RAND Journal of Economics
Vol. 52, No. 2, Summer 2021
pp. 334–358

# Team incentives under private contracting

**Maris Goldmanis***

**and**

**Korok Ray****

*We model a moral hazard in teams problem in which a profit-maximizing principal offers private contracts to multiple agents. Public contracts are common knowledge to all agents, but private contracts are known only by the principal and each individual agent. Public contracts can induce efficient outcomes but are subject to effort-reducing collusion between the principal and any given agent. Private contracts, by construction, are immune to such collusion but necessarily inefficient, as the principal is forced to make the team collectively the residual claimant (on margin), whereas efficiency requires that each individual agent be the residual claimant on his own.*

## 1. Introduction

■ There is no question that teamwork is vital to modern organizations, as teams permeate all levels of the corporate hierarchy (boards, executive teams, partnerships, project teams). Much of the prior literature has modeled these teams as a collection of agents who sign individual or joint contracts on their output with a principal. Yet an underlying assumption is that the contracts are public, namely, that all team members know the contracts offered to the others. In practice, contracts are largely private, and agents rarely know the full details offered to other members of the team.[1] We explore the economic effects on teamwork of private contracts in a setting where team members are contracted by a principal. We find that in order for private contracts to be in equilibrium (or, alternatively, for public contracts to be immune to bilateral renegotiation between the principal and the agents), the team as a whole must be made the residual claimant: that is, the marginal unit of output must be shared exactly between the agents, and the principal's marginal profit must be zero. This condition, which in our setting emerges endogenously due to the need to prevent secret renegotiation of contracts, can be seen as a marginal equivalent of the "budget balance"' condition exogenously imposed on teams in much of the literature on team

* Royal Holloway, University of London; maris.goldmanis@rhul.ac.uk.
** Texas A&M University; korok@tamu.edu.

[1] For example, the Big Four auditors keep their partner distributions secret, as compensation can be a sensitive topic that reveals performance, which both the firm and the partner may prefer to keep between themselves, not announced to all parties.

incentives in the absence of a principal. The similarity, however, should not be taken too far, as the collective residual claimant property must apply only for the marginal unit of output at the equilibrium and not for the total output or even for the marginal units of output at non-equilibrium levels of production.

Our setup is characterized by three key assumptions. First, contracts are (at least to some extent) private. This is common in economic settings in which there is asymmetric information or a large number of participants. Second, the agents are contracting with a profit-maximizing principal. Note that this excludes professional partnerships where the partners contract directly with each other as a group. Third, each individual agent's contribution to the output of the team is either unobservable or not contractible, so that contracts must be written on aggregate output.

There are a number of important applications that satisfy all of these assumptions. One example is government procurement in which the government acts as the buyer of a product that must require the collective work of several independent parties, such as in defense procurement for tanks or jets. Another example is the management team of a large international corporation, where each manager/executive has a private contract with the firm and each team member's contribution to the success of the firm is very hard to measure. A similar situation occurs in a large project team in software development or the film industry. Finally, even in simple settings, such as a home builder constructing a house, the individual subcontractors do not know of the offers granted to the other parties, so the painter and plumber privately contract with the home builder.

In all these settings, there is a team production problem, where the individual team members are producing some collective output, but asymmetric information, arising from the scale of operations, prevents full knowledge of the contract set to each individual and direct measurement of each individual's contribution. Although it may be possible that small teams have knowledge of all contracts, such as the founding team of a start-up venture, as the company grows, the new holders of equity will not know the full equity shares offered to other and prior executives. In this sense, private contracting is a natural outgrowth of the size and complexity of the firm. When the firm becomes sufficiently large and diverse, it is unlikely that all people within the firm know about the full contract set.

Public contracting is the benchmark case that has occupied most of the prior literature on team production. The principal simultaneously makes offers that are public (known to all parties); this gives rise to the bonding contract of Holmström (1982), in which every agent becomes a residual claimant on the firm's output and the principal charges each agent the efficient output level up front. Although this achieves efficiency, one chief problem of this "bonding" contract is that the principal has an incentive to reduce output, even though she[2] captures total surplus. She effectively pays more than one dollar in wages for every dollar in output and, therefore, has an incentive to strike a private contract with any one agent to reduce effort. This private contract would be acceptable for both parties: The agent is held to his reservation utility anyway, and the principal gains if output falls. The possibility of private contracts breaks the bonding contract.[3]

This begs the question of what the equilibrium looks like under private contracting. We propose a model of private contracts in which the principal makes contracts with a team of agents, but each agent only knows the terms of his own contract. Each agent therefore must speculate on the contract given to other agents. This creates a complex game of imperfect information, which has multiple equilibria. To narrow the scope of the problem, we follow the literature to consider only passive beliefs:[4] If the principal deviates from an agent's equilibrium contract, the agent believes that all other agents continue to face their equilibrium contracts. Under this assumption, we are able to derive our main result: that the equilibria under private contracting

[2] Throughout the article, we will refer to the agents as male and the principal as female.

[3] This vulnerability of the efficiency-inducing contracts of Holmström (1982) has been known at least since Eswaran and Kotwal (1984), who demonstrated that private contracting would break the forcing contract, which pays all agents a prize if the efficient outcome is obtained.

[4] This term is due to McAfee and Schwartz (1994). We will discuss the role of beliefs further when formally specifying our model. We will explore what happens when this assumption is relaxed in Section 5.

are characterized by a condition we call the collective residual claimant property. This states that the marginal unit of output at the equilibrium level must be exactly distributed among the agents, leaving the principal with zero marginal profit. (Note, however, that the total profit will be positive, whereas the agents will receive zero surplus. The principal will take all the surplus in via fixed participation fees.)

The intuition behind this result fundamentally stems from externalities. Under team production, because payment is made on group output, there is a positive externality from any agent's effort choice on other agents. Once any given agent $i$ has already committed to exert his efficient effort, he creates a positive externality on any other agent $j$, who implicitly has already received some benefit from the first agent's work. At that point, the principal and agent $j$ are both better off if agent $j$ shirks. Private contracts can thus break the public contracting equilibrium. Ultimately, the principal's profit function is invariant to marginal changes in output if and only if the collective residual claimant property holds exactly. In that case, the principal will have no incentives to privately contract to either increase or decrease any individual agent's effort.

In this sense, the literature has come full circle. The initial literature assumed budget balance as an exogenous constraint imposed on team production because it resembled a feature observed in some specific teams (like partnerships), but collective residual claimancy, which can also be thought of as "marginal budget balance" among the agents, has real economic content, as is apparent through the private contracting game. This marginal budget balance kills the principal's incentives to privately contract because it makes its payoff function insensitive to marginal changes in output.

An interesting implication of our central result is that the optimal private contracting outcome can always be implemented with a linear contract. This essentially happens because the conditions for optimality in the case of smooth contracts only pin down the optimal contract at a single point (the equilibrium output), and that jumps in contracts can never occur at the equilibrium point.

One limitation of our main model is its restriction to risk neutrality for the agents. We relax this assumption in Section 5, showing that in the case of risk-averse agents, the public equilibrium is still not immune to private modification by private contracts, but the collective residual claimant property is now modified by a risk premium term, making the team less than a full residual claimant on the margin.

Our analysis is related to two broad strands of literature: first, the literature on moral hazard in teams (pioneered by Holmström [1982]) and, second, the literature on bilateral contracting in multiagent settings with externalities, mostly developed in the context of vertical contracting in IO and pioneered by Crémer and Riordan (1987), Hart and Tirole (1990), and O'Brien and Shaffer (1992).

The team incentives literature has, until quite recently, focused on public contracts. Working in a setting without risk aversion, Holmström (1982) showed that the efficient outcome can be induced using either a discontinuous "forcing" contract that pays the agents only if the target outcome has been reached or an affine (linear) "bonding" contract, where each agent pays a participation fee equal to the entire output of the firm and then becomes the full residual claimant. Holmström (1982) also observed that efficient contracts would break the team's budget and would therefore need very strong commitment to implement, unless the team contracted with a non-productive principal. Rasmusen (1987) developed a version of Holmström's model under risk aversion using randomized contracts that balanced the team's budget even without a principal. Holmström and Milgrom (1987) characterized optimal contracts in a dynamic setting where agents, having constant absolute risk aversion, are provided incentives over time. They showed that the outcome is equivalent to a static setting with normally distributed noise and linear contracts. McAfee and McMillan (1991) showed that in a setting with both risk aversion and moral hazard, the optimal contracts are linear (affine). Finally, we should note that the team incentives problem is formally equivalent to the problem of cost sharing and cost allocation, which has been studied separately (Demski, 1981; Rajan, 1992; Baldenius, Dutta, and Reichelstein, 2007; Ray and Goldmanis, 2012).

Whereas only two years after Holmström's original contribution Eswaran and Kotwal (1984) showed that the efficient contracts of Holmström (1982) are vulnerable to private renegotiation, the team production literature never really turned to characterizing the set of contracts that would be immune to such renegotiation. The framework for private contracts was developed elsewhere—in the IO literature on bilateral contracting in a multiagent setting with externalities. The canonical application there is contracting between a manufacturer (who takes the role of a principal) and a number of retailers (who are the agents).

This vertical contracting literature clearly exposes the moral hazard problem in settings where the principal contracts bilaterally and privately with multiple agents who exert externalities on each other. The literature also establishes conditions under which contracts become immune to the principal's moral hazard problem in a variety of settings. Importantly for us, this literature also establishes the solution concepts on which we build our equilibrium definitions. These concepts include the notion of contract equilibrium consisting of those contracts that are immune to pairwise deviations (Crémer and Riordan, 1987; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994) and the assumption of passive beliefs (Hart and Tirole, 1990; McAfee and Schwartz, 1994). Several authors address the possible problems associated with the now-standard assumption of passive beliefs (including potential nonexistence) and provide alternatives (McAfee and Schwartz, 1995; Rey and Vergé, 2004; Eguia, Llorente-Saguer, Morton, and Nicolò, 2018). Segal (1999) and Segal and Whinston (2003) have developed generalized models that encompass much of the rest of the earlier literature.

Segal (1999) is the closest to our setting. On the face of it, one major difference is that his model does not include moral hazard on the part of the agent (although it does allow for moral hazard on the side of the principal, including the common agency problem of Bernheim and Whinston [1986]). However, as a given contract effectively induces a unique effort choice by the agent in our setup, our model is easy to transforminto one where the only decision variables are the contract choice by the agent and the acceptance/rejection decision by the principal, effectively allowing our model to fit in Segal's setup. However, due to its generality, Segal's model is not suited to develop a sharp characterization of the optimal wage contract in our specific setting (such as the collective marginal claimant property). Indeed, although our model is technically a subset of the models covered by Segal (1999), the insights we produce are new and not directly derivable from his setup.

An interesting and somewhat unexpected connection exists between our article and Dequiedt and Martimort (2015), who examine the vertical contracting problem of Industrial Organization when agents have private information. Although their contracts are public, the fact that information is private and reports manipulable by the principal makes the true terms of each contract essentially private. This creates a temptation for the principal to deviate from the efficient public contract (by manipulating the reports of the agents). The solution to the principal's moral hazard problem is, just as in our article, to impose a constant-profit condition on the principal to make her indifferent between the various manipulations. Thus, whereas in our article information is perfect and contracts private, which is the exact opposite of Dequiedt and Martimort (2015), fundamentally, the two settings are very close.

We do not consider private contracts between the agents themselves, as in Holmström and Milgrom (1990), Macho-Stadler and Pérez-Castrillo (1993), Ramakrishnan and Thakor (1991), Itoh (1993), and Varian (1990). Those articles all require some kind of individual performance measure or the ability for agents to observe action choices of other agents that the principal cannot, which does not fit the setting of this article, but more importantly, the temptation to privately contract rests with the principal, not with each individual agent. Even though the principal captures total surplus at the efficient effort profile, she still has the temptation to reduce aggregate output. It is this temptation that leads to privately contracting with any individual agent in order to reduce output.

The literature on repeated games has relevance for our results as well, such as Abreu, Dutta, and Smith (1994). For example, Baker, Gibbons, and Murphy (1994) show that implicit incentives can serve as a complement to explicit incentives in a repeated game between a single principal and single agent. Che and Yoo (2001) show that the optimal incentive scheme uses low-powered group incentives in a repeated game with a principal and a team. Both of these articles, like much of the repeated game literature, lean on the repetition of a static game to sustain equilibria that would vanish in a one-shot game, using folk-theorem-style arguments. Although we do not consider repeated games here, the notion of private contracting conceptually takes place in a dynamic setting. Indeed, the broad literature on repeated games in contract theory provides additional context for the more traditional explicit incentives considered here. Future research will provide more explicit characterizations of this dynamic interaction, and it will illustrate more explicitly how the possibility of building a reputation may make available equilibria that were previously unavailable in a one-shot game.

