\big(y(p)\big) = E(\hat{y}) = y^* = y(p^*),$

where the inequality follows from Jensen's inequality since the firm's income function is strictly concave. But p^* maximizes y, so $y(p^*) \geq y(z)$ for all z, and in particular for $z = Ep$. Contradiction. ■

Proof of Proposition 5

The first derivative of the profit function is:

$$s(p) := q(p) + (p - \upsilon)q'(p) \tag{10}$$

By (FOC), $s(p^*) = 0$. Recall $y''(p) = s'(p) < 0$. Given some starting value p_0, the first order algorithm at some price p_t is:

$$h(p_t) := p_{t+1} = p_t + s(p_t) \tag{11}$$

The steady-state condition is $h(\hat{p}) = \hat{p}$. Solving into the equation above shows that $s(\hat{p}) = 0$. Because the profit function is globally concave and has a unique maximizer, $s(p^*) = 0$ means $\hat{p} = p^*$. Thus, if the algorithm converges, it converges to the optimal price. Now examine convergence.

Under linear demand, we know $q(p) = a - bp$. The optimal price is $p^* = \frac{a+b\upsilon}{2b}$. Impose the regularity condition $a > b\upsilon$, so $p^* > \upsilon$. Rewriting the first derivative with respect to linear demand:

$$s(p) = a - 2bp + b\upsilon \tag{12}$$

Thus, the algorithm is:

$$h(p) = p_{t+1} = p_t(1 - 2b) + a + b\upsilon \tag{13}$$

This is a first order linear difference equation. It is a well-known result that this sequence p_t converges if and only if $|1 - 2b| < 1$, or $0 < b < 1$. Observe that the convergence occurs if the price path p_t tends to 0 as expected, since the first order condition requires $s(p^*) = 0$. ■