Multi-period Correlations across Public and Private Asset Classes

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Overview

- The significance of asset class correlations
- The challenges of multi-period correlations
- The challenges of multi-period correlations inclusive of private asset classes
- The correlations of returns vs. correlations of cumulative cash flows
- A distribution based framework of capturing cash flow correlations across asset classes
- Applications – Liability/Liquidity Driven Optimization (LDO), and analysis of higher moments

Introduction

- There has been extensive focus in the literature and industry on estimating multi-period volatility of asset classes and it's relationship to single-period volatility. Northfield discussion: diBartolomeo, September 2019, and Belev, June 2014

- Robust risk models can estimate single period volatility and based on appropriate assumptions of how that volatility compounds over many periods, long-terms investors in these asset classes can gauge their risk-return characteristics

- Less focus has been placed on estimating the long term correlations across asset, due to a number of challenges which we will describe in this presentation

- An observation can be made that correlations are even more important than volatilities to know with high confidence, because they indicate the efficiency of asset allocation. You can make up volatility with leverage. You can't make up diversification.

The Challenge of Multi-Periodicity

- In a single period the covariance between two assets calculated from a risk factor model is given as:

$$COV(A,B) = \sum_{i}^{K}\sum_{j}^{K} \beta_{A\_F_i}\beta_{B_{F_j}} * COV(F_i, F_j)$$

- Extending this algebraic expression to two periods becomes a very involved combination of correlations between factors and periods. Such permutations amount roughly to the number of factors to the power of number of periods times two. For 100 factors and 10 periods this is 20,000 correlation terms for just *two asset classes*. A number of (non-Northfield) commercial risk models have 2000+ factors.

- If we assume no stationarity and e.g. the asset expected return changes over time (or has an error associated with it) the expression becomes even more involved and intractable.

Multi-period Correlations with Private Assets

- Most factor risk models assume the dispersion of an asset return over a single period

- This entails that they forecast the asset price at the end of the period in a statistical distribution

- The market provides a robust arm's length pricing function for publicly traded assets. This is not the case for private assets

- For a LPA fund the GP provides an estimate of the NAV. For a private company or a building, NAV is provided by an appraiser

- In either case, these are very subjective, non-standard, non-arm's-length estimates of value that in addition hold very little information about their own statistical distribution due to smoothing

Correlations with Private Assets (cont'd)

- If the GP or appraiser "pricing function" is harnessed in the multi-period analysis, all of the above biases and errors will be compounded over the long run in the estimation of both volatility of and correlation with private assets

- One additional complicated dynamic is that by definition illiquid assets tend to exhibit skewed distribution in interim periods of extraordinarily high demand or high supply. For a long term investor with the "option to wait" those dynamics are irrelevant in the long run.

- Unlike pricing, cash flows of private assets are observable and occur at "arm's length"

- We shall demonstrate that by focusing on cash flows the requirement for an interim "pricing function" and its diffusion process parameters become unnecessary to capture private asset class volatilities or correlations of those assets long term value, and, ironically, also long term return.

Intuition of Long Term Correlations

- We can glean some intuition into the direction of the long term correlation between assets by looking into their one period relationship.

- Using a risk model we can find the beta of one asset class against another. It is equal to the COV(B,A) / ${\sigma_A}^2$. We showed how COV(B,A) is derived previously.

- Under a certain compound realization where there is a cross-asset driven component and a residual component $r_{\_B\_s}$

$$R_B = \left(1 + \beta_{B_{to_A}} * r_{A_{t1}} + r_{B_{s_{t1}}}\right) * (1 + \beta_{B_{to_A}} * r_{A_{t2}} + r_{B_{s_{t2}}})^*(1 + \beta_{B_{to_A}} * r_{A_{t3}} + r_{B_{s_{t3}}})$$

- Any A-driven term that gets multiplied by a B-specific term, even for one period only, makes the product uncorrelated with A. That is why *the terms that remain tied to A are a much smaller component* in the permutation of the product-sum than in the single period expression. So our expectation is that the correlation would go down over longer horizons compared to a single period.