## 2. The model

■ Consider a risk-neutral principal contracting with $n \geq 2$ risk-neutral agents, each of whom has an outside option of zero. Each participating agent $i$ exerts effort $e_i \geq 0$ at cost $C_i(e_i)$, where $C_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing and strictly convex $C^2$ function satisfying $C_i(0) = 0$. Let $N = \{1, 2, \ldots, n\}$ be the team and let $\boldsymbol{e} \equiv (e_i)_{i \in N}$ be the effort vector of all agents. Let the team production function be $q : \mathbb{R}^n_+ \to \mathbb{R}_+$. Each agent observes only his own effort, whereas all parties (the principal and all agents) observe joint output $q = q(\boldsymbol{e})$.[5] We assume that $q$ is a weakly concave $C^2$ function that is strictly increasing in each agent's effort.[6] Note that this implies that $q_i(\boldsymbol{e}) \equiv \frac{\partial q}{\partial e_i}(\boldsymbol{e}) > 0$, and $q_{ii}(\boldsymbol{e}) \equiv \frac{\partial^2 q}{\partial e_i^2}(\boldsymbol{e}) \leq 0$ for all $i$ and $\boldsymbol{e}$, so that from each agent's perspective the production function is strictly increasing and has nonincreasing returns. We remain agnostic about the cross-partial $q_{ij}(\boldsymbol{e}) \equiv \frac{\partial^2 q}{\partial e_i \partial e_j}(\boldsymbol{e})$ for $i \neq j$, which could be positive (strictly supermodular production technology), zero (additive production technology), or even negative (strictly submodular production technology). We do note though that the case of supermodularity is probably the most natural, as it implies synergies among the team: when an agent works harder, the other agents' marginal productivities also increase. Also observe that supermodularity of the production function is likely to imply strategic complementarity between the agents' efforts when each agent's compensation is proportional to output: as agent $i$ works harder, the marginal return from agent $j$'s effort increases, causing $j$ to also work harder. Similarly, submodularity is likely to cause efforts to be strategic substitutes.

We assume that there exists a first-best effort vector $\boldsymbol{e}^*$, which maximizes total surplus:

$$\boldsymbol{e}^* \in \arg\max_{\boldsymbol{e} \in \mathbb{R}^n_+} q(\boldsymbol{e}) - \sum_{i \in N} C_i(e_i). \tag{FB}$$

Our assumptions on $q$ and $C_i$ guarantee that $\boldsymbol{e}^*$ is the unique solution to the system of first-best conditions given by

$$q_i(\boldsymbol{e}^*) = C_{i'}(e_{i*}) \forall i \in N. \tag{FB $-$ FOC$_i$}$$

In words, the marginal cost of effort equals its marginal return, given by the marginal productivity of any given agent's effort on team production. First best output is $q^* \equiv q(\boldsymbol{e}^*)$.

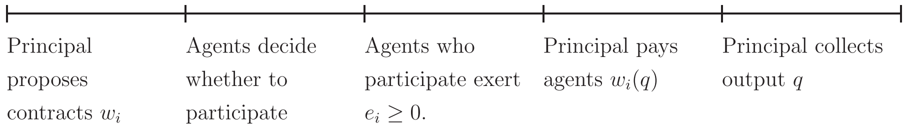
A bilateral contract is a weakly increasing and weakly concave $C^2$ function $w_i : \mathbb{R}_+ \to \mathbb{R}$, giving the total compensation paid by the principal to agent $i$ as a function of total output. We

[5] There is no measurement error or other stochastic variation. Because joint output is pooled across agents, there is still inherent difficulty in measuring performance. Even if performance were observable, it is still not necessarily contractible.

[6] Except possibly at points where $e_i = 0$ for some $i$, in which case $q_j(\boldsymbol{e}) = 0$ is allowed for $j \neq i$. This allows for production functions that require positive effort from all agents for positive output.

FIGURE 1

TIMELINE OF THE MODEL.




denote $s_i \equiv w_i(0)$ and, for each $q \in \mathbb{R}_+$, $b_i(q) \equiv w_i(q) - s_i$. We can interpret $s_i$ as agent $i$'s fixed salary (if $s_i > 0$) or participation fee (if $s_i < 0$) and $b_i$ as agent $i$'s output-dependent bonus.

We denote the set of permissible bilateral contracts by $W$. Generally, $W$ is taken to be the set of all weakly increasing and weakly concave $C^2$ functions, but in some examples and in the section on risk aversion, we will restrict $W$ to contain only linear contracts, given by $b_i(q) = b_i q$. This fits many common applications in practice, in which partnerships receive a fixed salary (possibly negative to allow a buy-in to the partnership).

The restriction to continuous contracts excludes the class of contracts where remuneration jumps abruptly when an output threshold is reached, such as the forcing contract of Holmström (1982), which implements first-best by paying the agents a prize if and only if the efficient level of output has been reached. Contracts of this kind are problematic due to their vulnerability to small trembles and information imperfections. For such contracts to work, the principal would need to know the solution she seeks to induce and would have to commit to levying harsh penalties if output varies from that solution. Agents, in turn, would accept such contracts only if they were absolutely sure that no other agent would deviate even slightly and that no random event would affect the output. Continuous contracts do not have this type of vulnerability.

We should note though that evidence on the prevalence of various compensation schemes in practice is scarce (Prendergast, 1999; Brown, 1992). Thus, despite the theoretical appeal of smooth contracts due to their tractability and lower commitment requirements, we should be wary of results that are restricted exclusively to continuous compensation schemes. Fortunately, in Section 5, we are able to show that allowing jump-discontinuous contracts, such as the forcing contract, does not change the set of implementable outcomes, so not much generality is lost by excluding them.[7]

The requirement that the contract be increasing and concave will guarantee the existence of unique incentive-compatible effort levels for all agents. We assume this globally for convenience, but it is clear that in general this property must hold only locally, in a neighborhood of the optimal solution.

## 3. Public Contracting

■ The principal *publicly* offers a bilateral contract $w_i$ to each agent. Importantly, the principal can commit to this public offer so she can be trusted not to privately contract with any agent. The game proceeds as follows: (1) the principal publicly makes bilateral contract offers $w_i$; (2) each agent, simultaneously with all others, accepts or rejects the contract offered to him; (3) agents who accept their contracts work, all picking their effort levels at the same time; (4) the principal pays out wages $w_i$ and collects $q$. Figure 1 outlines the timeline of the model.

As is common with team production problems, this public contracting game has multiple Subgame Perfect Nash Equilibria (SPNEs) in pure strategies. To ease analysis, we will follow most of the literature by examining the principal's preferred SPNE, which allows to formulate the problem as one where the principal chooses both the contract and the effort level to maximize

[7] The randomized equivalents introduced by Rasmusen (1987), for example, the scapegoat or massacre contracts, may not be subject to renegotiation, but are not applicable to our risk-neutral setting, as their punishment works only on risk-averse agents.

her profit subject to incentive compatibility and individual rationality (participation) constraints for the agents.

Incentive compatibility requires that the effort level must be consistent with rational choice by a participating agent, given the agent's contract and the equilibrium effort levels of all other agents. Thus, effort vector $\boldsymbol{e}$ is incentive compatible (IC) given contracts $\boldsymbol{w}$ if and only if, for all $i$,

$$e_i \in \arg\max_{e'_i} w_i(q(e'_i, \boldsymbol{e}_{-i})) - C_i(e'_i), \qquad \text{(IC}_i\text{)}$$

whose first-order conditions are (for all $i$)

$$w'_i(q(\boldsymbol{e}))q_i(\boldsymbol{e}) = C'_i(e_i). \qquad \text{(IC-FOC}_i\text{)}$$

Observe that our assumptions on the production and cost functions, along with the restriction to weakly concave contracts, guarantee that each agent's objective is everywhere strictly concave[8] so that the first-order conditions are necessary and sufficient for a maximum, and we can use (IC-FOC$_i$), instead of (IC$_i$), without loss of generality.

Individual rationality requires that each agent prefers to work for the principal, instead of taking his outside option (which we have normalized to zero):

$$w_i(q(\boldsymbol{e})) - C_i(e_i) \geq 0. \qquad \text{(IR}_i\text{)}$$

Let us call a contract-effort tuple $\langle \boldsymbol{w}, \boldsymbol{e} \rangle \in W^n \times \mathbb{R}^n_+$ a **proposal**. We can now formally define equilibrium in the setting of public contracts:

*Definition 1.* The proposal $\langle \widehat{\boldsymbol{w}}, \widehat{\boldsymbol{e}} \rangle$ is a **public contracts equilibrium** if

$$\langle \widehat{\boldsymbol{w}}, \widehat{\boldsymbol{e}} \rangle \in \arg\max_{\langle \boldsymbol{w}, \boldsymbol{e} \rangle \in W^n \times \mathbb{R}^n_+} q(\boldsymbol{e}) - \textstyle\sum_{i \in N} w_i(q(\boldsymbol{e})) \qquad (1)$$

subject to the constraints (IC$_i$) and (IR$_i$) for all $i$.

Next, note that the principal will select each $w_i$ such that (IR$_i$) binds:

$$w_i(q(\boldsymbol{e})) - C_i(e_i) = 0. \qquad \text{(IR}'_i\text{)}$$

As usual, this occurs because shifting $w_i$ by a constant only transfers rent between the principal and agent $i$ but does not affect the incentive constraint (IC-FOC$_i$). Thus, the principal can always shift the contracts so as to extract the whole surplus, holding all agents to their outside options.

Substituting the binding individual rationality constraint (IR$'_i$) into the objective function, we obtain

$$q(\boldsymbol{e}) - \sum_{i \in N} C_i(q(\boldsymbol{e})), \qquad (2)$$

which is identical to total surplus in (FB). Comparing the incentive compatibility constraints (IC-FOC$_i$) to the first-order conditions for first best (FB-FOC$_i$), we see that the principal can obtain the unconstrained maximum and implement efficient effort, $\widehat{\boldsymbol{e}} = \boldsymbol{e}^*$, by choosing $\widehat{w}_i$ such that

$$\widehat{w}'_i(q(\boldsymbol{e}^*)) = 1. \qquad \text{(PUB-FOC}_i\text{)}$$

Equation (PUB-FOC$_i$) states that at the equilibrium each agent is made the full residual claimant (on the margin): A unit increase in total output results in a unit increase in each agent's remuneration. Note, however, that this residual claimant property does not necessarily have to hold globally; it is sufficient for it to hold locally, at the equilibrium level of output.

[8] The second derivative of the objective is $w''_i q_i^2 + w'_i q_{ii} - C''_i < 0$, as $w'_i \geq 0$, $w''_i \leq 0$, $q_i > 0$, $q_{ii} \leq 0$, and $C''_i > 0$.

The optimal effort vector is unique because $\boldsymbol{e}^*$ is the unique maximizer of total surplus. The set of optimal contracts, however, is far from unique. It is fully characterized by equation (PUB-FOC$_i$) and the binding individual rationality constraints (IR$'_i$), and there is a continuum of contracts that satisfy these conditions.

In particular, observe that the optimum set includes the linear contracts $\widehat{w}_i(q) = \widehat{s}_i + q_i$, where $\widehat{s}_i = C_i(e_i^*) - q^* < 0$. Note that all agents' fixed salaries are negative, $\widehat{s}_i < 0$, because the total surplus is $q^* - \sum_{i \in N} C_i(e_i^*) > 0$.[9]

Thus, under public contracts, the principal would be able to implement first best even when restricted to linear contracts. The optimal linear contract is the standard bonding contract, in which the principal compensates each agent for his individual cost of effort, and each agent pays $q^*$ upfront. The principal makes each agent the residual claimant to guarantee efficient labor supply (note that with linear contracts the residual claimant property holds globally, at all levels of output). As the agents are held to their participation constraints, the principal earns all the rents, so the principal's profit is $\widehat{\pi} > 0$. Every agent faces a negative salary,[10] similar to the feature of many professional partnerships that require partners to buy into the partnership, such as in accounting, law, or medicine. Moreover, the large tournaments literature pioneered by Lazear and Rosen (1981) uses these bonding contracts: The agents are risk neutral, the win/loss prizes are large, and the principal collects the surplus from the agents through a bonding fee upfront.

The principal acts as an independent party who shares in the output of the team but bears no cost of production. As Holmström (1982) noted, introducing such a third party is necessary to achieve efficiency even under "budget balance" (i.e., without the need for the productive agents to commit to discarding all output if the target level has not been reached). Of course, the budget is balanced in a trivial sense because the third party plays no role in production, and *any* choice of sharing rules would balance the budget because the third party acts as a sink (Miller, 1997).[11]

In a public contracts equilibrium, every agent earns the full return to their labor on the margin but pays for this through a large negative salary. In practice, limited liability constraints of agents will prohibit the use of large negative salaries, but this is not the only problem with the efficient equilibrium contracts, as such contracts lead to collusion between the principal and agents, as we show in the next section.

