Risk and Volatility: *A Differential View*

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Over the past century, there has been a raging debate in the investment and academic world on the proper definition of risk. The first view emerges from practice, from investors like Benjamin Graham and Warren Buffett, who define risk as the loss of capital. The second view stems from academic finance, which ultimately defines risk as volatility of asset returns. This article seeks to make explicit these different approaches, to understand their relationship, and to reconcile them when possible. My aim is to shed light on the vague and somewhat ambiguous treatment of the most central component of security analysis: the proper definition of risk.

Ever since the early works of Graham and Dodd, legions of (value) investors have defined risk as the permanent loss of capital (Brandes [2004] and Graham and Zweig [2003]). In *The Intelligent Investor*, Graham writes about the need to focus on price and value, and thereby advocates for a significant margin of safety when investing (Graham and Zweig [2003] and Graham et al. [1988]). Graham's definition leans only on the purchase price (which is observable) and the true value of the firm (which is unobservable, but can be estimated by the analyst). Warren Buffett tells the story of the Washington Post Company in 1973, which sold for $80 million, even though its book value was at least $400 million (Buffett and Cunningham [2001]). Had the stock price dropped to $40 million, measured past volatility would have increased; yet, the chance of future permanent loss of capital would be even smaller, assuming a true $400 million valuation of the company. In practice, the estimate of the intrinsic value of the company is vital to this assessment of risk.[1] Yet, the academic conception of risk did not focus on the valuation problem, but instead went toward variability of outcomes, to which I now turn.

The early conception of risk came from academic economics and decision theory (Von Neumann [2001]). This concept of risk connects closely to utility theory, a mathematical representation for how a rational agent values uncertain gambles. A key aspect of expected utility theory is the concept of states of the world, or states of nature. The outcome space considers different events, each with a well-defined probability. For example, Ford can have a boom or bust year—these are the different states of the world; they are mutually exclusive and occur with different probabilities. That probability is either objective (based on past data or repeated observations) or subjective (the personal assessment of an individual investor). Expected utility theory takes the average utility of each outcome, weighted by their probabilities, across these different states of the world.

With these tools developed with the help of mathematicians, economists developed a theory of uncertainty, which became the theory of risk.[2] Previously, risk was uncertainty over the gambles described previously. The mathematical apparatus included the notion of a mean–variance preference. In this case, computing the mean and variance of underlying distribution conveniently represented the preference for a risky asset.

Academic finance began with basic tools of expected utility put forth by economists (Bodie, Kane, and Marcus [2005] and Ross, Westerfield, and Jaffe [2002]). There are different states of the world, each of which occurs with exogenous probabilities. The expected payoff from a security is the weighted average across these states of the world.

Expected utility theory had one basic flaw. There was no clear measure of these states of the world, nor of their probabilities. Ultimately, uncertainty was in the eye of the beholder, and two different investors could assess different probabilities for different states of the world. As such, actually computing forward-looking expected return in an uncontroversial way was impossible. Enter the empirical, data-driven financial economist. What can we observe about stock prices? We can certainly observe historical returns computed annually, monthly, or daily. Once we have that return data, we can easily compute statistical moments. Thus, the notion of volatility—the statistical variance around past return data and stock prices—was born.

As an example, Ibbotson [2013] considers total annual returns from U.S. stocks from 1926–2002, and computes the average return of large company stocks at 12.2% and small company stocks at 16.9%. The author then calculates the standard deviation of 20.5% on large caps and 33.2 % on small caps. Over a long past period, small company stocks had higher average returns, but also higher statistical *variance*. This is not necessarily a statement about risk. In isolation, this says nothing about the underlying uncertainty of the business, or even a rational agent's valuation of different states of the world. Rather, it is a measure of the dispersion of past return data.

