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Staged investments in entrepreneurial financing[☆]

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ABSTRACT

Venture capitalists deliver investments to entrepreneurs in stages. This paper shows staged financing is efficient. Staging lets investors abandon ventures with low early returns, and thus sorts good projects from bad. The primary implication from staging is that it is efficient to invest more in later rounds. The model yields a number of predictions on how the ratio of early to late round financing varies with uncertainty, the outside options of both parties, the value of the venture, the costs of investment, and project difficulty. We test these predictions against data on venture capital financings and find significant empirical support for the theory.

1. Introduction

> "You got to know when to hold 'em, know when to fold 'em,Know when to walk away and know when to run."–*Kenny Rogers*, *The Gambler*

This paper gives an efficiency-based explanation of staged financing in venture capital. Sahlman (1990), Gompers (1995), and Gompers and Lerner (1999) all document the extensive practice of venture capitalists delivering investments to new firms in stages. The current view in the venture capital literature is that staging mitigates moral hazard. Here, we argue that venture capitalists use staging as a sorting instrument. Staging investments provide the venture capitalist (VC) with the option of ending projects with low early returns. This sorts ventures into two groups: stay or quit. It is efficient to quit if the early returns are weak, and to stay otherwise. If the entrepreneur's early output is low, it is in the interests of both the VC *and* the entrepreneur to discontinue work and collect their respective outside options.

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The main result shows that it is efficient to assign more resources to the later stages of the project. Staging creates the possibility of termination after the early stage, and this reduces the project's expected return. This lowers the marginal return from investment, and so, the VC shades his investment downward in the early stage. Once the entrepreneur advances, the possibility of termination vanishes and the marginal return to investment rises, so the VC invests more. Those who make it to the second stage are more valuable precisely because their first stage output was sufficiently high. The VC invests more in the later stage because the new venture is "in the running" to becoming highly successful. Said differently, it is inefficient for the VC to bet big on a horse that won't finish the race.

The distinguishing feature of this paper is that the analysis operates entirely within a first-best setting. This pure investment model abstracts from conflicts of interest and agency problems between the VC and entrepreneur. While the relationship between a VC and an entrepreneur is of course rife with moral hazard, optimal investment can explain staged financing. This suggests that optimal decision making under uncertainty gives an alternative explanation of staged financing.

The model consists of a risk-neutral venture capitalist funding an entrepreneur over two stages. Each party invests resources (capital and labor) into each stage and the output from the project is the total investment plus a noise term in each stage. Both parties are symmetrically uninformed on the project's uncertainty. The project earns a positive return if the total output across both stages clears an exogenous hurdle. For example, consider a software company developing a new search engine. If the search engine is of sufficiently high quality, it has positive value and the company has potential to be taken public; otherwise, the product is worth nothing.

Both the VC and the entrepreneur have outside options in each stage. For example, the VC can fund other ventures and the entrepreneur can work on other projects. Staging investments gives the VC the option to discontinue the project, at which point both parties collect their respective outside options. The primary implication from staging investments is that it skews the efficient allocation of resources towards the later stages. The VC deliberately withholds investments in the early stages precisely because of the uncertainty from the early stage. In particular, the model shows that the VC will set a milestone after the first stage, and if the project's output clears this milestone, the VC knows the project is sufficiently successful and therefore, invests more.

In addition to this primary implication that investments increase in later rounds, the model generates several testable predictions. First, as the outside options of both parties increase, the VC will skew its investments even more into later rounds. Intuitively, as the parties' outside opportunities improve, the VC has a high opportunity cost from investing, and therefore can adopt a "wait and see" approach and can postpone investments into the future. Second, as uncertainty increases, it is efficient for the VC to invest more towards the *early* stage. While this may seem counterintuitive, the logic follows from the option value of continuing. Because the stages are sequential, an increase in uncertainty increases the upside benefit from continuing. This gives an extra benefit to investing in the early stage rather than the late stage. Finally, as the difficulty of project completion increases (because of market or technology factors), the VC will invest more resources into later rounds. This occurs because the VC is reluctant to invest too much money in early stage projects which are unlikely to "make it." All of these comparative statics give testable predictions on the ratio of early to late round financing.

We test the model against data on firms that obtained funding from VCs. We use a sample of VC financing rounds from the VentureXpert database maintained by Thomson Financial. This data classifies each investment round into four categories based on the operating stages of the firm at the time it received VC financing (seed stage, early stage, expansion stage, and late stage). Using these categories, we find robust empirical support for Theorem 1, which states the investments increase in later stages. We also find empirical support for Propositions 2, 3, and 4, which all posit relationships between the size and ratio of the investment rounds, and the various exogenous parameters of the model (the outside options of the VC, the variance and volatility of the output, and the value of the venture). These simple empirical tests provide preliminary support for the theoretical predictions of the model.

Existing literature on staged financing exists exclusively within agency models of asymmetric information. A landmark paper is Neher (1999), which claims entrepreneurs threaten to hold up VCs by reneging on investments, so VCs stage payments to reduce their bargaining power. Dividing investments into a number of stages creates inefficiencies but is necessary in overcoming the commitment problem. Landier (2002) argues that staging is one way of protecting an investor from risk when entrepreneurs have a high exit option, i.e. when bankruptcy laws are lenient and when there is little stigma associated with business failure. Bergemann and Hege (1998, 2005) study the dynamics of the optimal contract and equilibrium funding decisions in arm's length versus relationship financing. In other work, Bergemann and Hege (2003) show that the duration of funding, though not necessarily the level of funding, increases in later stages. Bergemann et al. (2009) build a continuous-time investment model where the critical threshold for success is exogenous and uncertain; they confirm our result that investment increases over later rounds, and even test their model against VC data, as we do. Their paper is similar to ours in that it relies on an investment model, though our two-period discrete model generates a web of comparative statics, which complement the prediction from their continuous-time model.

Cornelli and Yosha (2003) look at "window dressing," the manipulation of information on project performance, which entrepreneurs may practice in order to continue to receive funds. Wang and Zhou (2004) find that there are cases in which up-front financing may be superior to staging; under staged financing, VCs will underinvest in low quality projects and potentially doom them to failure. Elitzur and Gavious (2003) model a contracting problem between a venture capitalist and an entrepreneur. They show that the optimal incentive scheme backloads all incentive payments to the entrepreneur, therefore backloading optimal effort allocation as well. Fluck et al. (2004) use computational methods to demonstrate that resources increase in later rounds. In Yerramilli (2006), each party can hold up the other and threaten to walk away in order to press for a renegotiation of the contract. Finally, without the ability for investors to unilaterally cancel projects, Admati and Pfleiderer (1994) argue that entrepreneurs with outside financing will be reluctant to quit unproductive ventures. All of these models take place in moral hazard and asymmetric information settings; therefore, staging is an instrument to minimize agency costs. None of the prior theoretical work explores the efficiency properties of staged financing.

The existing theoretical work give different predictions on the evolution of investment over stages, few or no predictions on connecting staging and changes in the environment (uncertainty, outside opportunities, value of venture), and have yet to test their predictions against data (except Bergemann et al., 2009). In Neher (1999), investments increase over time because the VC is willing to invest more as the firm's collateral grows. Yet Giat et al. (2009) find that staged investments can increase over time, decrease over time, or rise and then fall. Hsu (2002) finds, computationally, that staging tends to be more profitable to investors when ventures are in early stages and will need greater amounts of capital in the future.[1] Yet, none of these papers make predictions on how the ratio of early to late stage financing changes with exogenous parameters of the environment, such as increase in uncertainty, project difficulty, or outside opportunities.

The paper is organized as follows. Section 2 presents the benchmark model and shows that staging investments increases total surplus. Section 3 explores the effects of staged financing on the ratio of early to late stage funding levels. Section 4 contains the comparative statics with respect to the model parameters, and delivers secondary implications on how the funding level changes with the uncertainty in the model, technology or market risk, or the outside options of the VC or entrepreneur. Section 5 tests the implications of the model against VC data. Section 6 concludes.

2. The model

Consider an entrepreneur working on a project (a new venture) over time. The entrepreneur seeks funding for the project from a venture capitalist (VC). Both parties are risk neutral. Production takes place across two stages, and there is no discounting. It takes time to establish a business, and the stages represent distinct phases in production. For example, the early stage involves establishing the founder's initial business plan, while the later stage involves marketing the plan and generating advertising revenue. Let k_t be the *total* resources invested in the project at stage t. This reflects the sum of both the entrepreneur's and the VC's resources (labor and capital) invested in the project. Though we call k_t investment, it includes human resources as well as financial resources. Since the focus of the analysis is on *efficient* resource allocation, it is not necessary to specify the entrepreneur's and venture capitalist's resources separately.

The total resources k_t in stage $t=1,2$ face a cost of resource function $C(k_t)$. This is the total social cost of resources in stage t. Assume C',C'' are strictly positive, so costs are separable across stages, increasing, and convex.[2] The convexity of the cost function reflects a convex cost of investment for the venture capitalist and a convex cost of effort for the entrepreneur. A convex cost of effort is a standard assumption, while a convex cost of investment simply reflects that the VC cannot invest arbitrarily large amounts without cost.[3] The convexity of the supply curve represents all the costs of raising capital to deliver funds to the entrepreneur. Output from the project is

$$q_t = k_t + \varepsilon_t. \tag{1}$$

The noise terms ε_t are i.i.d., and distributed symmetrically around a mean of zero and over infinite support, with cdf $G(\cdot)$ and density function $g(\cdot)$. Interpret ε_t as a stage-specific shock unknown to anyone. The ε_t captures all of the market and technological uncertainty in raising profits: novelty of the founder's idea, viability of the business plan, existence of a potential market, quality of human and physical capital, etc.

A project is a pair $(V, \bar{q})$, where $V>0$ is the value of the project and $\bar{q}>0$ is the final hurdle. After stage two, the VC takes the firm public if it is of sufficiently high quality. Therefore, the final hurdle represents the minimum quality necessary for a new venture to capture a positive market price when its shares are traded on public stock markets. The value of the venture is

$$V(q_1, q_2) = \begin{cases} V & \text{if } q_1 + q_2 > \bar{q} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Output (for e.g. profits, quality, sales) has no value unless it is sufficiently high. In most new ventures, the venture is worth little unless it can eventually be taken public, or at least generate profits. Assets of firms that have either failed to go public or have not successfully obtained later round financing are usually sold at low (firesale) prices; we simply normalize these low prices to zero. Thus, q_t is the project's *internal* output (prototypes, beta versions, etc.), while $V(q_1,q_2)$ measures the project's *external* value based on market valuation. Throughout, call q_t the project's *output*, and call $V(q_1,q_2)$ the project's *value*. Since information is symmetric in this model, both parties know the true value V but do not know whether output from the project is sufficiently high to clear the hurdle $\bar{q}$. Observe that output levels across stages are perfect substitutes. This isolates the effects of staging on investment from the effects of technology on investment.

Suppose that both the VC and the entrepreneur have outside options in each stage. These outside options capture the value of the outside opportunities of both parties. For example, the VC has many competing investments to fund and can allocate his

[1] Other empirical work in venture capital documents different features of the venture capital environment. Krohmer and Lauterbach (2005) find empirically that in the final stages of a project, investment managers may be too unwilling to pull the plug on failing projects. Cuny and Talmor (2005) and Bienz and Hirsch (2009) look at the differences between the two types of staged financing that are commonly observed, staging with milestones or with rounds.

[2] Separability of the cost function eases exposition and analysis in the later results of the paper. However, Proposition 1 still holds under non-separable cost functions. Details are available from the authors upon request.

[3] VCs draw from dedicated pools of capital that institutional investors supply. In particular, the VC raises capital in blocks ("funds"), usually targeted towards investments in a specific industry or technology. If the VC exhausts the fund and wants to invest more, he must raise a new fund, which involves soliciting interest from limited partners (institutional investors), advertising the fund through business networks, or transferring capital from other preexisting funds. See Prowse (1998) for a full description on the capital raising process.

capital and his time elsewhere. Similarly, the entrepreneur can either work on other new ventures, or even collect a wage as an employee for another organization. Let $\bar{u}_t$ be the sum of the outside options of the VC and entrepreneur in stage t. So $\bar{u}_t$ measures the opportunity cost of the project (time, labor, capital) to both parties.[4] The venture capitalist may conduct an evaluation of the venture after stage one. In fact, the purpose of staged financing is to give the venture capitalist an intermediate reading on the new venture, with the option of ending the venture if the early returns are weak.