Correlations: Returns vs. Cumulative Cashflows

- Return diffusion process requires reliance on a continuous “asset pricing function”. Cash flows do not pose this requirement for private assets

- Cash flow diffusion analysis sidesteps the interim distributional skew anomalies in extremely high or extremely low markets which are irrelevant to a long term investor

- Cash flow diffusion analysis sidesteps the interim distributional effects of transaction costs which are significant and highly uncertain for truly illiquid assets

- Cumulative cash flow analysis provides the flexibility to change the reinvestment rate assumption. Returns can be calculated from cumulative cash flows at any horizon

- This paradigm shift puts illiquid assets at center stage. Because from the perspective of a long term investor interim “market pricing” is of no consequence, rather than calling them illiquid we can think of them as non-tradable, but periodically liquid

Illiquid Private Asset Vehicles

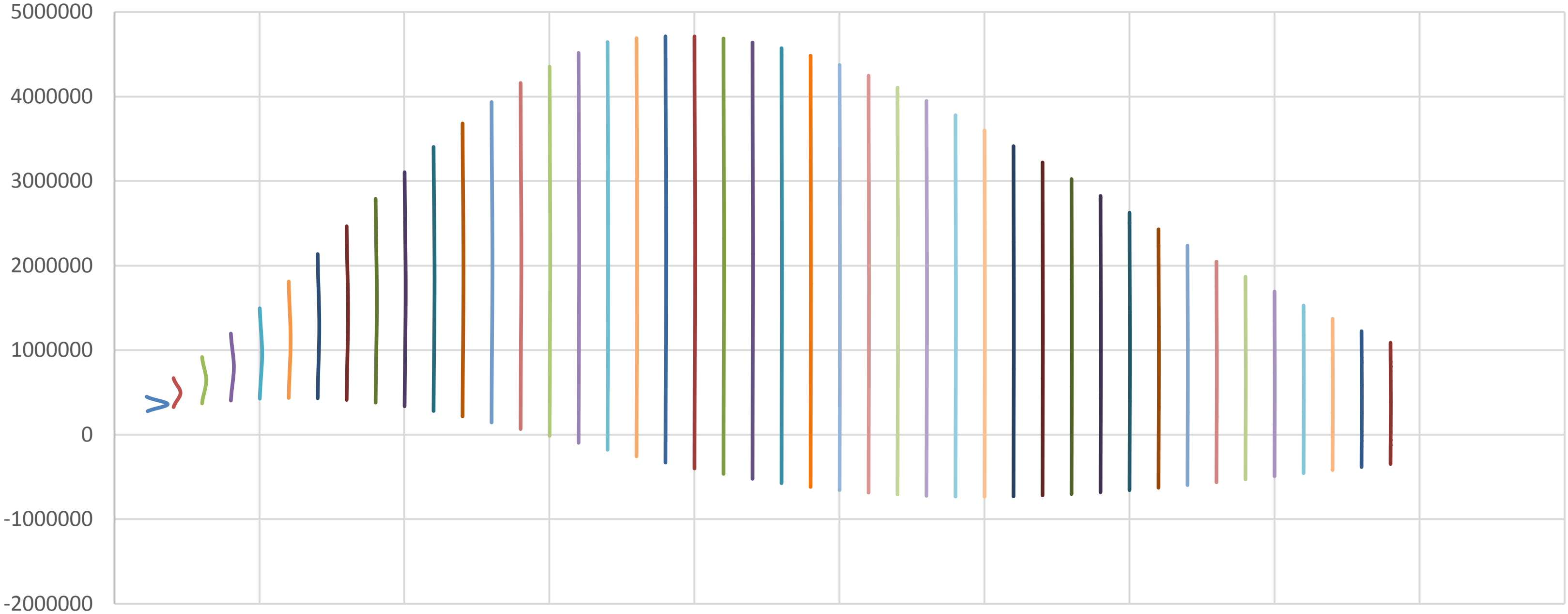
- The principle ways in which asset owners invest in private illiquid assets are two:
  - Through limited partnerships – i.e. private funds
  - Through direct investing / co-investing with private asset fund managers

- Due to its composite nature, private fund investing is more complex and that is why we will focus on it

- The typical underlying assets in which limited partnerships invest are:
  - Equity in private companies – common, preferred, hybrid. These investments can be early stage “venture capital”, growth, or late stage, i.e. “buyouts”.
  - Debt in private companies – convertible, or not. These investments can be in companies with stable financial status, or distressed companies.
  - Real Assets – commercial real estate, real assets.
  - Natural resources – farmland, timberland, etc.