THE MODEL

Consider how a value investor assesses risk. This investor seeks to preserve capital. Therefore, a risky investment is one that erodes capital. To fix ideas, I will consider a sample investment. This asset can be of any nature, though it is easiest to consider it as equity, rather than debt. The price of this asset is determined stochastically with the following form:

$$p = (1 + g)p_0 + \varepsilon, \tag{1}$$

where g is the equilibrium growth rate of the asset, p_0 is the intrinsic value, and ε is a noise term that follows a symmetric distribution f, with mean 0 and variance σ^2. Impose the regularity condition that $f > 0$ everywhere and $f'(x) > 0$ if and only if $x < 0$. The growth rate of the asset is exogenous and known to the investor.[3] It can also represent growth in residual earnings, or earnings per share, although I assume this is impounded into the price of the asset; it is unobservable from the data, and the investor must estimate it. The noise term, ε, captures all that is unknown to the investor and out of his control. Because the noise has a mean of 0, the expected price is $Ep = (1 + g)p_0$. To ease analysis, there is no explicit time dimension. The term σ^2 is the statistical variance on the random variable price. As such, σ^2 is the volatility of the asset price.

The investor has the option of purchasing the asset. If so, he buys the asset at an entry price, p_b (for buy price). This investor seeks to minimize the chance of a loss, which occurs if he decides to sell below his purchase price. Define risk as the probability of a loss; for a given purchase price, p_b, the risk at that price is:

$$R(p_b) = \textit{Probability of } [p < p_b] \tag{2}$$

Importantly, risk is a function of purchase price. Defining risk generally makes no sense, since risk depends on the price something is bought.[4] Thus, we arrive at our first result:

Proposition 1. *To a value investor, risk decreases as the purchase price falls, as the growth rate rises, and as the intrinsic value rises.*

The intuition to this result is simple. As the purchase price increases, there is a higher probability that the asset price will lie below the purchase price for a given growth rate and intrinsic value. Because the price of the asset is stochastic, there is uncertainty in whether $p < p_b$. Nonetheless, for virtually any distribution, the probability that $p < p_b$ will always increase in p_b. As for the growth rate, the proposition states that high-growth firms are actually *less* risky. A high-growth firm is more likely to ultimately lead to a higher price, since expected price is increasing in growth. There is a higher chance of a capital gain and, therefore, lower risk. Thus the classic distinction between growth and value is not as large as it seems. Here, growth and value are concepts that simultaneously affect investment risk. Higher growth and higher value both decrease risk. We can now state a more subtle result on the relationship between purchase price and growth:

Proposition 2. *For investments with positive expected return, the sensitivity of risk to purchase price* $\left(\frac{\partial R}{\partial p_b}\right)$ *decreases in the growth rate* (g).

The condition in the hypothesis occurs when the expected price Ep exceeds the purchase price p_b. This is similar to a positive net present value (NPV) investment.[5] Recall from Proposition 1 that risk decreases in purchase price. The sensitivity of risk to purchase price is given by the partial derivative. The Proof of Proposition 2 shows that this partial derivative decreases in g, the growth rate of the asset.

Volatility from Financial Economics

How would a financial economist value the asset and its risk? Surely this economist would not use the risk metric $R(p_b)$ developed previously, but rather the standard tool kit used by financial economics. Using the same model, the asset price continues to be $p = p_0(1 + g) + \varepsilon$, where ε is distributed according to f.

At this point, the economist would need to specify the preferences of the agent. A tractable, common, and noncontroversial utility function is constant absolute risk aversion, so that there are no wealth effects (wealth does not affect the evaluation of risk).[6] These preferences under normally distributed wealth are equivalent to mean–variance preferences, where the preferences are a linear function of the mean and variance of returns. Therefore, the rational agent will value the asset

$$EU = E(Returns) - \frac{r}{2}Var(Returns) \qquad (3)$$

where r is the coefficient of absolute risk aversion (independent of wealth) and where $Returns = (p - p_b)/p_b$. Observe that the agent does not simply maximize expected return, but also maximizes expected payoff, which includes the variance on returns. I use expected payoffs as synonymous with expected utility. This standard framework generates the following.