2.1. Upfront financing

As a benchmark, suppose the VC does not conduct an evaluation after the first stage. Importantly, there are no grounds for terminating the project after the first stage. So the VC gives all the funds for the project upfront; call this "upfront financing." To calculate the social payoff, observe that both parties receive positive surplus only if the project is a success, i.e. that $q_1+q_2>\bar{q}$. The probability of success is

$$P_0=\Pr(q_1+q_2>\bar{q})=\Pr(\varepsilon_1+\varepsilon_2>\bar{q}-k_1-k_2)=\int_{-\infty}^{\infty}\int_{\bar{q}-k_1-k_2-\varepsilon_1}^{\infty}g(\varepsilon_1)g(\varepsilon_2)d\varepsilon_2 d\varepsilon_1 \tag{3}$$

by the independence of the errors. After integrating and using the symmetry of the errors around zero,

$$P_0=\int_{-\infty}^{\infty}g(\varepsilon_1)[1-G(\bar{q}-k_1-k_2-\varepsilon_1)]d\varepsilon_1=\int_{-\infty}^{\infty}g(\varepsilon_1)G(\varepsilon_1+k_1+k_2-\bar{q})d\varepsilon_1. \tag{4}$$

Therefore, the marginal effect of increasing investment on improving the probability of success is

$$\frac{\partial P_0}{\partial k_t}=\int_{-\infty}^{\infty}g(\varepsilon_1)g(\varepsilon_1+k_1+k_2-\bar{q})d\varepsilon_1. \tag{5}$$

This expression is positive, so increasing investment makes it more likely that the project will clear the final hurdle. Moreover, observe that the right-hand side of the equality above is independent of t, and therefore so is the left-hand side. The VC can fund either in stage one or stage two, as it has the same effect on the project clearing the final hurdle. Thus, the probability of success increases by the same amount with investment in either stage. Since total investment is additive, stage one and stage two investment are perfect substitutes.

Since the objective of the analysis is to understand the efficient allocation of resources, it is necessary to consider the social planner's problem, i.e., the joint payoff of the entrepreneur and the VC combined. This is the expected benefit from investments, less the cost of investment in each stage.[5] The social planner maximizes total surplus, so the problem is

$$\max_{k_t} P_0V-C(k_1)-C_{k_2}, \tag{6}$$

which yields the first-order condition

$$V\frac{\partial P_0}{\partial k_t}\bigg|_{k_t=\hat{k}_t}=C'\left(\hat{k}_t\right), \tag{7}$$

where $\hat{k}_t$ denotes the optimal effort level.

The marginal cost of investment is equal to its marginal return, which is the marginal probability of success times the value of the project. Since the left-hand side is independent of t, the right hand side must be as well. Hence $\hat{k}_1=\hat{k}_2\equiv\hat{k}$; this is the efficient investment under upfront financing, and is the same in each stage. It is efficient to split investment evenly across stages since the cost of investment per stage is the same. Since the model is symmetric with respect to the VC and entrepreneur, it is possible to implement this first best solution, so the VC will split its investment evenly across stages, and the entrepreneur will exert effort and deploy resources evenly across stages. For example, this is the outcome under a contracting game where the venture capitalist is the principal who proposes a contract to the agent, the entrepreneur. In this setting, since both parties are risk neutral, it is straightforward to construct a contract that implements the first-best.[6]

Note that convexity of the cost function is not what guarantees that investment in both stages is the same. Investment is the same because (1) convexity of the cost function guarantees a unique solution, (2) the marginal return to investment in each period is the same, and (3) the cost function is separable and identical across stages. Convexity does, however, guarantee that efficient investment increases with V. Collecting terms, the efficient per-stage investment level $\hat{k}$ solves

$$C'\left(\hat{k}\right)=V\int_{-\infty}^{\infty}g(\varepsilon_1)g\left(\varepsilon_1+2\hat{k}-\bar{q}\right)d\varepsilon_1. \tag{8}$$

[4] The outside options are independent of early stage output. The results of the model generalize easily if outside options increase linearly in output.

[5] Observe that even though the production function $V(q_1,q_2)$ is discontinuous at the point $q_1+q_2=\bar{q}$, the planner's expected payoff PV is continuous in k_t.

[6] Since contracting issues are not central to this analysis, we do not outline the details of the contracting game, such as the contract space, the bargaining power between the two parties, etc. Such a game is straightforward to construct, as the principal will pay the agent W for success $(q_1+q_2>\bar{q})$ and L for loss $(q_1+q_2<\bar{q})$. To guarantee full incentives to exert first best, the principal will set $W-L=V$. Further details on this contract are available from the authors upon request.

The remaining constraint is a bound on the reservation utilities. The total surplus from having the entrepreneur undertake the project must be at least as large as the total outside options across both stages. So,

$$P_0V - 2C\left(\hat{k}\right) \geq \bar{u}_1 + \bar{u}_2. \tag{9}$$

Call this the project feasibility constraint.

2.2. Efficiency of staging investments

The main reason to conduct an evaluation halfway through a project is that it provides the option to abandon the project if the early returns are low. The two parties will use first stage output to compute the expected project value $V(q_1,q_2)$ after the second stage. This yields an expected value of continuing. Because of the outside options, it is efficient to continue only if this value exceeds these outside options.

If the VC and entrepreneur observe q_1 after the first stage and must decide whether to continue or not, their decision will depend on the observed q_1. Therefore the probability of continuing and the total surplus from continuing will also depend upon this observed q_1. The probability of clearing the final hurdle, conditional on a realized value q_1, is

$$P(q_1) \equiv \Pr(q_1 + q_2 > \bar{q} | q_1) = \Pr(\varepsilon_2 > \bar{q} - q_1 - k_2) = G(q_1 + k_2 - \bar{q}). \tag{10}$$

So the total surplus conditional on a realized q_1 is

$$S(q_1, k_2) = \mathbb{E}_2 V(q_1, k_2 + \varepsilon_2) - C(k_2) = P(q_1)V - C(k_2), \tag{11}$$

where $\mathbb{E}_t$ denotes the expectation taken over ε_t. Call this the continuation surplus function. For clarity, let $S(q_1) \equiv S(q_1, k_2^*)$ be the continuation surplus evaluated at the efficient investment level k_2^*.[7] This continuation surplus function reflects the expected total surplus from continuing after a realization of first stage output q_1. The continuation decision rests entirely on this function. In particular, it is efficient to continue if and only if $S(q_1) \geq \bar{u}_2$. The first result below shows that the continuation surplus function is strictly increasing. This means there exists a unique cut-off output level q^* such that $S(q^*) > \bar{u}_2$ if and only if $q_1 > q^*$. In words, the planner sets the optimal target q^* such that he is indifferent between advancing and retaining the entrepreneur. All proofs are in the Appendix.

Proposition 1. *There exists a target q^* such that it is efficient only for entrepreneurs with $q_1 > q^*$ to advance to the second stage.*

Because $q_1 + q_2 > \bar{q}$ in order to collect positive surplus, the stages are connected; output in the early stage signals final project value. Said differently, a successful early stage (high q_1) means that the project will have an easier time clearing the final hurdle, and therefore a higher chance of both parties collecting surplus. Proposition 1 shows that the continuation surplus is monotonic, and this generates the cutoff target q^*. In practice, this q^* represents the milestone in between rounds of venture financing.[8] If the quality of the project clears this milestone, then the entrepreneur qualifies for the next round of funds. The assumption on outside options is key. Without outside options, it would be efficient to continue for any q_1 since all parties get nothing by quitting and are at least as well off continuing (recall that V is always nonnegative).

Proposition 1 shows that staged financing generates more surplus than upfront financing. Under upfront financing, the VC does not collect information on early stage output, and therefore advances all projects regardless of their early performance. Under staged financing, the investor sorts projects into two groups: stay or quit. The target q^* conducts the sorting, in that it allows only entrepreneurs with high output to proceed. The VC can always set the target arbitrarily low, which permits continuation for all output levels, and hence replicates upfront financing. By setting the target optimally, the VC has an additional instrument to maximize total surplus, and therefore must be weakly better off. This suggests that stage financing does more than simply minimize agency costs, as the prior literature has argued. Instead, staging is a tool to make both VCs and entrepreneurs better off.

3. Effects of staged financing

Now that we know staged financing increases surplus, what is the efficient investment level per stage under the staged financing regime? The previous section shows that the continuation decision will take the form of a cut-off rule. Precisely, the VC sets some target (or milestone) q^* after the first stage, and advances the entrepreneur only if $q_1 > q^*$. The probability of clearing the target is

$$P_1 = \Pr(q_1 > q^*) = G(k_1 - q^*). \tag{12}$$

[7] Note that in general k_t^*, which is efficient under staged financing, differs from the $\hat{k}_t$ from the previous section, which is efficient under upfront financing.

[8] For example, milestones separate early round financing (series A) from later round financing (series B). New ventures must meet certain targets, such as number of employees hired, free cash flow, research and development investments, progress on business plan, etc. These targets constitute the milestone q^*.

As expected, this probability increases in first-stage investment since $\partial P_1/\partial k_1 = g(k_1 - q^*) > 0$. The ex-ante probability of success is

$$P \equiv \Pr(q_1 + q_2 > \bar{q}, q_1 > q^*) = \int_{q^*}^{\infty} P(q_1) g(q_1 - k_1) dq_1, \tag{13}$$

where $P(q_1) = G(q_1 + k_2 - \bar{q})$ is the interim probability of clearing the final hurdle and capturing V for each realization of q_1. Notice that

$$\begin{aligned} \frac{\partial P}{\partial k_2} &= \int_{q^*-k_1}^{\infty} g(\varepsilon_1) g(\varepsilon_1 + k_1 + k_2 - \bar{q}) d\varepsilon_1 > 0, \\ \frac{\partial P}{\partial k_1} &= \frac{\partial P}{\partial k_2} + \frac{\partial P_1}{\partial k_1} G(q^* + k_2 - \bar{q}) \\ &> \frac{\partial P}{\partial k_2}. \end{aligned} \tag{14}$$

The returns to investment are positive for both stages, but are higher for the *first* stage. Additional first stage investment increases the probability of success in two ways. First, it increases $q_1 + q_2$ and thus directly increases the probability of final success. Second, it increases first stage output ($q_1 = k_1 + \varepsilon_1$) and so improves the chance of advancing to the second stage. Hence, the marginal benefit of first stage investment exceeds the marginal benefit of second stage investment. It is incorrect to conclude from this, however, that it is efficient to invest more in the first stage, since this analysis both ignores the *cost* of investment, and takes q^* as given, whereas, in fact, q^* is determined simultaneously with the optimal k_t. Increasing first stage investment increases the chances of advancing to the second stage, and thus increases the probability of bearing the cost of a second stage investment. At the optimum, this cost is enough to push k_1 below k_2. To see this, it is necessary to solve the social planner's problem.

The conditional probability of clearing the final hurdle, given that the entrepreneur has reached the target, is

$$Q = \Pr(q_1 + q_2 > \bar{q} | q_1 > q^*). \tag{15}$$

So the ex-ante probability of clearing the hurdle $\bar{q}$ is $P = P_1 Q$. If the entrepreneur passes the intermediate target q^*, the planner gets V if he clears $\bar{q}$ and zero otherwise, and bears cost $C(k_2)$. If he doesn't pass the intermediate target, the planner gets only $\bar{u}_2$. So the total surplus is

$$P_1(QV - C(k_1) - C(k_2)) + (1 - P_1)(\bar{u}_2 - C(k_1)). \tag{16}$$

Rearranging terms gives the planner's problem

$$\max_{k_{t,q}} PV - C(k_1) + (1 - P_1)\bar{u}_2 - P_1 C(k_2), \tag{17}$$

subject to project feasibility. The last term above is the cost of advancing to the second stage. This cost is increasing in first-stage investment. As the VC invests more in stage one, he increases the *expected* second-stage cost, since larger first-stage investments increase the probability of making it to the second-stage. This cost forces first-stage investment downward, ultimately, below even second-stage investment. More generally, it is possible to write the planner's objective function in terms of the continuation surplus function. So the planner solves

$$\max_{k_{t,q}} \int_q^{\infty} S(q_1, k_2) g(q_1 - k_1) dq_1 + (1 - P_1)\bar{u}_2 - C(k_1), \tag{18}$$

where investment levels k_t and the target q are the planner's choice variables, and (k_t^*,q^*) denote the efficient choices. The first term is the expected value of continuing: the continuation surplus function integrated over all realizations of $q_1 > q^*$. The middle term $(1 - P_1)\bar{u}_2$ is the expected value of abandoning the project. Both parties collect their outside options if the project does not clear the target, which occurs with probability $1 - P_1$. Note that $C(k_2)$ does not appear in the objective function explicitly because it is embedded in $S(q_1$,$k_2)$. The planner bears the cost of $C(k_2)$ only in the event that the entrepreneur advances.

3.1. Primary implication: dynamic capital allocation

The following proposition solves the planner's problem (19) for the efficient allocation of resources across stages, and is the main result.

Theorem 1. *It is efficient to invest more in the second stage* ($k_1^* < k_2^*$).