## 4. Private contracting

■ In many contracting scenarios (such as employment), the principal contracts privately with each agent, even when the agents work together on a joint project. If the principal cannot commit to public contracts, but instead can make private contracts with each agent, the equilibrium contract above can no longer implement the first-best effort allocation. To see this, consider a simple example:

*Example 1.* Let $n = 2$. Let output be $q(e_1, e_2) = e_1 + e_2$. Let $C_i(e_i) = \frac{1}{2}e_i^2$, so $C_i'(e_i) = e_i$. First-best effort is $e_i^* = 1$, first-best output is $q^* = 2$, and cost of effort is $C_i(e_i^*) = \frac{1}{2}$. Total surplus is $q^* - \sum C_i(e_i^*) = 1$. Let the contract space be restricted to linear contracts, $w_i(q) = s_i + b_i q$. The efficient bonus that implements first-best is $\widehat{b}_i = 1$ with a "salary" of $\widehat{s}_i = \frac{1}{2} - 2 = -\frac{3}{2}$. Each agent earns $\widehat{u}_i = 0$. The principal's profit is $\widehat{\pi} = (1 - \widehat{b}_1 - \widehat{b}_2)q^* - (\widehat{s}_1 + \widehat{s}_2) = 1$. Suppose that both agents agree to this contract. However, after signing the contract but before working, the principal makes a private contract with the second agent. She renegotiates by proposing a new

[9] This observation extends to general weakly concave smooth contracts, for which $\widehat{w}_i(q^*) = \widehat{w}_i(0) + \int_0^{q^*} \widehat{w}_i'(q)\,dq \geq \widehat{w}_i(0) + \int_0^{q^*} \widehat{w}_i'(q^*)\,dq = \widehat{w}_i(0) + q^*$, where the inequality follows from concavity. Now, $\widehat{s}_i = \widehat{w}_i(0) \leq \widehat{w}_i(q^*) - q^* = C_i(e_i^*) - q^* < 0$.

[10] If agents faced a general outside option $\bar{u} > 0$, the efficient salary would be $\bar{u} + C_i(e_i^*) - q^*$, a linear increasing function of $\bar{u}$. These outside options do not change the bonus, so the normalization of $\bar{u} = 0$ is without loss.

[11] To see that the budget is balanced, let $w_p = q - \sum_{i \in N} w_i$ be the principal's wage. Then the sum of all wages is trivially equal to output: $w_p + \sum_{i \in N} w_i = q$.

contract $\widetilde{w}_2(q) = 0$. Facing this contract, the agent will select effort $\widetilde{e}_2 = 0$ earning utility $\widetilde{u}_2 = 0$, making him indifferent between this new contract and his original one. If the agent accepts this new contract, he effectively does not participate in the team, receiving no payments and making no contribution to the output. The first agent, however, does not observe this deviation and, therefore, exerts the efficient level of effort and (expecting the other agent to do the same) is happy to pay the previously agreed salary. The principal's profit is now

$$\widetilde{\pi} = e_1^* - \widehat{b}_1 e_1^* - \widehat{s}_1 = \frac{3}{2} > 1 = \widehat{\pi}. \tag{3}$$

In the example, the principal has an incentive to privately contract with one agent to reduce this agent's output to zero. The mechanics of this is straightforward. When agent two privately drops out, there are two types of losses and two types of gains to the principal: $(L1)$ reduction of total output by $q^* - \widetilde{q}$; $(L2)$ loss of agent two's fixed fee, $-\widehat{s}_2$; $(G1)$ savings in the form of bonus no longer paid to agent two, $\widehat{b}_2 q^*$; $(G2)$ savings due to reduced bonus for agent one, $\widehat{b}_1(\widetilde{q} - q^*)$. Because agent one is a residual claimant ($b_1 = 1$), the loss of output is exactly absorbed by the reduction in this agent's bonus, $G2 = L1$. Thus, the total gain to the principal from the renegotiation is simply the withheld net payment to the agent who dropped out, $\widehat{b}_2 q^* + \widehat{s}_2$, which, by the individual rationality constraint, is equal to the positive effort cost of this agent when participating, $C_1(e_1^*) > 0$.

The driving force behind the example is as follows. Because the efficient contract makes both agents residual claimants on the margin, any output losses from one agent's reduced effort are absorbed by the remaining agents, so the principal can enjoy the cost savings from having to pay the now-shirking agent less without ill effects to herself. Another way to look at this is that the principal benefits from effort (and hence output) reductions because the sum of the additional bonuses she needs to pay out under the efficient contract for each marginal unit of output is double the size of the increase of output. This logic is general: We will see that private contracting always breaks a public contracts equilibrium under public offers. First, however, we need to formalize our notion of private contracting.

Ultimately, we would like to analyze a game of imperfect information that is an exact analogue of the game in the previous section, except for the information environment. Figure 1 would still describe the timeline of the game, but each agent would only observe his own contract $w_i$ and would need to form beliefs over the possible contracts offered to others, $\boldsymbol{w}_{-i}$. We would be looking for the principal's most preferred Perfect Bayesian Equilibrium (PBE) in pure strategies. Unfortunately, such a game does not lend itself to very elegant direct analysis, due to the large number of classes of possible deviations from the equilibrium path by the principal (one class for each subset of agents). Consequently, we will define our private contracts equilibrium as the set of proposals that are immune to private renegotiation between the principal and **at most one agent**.[12]

We could interpret our equilibrium literally, as the principal's most preferred PBE of the game obtained from the public contracting one (as shown in Figure 1) by inserting a private renegotiation stage (where the principal is allowed to renegotiate with at most one agent) immediately after the initial public proposal. Such an interpretation, however, is slightly dissatisfying, as it is not clear why the exogenous restriction to renegotiation with at most one agent would exist. Alternatively, like McAfee and Schwartz (1995), we can think of this equilibrium as giving an upper bound on what the principal could achieve in the full game, where the principal privately contracts with all agents. In particular, note that our pairwise-proofness constraints ($\text{PP}_i$) must also hold in the general game (in which the equilibrium contract must be immune to renegotiation with any subset of agents, and, therefore, also to renegotiation with any single agent). Thus, any

[12] This approach has become standard in the vertical contracting literature in industrial organization, with notable examples including the "contract equilibria" of Crémer and Riordan (1987), O'Brien and Shaffer (1992), and the "pairwise-proof equilibrium" of McAfee and Schwartz (1995).

direct implications of this constraint would also hold in the general game. Finally, in Section 5 (Proposition 4), we show that in our setting, at least in the space of linear contracts, a contract's being immune to renegotiation with a single agent is not only necessary, but also sufficient for being immune to bilateral renegotiation with an arbitrary number of agents.[13] When Proposition 4 applies, our private contracts equilibrium will in fact be equal to the set of the principal's most preferred equilibria of the fully private contracting game described in the previous paragraph.

Before proceeding with the definition, we need to specify one remaining element of the underlying game: beliefs. Each agent's participation decision and effort choice (in the case of strict super- or submodularity of the production function) will depend on his beliefs about offers extended to the other agents (as these determine their effort choices). An agent who has received an equilibrium offer must of course believe that other agents have received an equilibrium offer (by consistency of beliefs in PBE). Off the equilibrium path, however, arbitrary beliefs are possible, generating many PBEs. To avoid this multiplicity of equilibria, we will follow McAfee and Schwartz (1994) and Segal (1999) by restricting attention to "passive beliefs": after observing an out-of-equilibrium offer from the principal, an agent believes that the other agents continue to face their equilibrium offers.[14] Incidentally, these are the only beliefs that are consistent with the literal interpretation of our equilibrium because, in this game, the principal can renegotiate with at most one agent. An agent receiving an alternative offer knows that the other agents have not received one. However, keeping in mind the fully private contracting game, we do explore alternative beliefs in Section 5.

We are now ready to formally state the private contracting problem. If we start with the proposal $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$, the set of alternative proposals $\langle \widetilde{w}_i, \widetilde{e}_i \rangle$ that agent $i$ will accept are precisely those that are incentive compatible and individually rational for agent $i$, given his belief that other agents still receive their original proposals $\langle \boldsymbol{w}_{-i}, \boldsymbol{e}_{-i} \rangle$. Incentive compatibility for a deviating offer is given by

$$\tilde{e}_i \in \arg\max_{e_i'} \tilde{w}_i(q(e_i', \boldsymbol{e}_{-i})) - C_i(e_i'), \qquad \text{(IC-D}_i\text{)}$$

which is equivalent to the first-order condition

$$\tilde{w}_i'(q(\tilde{e}_i, \boldsymbol{e}_{-i}))q_i(\tilde{e}_i, \boldsymbol{e}_{-i}) = C_i'(\tilde{e}_i). \qquad \text{(IC-D-FOC}_i\text{)}$$

Individual rationality for a deviating offer is given by:

$$\tilde{w}_i(q(\tilde{e}_i, \boldsymbol{e}_{-i})) - C_i(\tilde{e}_i) \geq 0. \qquad \text{(IR-D}_i\text{)}$$

Note also that for the original proposal (that is, $\langle \widetilde{w}_i, \widetilde{e}_i \rangle = \langle w_i, e_i \rangle$) (IC-D$_i$) and (IR-D$_i$) are equivalent to (IC$_i$) and (IR$_i$).

Thus, given $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ that satisfies (IC$_i$) and (IR$_i$), the principal cannot deviate in a profitable way for agent $i$ if and only if

$$\langle w_i, e_i \rangle \in \arg\max_{\langle \tilde{w}_i, \tilde{e}_i \rangle \in W \times \mathbb{R}_+} q(\tilde{e}_i, \boldsymbol{e}_{-i}) - \tilde{w}_i(q(\tilde{e}_i, \boldsymbol{e}_{-i})) - \textstyle\sum_{j \neq i} w_j(q(\tilde{e}_i, \boldsymbol{e}_{-i}))$$
$$\text{subject to the constraints (IC-D}_i\text{) and (IR-D}_i\text{).} \qquad \text{(PP}_i\text{)}$$

We will call such $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ **pairwise-proof for agent *i***. If a proposal is pairwise-proof for all $i \in N$, we will call it simply **pairwise-proof**. Note that we have defined pairwise-proofness only for proposals that are incentive compatible and individually rational: It would make no sense to talk about modifying a contract that would never have been accepted in the first place.

[13] As the argument proving this relies only on the local behavior of the contracts near the equilibrium, it is likely that it can be extended to general smooth contracts.

[14] The term "passive beliefs" was coined by McAfee and Schwartz (1994), but such beliefs had been adopted earlier by the vertical contracting IO literature pioneered by Hart and Tirole (1990) ("market-by-market" bargaining) and O'Brien and Shaffer (1992).

A private contracts equilibrium is the most profitable incentive compatible and individually rational proposal that is also pairwise-proof:

*Definition 2.* The proposal $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ is a **private contracts equilibrium** if

$$\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle \in \underset{\langle \boldsymbol{w}, \boldsymbol{e} \rangle \in W^n \times \mathbb{R}^n_+}{\arg\max} \; q(\boldsymbol{e}) - \textstyle\sum_{i \in N} w_i(q(\boldsymbol{e})) \quad \text{subject to the constraints (IC}_i\text{), (IR}_i\text{), and (PP}_i\text{) for all } i. \tag{4}$$

To determine the set of private contracts equilibria, we begin by characterizing the set of pairwise-proof proposals. By the usual logic, for the optimal deviation (for each $i$), the individual rationality constraint (IR-D$_i$) must bind. Substituting this into the objective function of (PP$_i$) turns that objective into

$$q(\widetilde{e}_i, \boldsymbol{e}_{-i}) - C_i(\widetilde{e}_i) - \sum_{j \neq i} w_j(q(\widetilde{e}_i, \boldsymbol{e}_{-i})). \tag{5}$$

Because the adjusted objective no longer contains $\widetilde{w}_i$, the effort in the optimal deviation is found as the unconditional maximizer of the objective, and the contract is then set to satisfy (IC-D$_i$) and (IR-D$_i$) with this effort level. For any $i$, $\widetilde{e}_i = e_i$ is a maximizer of the objective of (PP$_i$) if and only if the following first-order and second-order conditions hold:

$$q_i(\boldsymbol{e}) \left( 1 - \sum_{j \neq i} w_j'(q(\boldsymbol{e})) \right) - C_i'(e_i) = 0; \quad \text{(PP-FOC}_i\text{)}$$

$$q_{ii}(\boldsymbol{e}) \left( 1 - \sum_{j \neq i} w_j'(q(\boldsymbol{e})) \right) - q_i(\boldsymbol{e}) \sum_{j \neq i} w_j''(q(\boldsymbol{e})) - C_i''(e_i) < 0. \quad \text{(PP-SOC}_i\text{)}$$

Recalling that for $\langle \widetilde{w}_i, \widetilde{e}_i \rangle = \langle w_i, e_i \rangle$ (IC-D$_i$) and (IR-D$_i$) are equivalent to (IC$_i$) and (IR$_i$), we now have a full characterization of the set of pairwise-proof proposals: These are precisely the $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ that satisfy (IC$_i$), (IR$_i$), (PP-FOC$_i$), and (PP-SOC$_i$) for all $i$. Furthermore, substituting (IC$_i$) into (PP-FOC$_i$), we see that for any $i$ (PP-FOC$_i$) reduces to a simple condition on the sum of the first derivatives of the contracts:

$$q_i(\boldsymbol{e}) \left( 1 - \sum_{j \in N} w_j'(q(\boldsymbol{e})) \right) = 0 \quad \text{(PP-FOC}_i'\text{)}$$

$$\Leftrightarrow \quad \sum_{j \in N} w_j'(q(\boldsymbol{e})) = 1. \qquad \text{(CRC)}$$

The condition (CRC) is paramount for understanding private contracts equilibria. It says that at the equilibrium each additional unit of output produced on the margin is exactly distributed among the agents. In other words, the team is collectively made the residual claimant. This leads to the following definition:

*Definition 3.* The proposal $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ satisfies the **collective residual claimant** property if condition (CRC) holds.