Proposition 3. *The expected payoff from an investment increases in the growth rate and decreases in volatility. Expected payoff decreases in purchase price only for extreme values.*

The first result derives from the definition of expected utility. The expected payoff function is increasing in expected price and growth rate, while it is decreasing in volatility, by assumption. It is easy to see that increasing the purchase price decreases expected returns (whenever expected return is positive, i.e., the investment is profitable). In this sense, the model aligns with the payoff to the value investor, since increasing purchase price will erode returns.

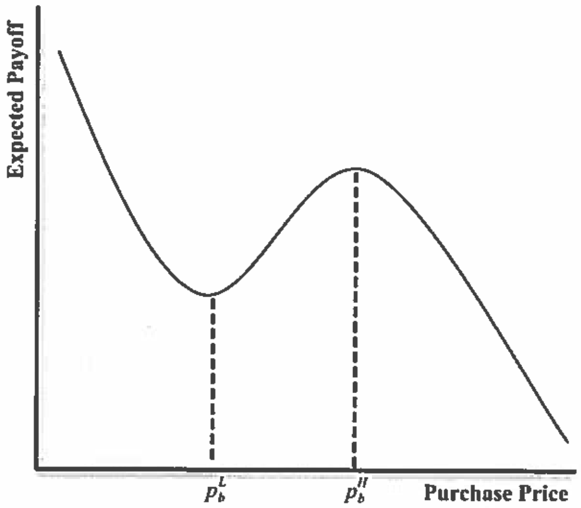
But the countervailing effect is on volatility. The variance of returns decreases in purchase price, so when the purchase price increases, the variance on return is reduced.[7] This is an artifact of the mean–variance preferences of the agent. If the agent truly likes expected return and dislikes variance in those returns, then buying at a lower price will raise the variance on returns. Because the agent dislikes variance, this will have a negative effect on the agent's payoff. In the proof of the proposition, I provide the range within which the disutility from the increased variance is so large that the agent actually prefers to increase the purchase price just to avoid this variance. This occurs within a bounded interval; outside of this interval (for very high or very low purchase prices), the payoff functions behave identically to the value investor's: higher entry prices cause lower expected payoffs.

Finally, Exhibit 1 gives a picture of the agent's expected payoff as a function of this purchase price. Observe that the relationship between payoff and purchase price is not monotonic. The interval in the picture shows where exactly expected payoff increases in purchase price, because of the high disutility from variance. If we assume mean–variance preferences, this picture portrays how a rational agent's payoff varies with the purchase price of the security.

If you assume that volatility captures risk, then indeed, investors dislike volatility. However, if risk is a separate concept altogether, then volatility has nothing to do with true risk, and from the preferences above, it is clear that the textbook economic agent cares nothing about the purchase price and does not assess risk in a similar way as the investor.

EXHIBIT 1

Expected Payoff vs. Purchase Price



RECONCILIATION

What is the relationship between risk and volatility? Recall that we have defined risk formally in Equation 1. We represent volatility with σ^2. Many observers have noticed that while there may be a negative relationship between *volatility* and returns, there may not necessarily be a relationship between *risk* and return, where risk is defined as capital loss. I formalize this in the next proposition:

Proposition 4. *Volatility = risk if and only if the stock has a positive expected return.*

Risk and volatility are different notions, and this proposition gives conditions under which they are the same. The condition is that the expected return must be positive. If so, then a highly volatile asset is also high risk, and if not, then a highly volatile asset is low risk. This should conform with intuition. The proof of the result leans on a mean preserving spread of the distribution, which essentially places more weight in the tails for a high-variance probability distribution. Of course, what matters for the value investor's definition of risk is the cumulative distribution function, as well as its comparison with low variance versus high variance. This generates the condition that the expected return must be positive for the two concepts to align.