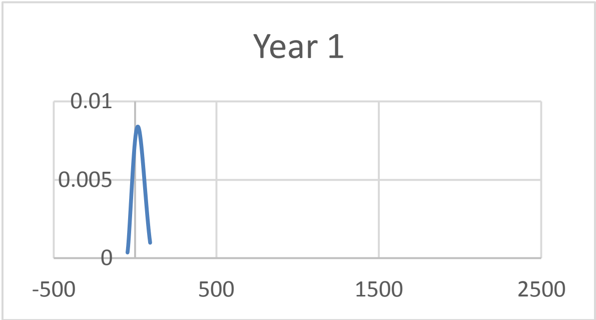
Forecasting Private Fund Cashflows

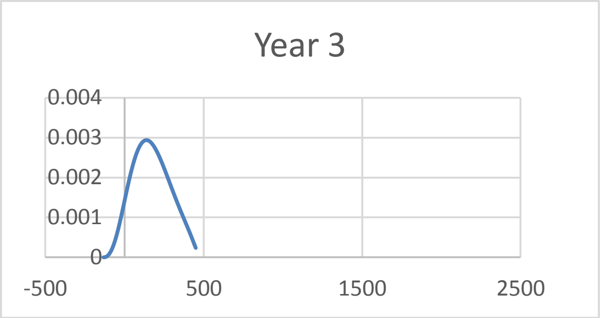
Periodic Non-cumulative Cash Flow Statistical Distributions over Remaining Quarters*

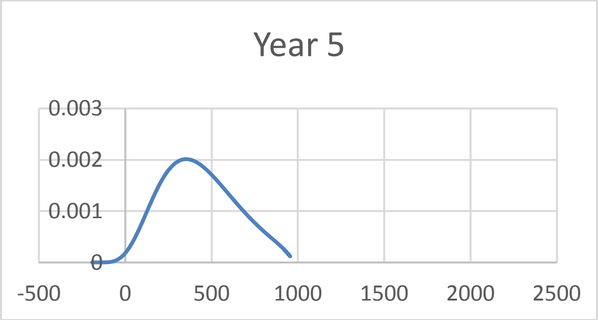


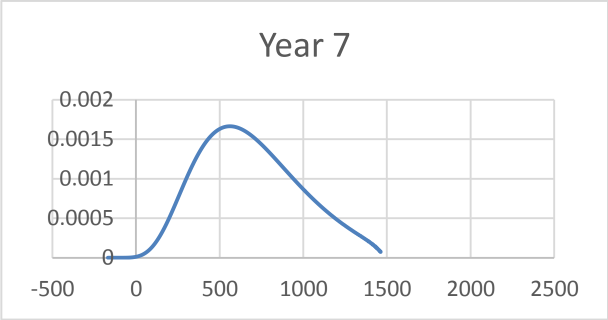
* Powered by ***Aspequity*** Cash Flow Model and ***Northfield*** Risk Model

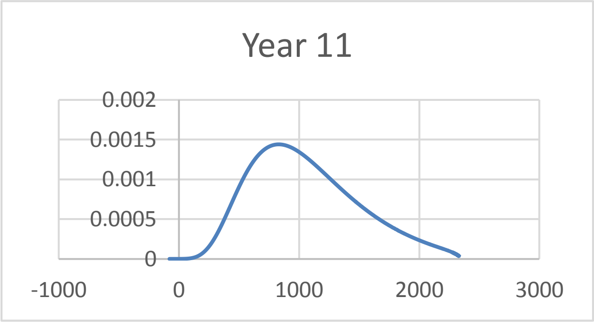
Cumulative Statistical Distribution of a Private Fund CFs