Note that this is a local property of $\boldsymbol{w}$ at $\boldsymbol{e}$, which does not necessarily have to hold globally for $\boldsymbol{w}$ (similarly to the individual residual claimancy at the public equilibrium). We could alternatively call this the *zero marginal profit* property, as it is equivalent to the requirement that the principal's marginal profit from an additional unit of output at the equilibrium is zero.

The role of the collective residual claimant property is to prevent private deviations by the principal from being profitable. When the principal secretly contracts with agent $j$ to reduce his effort, the resulting output loss on the margin must be exactly offset by marginal savings in wages. If the marginal output loss is more than offset by marginal decreases in wages (the sum of the marginal wages exceeds unity), the principal wants to secretly reduce $j$'s effort. If it is not fully offset, the principal wants to secretly increase $j$'s effort.

At this point, the individual rationality constraint is the only constraint affected by level shifts of the remuneration (i.e., shifts in $s_i$). Therefore, the principal can, as usual, shift the wage schedules to make ($\text{IR}_i$) bind for all $i$. Substituting the binding individual rationality constraints into the objective function and applying the conclusions about pairwise-proofness obtained so far, we can state the central result of this article: Private contracts equilibria are the proposals that maximize total surplus subject to individual rationality, incentive compatibility, collective residual claimancy, and the regularity condition ($\text{PP-SOC}_i$):

*Proposition 1.* The proposal $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ is a private contracts equilibrium if and only if

$$\begin{gathered}\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle \in \underset{\langle \boldsymbol{w}, \boldsymbol{e} \rangle \in W^n \times \mathbb{R}^n_+}{\arg\max} \; q(\boldsymbol{e}) - \textstyle\sum_{i \in N} C_i(e_i) \\ \text{subject to the constraints (CRC) and } (\text{IC}_i),\ (\text{IR}_i),\ \text{and } (\text{PP-SOC}_i) \text{ for all } i.\end{gathered} \tag{6}$$

Comparing Proposition 1 to the principal's program for public contracts equilibria, we see that the difference lies precisely in the collective residual claimant property.[15]

The collective residual claimant property is at odds with the optimality condition defining a public contracts equilibrium, equation ($\text{PUB-FOC}_i$), which requires that each individual team member be made the residual claimant. The public contracts equilibrium condition implies that $\sum_{j=1}^{n} w_j'(q(\widehat{\boldsymbol{e}})) = n > 1$, which violates collective residual claimancy. It follows that all public contracts equilibria are outside the feasible set for private contracts equilibria:

*Corollary 1.* A public contracts equilibrium is never a private contracts equilibrium.

Thus, as already anticipated in Example 1, the option to contract privately always breaks the efficient public contracts equilibria. The intuition is the same as in Example 1. Under a public contracts equilibrium, the principal benefits from effort (and hence output) reductions because the sum of the additional bonuses she needs to pay out for each marginal unit of output exceeds the increase of output by a factor of $n$.

Corollary 1 complements Eswaran and Kotwal (1984), who find that if the principal cannot commit to public offers, group penalty schemes cannot implement first-best effort in a moral hazard in teams setup. Group penalty schemes are nonlinear forcing contracts, paying out bonuses to all agents only if all agents select first-best effort. Our result gives a similar impossibility in the world of continuous contracts on joint output. Both results rely on private contracting to break the public contracting equilibrium.

Note that when evaluated at a public contracts equilibrium, $\langle \widehat{\boldsymbol{w}}, \widehat{\boldsymbol{e}} \rangle$, the left-hand side of ($\text{PP-FOC}_i'$) is negative (because $1 - \sum_{j=1}^{n} w_j'(q(\widehat{\boldsymbol{e}})) = 1 - n < 0$). Recalling that the left-hand side of ($\text{PP-FOC}_i'$) is the marginal benefit to the principal from privately increasing the effort proposed to agent $i$, the fact that this expression is negative when evaluated at the public equilibrium proves that it is optimal for the principal to *decrease* this proposed effort, which we now state as the next corollary:

*Corollary 2.* At any public contracts equilibrium, the principal has an incentive to deviate by privately contracting with any agent to decrease the agent's effort.

Note, however, that this does *not* guarantee that the private equilibrium effort of all agents will be lower than in the public equilibrium. We will compare equilibrium efforts shortly.

[15] In addition, there is also ($\text{PP-SOC}_i$), the associated regularity condition on the shape of the wage schedule, but this, as we will see shortly, has no influence on the equilibrium effort and output.

We can gain additional insight into the principal's program for a private contracts equilibrium by substituting (IC-FOC$_i$) into (CRC) to obtain

$$\sum_{j \in N} \frac{C_j'(e_j)}{q_j(\boldsymbol{e})} = 1. \qquad \text{(CRC}'\text{)}$$

Having removed the $w_i$ from (CRC), we now see that a private contracts equilibrium can be found in two steps. First, solve

$$\mathring{\boldsymbol{e}} \in \underset{\boldsymbol{e} \in \mathbb{R}_+^n}{\arg\max}\, q(\boldsymbol{e}) - \textstyle\sum_{i \in N} C_i(e_i) \quad \text{subject to (CRC}'\text{)}, \qquad (7)$$

and then find $\mathring{\boldsymbol{w}}$ to satisfy (IC$_i$), (IR$_i$) and (PP-SOC$_i$) when $\boldsymbol{e} = \mathring{\boldsymbol{e}}$.

Just as in the case of public contracts equilibria, $\mathring{\boldsymbol{w}}$ is far from unique because for each $i$ the constraints pin down only the values of $w_i$ and $w_i'$ at a single point, $q(\mathring{\boldsymbol{e}})$, and put a bound on the permissible values of $w_i''$ at the same point. Note in particular that the principal can choose the linear contracts $\mathring{w}_i(q) = \mathring{b}_i q + \mathring{s}_i$, where $\mathring{b}_i = C_i'(\mathring{e}_i)/q_i(\mathring{\boldsymbol{e}})$ by (IC-FOC$_i$) and $\mathring{s}_i = C_i(\mathring{e}_i) - \mathring{b}_i q(\mathring{\boldsymbol{e}})$ by (IR$_i$). It is easy to verify that (PP-SOC$_i$) holds for these linear contracts.[16] We thus have our third corollary:

*Corollary 3.* The set of private contracts equilibria contains the proposal $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$, where $\mathring{\boldsymbol{e}}$ is defined by (7) and $\mathring{w}_i(q) = \mathring{b}_i q + \mathring{s}_i$, with $\mathring{b}_i = C_i'(\mathring{e}_i)/q_i(\mathring{\boldsymbol{e}})$ and $\mathring{s}_i = C_i(\mathring{e}_i) - \mathring{b}_i q(\mathring{\boldsymbol{e}})$.

The set of private contracts equilibria always contains a proposal with a linear contract and restricting attention to such contracts is without loss of generality.

Finally, let us compare the private equilibrium effort, $\mathring{\boldsymbol{e}}$, to the efficient public equilibrium effort, $\boldsymbol{e}^*$. Noting that all terms in the sum in equation (CRC$'$) are positive, we see that, for all $j$,

$$\frac{C_j'(\mathring{e}_j)}{q_j(\mathring{\boldsymbol{e}})} < 1 = \frac{C_j'(e_j^*)}{q_j(\boldsymbol{e}^*)}. \qquad (8)$$

When the production function is additively separable ($q_{ij} \equiv 0$), $q_j(\boldsymbol{e})$ is independent of $\boldsymbol{e}_{-j}$, so that $C_j'(e_j)/q_j(\boldsymbol{e})$ is in fact a function of $e_j$ only. Furthermore, because we have assumed that $C'' > 0$ and $q_{jj} < 0$, this function is increasing in $e_j$. Therefore, (8) implies that $\mathring{e}_j < e_j^*$, giving our next corollary:

*Corollary 4.* When the production function is additively separable ($q_{ij} \equiv 0$), effort in a private contracts equilibrium is distorted downward from the efficient effort of a public contracts equilibrium ($\mathring{e}_j < e_j^*$ for all $j \in N$).

To get further insight into the analytically more complex cases of positive or negative synergies (strictly super- or submodular production functions), let us now simplify the analysis by restricting ourselves to the special case of identical agents and a symmetric production function (so that $C_i = C_j \equiv C$, $q_i = q_1$, $q_{ii} = q_{11}$, and $q_{ij} = q_{12}$ for all $i$ and all $j \neq i$). In this case, the concavity of the program in equation (7) will guarantee the existence of a unique, symmetric solution, $\mathring{e}_i = \mathring{e}_j \equiv \mathring{e}$ for all $i$ and $j$. The efficient solution will also have $e_i^* = e_j^* \equiv e^*$. Now, define

$$f(e) \equiv \frac{q_1(\boldsymbol{e})}{C'(e)}, \qquad (9)$$

[16] The left-hand side of (PP-SOC$_i$) becomes $q_{ii}(\mathring{\boldsymbol{e}})\mathring{b}_i - C_i''(e_i) < 0$, as $\mathring{w}_j'' = 0$ and $1 - \sum_{j \neq i} \mathring{w}_j'(q(\mathring{\boldsymbol{e}})) = \mathring{w}_i'(q(\mathring{\boldsymbol{e}})) = \mathring{b}_i$.

where each component of $\boldsymbol{e}$ is equal to $e$. Note that (CRC$'$) now reduces to $f(\mathring{e}) = n$, whereas (FB-FOC$_i$) states $f(e^*) = 1 < n$. Thus, a sufficient condition for effort to be lower in a private contracts equilibrium is that $f$ be decreasing, but this condition is always true due to the concavity of $q$, as we show in the Appendix, obtaining the following result:

*Corollary 5.* When agents are identical and the production function symmetric, effort in a private contracts equilibrium is distorted downward from the efficient effort of a public contracts equilibrium.

Thus the decrease of effort under private contracting is present regardless of synergies. It stems simply from the fact (illustrated in Example 1) that an effort-reducing private contract allows the principal to save on the wages of agents not involved in the private contract, who in turn absorb all the costs of such reductions through the participation fees they pay when expecting all agents to stick with the public contract.

## 5. Extensions

□ **Discontinuous contracts.** In this section, we extend $W$ to include not only fully smooth contracts, but also weakly increasing contracts that are only right continuous and have a finite number of jump discontinuities. Between any two discontinuities (and below the lowest and above the highest discontinuity), the contract still satisfies the original assumptions (i.e., is a weakly increasing and weakly concave $C^2$ function). Note that this class includes the forcing contract of Holmström (1982) as well as all other step functions and, more generally, piecewise linear functions, as long as the value at the points of discontinuity is on the right-hand segment.

Suppose that $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ is a private contracts equilibrium in this extended contract space. We begin with the following observation:

*Lemma 1.* No agent's contract has a discontinuity at $\mathring{q} \equiv q(\mathring{\boldsymbol{e}})$.

The proof of Lemma 1 (in the Appendix) shows that if a discontinuity existed for some agent $i$ at the equilibrium output level, the principal could benefit from privately colluding with another agent $j$ to slightly reduce $j$'s output. The principal would thus make fixed savings equal to the jump in $i$'s contract (due to not having to pay $i$ a prize), whereas the loss of output could be made arbitrarily small due to continuity of the output function.

Now, because no agent's contract has a discontinuity at $\mathring{q} = q(\mathring{\boldsymbol{e}})$, *all* of our earlier analysis still applies in a neighborhood around $\mathring{q}$, so that $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ must satisfy all the properties established in Section 4. Consequently, the set of attainable outcomes (i.e., effort vectors and output levels) is the same as in the main article, and all equilibria from the earlier analysis would still be in the equilibrium set even with the expanded contract space. Infinitely many additional, discontinuous contracts are now added to the equilibrium set, but they all attain the same outcome. Thus, allowing jump-discontinuous contracts makes no meaningful difference. We state this result in the proposition below:

*Proposition 2.* Allowing jump-discontinuous contracts: (1) does not change the sets of effort vectors $\mathring{\boldsymbol{e}}$ and output levels $\mathring{q}$ that can be achieved in a private contracts equilibrium and (2) strictly expands the set of equilibrium contracts $\mathring{\boldsymbol{w}}$.