To see a concrete example, consider the Ford Motor Company in the decade from January 1, 2004 to January 1, 2014. Panel A of Exhibit 2 plots its daily closing price, and Panel B of Exhibit 2 plots its volatility measured by the variance of daily returns of the past 30 days. The date of maximum volatility occurs on November 12, 2008, at a closing price of $1.84, just a week shy of the 10-year low on November 19, 2008 at $1.26. On this day of maximum volatility, the expected daily return (based on historical price data) is negative. This conforms exactly to the Proposition 4. The fall in the stock price yields a negative expected return and high volatility. However, if an investor were to purchase at this price, the entry price would be extremely low (given the observed price history). Most value investors would argue that even though Ford was hurting at the time, it was undervalued, given its true intrinsic worth. Due to the low risk, this would be an excellent time to buy the security, despite its high volatility. This example illustrates the differences between these two notions of risk.

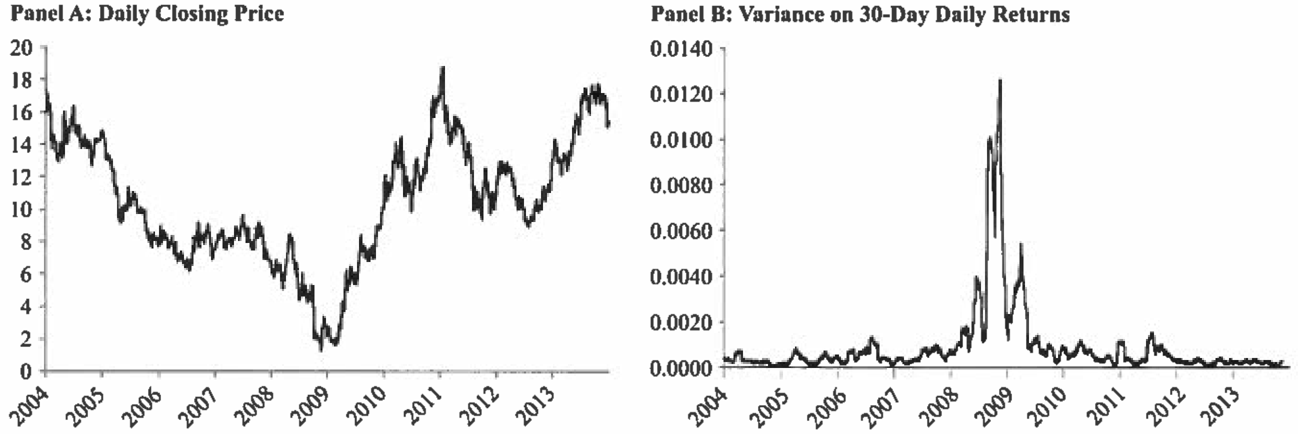
Volatility often arises from large price decreases, which are exactly the low risk opportunities, according to the value paradigm. The value investor cares about preserving capital. Warren Buffett has often talked about how he aims to find "great companies" at "good prices" (Buffett and Cunningham [2001]). The "good price" refers to buying at a low purchase price and the "great company" refers to a high growth rate in my model.

The economist cares about expected return, but problems arise because it is difficult to compute expected return based on states of nature, following its theoretical foundation. To calculate expected return, an economist would need to define all states of nature at any point, and then compute the weighted average of forecasted returns over those states of nature. However, this requires strong assumptions, namely, an identification of the states of nature and assessment of probabilities. Empirical financial economists lean on past data to compute expected returns and variances of the statistical distribution. The failure here is not that mean–variance preferences are unrealistic; the failure is that expected return is calculated in a convenient, but theoretically imperfect, way. Expected return should be based on the state of nature, but we are limited to available data; therefore, its computation rests on prior prices and returns.

Ultimately, this is a debate about market efficiency. If markets are efficient, then measuring expected return from the past return data is the best estimate of the

Exhibit 2

Ford Motor Company, January 1, 2004–January 1, 2014



true expected return, since those prices will capture all available information. But if ever this assumption fails, there is a gap between measurement and truth. The prior returns may say little or nothing about future returns, and it is the *future* returns that the investor cares for. If an alternative measure of expected returns that more closely aligns with the true states of nature and their underlying probabilities was available, I believe the debate on these two definitions of risk would subside.