Since the planner sets the target optimally, the marginal return of an entrepreneur who cleared the target exceeds the marginal return of an entrepreneur in the first stage. Formally,

$$C'(k_1^*) = E\left[S'(q_1)\right] < E\left[S'(q_1) | q_1 > q^*\right] = C'(k_2^*). \tag{19}$$

The mean marginal return conditional on $q_1 > q^*$ exceeds the unconditional mean. Since marginal costs are increasing, this implies that $k_1^* < k_2^*$. The marginal return to investment is lower in stage one precisely because the entrepreneur may not advance to the second stage. In this case, he bears the cost $C(k_1)$ but acquires the benefit V not with certainty, but with probability less than one. This lowers the marginal return in stage one relative to stage two. At the optimum the VC selects k_t to set the marginal costs equal to the marginal returns, and so he shades investment downward in the early stages. He will allocate more resources in the later stages of the project, where the marginal return is higher. Rewriting the first order conditions in terms of the specific production function here yields

$$C'(k_1^*) = V\frac{\partial P}{\partial k_2} < V\frac{\partial P/\partial k_2}{P_1} = V\frac{\partial Q}{\partial k_2} = C'(k_2^*). \tag{20}$$

Thus, those who make it to the second stage are more valuable precisely because their first stage output was sufficiently high. The VC invests more in the later stage because the new venture is "in the running" to becoming highly successful. It is inefficient for the VC to dump too many resources into a horse that won't finish the race.

Gompers (1995) provides empirical analysis of 794 firms that received VC funding and reports that later rounds obtain larger amounts compared to earlier rounds. However, the premise of Gompers (1995) is to explain staged financing as an instrument to mitigate agency conflicts. While the data is consistent with such an interpretation, there is no formal theoretical model in his paper. In contrast, we develop a formal model that not only generates the same prediction that venture capitalists will stage investments and such investments rise with time, but it also generates new comparative statics on the outside options and output variance. The agency models on staged financing cited above say nothing on these last two points. We now turn to these comparative statics of the model.

4. Secondary implications: comparative statics

While some prior theoretical work has made explicit predictions on the ratio of investment levels over stages (Giat et al., 2009; Hsu, 2002; Neher, 1999), these papers have not made predictions on how this ratio $\frac{k_1}{k_2}$ varies in different environments, such as industries with different levels of market risk, projects with different levels of technological feasibility, or more prestigious venture capitalists with better outside options. The objective of this section is to explore how the endogenous variables (k_t^*, q^*) vary with the exogenous parameters $(\bar{u}_2, v, \bar{q})$. In particular, the aim is to produce a number of secondary implications that can be tested against observed data on venture capital financings. If this ratio k_1^*/k_2^* increases, the VC skews the investment mix towards the early stage, and vice versa. To get traction on the model, er parameterize the cost function as $C(k_t) = \lambda k_t^{\gamma}$ for some constants $\lambda > 0$, $\gamma > 1$. The next result predicts how the ratio of investment levels varies with the outside options of both parties.

Proposition 2. *The ratio of early to late round financing k_1^*/k_2^* decreases in the outside options $\bar{u}_2$.*

As the outside options improve, it is efficient to invest even *more* money in later rounds. The outside options represent the opportunity cost of alternative investments in the second stage. As this opportunity cost increases, investors are more reluctant to fund ventures since alternative opportunities are promising. This causes the efficient investment level k_1^* to sink. However, once those ventures do clear the hurdle it is efficient to invest more, so k_2^* rises. The net effect is that the ratio k_1^*/k_2^* sinks. Ultimately, good outside opportunities allow VCs to withhold early round investments relative to later round investments. Similarly, good outside opportunities for the entrepreneur make it tempting for him to abandon projects with low early returns, and therefore this will cause him to invest less effort and resources into the project in the early stage. The outside options capture the opportunity cost of investment, and therefore, measure the tolerance for poor projects. With high outside options, this tolerance is low, and both parties invest less in the first stage.

In practice, there is wide heterogeneity among venture capitalists and entrepreneurs in terms of their outside options. For example, VC firms with successful records in bringing new ventures to an IPO and generating outside profits for their limited partners will often enjoy high outside options. These VC firms are routinely flooded with capital from limited partners as well as with proposals from many different entrepreneurs.[9] Similarly, entrepreneurs vary in their outside options as well. Successful managers at existing companies, or entrepreneurs with a prior record of performance in new companies, will no doubt enjoy multiple offers from management teams and VCs alike.[10]

Now consider what happens with an increase in the output variance, i.e. the variance on the error distribution g. For the remaining implications, let the cost of investment be quadratic ($\gamma = 2$) and the error distribution be uniform.

Proposition 3. *As the output variance increases, k_t^* decreases while k_1^*/k_2^* increases. If $\bar{u}_2 > \frac{V}{2}$, then q^* increases.*

As the variance of g increases, it is clear that this will choke off investment in both stages. This is the same intuition from the Lazear and Rosen (1981) tournament model, in which increased noise reduces effort incentives. What is not obvious is whether

[9] The VC firms that funded the most successful firms such as Google in the late 1990s, such as Kleiner Perkins or Sequoia Capital, have higher opportunity costs than less successful VC firms. Lerner and Schoar (2004) document considerable persistence in performance for private equity fund managers. Thus, historically successful VCs tend to be highly sought after.

[10] Sorensen (2007) structurally estimates a two-sided matching model of Silicon Valley VCs and entrepreneurs. He finds that high quality VCs match with high quality entrepreneurs, i.e. he finds evidence of positive sorting. This suggests that the outside options of the entrepreneur and VC "move together" — when the VC has high outside options (high quality), so does the entrepreneur.

the decrease in investment is larger in stage one versus stage two. It seems plausible that an increase in noise will cause the entrepreneur to withhold investment in the early round, and work harder in the later rounds. While this logic is compelling, it is misleading.

An increase in output variance affects later stage investment more than first stage investment. This occurs because the marginal benefit to first stage investment exceeds the marginal benefit to second stage investment (see (15)), since investment in the first stage not only affects the probability of clearing the final hurdle $\bar{q}$, but also of clearing the milestone q^*. Because the investor and the entrepreneur can quit the project if output does not clear q^*, an increase in the output variance increases the upside benefit from continuing. A larger first stage investment increases the chance of capturing this upside, and this gives an extra benefit to investing in the early stage rather than the later stage. Therefore, an increase in output variance decreases investment in both stages, but decreases late stage investment *more* than early stage investment.

Recall that the stage specific noise terms represent market and technological uncertainty at each stage. In reality, there is clearly a variation between different industries on this uncertainty. For example, some industries may have high uncertainty at the market level, possibly reflecting difficulty in bringing a new firm to market because of the strategic position of incumbents. On the other hand, some industries may exhibit high technological uncertainty, deriving from the production function itself; for example, the biological process of drug development may impose higher uncertainty on new biotech firms than technological uncertainty in other industries. This variation in market and technological uncertainty can be exploited to predict variation in the ratio of investment levels over stages. Finally, observe from Proposition 3 that if the investor and entrepreneur have sufficiently good outside options ($\bar{u}_2 > V/2$), then they will set higher targets when the output variance increases. So in more risky industries, it is efficient to set a higher milestone to justify later round financing.

Proposition 4. *As the value of the venture V increases, q^* decreases while k_1^* and k_2^* both increase.*

This comparative static is perhaps the most straightforward. As the venture becomes more valuable, it is efficient to invest more in each round. Said differently, each party is more willing to invest more and bear a higher cost of investment if the resulting benefit increases. The proof of the proposition shows that as the variance in the noise terms becomes sufficiently large, the ratio of early to late investments k_1^*/k_2^* does not vary with V. Therefore, even though the VC invests more in each stage, the ratio of investments across stages eventually stays constant. On top of this, higher valuation ventures should exhibit lower milestones (q^*) between early and late stages. Since q^* and k_t^* both increase, this increases the probability of success, since $P_1 = G(k_1^* - q^*)$. Intuitively, the venture is more valuable, and so, it becomes more desirable to pass at the interim stage, as this generates surplus for both parties. Passing the interim hurdle is made easier by simultaneously increasing investments in each stage and decreasing the milestone, thereby increasing the probability of investment. It is efficient to do this precisely because the end-game prize V is worth more.

In practice, measuring q^* directly may be difficult, as VCs may not have hard, objective criteria when deciding whether to continue funding projects or not. For example, part of the evaluation may be based on instinct for whether the project will be successful or not. Nonetheless, higher milestones are harder to clear than lower milestones, and therefore, VCs who set high milestones will abandon many ventures at the interim stage. Similarly, VCs with low milestones will tell most of its entrepreneurs to continue. Therefore one such empirical proxy for q^* is the number of firms abandoned at the interim stage divided by the total number of firms funded at the outset. In this sense, the milestone q^* reflects the quit rate, or abandonment rate, of the VC and entrepreneur.

Proposition 5. *As the final hurdle $\bar{q}$ increases, k_1^* decreases, q^* increases, and k_2^* is unchanged.*

Recall that $\bar{q}$ is the final hurdle that output must clear in order for both parties to receive value from the venture, and therefore reflects the fundamental difficulty of project completion (because of market or technology factors). As the hurdle increases, it is efficient to decrease first stage investments and leave late stage investments unchanged, thus decreasing the ratio of early to late stage investments. Formally, $\bar{q}$ affects the planner's payoffs only through the probability of success P. Specifically, for every q_1, $P(q_1) = G(q_1 + k_2 - \bar{q})$ decreases in $\bar{q}$. As a result, the expected benefit PV decreases in $\bar{q}$. As the benefit sinks, the VC lowers costly investment k_1. In fact, a marginal increase in $\bar{q}$ has the opposite effect of a marginal increase in V. As $\bar{q}$ increases, the VC simultaneously decreases k_1 and increases q^*, thus lowering the probability of clearing the target, since $P_1 = G(k_1 - q^*)$. In other words, when the project's difficulty increases, this lowers the expected benefit to the VC, so he reduces the probability of advancing the entrepreneur at the intermediate stage.

The concrete empirical prediction is that industries with higher final hurdles should observe more funding in later stages (lower $\frac{k_1}{k_2}$). Observe that this is the opposite prediction from an increase in variance, as predicted by Proposition 3. As an empirical matter, it will be important to distinguish high hurdles from high risk industries. The empirical measures of these variables may be close even though the theoretical concepts are quite different.[11] Finally, the comparative static with respect to the intermediate target is particularly elegant: $\frac{\partial q^*}{\partial \bar{q}} = 1$. For every unit increase in the final hurdle, it is efficient to increase the milestone by exactly that amount.

The last comparative static of this section involves the cost of investment.[12] On the VC side, the cost of investment includes the transaction costs of deploying capital from existing funds, as well as time and labor spent attracting additional capital from

[11] For example, consider the market for an AIDS vaccine. It is plausible that AIDS research is both highly risky (high variance on g), and that it is very difficult to discover an actual vaccine (high final hurdle $\bar{q}$).

[12] Recall that the cost of investment is $C(k_t) = \frac{\lambda}{2}(k_t)^2$, so λ is a parameter that scales the cost of investment.

institutional investors through raising a new fund. For example, the large influx of capital from public equity into private equity over the last twenty years (Prowse, 1998) has made it easier for VCs to raise new funds, and constitutes a reduction in the cost of investment λ. As such, Proposition 6 predicts that second stage investments will increase.[13] For the entrepreneur, the cost of investment includes his cost of effort as well as the cost of deploying his own capital in the firm. For example, suppose in the very early stage of the venture, the entrepreneur finances the project with his own savings and a small loan from the bank. Interest rates that govern the bank loan will affect his cost of investment, and hence higher interest rates may correspond to higher λ. According to Proposition 6, this results in the VC setting a higher milestone and making a larger second round investment.

Proposition 6. *As the cost of investment λ increases, k_2^* decreases and q^* increases.*

Market factors which increase the cost of investment will decrease later round investments and increase the milestone q^*.

5. Preliminary empirical tests

The theory of staged financing by venture capitalists presented in this paper has a number of empirical implications. In this section, we describe some preliminary evidence to support these implications. We find that the main theoretical result (Theorem 1, that funding increases in later rounds) is borne out robustly in our tests. The theoretical model also makes predictions about how the optimal amount invested in a particular round, and the ratio of early to late round financing, varies with changes in uncertainty, outside options for the VC (and entrepreneur), and the valuation of the venture. We derive empirical proxies for the various parameters in the model (such as outside options, output uncertainty, etc.) to test some of the propositions derived in Section 4. Some of the other propositions rely on parameters that are nearly impossible to measure (e.g. q^*, the hurdle rate that VCs use internally) and we are unable to test those directly.

5.1. Data and sample selection

For our empirical test, we use a sample of VC financing rounds obtained by US based firms. The source of our sample is the VentureXpert database maintained by Thomson Financial.[14] We collect data on all individual financing rounds classified as “Venture Capital Deals” by VentureXpert for the 11 year period starting in January 2000 and ending in December 2010. The basic data consists of: date of each financing round, names of VC firms investing in that round, dollar amount invested, and firm's industry. For some of the firms, the database also provides the direction of valuation for later rounds. For these rounds, VentureXpert notes if the financing occurred at a valuation higher or lower than the valuation at the immediately preceding financing round. Finally, VentureXpert database also reports the operating stage of the firm receiving financing at the time of each financing round.