□ **Risk.** The biggest limitation of our main model is the absence of risk and risk aversion. It is easy to see that risk *per se* changes very little if agents remain risk-neutral. Simple manipulation of the equations shows that, in such a case, the individual residual claimancy condition (PUB-FOC$_i$) that characterizes public contracts equilibria holds in expectation (i.e., $\mathbb{E}\widehat{w}_i'(q(\widehat{\boldsymbol{e}})) = 1$), and so does the collective residual claimancy condition (CRC) that characterizes private contracts equilibria, $\sum_{j\in N}\mathbb{E}\mathring{w}'_j(q(\mathring{\boldsymbol{e}}))=1$. The conflict between the two conditions remains, so that public contracts equilibria still fail to hold up under private contracting, and private contracts equilibria result in suboptimal effort. In this section, we extend the model to include risky production technology and risk-averse agents. Unfortunately, the general case does not lend itself to elegant analysis, so we restrict ourselves to the special case of the LEN framework: linear contracts, exponential utility, and normally distributed errors.

Output is now stochastic, $\widetilde{q}(\boldsymbol{e})=q(\boldsymbol{e})+\epsilon$, where $\epsilon$ is a white-noise term following a normal distribution with mean zero and variance $\sigma^2$: $\epsilon\sim N(0,\sigma^2)$. The function $q$ giving the deterministic component of output is as defined in the main section.

Although the principal remains risk neutral, the agents are now risk-averse expected utility maximizers with exponential (CARA) utility functions $u_i(x)=-\exp(-r_ix)$.[17] The main attraction of exponential utility is that for normally distributed variables, its certainty equivalent is linear in the mean and variance: if $\widetilde{x}\sim N(\mu,\sigma^2)$, then $\mathrm{CE}(\tilde{x})=\mu-r_i\sigma^2/2$. This allows us to replace nonlinear expected utility maximization with the equivalent linear certainty equivalent maximization.

The contract space is restricted to linear (affine) contracts, $w_i(q)=b_iq+s_i$ for some $b_i\in\mathbb{R}_+$ and $s_i\in\mathbb{R}$.

The principal is now maximizing her expected profit $\Pi$:

$$\Pi(\boldsymbol{w},\boldsymbol{e})=\mathbb{E}\left[\widetilde{q}(\boldsymbol{e})-\sum_{i\in N}(b_i\widetilde{q}(\boldsymbol{e})+s_i)\right]=q(\boldsymbol{e})\left[1-\sum_{i\in N}b_i\right]-\sum_{i\in N}s_i, \tag{10}$$

whereas agent $i$ is maximizing the certainty equivalent $U_i$ of his payoff:

$$U_i(\boldsymbol{w},\boldsymbol{e})=\mathrm{CE}[b_i\widetilde{q}(\boldsymbol{e})+s_i-C_i(e_i)]=b_iq(\boldsymbol{e})+s_i-r_ib_i^2\sigma^2/2-C_i(e_i). \tag{11}$$

The individual rationality constraint is now $U_i(\boldsymbol{w},\boldsymbol{e})\geq 0$:

$$b_iq(\boldsymbol{e})+s_i-\frac{1}{2}r_ib_i^2\sigma^2-C_i(e_i)\geq 0. \tag{$\widetilde{\mathrm{IR}}_i$}$$

The incentive compatibility constraint is identical to the first-order condition of the risk-free case (i.e., (IC-FOC$_i$)). This happens because effort choice does not affect the risk penalty term. It will be useful to reformulate (IC-FOC$_i$) by expressing $b_i$ as a function of $\boldsymbol{e}$:

$$b_i(\boldsymbol{e})=\frac{C_i'(e_i)}{q_i(\boldsymbol{e})}. \tag{$\widetilde{\text{IC-FOC}}_i$}$$

Note that

$$\forall i,\ \frac{\partial b_i}{\partial e_i}(\boldsymbol{e})=\frac{C_i''(e_i)q_i(\boldsymbol{e})-C_i'(e_i)q_{ii}(\boldsymbol{e})}{q_i(\boldsymbol{e})^2}>0; \tag{12}$$

$$\forall j\neq i,\ \frac{\partial b_j}{\partial e_i}(\boldsymbol{e})=-\frac{C_j'(e_j)q_{ij}(\boldsymbol{e})}{q_j(\boldsymbol{e})^2}. \tag{13}$$

By the same logic as before, ($\widetilde{\mathrm{IR}}_i$) must hold with equality at the public contracts equilibrium. Plugging ($\widetilde{\text{IC-FOC}}_i$) and the binding version of ($\widetilde{\mathrm{IR}}_i$) into the objective function for public contracts equilibrium shows that public contracts equilibria are now found by finding the $\boldsymbol{e}$ that maximizes

$$q(\boldsymbol{e})-\sum_{i\in N}\left[C_i(e_i)+\frac{r_ib_i(\boldsymbol{e})^2\sigma^2}{2}\right]. \tag{14}$$

[17] Note that $u_i'(x)=r_i\exp(-r_ix)$ and $u_i''(x)=-r_i^2\exp(-r_ix)$, so that agent $i$'s coefficient of absolute risk aversion, $-u_i''(x)/u'(x)=r_i$, is constant at $r_i$.

The first-order conditions for the $\boldsymbol{e}$ of a **public contracts equilibrium** is now, for all $i$:

$$q_i(\boldsymbol{e}) - C_i'(e_i) - r_i b_i \sigma^2 \frac{\partial b_i}{\partial e_i}(\boldsymbol{e}) - \sum_{j \neq i} r_j b_j \sigma^2 \frac{\partial b_j}{\partial e_i}(\boldsymbol{e}) = 0. \qquad (\widetilde{\text{PUB-FOC}_i})$$

As before, the private contracts equilibrium program is driven by the pairwise-proofness conditions. The objective of the $i$-th PP condition is now

$$q(\widetilde{e}_i, \boldsymbol{e}_{-i}) - \left[\widetilde{b}_i q(\widetilde{e}_i, \boldsymbol{e}_{-i}) + s_i\right] - \sum_{j \neq i} \left[b_j q(\widetilde{e}_i, \boldsymbol{e}_{-i}) + s_j\right]. \qquad (15)$$

The first-order condition for the incentive compatibility constraint is the same as in the risk-free case, (IC-D-FOC$_i$), whereas the individual rationality constraint differs from (IR-D$_i$) only by the addition of a risk penalty term, $-r_i \widetilde{b}_i^2 \sigma^2/2$, to the left-hand side. As usual, the individual rationality constraint must bind. Substituting the binding IR constraint and the IC constraint into the objective, taking the derivative with respect to $\widetilde{e}_i$, and evaluating at $\langle \widetilde{w}_i, \widetilde{e}_i \rangle = \langle w_i, e_i \rangle$, gives us the following first-order conditions (for all $i$) for **pairwise-proofness**:

$$q_i(\boldsymbol{e}) - C_i'(e_i) - r_i b_i \sigma^2 \frac{\partial b_i}{\partial e_i}(\boldsymbol{e}) - \sum_{j \neq i} b_j q_i(\boldsymbol{e}) = 0. \qquad (\widetilde{\text{PP-FOC}_i})$$

Comparing ($\widetilde{\text{PUB-FOC}_i}$) and ($\widetilde{\text{PP-FOC}_i}$) shows that the two conditions differ in a more complex way than their risk-free analogues. In the risk free case, the only way in which the PP conditions differed from the public equilibrium FOCs was the additional downward pressure on $e_i$ in the PP conditions, due to savings on the wages of players other than $i$, which unambiguously created an incentive for the principal to deviate from the public equilibrium by privately reducing $e_i$. In the present case, this downward pressure is still there (observe the term $-\sum_{j \neq i} b_j q_i(\boldsymbol{e}) < 0$ in ($\widetilde{\text{PP-FOC}_i}$)), but there is now an additional difference of ambiguous sign: The effect that an increase in $e_i$ has on the risk penalties of other agents in the public contracts setting, which is absent in the private setting (as other agents do not observe changes in $\widetilde{e}_i$ and, therefore, cannot demand additional risk premia). This effect is captured in the term $-\sum_{j \neq i} r_j b_j \sigma^2 \frac{\partial b_j}{\partial e_i}(\boldsymbol{e})$ in (PUB-FOC$_i$). Observe that the sign of this term is determined by the signs of $q_{ij}(\boldsymbol{e})$. In particular, if the sign of $q_{ij}$ is the same for all $i$ and $j$, then $-\sum_{j \neq i} r_j b_j \sigma^2 \frac{\partial b_j}{\partial e_i}(\boldsymbol{e})$ also has the same sign. It is most natural to assume that $q_{ij}(\boldsymbol{e}) \geq 0$ for all $i$ and $j$, which encompasses both additively separable production technology ($q_{ij}(\boldsymbol{e}) = 0$) and strictly supermodular production ($q_{ij}(\boldsymbol{e}) > 0$). In this case, $-\sum_{j \neq i} r_j b_j \sigma^2 \frac{\partial b_j}{\partial e_i}(\boldsymbol{e}) \geq 0$, so the risk penalties effect is either absent or drives $e_i$ in the public case in the opposite direction of what the previously-discussed wage-saving effect did in the pairwise-proofness condition. Hence, both effects act to create a negative difference between agent $i$'s efforts in public and private contracts equilibria. This creates an incentive for the principal to break the public contracts equilibrium by privately renegotiating with any agent to reduce his effort. We confirm this intuition in the Appendix, proving the following:

*Proposition 3.* In the LEN setting, if $q_{ij}(\boldsymbol{e}) \geq 0$ (for all $i, j \in N$ and $\boldsymbol{e} \in \mathbb{R}_+^n$), then no public contracts equilibrium is a private contracts equilibrium. In particular, the principal has an incentive to privately collude with any agent $i$ to reduce $e_i$ from the public equilibrium level.

Unlike in the riskless case, the team is no longer the collective residual claimant: Their marginal wages at the equilibrium are reduced from full collective residual claimancy by a risk premium term. To see this, we use (IC-FOC$_i$) to eliminate $C_i'$ from ($\widetilde{\text{PP-FOC}_i}$) and then collect the bonus terms to obtain:

$$\sum_{j \in N} b_j = 1 - r_i \sigma^2 b_i \frac{\frac{\partial b_i}{\partial e_i}(\boldsymbol{e})}{q_i(\boldsymbol{e})} < 1. \qquad (16)$$

□ **Bilateral private contracting with multiple agents.** Now, suppose the principal can privately and separately contract with multiple agents instead of just one agent. We still maintain the assumption of passive beliefs: when receiving an alternative proposal, each agent still believes that others have received their equilibrium proposals.

Given $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ that satisfies ($\text{IC}_i$) and ($\text{IR}_i$), the principal cannot bilaterally renegotiate with a set of agents $J \subseteq N$ in a profitable way if and only if

$$\langle \boldsymbol{w}_J, \boldsymbol{e}_J \rangle \in \underset{\langle \tilde{\boldsymbol{w}}_J, \tilde{\boldsymbol{e}}_J \rangle \in W^{|J|} \times \mathbb{R}_+}{\arg\max} \; q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - \sum_{i \in J} \tilde{w}_i(q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})) - \sum_{i \notin J} w_i(q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}))$$
$$\text{subject to the constraints (IC-D}_i\text{) and (IR-D}_i\text{) for all } i \in J. \quad (\text{PP}_J)$$

We will call such $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ **pairwise-proof for set of agents $J$**. If a proposal is pairwise-proof for all sets of agents $J \subset N$, we will call it **strongly pairwise-proof**. A strong private contracts equilibrium imposes strong pairwise-proofness:

*Definition 4.* The proposal $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ is a **strong private contracts equilibrium** if

$$\begin{gathered} \langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle \in \underset{\langle \boldsymbol{w}, \boldsymbol{e} \rangle \in W^n \times \mathbb{R}_+^n}{\arg\max} \; q(\boldsymbol{e}) - \textstyle\sum_{i \in N} w_i(q(\boldsymbol{e})) \\ \text{subject to the constraints (IC}_i\text{) } \textit{and} \text{ (IR}_i\text{) for all } i \in N, \\ \text{and (PP}_J\text{) for all } J \subset N. \end{gathered} \tag{17}$$

As discussed before, the strong private contracts equilibria are the principal's most preferred equilibria in a game where all agents are privately offered individual contracts from the outset.

Observe that ($\text{PP}_J$) implies ($\text{PP}_i$), so that all conclusions stemming from ($\text{PP}_i$) still hold, including in particular ($\text{PP-FOC}_i$), ($\text{PP-SOC}_i$), ($\text{PP-FOC}'_i$), and (CRC). Thus, the proposals in a strong private equilibrium must satisfy the collective residual claimant property. However, this might not be enough though, as these conditions do not consider pairwise-proofness for non-singleton sets of agents. It turns out, however, that (CRC) is, in fact, sufficient as we now go on to show (we will establish the result formally only for linear contracts, but the logic does extend to the general case).

First, note that by the usual logic ($\text{IR-D}_i$) must hold with equality. Summing the binding individual rationality constraints for all agents in the set $J$ yields

$$\sum_{i \in J} \widetilde{w}_i(q(\widetilde{e}_i, \boldsymbol{e}_{-i})) = \sum_{i \in J} C_i(\widetilde{e}_i). \tag{18}$$

Now, adding the left-hand side and subtracting the right-hand side of this identity from the objective function of ($\text{PP}_J$) (so that the value remains unchanged), we obtain

$$q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - \sum_{i \in J} [\widetilde{w}_i(q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})) - \widetilde{w}_i(q(\widetilde{e}_i, \boldsymbol{e}_{-i}))] - \sum_{i \in J} C_i(\widetilde{e}_i) - \sum_{i \notin J} w_i(q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})). \tag{19}$$

Note that unlike the single-agent PP problem, the objective function after taking account of the individual rationality constraints still contains $\boldsymbol{w}_J$. This is because of the discrepancy of the points where the principal and the agents evaluate $\boldsymbol{w}_J$: although the principal (in her objective function) correctly observes that the actual output will be $q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})$, the agents' passive beliefs make them think the output is going to be $q(\widetilde{e}_i, \boldsymbol{e}_{-i})$, which, therefore, is the output that enters the individual rationality constraints.