Finally, observe that a long-term investor cares about minimizing the probability of loss, which is given by the cumulative distribution function. Of course, volatility is a feature of the distribution and can affect its shape. But it is not volatility per se that troubles a long-term investor. He really wants to preserve capital, and optimizing for volatility may or may not achieve that. In other words, low volatility is neither necessary nor sufficient to preserve capital. In mathematical terms, what matters for the value investor is the behavior of the cumulative distribution function. If there is any uncertainty on the distribution, or if it is not normal and cannot be represented with its first two moments, then optimizing over variance will lead to erroneous investment decisions. That there may be no convenient way to represent the cumulative distribution (and therefore, the true risk of the investment) in terms of a finite number of moments (like volatility) makes it difficult to argue that focusing on the moments is an appropriate decision-making criterion.

CONCLUSION

To a financial economist, the value investor's notion of risk conflates expected return and volatility into a single concept, as is evident from the formal analysis provided. If asset prices are random variables that follow well-defined probability distributions, then the variables will have statistical moments. The first two, and arguably the most important, moments are mean and variance; therefore, any notion of risk must build from these statistical foundations.

The value investor assesses risk primarily on the downside. Risk is the chance of a permanent loss of capital. A value investor need not care for volatility, so far as it does not lead to a loss. Value investors rarely talk of risk as the permanent *gain* of capital. I believe this stems from the casual notion that downside loss is more relevant for "risk" than upside gain. Thus, to avoid a permanent loss, it would require sufficiently high expected return, rather than a low volatility, per se. This is evident from the formal definition of risk (from $R(p_b)$), which is a function of the growth rate of the asset. As the asset grows (i.e., as its expected return increases), it is less likely to lead to a loss. To conclude, the disagreement between these two views on risk is more a difference in language than ideas.[8] Risk is a loaded term. If we simply agree to rename what the financial economists call risk to volatility, then I believe we would avoid many of these disputes.

Appendix

Proof of Proposition 1. For a given purchase price, p_b, we can write $R(p_b)$ as

$$R(p_b) = \Pr((1+g)p_0 + \varepsilon < p_b) = \Pr(\varepsilon < p_b - (1+g)p_0) = F(p_b - (1+g)p_0) \quad (4)$$

Taking partial derivatives of $R(p_b)$ gives the following comparative statics:

$$\frac{\partial R}{\partial g} = -p_0 f(p_b - (1+g)p_0) < 0 \quad (5)$$

$$\frac{\partial R}{\partial p_b} = f(p_b - (1+g)p_0) > 0 \quad (6)$$

$$\frac{\partial R}{\partial p_0} = -f(p_b - (1+g)p_0)(1+g) < 0. \quad (7)$$

QED

Proof of Proposition 2. By its definition, risk is given by

$$R(p_0) = F(p_b - (1+g)p_0) \quad (8)$$

Recall from Proposition 1 the derivative with respect to p_b:

$$\frac{\partial R}{\partial p_b} = f(p_b - (1+g)p_0) \quad (9)$$

Now differentiate with respect to g:

$$\frac{\partial^2 R}{\partial p_b \partial g} = f'(p_b - (1+g)p_0)(-p_0) \quad (10)$$

Now $f'(x) > 0$, if and only if $x < 0$. Therefore, the cross-partial is positive, if and only if

$$p_b < (1+g)p_0 = Ep \quad (11)$$

QED

Proof of Proposition 3. Recall that the pricing equation is $p = p_0(1+g) + \varepsilon$. Therefore, the expected price is $Ep = p_0(1+g)$ and the variance on price is σ^2. The expected return from the security is

$$E(Returns) = \frac{Ep - p_b}{p_b} \quad (12)$$

The variance on return is

$$Var(Returns) = \frac{\sigma^2}{p_b^2} \quad (13)$$

This holds because p_b is exogenous and therefore has zero covariance with price. Expected payoff under a mean-variance utility function is

$$EU = E(Returns) - \frac{r}{2} Var(Returns) \quad (14)$$

Let $D = Ep - p_b$. Rewriting,

$$EU = \frac{Dp_b - \frac{r}{2}\sigma^2}{p_b^2} \quad (15)$$

The agent maximizes expected payoff with respect to purchase price. The partial derivative of EU with respect to p_b is

$$\frac{\partial EU}{\partial p_b} = \frac{(p_b^2(D - p_b) - (Dp_b - \frac{r}{2}\sigma^2)2p_b)}{p_b^4} \quad (16)$$