Following Gompers (1995), we retain only those financing rounds where the firm is classified as one of the following four stages: seed stage, early stage, expansion, or later stage. This feature of the data allows us to design tests of Propositions 2 and 3. Our final sample consists of 40,685 financing rounds and a total of $368,061 million raised.

Table 1 provides calendar year distribution of our sample. Our sample includes the height of the “Dot Com” bubble years of 2000 and 2001. These two years were remarkable, as the amount invested in aggregate was almost $150 billion, which is almost 40% of the total financing over the entire 11 year sample period. The average financing round was similarly much higher, especially in the year 2000. The data reported is in nominal dollars, which understates the investment boom of the 2000–2001 period. VC investment shrank dramatically in 2002 and 2003 before it started recovering in 2004. However, the financial crisis in 2008 appears to have reversed this recovery in VC financing levels, as the aggregate investment fell by over 30% in 2009.

Panel A of Table 2 looks at the distribution of VC financing across the 10 industry groups reported by VentureXpert. We report the percentage of rounds in each year across different industry groups. Not surprisingly, there are strong industry and time trends. At the start of our sample period, 38.9% of financing rounds were for firms classified as “Internet-Specific.” This category shrank to roughly 14% in 2004, and was 23% at the end of our sample period. The dramatic decrease is underscored by similar sharp decline in “Communication/Media” industry, which touted itself as providing the infrastructure for the growth in internet traffic. Several fiber optics and telecom ventures were funded in early 2000, but the category fell out of favor by the end of the sample period. Another illustration is the boom in the “Alternative Energy” sector. The “Industrial/Energy” industry reported 1.5% of all financing rounds in 2000, but accounted for 7.2% of all deals in 2011. This represents an almost four-fold increase.

The results of Table 1 and the industry results described here underscore two widely accepted views about venture capital industry. First, there are significant boom and bust cycles, where high realized returns lead to larger funds being raised and invested. This leads to a period of low returns, which in turn causes the industry to shrink (Gompers and Lerner, 1999). Second, there is significant trend chasing by VC funds. Industry sectors become “hot” and attract disproportionately higher share of investments for a while before falling out of favor (Loughran and Shive, 2007). During our sample period, we see this in Communications, Internet and Alternative Energy sectors. These findings suggest that we need to control for industry and timetrends in our tests.

[13] According to Prowse (1998), the venture capital market grew five-fold from the years 1980 to 1984 itself. The volume of venture capital available post-1980 dwarfs the pre-1980 levels.

[14] This database is one of the two main VC financing databases that have been used extensively in existing empirical studies. The other main database is VentureOne, offered by Dow Jones.

Table 1
Calendar time distribution of funding rounds.
This table reports the distribution of financing rounds and descriptive statistics of funding amounts across all the years in our sample period (2000-2010). The sample consists of all Venture Capital related funding rounds in the ThomsonOne VentureXpert database, which are classified as (1) Seed Stage; (2) Early Stage; (3) Expansion; and (4) Late Stage. All the amounts are reported in $ millions and are in nominal terms.

Year	Rounds of VC financing	Total VC investments	Avg. investment per round	25 pctile	Median investment per round	75 pctile
2000	7567	104,957	13.9	2.5	7.2	17.0
2001	4341	41,752	9.6	1.8	5.0	12.0
2002	2950	22,333	7.6	1.5	4.4	10.0
2003	2721	19,632	7.2	1.8	4.5	9.4
2004	2907	23,165	8.0	2.0	5.0	10.0
2005	3042	23,643	7.8	2.0	5.0	10.0
2006	3531	27,142	7.7	1.6	4.5	10.0
2007	3850	31,788	8.3	1.5	4.6	10.0
2008	3886	30,248	7.8	1.3	4.0	10.0
2009	2777	20,289	7.3	1.0	3.5	8.4
2010	3113	23,112	7.4	1.2	3.6	9.6
Total	40,685	368,061	9.0	1.7	5.0	11.0

In Panel B of Table 2, we report the distribution of investment round by the operating stage of the firm receiving the financing. VentureXpert uses four categories to denote the operating stage: (1) Seed Stage; (2) Early Stage; (3) Expansion; and (4) Late Stage. There are no clear, definite rules for how a firm is classified in each stage. These categories should be viewed as relative measures of firm development rather than absolute measures. We denote the “Seed Stage” rounds as “First Stage” investments corresponding to k_1 in our model. Typically, such investments are the first external investment in a firm. These investments are frequently made to entrepreneurs when they are launching the venture. “Early Stage” financing occurs subsequent to Seed Stage, and firms receiving this type of funding are more developed. We denote such funding rounds as “Second Stage” financing, which corresponds to k_2 in our model. For robustness, we also retain the two categories of “Expansion” and “Late Stage” financings for firms with a fairly developed product and market strategies. We consolidate these two types of financings in a single category, which we denote as “Third Stage”; this should be analogous to k_3 in our model.[15] We run our primary tests for relationships between First and Second Stage funding amounts as well as for Second and Third Stage financings. Unlike Panel A, we do not see dramatic time trends in distribution of VC funding across different stages. There is some evidence that VC financing became more focused on late stage investing during the middle of our sample period. However, by the end of the sample period in 2010, the relative shares of different stage financings had returned to almost the same levels as at the start of the sample period in 2000.

5.2. Empirical results

Theorem 1 predicts that the observed funding levels will be significantly higher for the Second Stage compared to First Stage. Our first set of tests aim to analyze the amounts provided by VCs across different stage of development (of the firm receiving the funding). A straightforward intuitive test for Theorem 1 is to compare average (median) amounts that are invested in different stages, and to test if these differences are significant. Table 3 reports average funding amounts for the three stages defined earlier. The amounts are adjusted for inflation using the CPI deflator, and are reported in constant 2009 dollars. As we can see, on average, the amount invested in the Second Stage is $7.31 million compared to $4.56 million in First Stage. This difference is statistically significant at 1% level (*t*-statistic of 11.85). The median amount in the First Stage is $1.87 million and in the Second Stage is $4.24 million. The Wilcoxon rank-sum test static for difference in medians is 27.35, which is significant at the 1% level. The last row of Table 3 compares the funding amounts for Second Stage versus Third Stage. Again, the Third Stage funding amounts are almost 50% larger both in terms of average as well as median amounts, compared to the Second Stage funding amounts. These differences are also statistically significant at the 1% level.

While the evidence provided in Table 3 is consistent with the main theoretical prediction, the descriptive statistics in Tables 1 and 2 show that there are strong calendar and industry level trends. These effects are not controlled for in the simple difference of means (medians) tests reported in Table 3. Consequently, to better distinguish the effect of operating stage of firm on the amount of funding received from venture capitalists, we test an OLS specification of the following form:

$$\begin{aligned}(LnAmount)_{ij} = \beta_0 + \beta_1(Second\,Stage)_i + \beta_2(Third\,Stage)_i \\ +\beta_3(Industry\,Dummy)_j + \beta_4(Year\,Dummy) \\ +\beta_5(Other\,Controls)_{ij}\end{aligned} \tag{21}$$

The regression results are described in Table 4 and provide a more robust estimate of how operating stage affects the funding amount. The dependent variable is the natural log of VC funding provided in round *i* to firm *j*. The amounts are in constant 2009

[15] It is easy to show that the theoretical predictions derived for k_1^*/k_2^* will also hold for k_2^*/k_3^*.

Table 2
Percentage of funding rounds by industry and stage of development.
Panel A describes the industry composition of our sample of VC financings through the sample period. The industry groups are as reported by VentureXpert. Panel B provides the same information across the operating stage of the firm raising VC funds. The operating stage is set at the time of the financing round and is based on the operating stage classification reported by VentureXpert. We denote the "Seed Stage" rounds as "First Stage". The "Early Stage" financing is denoted as "Second Stage" financing. The two categories of "Expansion" and "Late Stage" financings are consolidated in a single category which we denote as "Third Stage".

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Total
	%	%	%	%	%	%	%	%	%	%	%	%
Panel A: Industry												
Biotechnology	3.2	5.6	7.1	9.0	8.4	8.9	8.9	10.1	9.8	12.1	11.9	8.0
Communications/media	10.5	12.2	11.2	11.0	10.7	10.5	9.2	8.3	6.0	5.8	4.2	9.2
Computer hardware	2.9	2.9	3.3	3.6	3.9	3.6	3.6	3.8	3.5	3.6	4.0	3.4
Computer software	22.9	24.8	28.8	29.1	27.0	24.9	23.1	21.0	20.9	20.3	22.4	23.8
Consumer related	3.0	3.6	2.4	2.2	2.9	3.1	3.3	3.5	3.8	3.2	3.0	3.1
Industrial/energy	1.5	2.9	2.9	2.8	3.6	3.4	4.6	5.8	7.3	7.3	7.2	4.2
Internet specific	38.9	26.3	19.3	15.4	14.3	16.9	19.1	19.9	21.9	21.2	23.0	23.6
Medical/health	6.3	8.7	12.1	12.6	14.1	13.7	13.9	14.0	13.6	15.2	13.5	11.7
Other products	5.0	5.1	4.0	4.3	4.1	4.4	4.3	5.4	5.7	4.9	5.1	4.8
Semiconductors	5.9	7.8	9.1	10.1	11.1	10.7	9.9	8.1	7.5	6.3	5.7	8.1
Panel B: Stage												
First Stage	8.4	5.6	5.3	6.1	6.2	7.3	9.6	11.4	12.4	11.5	10.8	8.6
Second Stage	34.8	26.7	26.4	26.6	27.0	25.9	25.2	25.6	26.0	30.4	34.1	28.7
Third Stage	56.8	67.7	68.3	67.4	66.8	66.8	65.3	63.0	61.6	58.2	55.1	62.7

dollars to adjust for inflation. The regression omits the First Stage rounds and includes dummy variables for the Second Stage and Third Stage financing rounds. The regression also includes controls for industry and year of financing. VC industry is characterized by strong boom and bust patterns, so the inclusion of year dummies aims to correct for this feature. We also control for it in a more direct fashion by including the aggregate fund-raising activity of the VC industry. It can be argued that subsequent to the periods when the VC industry has raised significantly higher amounts, VCs will be willing to invest larger amounts per round.

In column 1 of Table 4, we report the base model without any controls for industry effects, but include year fixed effects. The coefficient for Second Stage dummy variable is 0.70. This is significant at the 1% level. Since the First Stage is the omitted variable, the constant term provides an estimate of average amount (in terms of natural logarithm) invested in this stage, after controlling for year effects. The coefficient of 7.06 translates into approximately $1.16 million dollars. The coefficient for the Second Stage dummy variable is 0.70. This implies that holding all else constant, the amount invested in Second Stage is $2.34 million. Thus, after controlling for calendar time effects, the average amount invested in Second Stage is roughly twice as large as the First Stage. Thus, the operating stage of a firm receiving VC funding has a significant effect on the amount provided by the VC. The coefficient is also significant at the 1% level (*t*-statistic of 24.90). The coefficient for Third Stage is even higher at 1.17, and also significant at the 1% level (*t*-statistic of 43.05). This implies that holding all else equal, a firm receiving funding at the Third Stage, on average, raises $3.75 million. This is almost three times the funding raised in the First Stage and 60% larger than the funding raised in the Second Stage.

In column 2, we include controls for the firm's industry. The omitted industry category is "Internet Specific." All industry dummies except Computer Hardware have significant coefficients. This underscores significant industry level effects on amounts raised. Biotechnology, Medical/Health, Communications, and Semiconductor firms raise larger amounts compared to Internet Specific firms, while others raise smaller amounts. However, our main variables of interest are the dummy variables for Second Stage and Third Stage. The coefficients for these are 0.73 and 1.21, respectively. Comparing these coefficients to those reported in column 1 shows that both the magnitude as well as significance are now even greater. Thus, even after controlling for industry and year effects, the operating stage of a firm has a significant economic and statistical effect on the VC amount raised by that firm. In the last column of Table 3, we use the incremental aggregate amount raised by Venture Capital funds, instead of year fixed effects, to control for trends in VC fund raising. We obtain this data from National Venture Capital Association's year book (2011 edition).[16] The variable "VC Fundraising" is the total amount raised (in billions of 2009 dollars) in the year prior to the year in which the financing round took place. The results are essentially unchanged compared to those reported in column 2. Taken together, the results described in Tables 3 and 4 provide significant support for Theorem 1.