Powered by ***Aspequity*** Cash Flow Model and Simulation Technology (***effectively captures 10^{21} cash flow paths***), in combination with the ***Northfield*** Risk Model

Estimating Asset Class Covariance Matrices

- **Build Covariance Matrices among the Asset Classes over Different Horizons**
  - Project the cumulative performance of public assets under multi-period assumptions, effectively, as a "liquid equivalent" diffusion process.
  - Project expected periodic fund cashflows of "non-traded" assets using the *Aspequity* cash flow model. Introduce the volatility dimension to the expected CF using the *Northfield* risk model. Northfield provides risk models with asset-by-asset granularity for equity, debt, and *real assets* inclusive of CRE, infrastructure, and natural resources. This step produces periodic statistical distributions of the private fund portfolio cash flows.
  - Utilizing the Aspequity simulation algorithm build a statistical distribution of the cumulative performance of the private asset class over each time horizon.
  - Capture the *one period "beta"* of each asset class against each other one using the Northfield risk model. Cont'd…

Asset Class Covariance Matrices (cont'd)

- **Build Covariance Matrices over Different Horizons (cont'd)**
  - Utilizing the Aspequity simulation algorithm build a distribution of one asset class as "driven" solely by the other asset class, period by period.
  - Both in the standalone simulation, and the "another asset class driven" simulation, particular attention should be given to the fact that when periodic "non-traded" asset class cashflow occur, they compound at a reinvestment rate that is equal e.g. to the realized public assets in which they get reinvested. The assigned reinvestment asset class for "non-traded" asset cash flow should be specified in advance.
  - Based on the realized variance of the asset class over the particular horizon derived in this way, and comparing it to the variance of the standalone driver asset class, calculate *n-period beta* of that asset class against the "driver" asset class. Using the beta and the variances of both asset classes calculate a covariance between the two assets.