In the Appendix, we solve the program ($\text{PP}_J$) for linear contracts, arriving at the following proposition:

*Proposition 4.* Let $W$ be restricted to linear contracts. Then the set of strong private contracts equilibria equals the set of private contracts equilibria.

□ **Private contracting with groups of agents.** In this section, we let the principal collectively contract with groups of agents. The difference between this and the setting in the previous section is that when the principal offers competing proposals to a group, all members of the group observe each other's offers. Regarding agents outside the group, they still continue to hold passive beliefs.

Given $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$, let the principal make the alternative proposal $\langle \widetilde{\boldsymbol{w}}_J, \widetilde{\boldsymbol{e}}_J \rangle$ to a group $J \subset N$. Given any $i \in J$, we let $\widetilde{\boldsymbol{e}}_{J-i}$ and $\widetilde{\boldsymbol{w}}_{J-i}$ denote the vectors of the group with agent $i$ removed. Incentive compatibility for any agent $i \in J$ is now given by

$$\tilde{e}_i \in \arg\max_{e_i'} \tilde{w}_i(q(e_i', \tilde{\boldsymbol{e}}_{J-i}, \boldsymbol{e}_{-J})) - C_i(e_i'), \quad \text{(IC-D-G}_i\text{)}$$

which is equivalent to the first-order condition

$$\tilde{w}_i'(q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}))q_i(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) = C_i'(\tilde{e}_i). \quad \text{(IC-D-FOC-G}_i\text{)}$$

Individual rationality for a deviating offer is given by

$$\tilde{w}_i(q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - C_i(\tilde{e}_i) \geq 0. \quad \text{(IR-D-G}_i\text{)}$$

Given $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ that satisfies (IC$_i$) and (IR$_i$), the principal cannot profitably renegotiate with group of agents $J \subseteq N$ if and only if

$$\langle \boldsymbol{w}_J, \boldsymbol{e}_J \rangle \in \arg\max_{\langle \tilde{\boldsymbol{w}}_J, \tilde{\boldsymbol{e}}_J \rangle \in W^{|J|} \times \mathbb{R}_+} q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - \textstyle\sum_{i \in J} \tilde{w}_i(q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})) - \sum_{i \notin J} w_i(q(\tilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}))$$
$$\text{subject to the constraints (IC-D-G}_i\text{) and (IR-D-G}_i\text{) for all } i \in J. \quad \text{(GP}_J\text{)}$$

We will call such $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$ **group-proof for set $J$**. If a proposal is group-proof for all sets of agents $J \subset N$, we will call it simply **group-proof**. Equilibrium is defined in the usual way:

*Definition 5.* The proposal $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ is a **group-proof contracts equilibrium** if

$$\begin{gathered} \langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle \in \arg\max_{\langle \boldsymbol{w}, \boldsymbol{e} \rangle \in W^n \times \mathbb{R}^n_+} q(\boldsymbol{e}) - \textstyle\sum_{i \in N} w_i(q(\boldsymbol{e})) \\ \text{subject to the constraints (IC}_i\text{) } \textit{and} \text{ (IR}_i\text{) for all } i \in N, \\ \text{and (GP}_J\text{) for all } J \subsetneq N. \end{gathered} \tag{20}$$

Note that we exclude the grand coalition, $J = N$, as in that case (GP$_J$) would reduce to the definition of a public contracts equilibrium. A group-proof equilibrium would then be required to be both a public contracts equilibrium and a private contracts equilibrium (due to imposing (GP$_J$) for singleton sets), which we already know to be impossible.

In the proof of Proposition 5 in the Appendix, we show that the constraints in (GP$_J$) can be reduced to

$$\sum_{i \in N \setminus J \cup \{j\}} w_i'(q(\boldsymbol{e})) = 1 \tag{21}$$

for all $J \subsetneq N$ and for all $j \in J$. All agents outside the group, together with any given member of a group, must collectively be the residual claimants (on the margin). (Note that for singleton $J$, this reduces to the familiar collective residual claimant property for the whole team.) Just as the condition (CRC) in the single-agent deviation case, this condition is necessary to balance output losses and wage savings to disincentivize the principal from offering private contracts that increase or decrease an agent's effort. In the case of group negotiation, however, the agents inside the group will know what is happening and will, therefore, refuse to accept wage losses, and, therefore, the balancing act must happen between the agent receiving a deviating proposal and the unaware agents outside the group.

However, it is impossible for these conditions to hold for all groups and all agents simultaneously (see the proof for details), leading to the following negative conclusion:

*Proposition 5.* For any $n \geq 3$, the set of group-proof contracts equilibria is empty.

Note that for $n = 2$, the definition of group-proof contracts equilibrium reduces to private contracts equilibrium, as the only possible "coalitions" are singleton sets.

□ **Alternative beliefs.** We have assumed throughout the article that agents hold passive beliefs: when an agent receives a deviating offer, he thinks that all others are still getting their equilibrium offers. This is the most common belief assumption in the literature and is also the only one admissible in the game that corresponds to a literal interpretation of the restriction to renegotiation with a single agent at a time. However, as that literal interpretation is not particularly compelling, other belief structures are certainly plausible.

We begin this section with a very general observation: One of our central results, namely, that the efficient public contracts equilibrium is never immune to bilateral renegotiation, holds for any consistent belief structure whatsoever.[18] This happens because the efficient equilibrium can be broken by offering some agent $i$ the "zero proposal" (a constant wage of zero and no effort), whose value to the agent is independent of the actions taken by others. Essentially, this deviation works by the same logic as Example 1: because all agents are residual claimants, any output losses from the deviation are absorbed by agents other than $i$, and the principal can enjoy the wage savings from not having to compensate $i$ for his effort. We fill in the details in the Appendix, arriving at the following result:

*Proposition 6.* Given any consistent beliefs, a public contracts equilibrium is never a private contracts equilibrium.

Proposition 6 shows what cannot happen in a private contracts equilibrium with alternative beliefs. What *will* happen depends on the particular beliefs assumed. In their seminal article, McAfee and Schwartz (1994) discussed two alternatives to passive beliefs: "symmetric beliefs", where an agent receiving a deviating offer believes that all others have received the same offer, and "wary beliefs", where agent $i$ who observes a deviation believes that other agents have been given offers that are optimal for the principal, given the offer that has been made to $i$. Wary beliefs are the most appealing conceptually (Rey and Vergé, 2004), but, beyond highly simplified examples, the equilibrium program under such beliefs is not tractable.[19] We will examine only symmetric beliefs. Note that for the definition of symmetric beliefs to be meaningful, we also need to assume that the agents are identical and the production function symmetric; otherwise, the belief of identical offers would certainly be strange.

If we start with the proposal $\langle \boldsymbol{w}, \boldsymbol{e} \rangle$, an agent $i$ receiving a competing proposal $\langle \widetilde{w}, \widetilde{e} \rangle$ will now believe that others have received the same offer, $\widetilde{\boldsymbol{b}}'_{-i} = (\widetilde{b}, \ldots \widetilde{b})$ and $\widetilde{\boldsymbol{e}}'_{-i} = (\widetilde{e}, \ldots \widetilde{e})$. Consequently, his incentive compatibility constraint for the deviating offer will be

$$\tilde{e} \in \arg\max_{e'_i} \tilde{w}(q(e'_i, \tilde{\boldsymbol{e}}_{-i})) - C_i(e'_i), \qquad \text{(IC-D-SYM}_i\text{)}$$

which is equivalent to the first-order condition

$$\tilde{w}'(q(\tilde{e}, \tilde{\boldsymbol{e}}_{-i}))q_i(\tilde{e}, \tilde{\boldsymbol{e}}_{-i}) = C'_i(\tilde{e}). \qquad \text{(IC-D-FOC-SYM}_i\text{)}$$

Individual rationality for a deviating offer is given by

$$\tilde{w}(q(\tilde{e}, \tilde{\boldsymbol{e}}_{-i})) - C_i(\tilde{e}) \geq 0. \qquad \text{(IR-D-SYM}_i\text{)}$$

[18] Consistency restricts beliefs only on the equilibrium path; here, it simply requires that agents receiving equilibrium offers believe that others have also received equilibrium offers. There are no restrictions on the beliefs of agents who have received a deviating proposal.

[19] For example, in the setting of contracts in vertical chains, McAfee and Schwartz (1994) restrict themselves to two symmetric downstream firms.

The pairwise-proofness condition for agent $i$ is now

$$\langle w_i, e_i\rangle \in \underset{\langle \tilde{w},\tilde{e}\rangle\in W\times\mathbb{R}_+}{\arg\max}\; q(\tilde{e}, \boldsymbol{e}_{-i}) - \tilde{w}(q(\tilde{e}, \boldsymbol{e}_{-i})) - \sum_{j\neq i} w_j(q(\tilde{e}, \boldsymbol{e}_{-i}))$$

$$\text{subject to the constraints (IC-D-SYM}_i\text{) and (IR-D-SYM}_i\text{).} \qquad \text{(PP-SYM}_i\text{)}$$

Public contracts equilibrium is now defined as in Defintion 2 but with (PP$_i$) replaced by (PP-SYM$_i$).

As usual, IR-D-SYM$_i$ must bind. Adding the zero-valued left-hand side of the constraint to the objective function of PP-SYM$_i$ turns the objective into

$$q(\widetilde{e}, \boldsymbol{e}_{-i}) - [\widetilde{w}(q(\widetilde{e}, \boldsymbol{e}_{-i})) - \widetilde{w}(q(\widetilde{e}, \widetilde{\boldsymbol{e}}_{-i}))] - C_i(\widetilde{e}) - \sum_{j\neq i} w_j(q(\widetilde{e}, \boldsymbol{e}_{-i})). \tag{22}$$

Just as in the section on bilateral private contracting with multiple agents, the objective still contains $\widetilde{w}$; as in that case, this happens because the beliefs of the agent receiving a deviating offer are inconsistent with the principal's actual action of making a single deviating proposal. In the case of bilateral private contracting with multiple agents, this difference turned out to be immaterial on the margin at the equilibrium contract; here, on the contrary, it turns out to have a much more damaging effect: It prevents any symmetric proposal from being immune to private renegotiation.

As in Section 5, we now restrict the contract space to linear contracts. As in that section, this simplifies the analysis greatly, but because the analysis relies only on local arguments in a neighborhood around the optimum, we believe the same logic can be extended to the general case. Let us call a proposal symmetric if $\langle w_i, e_i\rangle = \langle w_j, e_j\rangle$ for all $i$ and $j$. In the Appendix, we derive the following result:

*Proposition 7.* Let $W$ be restricted to linear contracts. Under symmetric beliefs, no symmetric private contracts equilibria exist.

The restriction to symmetric equilibria is natural, as it would not make much sense for agents to hold symmetric beliefs about deviating offers starting from an asymmetric candidate equilibrium.

This negative result is surprising. In other settings, nonexistence has been found to be a problem with passive, not symmetric beliefs (McAfee and Schwartz, 1995). In such settings (e.g., McAfee and Schwartz (1994)), it has been established that symmetric beliefs allow the efficient public equilibrium to remain an equilibrium even in the face of private contracting because the nature of symmetric beliefs causes agents receiving deviating offers to act as if all agents had been given the same offer publicly. In our setting, however, there is a mismatch between the public equilibrium that the agents have in mind and the actual proposals offered, which the principal can use to extract additional surplus, either from the agent to which an alternative proposal is made or from the remaining agents. The former case happens after an effort-increasing deviation, when the agent who has received it is willing to pay a higher participation fee due to incorrectly expecting others to also work harder by symmetry. The latter case happens after an effort-decreasing deviation, because the agents who have not received deviating offers incorrectly expect everyone to still work hard. To gain more traction on this, let us revisit the original example we used to motivate our analysis at the beginning of Section 4.

*Example 2.* As in Example 1, let $n = 2$, $q(e_1, e_2) = e_1 + e_2$, and $C_i(e_i) = \frac{1}{2}e_i^2$, and let the contract space be restricted to linear contracts, $w_i(q) = s_i + b_i q$. The efficient bonus that implements first-best is $\widehat{b}_i = 1$ with a "salary" of $\widehat{s}_i = \frac{1}{2} - 2 = -\frac{3}{2}$. Each agent earns $\widehat{u}_i = 0$. The principal's profit is $\widehat{\pi} = (1 - \widehat{b}_1 - \widehat{b}_2)q^* - (\widehat{s}_1 + \widehat{s}_2) = 1$.

Unlike Example 1, let the agents now have symmetric beliefs.