Proposition 2 predicts that the ratio of funding in First Stage to Second Stage will decrease as the outside options for the Venture Capitalist and the entrepreneur increase. Sorensen (2007) finds that high quality VCs match with high quality entrepreneurs. This is consistent with industry folklore which attributes VC success to both the quality of VC executives as well as their access to quality "deal flow" (Hochberg et al., 2007). In short, we can abstract away from exact make-up of outside options for the entrepreneur, as long as we can find a good proxy for outside options enjoyed by the VC. Since the reputation of a VC is

[16] NVCA incrementally measures new commitments to funds raised during the calendar year. For example, a venture capital firm announces a $200 million fund in late 2007, raises $75 million in 2008, and subsequently raises the remaining $125 million in 2009. Nothing would be reflected in 2007, $75 million would be counted in 2008, and $125 million would be counted in 2009.

Table 3
Mean (median) amount invested in each stage.
This table reports the mean and median amount per funding round for different investment stages in our sample of VC financings. The operating stage of the firm raising VC funds is set at the time of financing and is based on the operating stage classification reported by VentureXpert. We denote the "Seed Stage" rounds as "First Stage". The "Early Stage" financing is denoted as "Second Stage" financing. The two categories of "Expansion" and "Late Stage" financings are consolidated in a single category which we denote as "Third Stage". All amounts are in thousands of constant 2009 dollars. (*** Significant at 1% level, ** Significant at 5% level,* Significant at 10% level).

	N	Amount (mean)	*t*-test	Amount (median)	Wilcoxon ranksum
First Stage	3508	4561		1869	
Second Stage	11,664	7310	–11.85***	4240	–27.35***
Third Stage	25,513	12,418	–28.12***	6760	–31.76***

considered to be its most significant characteristic in attracting entrepreneurs (i.e. generating good deal flow), we focus on this attribute. To keep our analysis simple and transparent, we use the two lists described by Metrick and Yasuda (2010) as our list of highly reputable VCs. While they admit that the lists are subjective, it is based on the following criteria:

- Firms that are able to raise their carried interest to 30%. This implies there is very high demand by investors to invest in funds sponsored by these VCs, since a typical "average" fund is only able to get a 20% carried interest.
- The firm had at least one fund with committed capital of $50 million or more, and was able to achieve a value multiple of 5 or greater. This condition ensures that the VC firm had at least one fund that generated extremely high returns for its investors.

Metrick and Yasuda (2010) list 15 VC firms that meet the two criteria listed above. They further narrow this list by using even stricter criteria: within this list of 15 firms, there are 6 firms that had at least one fund, which had a committed capital of $50 million or more, and achieved a value multiple of 10 or greater. This list is reproduced in Appendix B. In order to test Proposition 2, we first create the ratio of funds invested in different stages.[17] By construction, this limits our sample to only those firms that received financing at least once in two different and chronologically consistent stages.[18] For our study, we create two dummy variables, Top_VC1 and Top_VC2. The variable Top_VC2 equals one if at least one of the VCs in the later financing round was among the top 15 VC firms identified by Metrick and Yasuda (2010), and zero otherwise. The variable Top_VC1 for a financing round equals one if at least one of the VC firms in the later financing round is from the list of top six VC firms, and zero otherwise.

Table 5 provides a simple *t*-test result of the ratio of amounts invested in one stage divided by amounts invested in the following stage. In panel A, our sample is restricted to those firms that receive funding in the First Stage and go on to raise money in Second Stage. For each company that meets this requirement, we estimate the ratio[19] of amounts moved in each round to estimate the parameter k_1^*/k_2^* described in Proposition 2. We divide the sample in two subsamples. The first group only includes those firms that obtained their later round financing from a highly reputable VC, i.e., a VC firm listed in either the top 15 or top 6 by Metrick and Yasuda (2010). The k_1^*/k_2^* ratio for firms receiving funding from top VC is 0.88 if the VC is ranked in top 6 and 0.89 if the VC is ranked in top 15. The ratio increases to 0.93 if none of the VCs involved in the later round are ranked in the most reputable VC's list. The difference is statistically significant (*t* -statistic of 2.39 and 2.77). In Panel B of Table 5, we reproduce this analysis for firms that were able to raise money in both Second Stage and Third Stage. Conceptually, we test to see if the ratio k_2^*/k_3^* differs systematically depending on the reputation (i.e. outside options) of the VC providing the funds. Again, the ratio is 0.99 for non-ranked VCs and 0.93 (0.95) for the firms raising funds from a top 6 (top 15) VCs. This difference is significant at the 1% level. These results provide empirical support for Proposition 2, which predicts that the ratio of funds raised in earlier rounds to later rounds decrease in the outside options for the VC.

The results described in Table 5 are univariate and do not take into account industry and time effects. To control for these, we estimate an OLS model of the following form:

$$\left(\frac{k_t}{k_{t+1}}\right)_j = \beta_0 + \beta_1(Top\,VC) + \beta_3(Industry\,Dummy)_j + \beta_4(Year\,Dummy) \tag{22}$$

The dependent variable is either the ratio k_1^*/k_2^* or k_2^*/k_3^* for a firm *j* that received funding in two different stages. The variable Top_VC1 (Top_VC2) equals one if the later round financing involved at least one VC firm from the list of top 6 VCs (top 15 VCs) described in Appendix A. We also include industry and year fixed effects as in models estimated in Table 4. We report our results in Table 6. Columns 1 and 2 describe the results when the dependent variable is the ratio of amounts raised in First Stage and

[17] In cases where there are multiple rounds of financing classified under the same stage, we take the last round of financing classified in that stage and divide it by the first round of financing in the next stage. In our robustness test, we also used the average of all rounds in a particular stage divided by the average of all rounds in the subsequent stage. The results remain unchanged.
[18] To be included, a firm that receives funding in the first stage must also raise funding in the second stage.
[19] Since ratios are sensitive to extreme outliers, we use logarithm of amounts raised in each round, adjusted for inflation.

Table 4
Regression for funding amounts per round and investment stage.
This table reports the estimates of the following OLS model:

$$(lnAmount)_{ij} = \beta_0 + \beta_1(Second\,Stage)_i + \beta_2(Third\,Stage)_i + \beta_3(Industry\,Dummy)_j$$
$$+\beta_4(Year\,Dummy) + \beta_5(Other\,Controls)_{ij}$$

The dependent variable is the natural log of VC funding provided in round *i* to firm *j*. The amounts are in constant 2009 dollars to adjust for inflation. The industry groups are as reported by VentureXpert. Internet-specific is the omitted industry group. The operating stage is set at the time of financing and is based on the operating stage classification reported by VentureXpert. We denote the "Seed Stage" rounds as "First Stage". The "Early Stage" financing is denoted as "Second Stage" financing. The two categories of "Expansion" and "Late Stage" financings are consolidated in a single category which we denote as "Third Stage". First Stage is the omitted category. Heteroscedastic robust *t*-statistics for the coefficients are provided in the parentheses. (*** Significant at 1% level, ** Significant at 5% level, * Significant at 10% level).

	(1)	(2)	(3)
Constant	7.06***	6.96***	7.19***
	(183.72)	(172.97)	(237.94)
Second_Stage	0.70***	0.73***	0.75***
	(24.90)	(25.90)	(26.75)
Third_Stage	1.17***	1.21***	1.20***
	(43.05)	(44.46)	(44.44)
Biotech		0.33***	0.20***
		(11.12)	(6.83)
Communications_Media		0.27***	0.24***
		(10.15)	(8.84)
ComputerHardware		0.04	−0.03
		(0.94)	(−0.73)
ComputerSoftware		−0.06***	−0.11***
		(−3.06)	(−5.74)
Industrial_Energy		−0.02	−0.17***
		(−0.49)	(−3.98)
ConsumerRelated		−0.48***	−0.55***
		(−10.67)	(−12.08)
Medical_Health		0.16***	0.05**
		(6.03)	(2.06)
Other_Products		−0.17***	−0.23***
		(−4.50)	(−6.02)
Semiconductors		0.25***	0.18***
		(8.99)	(6.48)
VC_FundRaising			5.01***
			(22.83)
N	40,660	40,660	40,660
R^2	0.101	0.114	0.082

Second Stage (k_1^*/k_2^*). In column 1, we use Top_VC1 to measure the effect of VC reputation on the interstage funding ratio. The coefficient for Top_VC1 is −0.05 and it is significant at the 1% level. If we use Top_VC2 as a proxy for VC reputation, the results are essentially unchanged, with a coefficient estimate of −0.04 (significant at the 1% level). The negative and significant coefficients for the VC reputation variable can be interpreted as consistent with predictions of Proposition 2. In columns 3 and 4, we provide additional robustness test by using the funding ratio of second and Third Stage (k_2^*/k_3^*) as our dependent variable. Again, the coefficient is −0.06 for Top_VC1 and −0.05 for Top_VC2. In both models, the reputation of VC is significant at the 1% level. These tests provide additional empirical support for the Proposition 2.

Proposition 3 describes the effect of output variance on the amount invested (k_t^*) as well as on the funding ratio (k_1^*/k_2^*). We employ the monthly lagged level of implied volatility of S&P 500 (VIX) as a measure of market's anticipation of future uncertainty. VIX is reported by Chicago Board Options Exchange Market (CBOE) and is an implied Volatility Index, and can be interpreted as the market's expectation of movement for the S&P 500 index (Whaley, 2009). An elevated level of VIX implies a period of high *future* uncertainty, while a low level implies low future uncertainty. We use the daily VIX levels to compute mean monthly VIX values. Our basic OLS model is as follows

$$\begin{aligned}(LnAmount)_{ij} = \beta_0 &+ \beta_1(LagVIX)_i + \beta_2(Second\,Stage)_i + \beta_3(Third\,Stage)_i \\ &+\beta_4(Industry\,Dummy)_j + \beta_5(Year\,Dummy) \\ &+\beta_6(Other\,Controls)_{ij}\end{aligned} \tag{23}$$

The dependent variable is the log of amounts raised in round *i*. LagVIX is the level of average monthly VIX for the month immediately before the month in which the financing round took place. The other variables have been discussed earlier. Table 7A reports our estimation results. Column 1 provides a simple estimation with industry and year fixed effects. The coefficient of

Table 5
Ratio of funding amounts across different stages and VC reputation.
This table reports a simple *t*-test result of the ratio of amounts invested in one stage divided by amounts invested in the following stage. In panel A, our sample is restricted to those firms that receive funding in the First Stage and go on to raise money in Second Stage. For each company that meets this requirement we estimate the ratio of amounts in each round to estimate the parameter k_1^*/k_2^* described in Proposition 2. We divide the sample into sub samples. The first group only includes those firms that obtained their later round financing from a highly reputable VC , i.e., a VC firm listed in either the top 15 or top 6 by Metrick and Yasuda (2010). The second group did not receive funding from a highly ranked VC. Panel B reports the same test for the ratio of Second Stage funding to Third Stage funding amounts (k_2^*/k_3^*). (*** Significant at 1% level, ** Significant at 5% level, * Significant at 10% level).

	VC top ranked	VC not top ranked	*t*-test
Panel A: First Stage versus Second Stage (k_1^*/k_2^*)			
VC ranked as Top 6	0.88	0.93	2.39**
Number of observations (N)	81	1017	
VC ranked as Top 15	0.89	0.93	2.77***
Number of observations (N)	169	929	
Panel B: Second Stage versus Third Stage (k_2^*/k_3^*)			
VC ranked as Top 6	0.93	0.99	5.45***
Number of observations (N)	335	3708	
VC ranked as Top 15	0.95	0.99	6.24***
Number of observations (N)	721	3322	

LagVix is −0.56 and is significant at the 1% level. To get an economic intuition, if we assume that the firm being financed is from Internet Specific industry group (the omitted industry group) and the LagVIX is at its sample mean of 0.22, the predicted funding round amount is approximately \$2.82 million. A one standard deviation increase in LagVIX of 0.09 reduces this predicted funding

Table 6
Regression for ratio of funding across rounds and VC reputation.
This table reports the estimates of the following OLS model:

$$\left(\frac{k_t}{k_{t+1}}\right)_j = \beta_0 + \beta_1(Top\,VC) + \beta_3(Industry\,Dummy)_j + \beta_4(Year\,Dummy) + \beta_5(Other\,Controls)_{ij}$$

The dependent variable is either the ratio k_1^*/k_2^* or k_2^*/k_3^* for a firm *j* that received funding in two different stages. Columns 1 and 2 provide the results for the ratio of First Stage funding amount to Second Stage funding amount (k_1^*/k_2^*). Columns 3 and 4 report the same estimation using the ratio of Second Stage funding to Third Stage funding (k_2^*/k_3^*) as a dependent variable. The variable Top_VC1 (Top_VC2) equals one if the later round financing involved at least one VC firm from the list of top 6 VCs (top 15 VCs) described in Appendix B. We also include industry and year fixed effects as in models estimated in Table 4. Internet-specific is the omitted industry group. Heteroscedastic robust *t*-statistics for the coefficients are provided in the parentheses. (*** Significant at 1% level, ** Significant at 5% level, * Significant at 10% level).