Correlations from Cumulative Cashflow Distributions

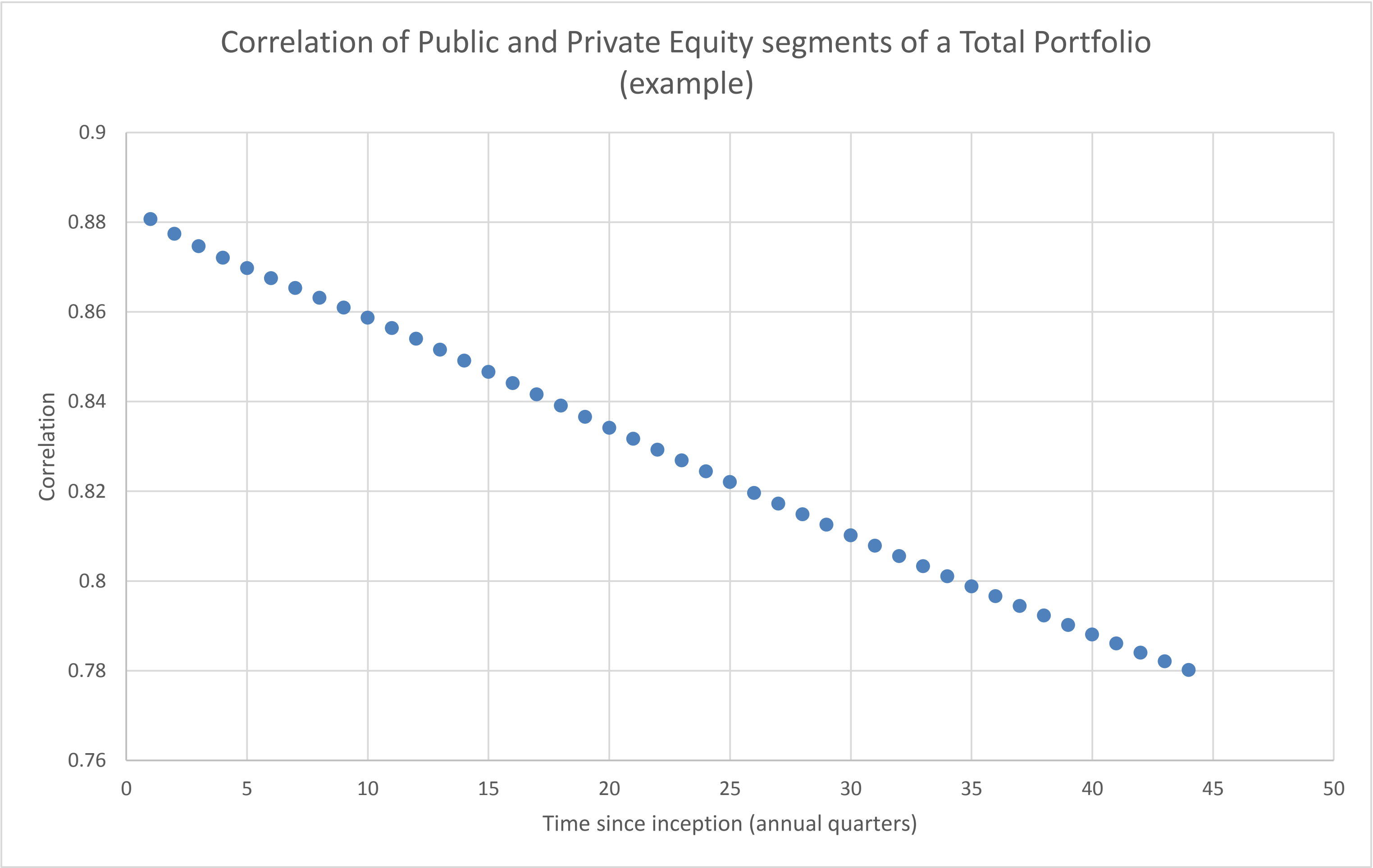
- **Can be estimated from the projected cumulative cash flow distribution processes for $\boldsymbol{B}$ and $\boldsymbol{B \sim f(A)}$**