Rearranging terms, this derivative is positive, if and only if $p_b^2 + Dp_b - r\sigma^2 < 0$.

This is a quadratic polynomial; therefore, it will have two roots, call them p_b^L and p_b^H. Therefore, the partial derivative is positive, if and only if p_b lies within (p_b^L, p_b^H). It is straightforward to see that the expected utility rises in p_0, g, and decreases in σ^2.

QED

Proof of Proposition 4. Consider a mean preserving spread of the distribution, f (Mas-Colell, Whinston, and Green [1995]). Let $\sigma_L < \sigma_H$ denote a low variance and a high variance for the distribution, f. Let σ_i^2, $f_i = f(0, \sigma_i^2)$ for $i = L, H$. Therefore, f_i and F_i are the PDF and CDF for variance σ_i^2. As with any mean preserving spread, increasing the variance keeps the distribution f fixed, but expands the tails. As such, it is clear that:

$$F_L(x) > F_H(x) \text{ for all } x > 0 \quad (17)$$

$$F_L(x) < F_H(x) \text{ for all } x < 0 \quad (18)$$

By definition, $R_i(p_b) = F_i(p_b - (1+g)p_0)$. Therefore, we have the following conditions:

$$R_H > R_L \text{ iff } p_b < (1+g)p_0 \quad (19)$$

$$R_H < R_L \text{ iff } p_b > (1+g)p_0 \quad (20)$$

QED

ENDNOTES

The author would like to thank Aaron Brown, Lee Pinkowitz, the Brandes Institute Board, and an anonymous reviewer for helpful comments.

[1]There is no question that this is an absolutely critical assumption. Simply inspecting book value against current price is insufficient to determine risk. The analyst must assess the true value of the company, as there are plenty of examples of companies whose book value collapsed but subsequently went bankrupt. A steering assumption in the model presented later is that the intrinsic value of the company is not known for certain but can be estimated and therefore predicted statistically.

[2]The other primary benefit of expected utility theory is its mathematical representation. Expected utility is a linear function of the underlying utility functions. As such, it has a mathematical form that is highly useful for a theoretical economist. Linear functions behave well and in a wide variety of conditions, which explains their success.

[3]Unknown growth rates would complicate the analysis, but not materially change the results.

[4]This already is a fundamental difference from the traditional economics/finance definition of risk. The level of the purchase price will be key in the analysis.

[5]Technically, NPV is not defined here because the analysis occurs in one period rather than in many, and there is uncertainty. However, the idea of a profitable investment is similar to a positive NPV investment.

[6]The most common expression of this is exponential utility. Details can be found in any standard economics textbook, such as Varian [1992, p. 198].

[7]Variance on a traded asset is not the function of an agent's action, usually. In this example, variance on return refers to the return based on a specific purchase decision made by the investor. While it is true that returns are computed over fixed time periods (daily, monthly, annually), the definition of return here, detailed in the appendix, is simply the difference between the exit price and entry price, divided by the entry price. Therefore, the phrase "variance on return" is specific to the asset and price it was bought and sold at.

[8]This mirrors the confusion over the word efficiency in economics and finance. To economists, efficiency means social optimality. To a financial economist, efficiency means incorporation of publicly available information into security prices. This double use of the word efficiency has led to much confusion among the public. Unfair and misleading criticism blamed the efficient market hypothesis for the financial crisis of 2008. If there was a crime committed by the founders of the efficient market hypothesis, it was that they chose the wrong name!

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