First, note that the efficient public contracts equilibrium can still be broken by privately offering the second agent $\widetilde{w}_2 = 0$. Agent 2 still picks $\widetilde{e}_2 = 0$ and gets utility $\widetilde{u}_2 = 0$, regardless of his beliefs about others, so he is (weakly) willing to accept. Agent 1 is still exerting efficient effort $e_1^* = 1$ and is happy with $\widehat{s}_1 = -3/2$, as consistency of beliefs requires that he still expects Agent 2 to also exert efficient effort. Consequently, just as in Example 1, the principal's profit is now

$$\widetilde{\pi} = e_1^* - \widehat{b}_1 e_1^* - \widehat{s}_1 = \frac{3}{2} > 1 = \widehat{\pi}, \tag{23}$$

so the deviation is profitable.

Now, suppose there is a symmetric public contracts equilibrium with $b_1 = b_2 = b$. By incentive compatibility, the effort is also $e_1 = e_2 = b$, and by individual rationality, each agent receives $s = -(3/2)b^2$. Now, suppose the principal deviates by offering Agent 2 the bonus $\widetilde{b}$. Then, Agent 2 exerts $\widetilde{e} = \widetilde{b}$ by (IC) and receives $\widetilde{s} = -(3/2)\widetilde{b}^2$ by (IR) and symmetric beliefs. The principal's profit is now

$$(b + \widetilde{b})(1 - b - \widetilde{b}) - s - \widetilde{s} = \frac{1}{2}\widetilde{b}^2 + (1 - 2b)\widetilde{b} + \frac{1}{2}b^2 + b, \tag{24}$$

which, as a function of $\widetilde{b}$, is a parabola opening **upward** and consequently has no maximum anywhere. In particular, it does not have a maximum at $\widetilde{b} = b$, so the assumption that $b_1 = b_2 = b$ was part of a symmetric equilibrium was incorrect.

Also, note that the first-order condition with respect to $\widetilde{b}$ gives the **minimum** at $\widetilde{b} = 2b - 1$. At $b^* = 1$, this minimum is at $\widetilde{b} = 1 = b^*$, so the efficient contract can be broken both by effort-increasing and effort-decreasing deviations.

## Appendix

*Proof of Corollary* 5. We need to prove that $f'(e) < 0$ for all $e$. Taking the derivative of $f$ with respect to $e$:

$$f'(e) = \frac{\left[q_{11}(\boldsymbol{e}) + \sum_{i \neq 1} q_{1i}(\boldsymbol{e})\right]C'(e) - q_1(\boldsymbol{e})C''(e)}{[C'(e)]^2} \tag{A1a}$$

$$\stackrel{\text{sym.}}{=} \frac{[q_{11}(\boldsymbol{e}) + (n-1)q_{12}(\boldsymbol{e})]C'(e) - q_1(\boldsymbol{e})C''(e)}{[C'(e)]^2} \tag{A1b}$$

Because $q_1 > 0$, $C' > 0$, and $C'' > 0$, a sufficient condition for this to be negative is that $q_{11}(\boldsymbol{e}) + (n-1)q_{12}(\boldsymbol{e}) \leq 0$. If synergies between the agents are negative or zero, $q_{12} \leq 0$, this is obviously true and we are done. For the rest of the proof, assume that $q_{12} > 0$.

Now, let us look at the Hessian matrix for $q$ evaluated at $\boldsymbol{e}$. By symmetry, this is (omitting the arguments for readability):

$$H = \begin{pmatrix} q_{11} & q_{12} & q_{12} & \cdots & q_{12} \\ q_{12} & q_{11} & q_{12} & \cdots & q_{12} \\ q_{12} & q_{12} & q_{11} & \cdots & q_{12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{12} & q_{12} & q_{12} & \cdots & q_{11} \end{pmatrix}. \tag{A2}$$

Subtracting the first column of $H$ from all the others and then adding to the first row each of the others, we obtain

$$H' = \begin{pmatrix} q_{11} + (n-1)q_{12} & 0 & 0 & \cdots & 0 \\ q_{12} & q_{11} - q_{12} & 0 & \cdots & 0 \\ q_{12} & 0 & q_{11} - q_{12} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{12} & 0 & 0 & \cdots & q_{11} - q_{12} \end{pmatrix}. \tag{A3}$$

Because the operations applied to obtain $H'$ do not affect the determinant, we have

$$\det H = \det H' = [q_{11} + (n-1)q_{12}][q_{11} - q_{12}]^{n-1}. \tag{A4}$$

Now, by Sylvester's Criterion for concavity applied to the last principal minor, $(-1)^n \det H \geq 0$. Because $q_{11} < 0$ and $q_{12} \geq 0$, we have $q_{11} - q_{12} < 0$, so that $[q_{11} - q_{12}]^{n-1}$ has the same sign as $(-1)^{n-1}$ and $(-1)^n \det H$ has the same sign as $[q_{11} + (n-1)q_{12}](-1)^{2n-1}$. Therefore the fact that $(-1)^n \det H \geq 0$ implies that $q_{11} + (n-1)q_{12} \leq 0$. This proves that $f'(e) > 0$ for all $e$, as required. □

*Proof of Lemma* 1. Suppose that agent $i$'s contract has a discontinuity at $\mathring{q} = q(\mathring{\boldsymbol{e}})$, so that $\epsilon \equiv \mathring{w}_i(\mathring{q}) - \lim_{q \to \mathring{q}_-} \mathring{w}_i(q) > 0$.

Now consider some agent $j \neq i$. Note that by continuity of $q$, there exists some $\delta > 0$ such that $\widetilde{q} \equiv q(\mathring{e}_j - \delta, \boldsymbol{e}^\circ_{-j}) > q(\mathring{\boldsymbol{e}}) - \epsilon$. Let $\widetilde{e}_j \equiv \mathring{e}_j - \delta$.

Define

$$\widetilde{w}_j(q) = \begin{cases} 0 & \text{if } q < \widetilde{q} \\ \mathring{w}_j(\mathring{q}) & \text{if } q \geq \widetilde{q}, \end{cases} \tag{A5}$$

and let the principal make agent $j$ the alternative proposal $\langle \widetilde{w}_j, \widetilde{e}_j \rangle$. The proposal is clearly individually rational, as the utility of agent $j$ is $\widetilde{u}_j = \widetilde{w}_j(\widetilde{q}) - C_j(\widetilde{e}_j) = \mathring{w}_j(\mathring{q}) - C_j(\widetilde{e}_j) > \mathring{w}_j(\mathring{q}) - C_j(\mathring{e}_j) = \mathring{u}_j \geq 0$, where the strict inequality holds because $C_j$ is increasing and the weak inequality holds because $\langle \mathring{\boldsymbol{w}}, \mathring{\boldsymbol{e}} \rangle$ is individually rational. The proposal is also clearly incentive compatible, as choosing $e'_j < \widetilde{e}_j$ would lead to a negative payoff $(-C_j(e'_j) < 0)$, and the payoff function to the right of $\widetilde{e}_j$ is decreasing because wage is constant and costs are increasing.

The principal's payoff has changed by

$$\widetilde{\pi} - \mathring{\pi} = (\widetilde{q} - \mathring{q}) - (\widetilde{w}_j(\widetilde{q}) - \mathring{w}_j(\mathring{q})) - (\mathring{w}_i(\widetilde{q}) - \mathring{w}_i(\mathring{q})) - \sum_{k \notin \{i,j\}} (\mathring{w}_k(\widetilde{q}) - \mathring{w}_k(\mathring{q})). \tag{A6}$$

The first term, $\widetilde{q} - \mathring{q}$, is greater than $-\epsilon$ by construction. The second term, $\widetilde{w}_j(\widetilde{q}) - \mathring{w}_j(\mathring{q})$, is zero by construction. The third term, $\mathring{w}_i(\widetilde{q}) - \mathring{w}_i(\mathring{q})$, is not more than $-\epsilon$ due to the discontinuity of $\mathring{w}_i$ at $\mathring{q}$. Each summand in the last term is nonpositive, as $\widetilde{q} < \mathring{q}$. It follows that $\widetilde{\pi} - \mathring{\pi} > (-\epsilon) + 0 - (-\epsilon) - 0 = 0$, so the alternative private offer breaks the proposed private equilibrium. A contradiction. □

*Proof of Proposition* 3. Let $(\widehat{\boldsymbol{w}}, \widehat{\boldsymbol{e}})$ be a public contracts equilibrium. Then, by $(\widetilde{\text{PUB-FOC}_i})$,

$$q_i(\widehat{\boldsymbol{e}}) - C'_i(\widehat{e}_i) - r_i \widehat{b}_i \sigma^2 \frac{\partial b_i}{\partial e_i}(\widehat{\boldsymbol{e}}) = \sum_{j \neq i} r_j \widehat{b}_j \sigma^2 \frac{\partial b_j}{\partial e_i}(\widehat{\boldsymbol{e}}) = -\sum_{j \neq i} r_j \widehat{b}_j \sigma^2 \frac{C'_j(\widehat{e}_j) q_{ij}(\widehat{\boldsymbol{e}})}{q_j(\widehat{\boldsymbol{e}})^2} \leq 0. \tag{A7}$$

Evaluating the left-hand side of the pairwise-proofness condition $(\widetilde{\text{PP-FOC}_i})$ at $(\widehat{\boldsymbol{w}}, \widehat{\boldsymbol{e}})$ yields

$$\left[ q_i(\widehat{\boldsymbol{e}}) - C'_i(\widehat{e}_i) - r_i \widehat{b}_i \sigma^2 \frac{\partial b_i}{\partial e_i}(\widehat{\boldsymbol{e}}) \right] - \sum_{j \neq i} \widehat{b}_j q_i(\widehat{\boldsymbol{e}}) \overset{(A7)}{\leq} -\sum_{j \neq i} \widehat{b}_j q_i(\widehat{\boldsymbol{e}}) < 0. \tag{A8}$$

Thus, $(\widehat{\boldsymbol{w}}, \widehat{\boldsymbol{e}})$ does not satisfy $(\widetilde{\text{PP-FOC}_i})$ and is, therefore, not a private contracts equilibrium.

In particular, recalling that the left-hand side of $(\widetilde{\text{PP-FOC}_i})$ is the marginal benefit to the principal from privately increasing the effort proposed to agent $i$ when he offers, the fact that this expression is negative proves that it is optimal for the principal to *decrease* this proposed effort. □

*Proof of Proposition* 4. Let $W$ consist of linear contracts, $w_i(q) = s_i + b_i q$. The objective of $(\text{PP}_J)$ now becomes

$$q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - \sum_{i \in J} \widetilde{b}_i [q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - q(\widetilde{e}_i, \boldsymbol{e}_{-i})] - \sum_{i \in J} C_i(\widetilde{e}_i) - \sum_{i \notin J} [b_i q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) + s_i], \tag{A9}$$

and $(\text{IC-D-FOC}_i)$ is equivalent to $\widetilde{b}_i(\widetilde{e}_i, \boldsymbol{e}_{-i}) = C'_i(\widetilde{e}_i)/q_i(\widetilde{e}_i, \boldsymbol{e}_{-i})$. Substituting this constraint into the objective function reduces the program $(\text{PP}_J)$ to an unconstrained optimization problem in $\widetilde{\boldsymbol{e}}_J$.

The first-order condition with respect to $\widetilde{e}_j$ is

$$\begin{aligned} 0 = q_j(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) &- \sum_{i \in J} \frac{\partial \widetilde{b}_i}{\partial \widetilde{e}_j} [q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - q(\widetilde{e}_i, \boldsymbol{e}_{-i})] \\ &- \sum_{i \in J} \widetilde{b}_i q_j(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) + \widetilde{b}_j q_j(\widetilde{e}_j, \boldsymbol{e}_{-j}) - C'_j(\widetilde{e}_j) - \sum_{i \notin J} b_i q_j(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}). \end{aligned} \tag{A10}$$

Evaluating at $\widetilde{\boldsymbol{e}}_J = \boldsymbol{e}_J$ (so that $\widetilde{\boldsymbol{b}}_J = \boldsymbol{b}_J$), this turns into

$$0 = q_j(\boldsymbol{e}) - \sum_{i \in J} \frac{\partial \widetilde{b}_i}{\partial \widetilde{e}_j} [q(\boldsymbol{e}) - q(\boldsymbol{e})] - \sum_{i \in J} b_i q_j(\boldsymbol{e}) + b_j q_j(\boldsymbol{e}) - C'_j(\boldsymbol{e}_j) - \sum_{i \notin J} b_i q_j(\boldsymbol{e}) \tag{A11a}$$

$$= q_j(\boldsymbol{e}) - \sum_{i \in N} b_i q_j(\boldsymbol{e}) + b_j q_j(\boldsymbol{e}) - C'_j(\boldsymbol{e}_j) \tag{A11b}$$

$$\stackrel{\text{(IC-D-FOC}_i\text{)}}{=} q_j(\boldsymbol{e})\left(1-\sum_{i\in N} b_i\right), \tag{A11c}$$

which is equivalent to $\sum_{i\in N} b_i = 1$. It is also easy to check that the second-order condition at $\widetilde{\boldsymbol{e}}_J = \boldsymbol{e}_J$ reduces to $q_{ii}(\boldsymbol{e})(1-\sum_{i\notin J} b_i) - C_i''(\boldsymbol{e}) < 0$, so the first-order condition is indeed necessary and sufficient a maximum. Thus, the strong pairwise-proofness condition is equivalent to collective residual claimancy. Consequently, the program for strong private contracts equilibrium becomes equal to the program for simple private contracts equilibrium. □

*Proof of Proposition* 5. By the usual logic, (IR-D-G$_i$) must bind. Substituting it into the objective of (GP$_J$) turns the objective into

$$q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - \sum_{i\in J} C_i(\widetilde{e}_i) - \sum_{i\notin J} w_i(q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})). \tag{A12}$$

The first-order condition for optimal $\widetilde{e}_i$ is

$$q_i(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) - C_i'(\widetilde{e}_i) - \sum_{j\notin J} w_j'(q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}))q_i(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}) = 0. \tag{A13}$$

Substituting (IC-D-FOC-G$_i$) into this and rearranging yields

$$q_i(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J})\left(1 - \sum_{j\in N\setminus J\cup\{i\}} w_j'(q(\widetilde{\boldsymbol{e}}_J, \boldsymbol{e}_{-J}))\right) = 0. \tag{A14}$$

Evaluating this at $\widetilde{\boldsymbol{e}}_J = \boldsymbol{e}_J$ and noting that $q_i > 0$ shows that (GP$_J$) is equivalent to

$$\sum_{j\in N\setminus J\cup\{i\}} w_j'(q(\boldsymbol{e})) = 1, \tag{$\star$}$$

for all $i \in J$.