- **A computational short-cut:**

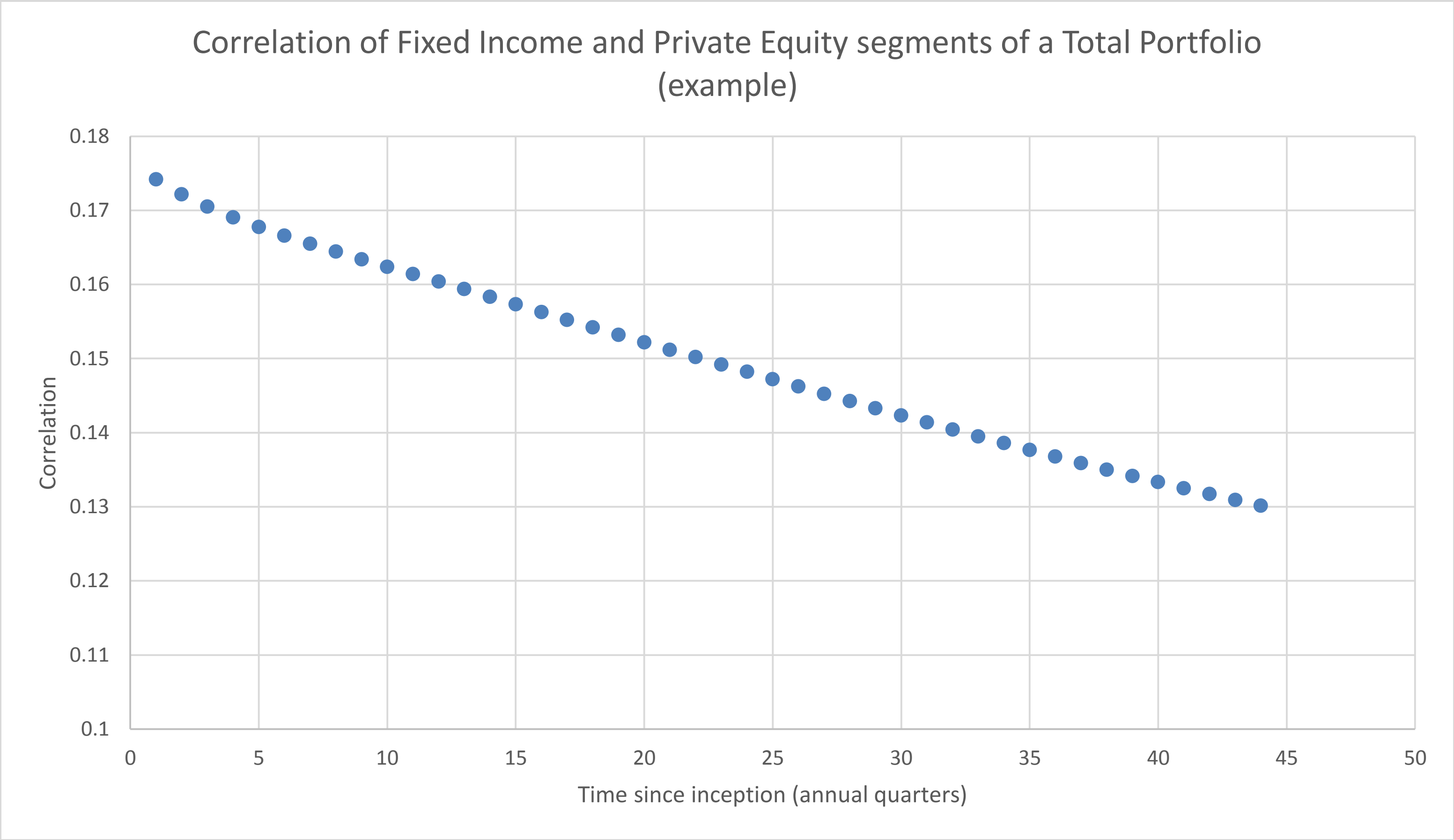
$$\boldsymbol{Corr_{Horizon\,T}\,(B,A) = \frac{\sigma_{B\sim f(A),T}}{\sigma_{B,T}} = \frac{\beta_{B-to-A,T} * \sigma_{A,T}}{\sigma_{B,T}} = \frac{COV(B,A)_T * \sigma_{A,T}}{\sigma_{A,T}{}^2 * \sigma_{B,T}}}$$
$$\boldsymbol{= \frac{COV(B,A)_T}{\sigma_{A,T} * \sigma_{B,T}}}$$

where $\boldsymbol{\sigma_{B\sim f(A),T}}$ and $\boldsymbol{\sigma_{B,T}}$ come directly from the projected cumulative cash flow distribution processes for $\boldsymbol{B}$ and $\boldsymbol{B\sim f(A)}$

Time Dependency of Correlation of Public and Private Equity



Time Dependency of Correlation of Fixed Income and Private Equity



Long Term Investors: Count your Blessings

- Long term investors are by definition more diversified than short term investors invested in an identical asset class mix
- For those investor that identify assets with "alpha" uncorrelated with the other asset classes there is an additional sweetener in the long run: apart from being a clear positive on its own, the uncorrelated alpha increases the diversification benefit in the long run as it increases (per slide 14) the correlation denominator $\sigma_{B,T}$ more than it decreases the numerator $\sigma_{B\sim f(A),T}$. This effect is a direct consequence of the computation of the variance of compound return (VCR):

$$VCR = [\sigma^2 + (1+\mu)^2]^k - (1+\mu)^{2k}$$

- As noted, LT investors are less subject to skewed distributions in stressed markets. *But…*

Count your Blessings (cont'd)

- …But Long Term Investor is not an absolute qualification. The Long Term Investor becomes a Short Term Investor when a liability emerges