	(1)	(2)	(3)	(4)
Constant	0.89***	0.90***	1.06***	1.06***
	(26.42)	(26.48)	(55.96)	(55.99)
Top_VC1	−0.05***		−0.06***	
	(−3.02)		(−7.19)	
Top_VC2		−0.04***		−0.05***
		(−3.65)		(−7.65)
Biotech	0.00	−0.00	−0.06***	−0.06***
	(0.00)	(−0.15)	(−5.42)	(−5.52)
Communications_Media	−0.03	−0.03	−0.01	−0.01
	(−1.42)	(−1.35)	(−0.97)	(−0.81)
ComputerHardware	0.01	0.01	−0.01	−0.01
	(0.24)	(0.25)	(−0.78)	(−0.72)
ComputerSoftware	−0.02	−0.02	−0.02**	−0.02**
	(−1.26)	(−1.35)	(−2.47)	(−2.49)
Industrial_Energy	−0.05*	−0.06**	0.02	0.02
	(−1.95)	(−2.04)	(0.73)	(0.68)
ConsumerRelated	0.06	0.06	−0.01	−0.01
	(1.33)	(1.30)	(−0.48)	(−0.58)
Medical_Health	0.02	0.02	−0.04***	−0.04***
	(0.86)	(0.75)	(−3.96)	(−3.98)
Other_Products	0.04	0.04	0.03	0.03
	(0.88)	(0.78)	(1.42)	(1.36)
Semiconductors	0.00	0.00	−0.05***	−0.04***
	(0.08)	(0.18)	(−4.63)	(−4.46)
N	1098	1098	4043	4043
R^2	0.063	0.066	0.040	0.043

Table 7A
Regression for funding amounts per round and output uncertainty.
This table reports the estimates of the following OLS model:

$$(lnAmount)_{ij} = \beta_0 + \beta_1(LagVIX)_i + \beta_2(Second\,Stage)_i + \beta_3(Third\,Stage)_i + \beta_4(Industry\,Dummy)_j$$
$$+\beta_5(Year\,Dummy) + \beta_6(Other\,Controls)_{ij}$$

The dependent variable is the natural log of VC funding provided in round i to firm j. The amounts are in constant 2009 dollars to adjust for inflation. VIX is the Chicago Board Options Exchange Market Volatility Index, which measures the implied volatility of S&P 500 index options in percentage. LagVIX is the level of average monthly VIX for the month immediately before the month in which the financing round took place. The industry groups are as reported by VentureXpert. Internet-specific is the omitted industry group. The operating stage is set at the time of financing and is based on the operating stage classification reported by VentureXpert. We denote the "Seed Stage" rounds as "First Stage". The "Early Stage" financing is denoted as "Second Stage" financing. The two categories of "Expansion" and "Late Stage" financings are consolidated in a single category which we denote as "Third Stage". First Stage is the omitted category. Heteroscedastic robust t-statistics for the coefficients are provided in the parentheses. (*** Significant at 1% level, ** Significant at 5% level, * Significant at 10% level).

	(1)	(2)	(3)
Constant	8.07***	7.16***	7.39***
	(153.74)	(126.43)	(215.20)
LagVIX	−0.56***	−0.61***	−0.96***
	(−4.52)	(−5.05)	(−11.09)
Second_Stage		0.73***	0.75***
		(25.95)	(26.72)
Third_Stage		1.21***	1.20***
		(44.53)	(44.32)
Biotech	0.22***	0.33***	0.21***
	(7.09)	(11.14)	(6.91)
Communications_Media	0.32***	0.27***	0.23***
	(11.69)	(10.20)	(8.59)
ComputerHardware	0.10**	0.04	−0.03
	(2.42)	(0.98)	(−0.74)
ComputerSoftware	−0.01	−0.06***	−0.11***
	(−0.65)	(-2.98)	(-5.79)
Industrial_Energy	−0.03	−0.02	−0.16***
	(−0.70)	(−0.46)	(-3.73)
ConsumerRelated	−0.41***	−0.48***	−0.55***
	(-8.93)	(-10.67)	(-12.13)
Medical_Health	0.13***	0.16***	0.05**
	(4.97)	(6.04)	(2.01)
Other_Products	−0.11***	−0.17***	−0.23***
	(-2.99)	(-4.48)	(-6.00)
Semiconductors	0.28***	0.25***	0.17***
	(9.59)	(8.97)	(6.15)
VC_FundRaising			5.52***
			(24.45)
N	40,660	40,660	40,660
R^2	0.056	0.115	0.085

amount by approximately $140,000 (almost 5% reduction). Thus, as predicted by Proposition 2, the observed funding amount declines when the overall uncertainty increases.

Columns 2 and 3 provide additional robustness checks. In column 2 we report the estimates with additional control for a firm's operating stage at the time of funding round. Consistent with our results in Table 4, operating stage continues to be a significant determinant of the amount raised in a round. The coefficient on LagVIX is still negative (−0.61) and continues to be significant at the 1% level. In column 3, we use the VC industry's annual fund raising level instead of year fixed effects. As expected, if the funding round happens following a year of high fund raising, the amount invested in such a round is significantly higher, as denoted by positive and significant coefficient for VC Fund-raising variable. The coefficient on LagVIX is even more negative (−0.91) and still significant at the 1% level. Thus, the first prediction arising from Proposition 3 is strongly supported by these results.

We next test the second prediction of Proposition 3, which states that the ratio of First Stage funding to Second Stage funding amount (k_1^*/k_2^*) will increase as the output variance increases. To test this, we estimate the following OLS model

$$\left(\frac{k_1}{k_2}\right)_j = \beta_0 + \beta_1(LagVIX) + \beta_2(Top_VC) + \beta_3(IndustryDummy)_j$$
$$+\beta_4(YearDummy) + \beta_5\left(Other\,Controls_{ij}\right) \tag{24}$$

The results are described in Table 7B. Columns 1 and 2 use the ratio k_1^*/k_2^* as the dependent variable. Our results in Tables 5 and 6 show that the reputation of the VC providing the funds is a significant determinant of the interstage funding ratio. To

Table 7B
Regression of ratio of funding amounts across subsequent stages and output uncertainty.
This table reports the estimates of the following OLS model:

$$\left(\frac{k_1}{k_2}\right)_j = \beta_0 + \beta_1(LagVIX) + \beta_2(TOp_VC) + \beta_3(Industry\,Dummy)_j$$
$$+\beta_4(Year\,Dummy) + \beta_5(Other\,Controls)_{ij}$$

The dependent variable is either the ratio k_1^*/k_2^* or k_2^*/k_3^* for a firm j that received funding in two different stages. VIX is the Chicago Board Options Exchange Market Volatility Index, which measures the implied volatility of S&P 500 index options in percentage. LagVIX is the level of average monthly VIX for the month immediately before the month in which the financing round took place. Columns 1 and 2 provide the results for the ratio of First Stage funding amount to Second Stage funding amount (k_1^*/k_2^*). Columns 3 and 4 report the same estimation using the ratio of Second Stage funding to Third Stage funding (k_2^*/k_3^*) as the dependent variable. The variable Top_VC1 (Top_VC2) equals one if the later round financing involved at least one VC firm from the list of top 6 VCs (top 15 VCs) described in Appendix B. We also include industry and year fixed effects as in models estimated in Table 4. Internet-specific is the omitted industry group. Heteroscedastic robust t-statistics for the coefficients are provided in the parentheses. (*** Significant at 1% level, ** Significant at 5% level, * Significant at 10% level).

	(1)	(2)	(3)	(4)
Constant	0.85***	0.86***	1.04***	1.04***
	(23.12)	(23.23)	(38.24)	(38.27)
LagVIX	0.33***	0.33***	0.05	0.05
	(3.33)	(3.33)	(0.90)	(1.03)
Top_VC1	−0.04***		−0.06***	
	(−2.86)		(−7.12)	
Top_VC2		−0.04***		−0.05***
		(−3.46)		(−7.63)
Biotech	−0.00	−0.00	−0.06***	−0.06***
	(−0.09)	(−0.23)	(−5.40)	(−5.51)
Communications_Media	−0.03	−0.03	−0.01	−0.01
	(−1.55)	(−1.49)	(−0.98)	(−0.82)
ComputerHardware	0.01	0.01	−0.01	−0.01
	(0.27)	(0.28)	(−0.79)	(−0.73)
ComputerSoftware	−0.02	−0.03	−0.02**	−0.02**
	(−1.56)	(−1.64)	(−2.52)	(−2.55)
Industrial_Energy	−0.05*	−0.05*	0.02	0.02
	(−1.77)	(−1.86)	(0.70)	(0.64)
ConsumerRelated	0.06	0.06	−0.01	−0.01
	(1.42)	(1.40)	(−0.50)	(−0.60)
Medical_Health	0.02	0.02	−0.04***	−0.04***
	(0.95)	(0.84)	(−3.94)	(−3.96)
Other_Products	0.04	0.04	0.03	0.03
	(0.89)	(0.80)	(1.44)	(1.38)
Semiconductors	0.00	0.00	−0.05***	−0.04***
	(0.12)	(0.21)	(−4.62)	(−4.46)
N	1098	1098	4043	4043
R^2	0.081	0.084	0.041	0.043

control for this effect, we include the Top_VC1 (Top_VC2) as additional control in column 1 (column 2). The key variable of interest for Proposition 2 is the coefficient for LagVIX, which is predicted to be positive and significant. This coefficient is indeed positive and significant at the 1% level when we use the ratio of First Stage funding amount to Second Stage funding amount (k_1^*/k_2^*). Independent of what VC reputation control we use, the coefficient is 0.33 and significant at the 1% level. In columns 3 and 4, we rerun our OLS model using the ratio of Second Stage funding to Third Stage funding amounts (k_2^*/k_3^*). While the coefficient for LagVIX is still positive (0.05), it is no longer statistically significant. Thus, while we are able to provide support for Proposition 3 in case of the first two funding rounds, our evidence is weaker as the ratio of Second and Third Stage funding does not appear to be associated with expected uncertainty.

Finally, we provide some empirical evidence for the Proposition 4, which states that as the valuation of the venture increases, any given funding round will be associated with larger amounts of capital raised. To test this proposition, we estimate the following model:

$$\begin{aligned}(lnAmount)_{ij} = {} & \beta_0 + \beta_1(ValuationUP)_i + \beta_2(Second\,Stage)_i + \beta_3(Third\,Stage_i) \\ & +\beta_4(IndustryDummy)_j + \beta_5(YearDummy) \\ & +\beta_6(OtherControls)_{ij}\end{aligned} \tag{25}$$

The dependent variable is the log of total amount raised by firm j in round i. VentureXpert provides a field titled "Valuation Direction", which describes whether the value of the venture in the current financing round has gone up or down, compared to the most recent round. We create a dummy variable ValuationUP which equals one if the current round is being raised at valuation higher than the most recent previous round, and zero otherwise. This information is only available for approximately 10% of the overall sample. The results of our tests are reported in Table 8. Column 1 provides the estimates for a base model that controls for the industry and time fixed effects. The coefficient for ValuationUp is 0.20. Thus, for an Internet-Specific industry firm (the omitted industry group), the expected funding amount increases from \$5.43 million, if the valuation direction is flat or down, to \$6.63 million if the valuation in the current round is higher than the previous round. This implies an almost 23% increase, which is economically significant. The coefficient is also significant at the 1% level. Column 2 includes the operating stage of the firm at the time of financing as additional controls. This specification makes the coefficient for the ValuationUP variable larger and more significant. In column 3, we replace year fixed effects by lagged annual fund raising by VC industry. The coefficient for ValuationUp is twice as large compared to column 1 (0.42) and strongly significant. The results described in Table 8 provide robust empirical support for Proposition 4.

Propositions 5 and 6 did not lend themselves to measurement, and therefore we did not test them. These propositions all involve variables that are not publicly available, but rather reside inside VC firms, if at all. For example, Proposition 5 postulates a relationship between the final hurdle $\bar{q}$ and first round investment k_1^*, the milestone q^*, and second round investment k_2^*. Of course, VCs do not disclose their final hurdle $\bar{q}$, and therefore it is impossible to test this proposition with publicly available data. Proposition 6 involves the cost of investment λ, which includes the transaction cost of deploying capital from existing funds, and

Table 8
Regression for funding amounts per round and valuation change.
This table reports the estimates of the following OLS model:

$$(lnAmount)_{ij} = \beta_0 + \beta_1(ValuationUP)_i + \beta_2(Second\,Stage)_i + \beta_3(Third\,Stage)_i + \beta_4(Industry\,Dummy)_j$$
$$+\beta_5(Year\,Dummy) + \beta_6(Other\,Controls)_{ij}$$

The dependent variable is the natural log of VC funding provided in round *i* to firm *j*. The amounts are in constant 2009 dollars to adjust for inflation. VentureXpert provides a field titled "Valuation Direction" which describes if the value of the venture in the current financing round has gone up or down compared to the most recent previous round. We create a dummy variable ValuationUP which equals one if the current round is being raised at a valuation higher than the most recent previous round and zero otherwise. The industry groups are as reported by VentureXpert. Internet-specific is the omitted industry group. The operating stage is set at the time of financing and is based on the operating stage classification reported by VentureXpert. We denote the "Seed Stage" rounds as "First Stage". The "Early Stage" financing is denoted as "Second Stage" financing. The two categories of "Expansion" and "Late Stage" financings are consolidated in a single category which we denote as "Third Stage". First Stage is the omitted category. Heteroscedastic robust *t*-statistics for the coefficients are provided in the parentheses. (*** Significant at 1% level, ** Significant at 5% level, * Significant at 10% level).