These conditions are easily seen to be mutually inconsistent. Denote $w_j'(q(\boldsymbol{e})) = b_j$.

First, consider any singleton "coalition". The condition then says

$$\sum_{j\in N} b_j = 1, \tag{$\star\star$}$$

which is just the usual collective residual claimant property.

Now take some Agent $i_1$ and let $J = N \setminus \{i_1\}$. Take another agent, $i_2 \in J$. Now, ($\star$) for $i_2$ says that

$$b_{i_1} + b_{i_2} = 1. \tag{$\star\star\star$}$$

Thus, ($\star\star$) and ($\star\star\star$) imply that

$$\sum_{j\notin\{i_1,i_2\}} b_j = 0, \tag{A15}$$

which is impossible, as the sum is over a nonmepty set ($n \geq 3$), and each summand is positive. □

*Proof of Proposition* 6. Recall that in a public contracts equilibrium, and $w_i'(q(\boldsymbol{e}^*)) = 1$ for all $i$. We will show that this is not a private contracts equilibrium because it can be broken by the zero proposal $\widetilde{w}_i = 0$ and $\widetilde{e}_i = 0$ for any agent $i$. Let the renegotiation happen with Agent 1 and let $q^* = q(\boldsymbol{e}^*)$ and $\widetilde{q} \equiv q(0, q_{-1}^*)$.

First note that the proposal is clearly strictly incentive compatible: Increasing effort above zero would be costly for the agent, and he would not be compensated for it. Next, observe that when accepting the new proposal, Agent 1 gets utility $\widetilde{u}_2 = 0$ regardless of his beliefs about others, so he is (weakly) willing to accept.

When Agent 1 accepts the zero contract instead of the efficient contract, the principal's profit is affected (compared to the efficient public equilibrium) in the following ways:

1. Output is reduced from $q^*$ to $\widetilde{q}$, reducing profit by $\Delta_1 = q^* - \widetilde{q}$;
2. The principal no longer has to pay Agent 1 $w_1(q^*)$, so profit is increased by $\Delta_2 = w_1(q^*) = C_1(e_1^*) > 0$ (where the last equality holds by the binding individual rationality constraint).
3. The wages the principal has to pay each agent $i$ other than Agent 1 are reduced by

$$\delta_i = w_i(q^*) - w_i(\tilde{q}) = \int_{\tilde{q}}^{q^*} w_i'(q)\,dq \stackrel{\text{concavity}}{\geq} \int_{\tilde{q}}^{q^*} w_i'(q^*)\,dq \stackrel{\text{(PUB-FOC}_i\text{)}}{=} q^* - \tilde{q}, \tag{A16}$$

resulting in an increase in profit by

$$\Delta_3 = \sum_{i\neq 1} \delta_i \geq \sum_{i\neq 1} (q^* - \widetilde{q}) = (n-1)(q^* - \widetilde{q}) \geq q^* - \widetilde{q} \tag{A17}$$

The total increase in profit is

$$\Delta = -\Delta_1 + \Delta_2 + \Delta_3 \geq C_1(e_1^*) > 0, \tag{A18}$$

so the deviation is profitable. □

*Proof of Proposition* 7. Let $W$ consist of linear contracts, $w_i(q) = s_i + b_i q$. The objective of the PP condition now becomes

$$q(\widetilde{e}, \boldsymbol{e}_{-i}) - \widetilde{b}[q(\widetilde{e}, \boldsymbol{e}_{-i}) - q(\widetilde{e}, \widetilde{\boldsymbol{e}}_{-i})s] - C_i(\widetilde{e}) - \sum_{j\neq i} \left[b_j q(\widetilde{e}, \boldsymbol{e}_{-i}) + s_j\right], \tag{A19}$$

and (IC-D-FOC-SYM$_i$) is equivalent to $\widetilde{b}(\widetilde{e}) = C_i'(\widetilde{e})/q_i(\widetilde{e}, \widetilde{\boldsymbol{e}}_{-i})$. Substituting this constraint into the objective function reduces the program PP-SYM$_i$ to an unconstrained optimization program in $\widetilde{e}$.

The derivative with respect to $\widetilde{e}$ is

$$\begin{aligned} &q_i(\widetilde{e}, \boldsymbol{e}_{-i}) - \widetilde{b}(\widetilde{e})\left[q_i(\widetilde{e}, \boldsymbol{e}_{-i}) - \sum_{j\in N} q_j(\widetilde{e}, \widetilde{\boldsymbol{e}}_{-i})\right] \\ &\qquad - \frac{\partial \widetilde{b}}{\partial \widetilde{e}}(\widetilde{e})[q(\widetilde{e}, \boldsymbol{e}_{-i}) - q(\widetilde{e}, \widetilde{\boldsymbol{e}}_{-i})] - C_i'(\widetilde{e}) - \sum_{j\neq i} b_j q_i(\widetilde{e}, \boldsymbol{e}_{-i}). \end{aligned} \tag{A20}$$

Now suppose $\langle \boldsymbol{w}, \boldsymbol{e}\rangle$ is a symmetric private equilibrium, with each agent's effort equal to $e$ and each agent's bonus equal to $b$. Evaluating the derivative above at $\widetilde{e} = e$, recalling that $q_i = q_j$ for all $i$ and $j$, and noting that $\widetilde{b}(e) = C_i'(e)/q_i(e) = b$ yields

$$q_i(\boldsymbol{e}) - b(1-n)q_i(\boldsymbol{e}) - C_i'(e) - (n-1)bq_i(\boldsymbol{e}) = q_i(\boldsymbol{e}) - C_i'(e). \tag{A21}$$

If $\widetilde{e} = e$ is an optimal deviation, this derivative must be zero (i.e., we must have $e = e^*$). In other words, the only candidate private contracts equilibrium is the efficient public contracts equilibrium, $e = e^*$ and $b = 1$. However, this candidate does not work either, by Proposition 6. □

## References


ABREU, D., DUTTA, P.K., AND SMITH, L. "The Folk Theorem for Repeated Games: a Neu Condition." *Econometrica*, Vol. 62 (1994), pp. 939–948.

BAKER, G., GIBBONS, R., AND MURPHY, K.J. "Subjective Performance Measures in Optimal Incentive Contracts." *The Quarterly Journal of Economics*, Vol. 109 (1994), pp. 1125–1156. Publisher: Oxford University Press.

BALDENIUS, T., DUTTA, S., AND REICHELSTEIN, S. "Cost Allocation for Capital Budgeting Decisions." *The Accounting Review*, Vol. 82 (2007), pp. 837–867.

BERNHEIM, B.D. AND WHINSTON, M.D. "Common Agency." *Econometrica*, Vol. 54 (1986), pp. 923–942.

BROWN, C. "Wage Levels and Method of Pay." *The RAND Journal of Economics*, Vol. 23 (1992), pp. 366–375. Publisher: RAND Corporation, Wiley.

CHE, Y.-K. AND YOO, S.-K. "Optimal Incentives for Teams." *The American Economic Review*, Vol. 91 (2001), pp. 525–541.

CRÉMER, J. AND RIORDAN, M.H. "On Governing Multilateral Transactions with Bilateral Contracts." *The RAND Journal of Economics*, Vol. 18 (1987), pp. 436–451.

DEMSKI, J. "Cost Allocation Games." In S. Moriarity, ed., *Joint Cost Allocations: proceedings of the University of Oklahoma Conference on Cost Allocations*, pp. 142–73. Center for Economic and Management Research, University of Oklahoma, 1981.

DEQUIEDT, V. AND MARTIMORT, D. "Vertical Contracting with Informational Opportunism." *The American Economic Review*, Vol. 105 (2015), pp. 2141–2182.

EGUIA, J.X., LLORENTE-SAGUER, A., MORTON, R., AND NICOLÒ, A. "Equilibrium Selection in Sequential Games with Imperfect Information." *Games and Economic Behavior*, Vol. 109 (2018), pp. 465–483.

ESWARAN, M. AND KOTWAL, A. "The Moral Hazard of Budget-Breaking." *The RAND Journal of Economics*, Vol. 15 (1984), pp. 578–581.

HART, O. AND TIROLE, J. "Vertical Integration and Market Foreclosure." *Brookings Papers on Economic Activity. Microeconomics*, Vol. 1990 (1990), pp. 205–276.

HOLMSTRÖM, B. "Moral Hazard in Teams." *The Bell Journal of Economics*, Vol. 13 (1982), pp. 324–340.

HOLMSTRÖM, B. AND MILGROM, P. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica*, Vol. 55 (1987), pp. 303–328.

HOLMSTRÖM, B. AND MILGROM, P. "Regulating Trade Among Agents." *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft*, Vol. 146 (1990), pp. 85–105.

ITOH, H. "Coalitions, Incentives, and Risk Sharing." *Journal of Economic Theory*, Vol. 60 (1993), pp. 410–427.

Lazear, E. P. and Rosen, S. "Rank-Order Tournaments as Optimum Labor Contracts." *Journal of Political Economy*, Vol. 89 (1981), pp. 841–864.

Macho-Stadler, I. and Pérez-Castrillo, J. D. "Moral Hazard with Several Agents: the Gains from Cooperation." *International Journal of Industrial Organization*, Vol. 11 (1993), pp. 73–100.

McAfee, R.P. and McMillan, J. "Optimal Contracts for Teams." *International Economic Review*, Vol. 32 (1991), pp. 561–577.

McAfee, R.P. and Schwartz, M. "Opportunism in Multilateral Vertical Contracting: nondiscrimination, Exclusivity, and Uniformity." *The American Economic Review*, Vol. 84 (1994), pp. 210–230.

McAfee, R.P. and Schwartz, M. "The Non-Existence of Pairwise-Proof Equilibrium." *Economics Letters*, Vol. 49 (1995), pp. 251–259.

Miller, N.H. "Efficiency in Partnerships with Joint Monitoring." *Journal of Economic Theory*, Vol. 77 (1997), pp. 285–299.

O'Brien, D.P. and Shaffer, G. "Vertical Control with Bilateral Contracts." *The RAND Journal of Economics*, Vol. 23 (1992), pp. 299–308.

Prendergast, C. "The Provision of Incentives in Firms." *Journal of Economic Literature*, Vol. 37 (1999), pp. 7–63.

Rajan, M.V. "Cost Allocation in Multiagent Settings." *The Accounting Review*, Vol. 67 (1992), pp. 527–545.

Ramakrishnan, R.T.S. and Thakor, A.V. "Cooperation versus Competition in Agency." *Journal of Law, Economics, & Organization*, Vol. 7 (1991), pp. 248–283.

Rasmusen, E. "Moral Hazard in Risk-Averse Teams." *The RAND Journal of Economics*, Vol. 18 (1987), pp. 428–435.

Ray, K. and Goldmanis, M. "Efficient Cost Allocation." *Management Science*, Vol. 58 (2012), pp. 1341–1356.

Rey, P. and Vergé, T. "Bilateral Control with Vertical Contracts." *The RAND Journal of Economics*, Vol. 35 (2004), pp. 728–746.

Segal, I. "Contracting with Externalities." *The Quarterly Journal of Economics*, Vol. 114 (1999), pp. 337–388.

Segal, I. and Whinston, M.D. "Robust Predictions for Bilateral Contracting with Externalities." *Econometrica*, Vol. 71 (2003), pp. 757–791.

Varian, H.R. "Monitoring Agents With Other Agents." *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft*, Vol. 146 (1990), pp. 153–174.