	(1)	(2)	(3)
Constant	8.60***	7.10***	7.10***
	(54.54)	(30.96)	(36.44)
Valuation_Up	0.20***	0.27***	0.42***
	(3.77)	(5.46)	(8.78)
Second_Stage		0.93***	0.96***
		(5.14)	(5.03)
Third_Stage		1.72***	1.76***
		(9.71)	(9.47)
Biotech	0.31***	0.38***	0.20***
	(4.98)	(6.41)	(3.45)
Communications_Media	0.48***	0.46***	0.38***
	(7.85)	(7.85)	(6.20)
ComputerHardware	−0.11	−0.15	−0.22
	(−0.83)	(−1.16)	(−1.64)
ComputerSoftware	−0.18***	−0.17***	−0.24***
	(−3.66)	(−3.59)	(−4.72)
Industrial_Energy	0.00	0.01	−0.15
	(0.01)	(0.08)	(−1.11)
ConsumerRelated	−0.88***	−0.93***	−1.01***
	(−5.03)	(−5.37)	(−5.98)
Medical_Health	0.13**	0.14**	−0.00
	(2.19)	(2.50)	(−0.04)
Other_Products	−0.39***	−0.43***	−0.53***
	(−2.82)	(−3.15)	(−3.78)
Semiconductors	0.27***	0.30***	0.17***
	(4.17)	(4.76)	(2.60)
VC_FundRaising			3.78***
			(8.42)
N	4742	4742	4742
R^2	0.111	0.187	0.138

the time and labor spent attracting new capital, via raising new funds. These are also costs, which are internal to the VC, and they are hard to measure.

6. Conclusion

Staged financing is a fundamental feature of the venture capital market. VCs do not fund new ventures all at once, but instead deliver the investments in stages, forcing the project to clear a sequence of milestones in order to guarantee future funding. Not only is staged financing efficient, but it skews the allocation of investment towards later stages. Staged financing creates the possibility of termination after the early stage, and this introduces uncertainty into the early stage. This uncertainty decreases the expected surplus in stage one, and therefore, it is efficient to invest less in stage one. Once the entrepreneur has proven his first stage output to be high ($q_1 > q^*$), this uncertainty vanishes, and expected surplus rises. As a result of this, it is efficient to invest more in the later stage.

The model produces a number of empirical implications. The secondary implications of the model all predict how the ratio of investment levels over stages (k_1^*/k_2^*) varies with the parameters of the model, such as the outside options of both parties, the variance in the error distribution, and the difficulty of project completion. We test these predictions against observed data on VC financings, and find preliminary support for our model. In particular, our initial empirical exercises show that the main result and several of these secondary implications are consistent with publicly available venture capital data. That the model here can be tested is both its distinguishing feature and its primary strength.

Future work in this area can extend this model in a number of promising directions. For example, the valuation of the project V is known by both parties at the outset, though in practice the firm's valuation is highly uncertain prior to the initial public offering. Also, it is an open question as to how syndicates of venture capitalists investing simultaneously in a firm will change the conclusions of this paper. We assumed throughout that the VC acts as a single entity, though in practice a lead venture capitalist provides the majority of the financing while secondary VCs share the risk by holding a minority share of the equity in the firm. We are optimistic about future theoretical work in the VC literature that connects theory with empirics, as we do here.

Appendix A

Proof of Proposition 1. Let $S(q_1) \equiv S(q_1, k_2^*)$ and $q_1^* = k_1^* + \varepsilon_1$. Since $g > 0$,

$$S'(q_1) = \frac{\partial S(q_1, k_2^*)}{\partial q_1} = P'(q_1)V = Vg(q_1 + k_2^* - \bar{q}) > 0.$$

So continuation surplus is strictly increasing and continuous in q_1. Recall that $V(q_1, q_2) \to 0$ as $q_t \to -\infty$ for some t. Since $\bar{u}_2 > 0$, there exists an x small enough such that $0 < S(x) < \bar{u}_2$. Now

$$\begin{aligned} PV - C(k_1^*) - C(k_2^*) &= \mathbb{E}_1 S(q_1^*) - C(k_1^*) \\ &= \mathbb{E}_1[\mathbb{E}_2 V(q_1^*, k_2^* + \varepsilon_2) - C(k_2^*)] - C(k_1^*) \\ &= \mathbb{E}V(q_1^*, q_2^*) - C(k_2^*) - C(k_1^*) \\ &\geq \bar{u}_1 + \bar{u}_2, \end{aligned}$$

where the inequality follows from project feasibility. Therefore $\mathbb{E}_1 S(q_1^*) > \bar{u}_2$. By the mean value theorem there exists a $y \in \mathbb{R}$ such that $S(y) = \mathbb{E}S(q_1^*)$, and hence $S(y) > \bar{u}_2 > S(x)$. By the intermediate value theorem there exists a $q^* \in (x, y)$ such that $S(q^*) = \bar{u}_2$. If $S(q_1) < \bar{u}_2$, it is efficient to terminate the project. Since $S(q_1)$ is monotonically increasing in q_1, this holds for $q_1 < q^*$ as well. ■

Proof of Theorem 1. Recall that

$$S(q_1) = P(q_1)V - C(k_2) \quad \textit{and} \quad P(q_1) \to 0 \quad \textit{as} \quad q_1 \to -\infty.$$

The planner solves

$$\max_{k_{t,q}} \int_q^\infty S(q_1, k_2) g(q_1 - k_1) dq_1 + (1 - P_1)\bar{u}_2 - C(k_1).$$

The first order conditions with respect to q,k_2,k_1 are

$$S(q^*) = \bar{u}_2\,;\int_{q^*}^{\infty}\frac{\partial S(q_1,k_2)}{\partial k_2}g(q_1-k_1^*)dq_1 = 0; C'(k_1^*) = -\int_{q^*}^{\infty}S(q_1)g'(q_1-k_1^*)dq_1-g(q^*-k_1^*)\bar{u}_2,$$

where $S(q_1)=S(q_1,k_2^*)$, and $S'(q_1)=\frac{\partial S(q_1,k_2^*)}{\partial q_1}$.

In what follows, we write k_t for k_t^* for visual clarity. Substituting $S(q^*)=\bar{u}_2$ into the last equation and integrating by parts gives

$$C'(k_1) = \int_{q^*}^{\infty}S'(q_1)g(q_1-k_1)dq_1.$$

From the continuation surplus function $S(q_1)=P(q_1)V - C(k_2)$,

$$S'(q_1) = g(q_1+k_2-\bar{q})V;$$
$$\frac{\partial S}{\partial k_2} = g(q_1+k_2-\bar{q})V-C'(k_2).$$

Combining these gives

$$\frac{\partial S}{\partial k_2} = g(q_1+k_2-\bar{q})V-C'(k_2).$$

Integrating both sides and combining with the FOC for k_2 yields

$$0 = \int_{q^*}^{\infty}\frac{\partial S(q_1)}{\partial k_2}g(q_1-k_1)dq_1 = \int_{q^*}^{\infty}S'(q_1)g(q_1-k_1)dq_1-P_1C'(k_2).$$

where $P_1=\Pr(q_1>q^*)$. Now combining with FOC for k_1 gives

$$C'(k_1) = \int_{q^*}^{\infty}S'(q_1)g(q_1-k_1)dq_1 = P_1C'(k_2)<C'(k_2).$$

And, since marginal costs are increasing, this means $k_1<k_2$. ■

Proof of Proposition 2. Consider possible values $\bar{u}_a$ and $\bar{u}_b$ for $\bar{u}_2$, with $\bar{u}_a$ corresponding to q_a^*, k_1^a, and k_2^a. Let $S_a(q_1)=S(q_1,k_2^a)$. And let $\bar{u}_b$ correspond to q_b^*, k_1^b, and k_2^b, where $S_b(q_1)=S(q_1,k_2^b)$.

Lemma 1. *If $\bar{u}_a<\bar{u}_b$, then $q_a^*-k_1^a<q_b^*-k_1^b$.*

Proof. Suppose the contrary, that $q_a^*-k_1^a\geq q_b^*-k_1^b$. For clarity, write ε for ε_1. Let $F(k_1{}^*,k_2{}^*,q^*|\bar{u}_2)$ be the value function of the social planner's objective function, so

$$F\Big(k_1^*,k_2^*,q^*|\bar{u}_2) = \max_{k_t,q}\int_q^{\infty}S(q_1,k_2)g(q_1-k_1)dq_1+(1-P_1)\bar{u}_2-C(k_1),$$

where $P_1=1 - G(q^* - k_1^*)$.

By optimality of (q_a^*, k_1^a, k_2^a) and (q_b^*, k_1^b, k_2^b),

$$F\Big(k_1^a,k_2^a,q_a^*|\bar{u}_a) > F\Big(k_1^b,k_2^b,q_b^*|\bar{u}_a)\,and\,F\Big(k_1^b,k_2^b,q_b^*|\bar{u}_b) > F\big(k_1^a,k_2^a,q_a^*|\bar{u}_b).$$

Expanding,

$$\bar{u}_a\int_{-\infty}^{q_a^*-k_1^a}g(\varepsilon)d\varepsilon+\int_{q_b^*-k_1^b}^{\infty}S_b\Big(\varepsilon+k_1^b\Big)g(\varepsilon)d\varepsilon-C\Big(k_1^b\Big)<\bar{u}_a\int_{-\infty}^{q_a^*-k_1^a}g(\varepsilon)d\varepsilon+\int_{q_a^*-k_1^a}^{\infty}S_a\big(\varepsilon+k_1^a\big)g(\varepsilon)d\varepsilon-C\big(k_1^a\big); \tag{A1}$$

$$\bar{u}_b\int_{-\infty}^{q_a^*-k_1^a}g(\varepsilon)d\varepsilon+\int_{q_a^*-k_1^a}^{\infty}S_a\big(\varepsilon+k_1^a\big)g(\varepsilon)d\varepsilon-C\big(k_1^a\big)<\bar{u}_b\int_{-\infty}^{q_b^*-k_1^b}g(\varepsilon)d\varepsilon+\int_{q_b^*-k_1^b}^{\infty}S_b\Big(\varepsilon+k_1^b\Big)g(\varepsilon)d\varepsilon-C\Big(k_1^b\Big). \tag{A2}$$

Now, (A1) implies

$$\int_{q_b^*-k_1^b}^{q_a^*-k_1^a}\left(\bar{u}_a-S_b\left(\varepsilon+k_1^b\right)\right)g(\varepsilon)d\varepsilon+\int_{q_a^*-k_1^a}^{\infty}\left(S_a\left(\varepsilon+k_1^a\right)-S_b\left(\varepsilon+k_1^b\right)\right)g(\varepsilon)d\varepsilon-C\left(k_1^a\right)+C\left(k_1^b\right)>0$$
$$\Rightarrow\int_{q_b^*-k_1^b}^{q_a^*-k_1^a}\left(S_b\left(\varepsilon+k_1^b\right)-\bar{u}_a\right)g(\varepsilon)d\varepsilon+\int_{q_a^*-k_1^a}^{\infty}\left(S_b\left(\varepsilon+k_1^b\right)-S_a\left(\varepsilon+k_1^a\right)\right)g(\varepsilon)d\varepsilon+C\left(k_1^a\right)-C\left(k_1^b\right)<0.$$

And, if $q_b^*-k_1^b \le q_a^*-k_1^a$), then it also holds that the left-hand side is negative when $\bar{u}_a$ is replaced by $\bar{u}_b$, since $\bar{u}_a<\bar{u}_b$. So

$$\int_{q_b^*-k_1^b}^{q_a^*-k_1^a}\left(S_b\left(\varepsilon+k_1^b\right)-\bar{u}_b\right)g(\varepsilon)d\varepsilon+\int_{q_a^*-k_1^a}^{\infty}\left(S_b\left(\varepsilon+k_1^b\right)-S_a\left(\varepsilon+k_1^a\right)\right)g(\varepsilon)d\varepsilon+C\left(k_1^a\right)-C\left(k_1^b\right)<0.$$

But a similar calculation subtracting the left-hand side of Eq. (A2) from the right shows that the term above is positive. Contradiction. Thus, $q_b^*-k_1^b>q_a^*-k_1^a$. ■

By the lemma, if $\bar{u}_a<\bar{u}_b$, then

$$\frac{\left(q_b^*-k_1^b\right)-\left(q_a^*-k_1^a\right)}{\bar{u}_b-\bar{u}_a}>0.$$

Taking the limits gives

$$\frac{\partial(q^*-k_1)}{\partial\bar{u}_2}=\lim_{\bar{u}_b\to\bar{u}_a}\frac{\left(q_b^*-k_1^b\right)-\left(q_a^*-k_1^a\right)}{\bar{u}_b-\bar{u}_a}\ge 0.$$

Now $P_1=1-G(q^*-k_1)$, so

$$\frac{\partial P_1}{\partial\bar{u}_2}=-g(q^*-k_1)\frac{\partial(q^*-k_1)}{\partial\bar{u}_2}<0.$$

Since $\frac{C'(k_1)}{C'(k_2)}=P_1$, this means

$$\frac{\partial\left(C'(k_1)/C''(k_2)\right)}{\partial\bar{u}_2}=\frac{\partial P_1}{\partial\bar{u}_2}<0.$$

For $C(k)=\lambda k^{\gamma}, \frac{C'(k_1)}{C'(k_2)}=\left(\frac{k_1}{k_2}\right)^{\gamma-1}$, so

$$\frac{\partial P_1}{\partial\bar{u}_2}=\frac{\partial(k_1/k_2)}{\partial\bar{u}_2}(\gamma-1)\left(\frac{k_1}{k_2}\right)^{\gamma-2}<0\Rightarrow\frac{\partial(k_1/k_2)}{\partial\bar{u}_2}<0,$$

since $k_t>0$, $\gamma>1$. ■

Proof of Proposition 3. Let g be uniform over $[-\beta, \beta]$ for $\beta>0$, and take the cost function to be quadratic, so $C(x)=\frac{\lambda x^2}{2}$.

Then,

$$S(q_1-k_2)=V\int_{\bar{q}}-q_1{}^{\infty}g(q_2-k_2)dq_2-\frac{\lambda}{2}k_2^2;S'(q_1)=Vg(\bar{q}-q_1k_2);\frac{\partial S(q_1)}{\partial k_2}=S'(q_1)-\lambda k_2.$$

This gives first order conditions

$$S(q^*) = \bar{u}_2 \Leftrightarrow V\int_{\bar{q}} - q^{*\infty} g(q_2-k_2)dq_2 - \frac{\lambda}{2}k_2^2 = \bar{u}_2\,.$$

$$\int_{q^*}^{\infty} \frac{\partial S(q_1)}{\partial k_2} g(q_1-k_1)dq_1 = 0 \Leftrightarrow \int_{q^*}^{\infty} Vg(\bar{q}-q_1-k_2)g(q_1-k_1)dq_1 = \int_{a^*}^{\infty} \lambda k_2 g(q_1-k_1)dq_1.$$

Since g' is not defined, use the equivalent formulation

$$C'(k_1) = \int_{q^*}^{\infty} S'(q_1)g(q_1-k_1)dq_1 \Leftrightarrow \lambda k_1 = \int_{q^*}^{\infty} Vg(\bar{q}-q_1-k_2)g(q_1-k_1)dq_1.$$

Take candidate $\tilde{q}^*$, $\tilde{k}_1$, $\tilde{k}_2$ values as:

$$\tilde{q}^* = \frac{8\beta\bar{u}_2 - 4\beta V + 4\bar{q}V - \frac{V^2}{\beta\lambda}}{4V}; \tag{A3}$$

$$\tilde{k}_1 = \frac{-8\beta\bar{u}_2 + 8\beta V - 4\bar{q}V + \frac{V^2}{\beta\lambda}}{4(4\beta^2\lambda - V)}; \tag{A4}$$

$$\tilde{k}_2 = \frac{V}{2\beta\lambda}. \tag{A5}$$

We claim that for β large enough, these values satisfy the three FOCs. For sequences $x(n)$, $y(n)$, say $x(n)$ is asymptotically equal to $y(n)$ (i.e. $x(n)\sim y(n)$) if

$$\lim_{n\to\infty} \frac{x(n)}{y(n)} = 1.$$

Observe first that as $\beta \to \infty$,

$$\lim_{\beta\to\infty} \tilde{k}_1 = 0, \qquad \lim_{\beta\to\infty} \tilde{k}_2 = 0, \qquad \tilde{q}^* \sim \left(\frac{2\bar{u}}{V} - 1\right)\beta.$$

Since $\bar{u}<V, \frac{2\bar{u}}{V} - 1 \in (-1, 1)$. Observe that

$$\tilde{k}_1 - \beta < \tilde{q}^* \textit{ for large enough } \beta. \tag{A6}$$

This holds because $\tilde{k}_1 - \beta \sim -\beta$, but $\tilde{q}^* \sim \alpha\beta$ for $\alpha \in (-1, 1)$. And

$$\tilde{k}_1 < \bar{q} - \tilde{k}_2 \textit{ for large enough } \beta, \tag{A7}$$

since $\bar{q} > 0$, and $\tilde{k}_1 \to 0$, $\tilde{k}_2 \to 0$. Moreover,

$$q^* > \bar{q} - \tilde{k}_2 - \beta \textit{ for large enough } \beta, \tag{A8}$$

since, $q^* \sim \alpha\beta$ for $\alpha > -1$, $\bar{q} - \tilde{k}_2 - \beta \sim -\beta$. Finally,

$$\bar{q} - \tilde{q}^* > \tilde{k}_2 - \beta \textit{ for large enough } \beta, \tag{A9}$$

since $\bar{q} - \tilde{q}^* \sim (-\alpha)\beta$ for $\alpha < 1$, and $\tilde{k}_2 - \beta \sim -\beta$.

Now observe that for sufficiently large β

$$\begin{aligned} V\int_{\bar{q}-\tilde{q}^*}^{\infty} g\left(q_2 - \tilde{k}_2\right)dq_2 - \frac{\lambda}{2}k_2^2 &= \frac{V}{2\beta}\left(\tilde{k}_2 + \beta - \max\left(\tilde{k}_2 - \beta, \bar{q} - \tilde{q}^*\right)\right) - \frac{\lambda}{2}\tilde{k}_2^2 \\ &= \frac{V}{2\beta}\left(\tilde{k}_2 + \beta - \bar{q} + \tilde{q}^*\right) - \frac{\lambda}{2}\tilde{k}_2 \textit{ by } = \bar{u}_2, \end{aligned}$$

where the last equality follows from plugging in $\tilde{k}_2$, $\tilde{q}^*$.

Claim 1. For large enough β,

$$\int_{\tilde{q}^*}^{\infty} Vg\left(\bar{q}-q_1-\tilde{k}_2\right)g\left(q_1-\tilde{k}_1\right)dq_1 = \int_{\tilde{q}^*}^{\infty} \lambda k_2 g(q_1-k_1)dq_1.$$

Proof. This holds iff

$$\frac{V}{4\beta^2}\left(min\left(\bar{q}-\tilde{k}_2+\beta,\beta+\tilde{k}_1\right)-\max\left(\bar{q}-\tilde{k}_2-\beta,-\beta+\tilde{k}_1,\tilde{q}^*\right)\right)=\frac{\lambda\tilde{k}_2}{2\beta}\left(\tilde{k}_1+\beta-\max\left(\tilde{q}^*,\tilde{k}_1-\beta\right)\right),$$

which, by Eqs. (A6), (A7), and (A8) holds iff

$$\tilde{k}_2=\frac{V}{2\beta\lambda}.\ \blacksquare$$

Claim 2. For large enough β,

$$\lambda\tilde{k}_1=\int_{\tilde{q}^*}^{\infty} Vg\left(\bar{q}-q_1-\tilde{k}_2\right)g\left(q_1-\tilde{k}_1\right)dq_1.$$

Proof. As before in Claim 1, this holds iff

$$\frac{V}{4\beta^2}\left(\tilde{k}_1+\beta-\tilde{q}^*\right)=\lambda\tilde{k}_1.$$

Plugging in $\tilde{k}_1$ and $\tilde{q}^*$ confirms that this holds. ■

So, for large enough β, $\tilde{k}_1,\tilde{k}_2,\tilde{q}^*$ satisfy the three FOCs. Observe that $\tilde{k}_2\sim\frac{\gamma}{\beta}$, $\tilde{k}_1\sim\frac{\delta}{\beta}$, and $\frac{\tilde{k}_1}{\tilde{k}_2}$ approaches $\frac{V-\bar{u}}{V}$.

In fact,

$$\frac{\tilde{k}_1}{\tilde{k}_2}=\frac{\beta\lambda\left(8\beta(V-\bar{u}_2)-4\bar{q}V+\frac{V^2}{\beta\lambda}\right)}{2V(4\beta^2\lambda-V)};\frac{\partial\frac{k_1}{k_2}}{\partial\beta}=\frac{2\lambda\left(4\bar{u}_2\beta-6V\beta+\bar{q}\left(V+4\lambda\beta^2\right)\right)}{(V-4\lambda\beta^2)^2}>0.$$

As $\beta\rightarrow\infty$, this goes as $\frac{J}{\beta^2}$ where J is positive.

Eventually, increasing the width of the support of ε_t makes $\frac{k_1}{k_2}$ increase to some asymptote $\frac{1-\bar{u}}{V<1}$. ■

Proof of Proposition 4. The same argument from the proof of Proposition 3 shows that the candidate values $\tilde{q}^*$, $\tilde{k}_1$, and $\tilde{k}_2$, given by Eqs. (A3), (A4), and (A5), will satisfy the first order conditions. Straightforward computations show that

$$\frac{\partial q^*}{\partial V}\quad and\quad \frac{\partial k_2^*}{\partial V}>0.$$

Furthermore, for large β

$$\frac{\partial k_1}{\partial V}\sim\frac{1}{\beta\lambda}>0\quad and\quad \frac{\partial(k_1/k_2)}{\partial V}\rightarrow 0.\ \blacksquare$$

Proof of Proposition 5. Straightforward computations on the candidate values $\tilde{q}^*$, $\tilde{k}_1$, and $\tilde{k}_2$, defined in Eqs. (A3), (A4), and (A5) in the proof of Proposition 3, shows that

$$\frac{\partial k_2}{\partial\bar{q}}=0,\qquad and\quad \frac{\partial q^*}{\partial\bar{q}}=1,\qquad \frac{\partial k_1}{\partial\bar{q}}=\frac{V}{4\beta^2\lambda-V}$$

for sufficiently large β. ■

Proof of Proposition 6. Straightforward calculations on the candidate values $\tilde{q}^*$, $\tilde{k}_1$, and $\tilde{k}_2$, from Eqs. (A3), (A4), and (A5) from the proof of Proposition 3, show that

$$\frac{\partial q^*}{\partial \lambda} = \frac{V^2}{\beta\lambda^2} > 0 \qquad and \qquad \frac{\partial k_2}{\partial \lambda} = \frac{-V}{2\beta\lambda^2} < 0. \blacksquare$$

Appendix B. List of highly reputable venture capital firms

In this Appendix we list the top 6 and top 15 VC firms as ranked by Metrick and Yasuda (2010). While they claim that the list is subjective, all firms in this list meet the following two benchmarks.

- Firms that are able to raise their carried interest to 30%. This implies that there is very high demand by investors to invest in funds sponsored by these VCs.
- The firm had at least one fund with committed capital of $50 million or more and was able to achieve a value multiple of 5 or greater. This condition ensures that the VC firm had at least one fund that generated extremely high returns for its investors.

The top 6 VCs in panel A meet an even higher standard of performance. Apart from meeting the 30% carried interest, these firms have at least one fund of committed capital of $50 million or larger that achieved a value multiple of 10 or greater.

VC Firm	Location	Year of Founding
Panel A: Top 6		
Accel Partners	Palo Alto, CA	1983
Benchmark Capital	Menlo Park, CA	1985
Charles River Ventures	Waltham, MA	1970
Kleiner Perkins Caufield & Byers	Menlo Park, CA	1972
Matrix Partners	Waltham, MA	1982
Sequoia Capital	Menlo Park, CA	1971
Panel B: Top 15 (Including all VCs listed in Panel A)		
Battery Ventures	Wellesley, MA	1983
Doll Capital Management (DCM)	Menlo Park, CA	1996
Draper Fisher Jurveston	Menlo Park, CA	1986
Institutional Venture Partners	Menlo Park, CA	1974
InterWest Partners	Menlo Park, CA	1979
Menlo Ventures	Menlo Park, CA	1976
New Enterprise Associates	Baltimore, MD	1978
Summit Partners	Boston, MA	1984
Technology Crossover Ventures	Palo Alto, CA	1995

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