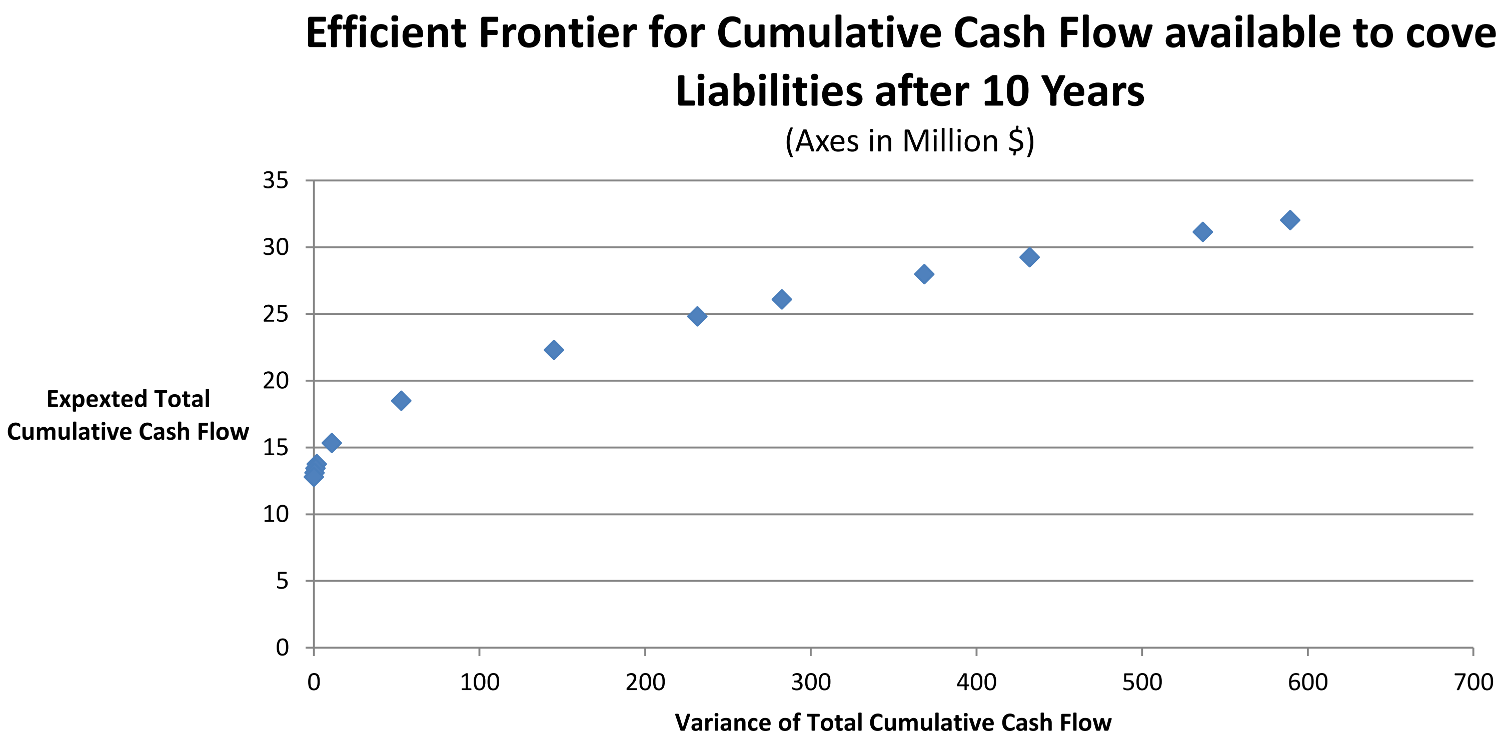
- No Investor is guaranteed that a liability won't emerge. In this sense instead of thinking of Long Term and Short Term investors we should be rather be thinking of:
  - Investors with a deterministic future stream of liabilities
  - Investors with a stochastic future stream of liabilities

- The Objective of a Liability Driven Investor :
  - ***what is the portfolio that maximizes long term performance, while ensuring that the liquid component of the portfolio performance never falls below a certain level of confidence at any given time horizon when periodic liabilities are due***

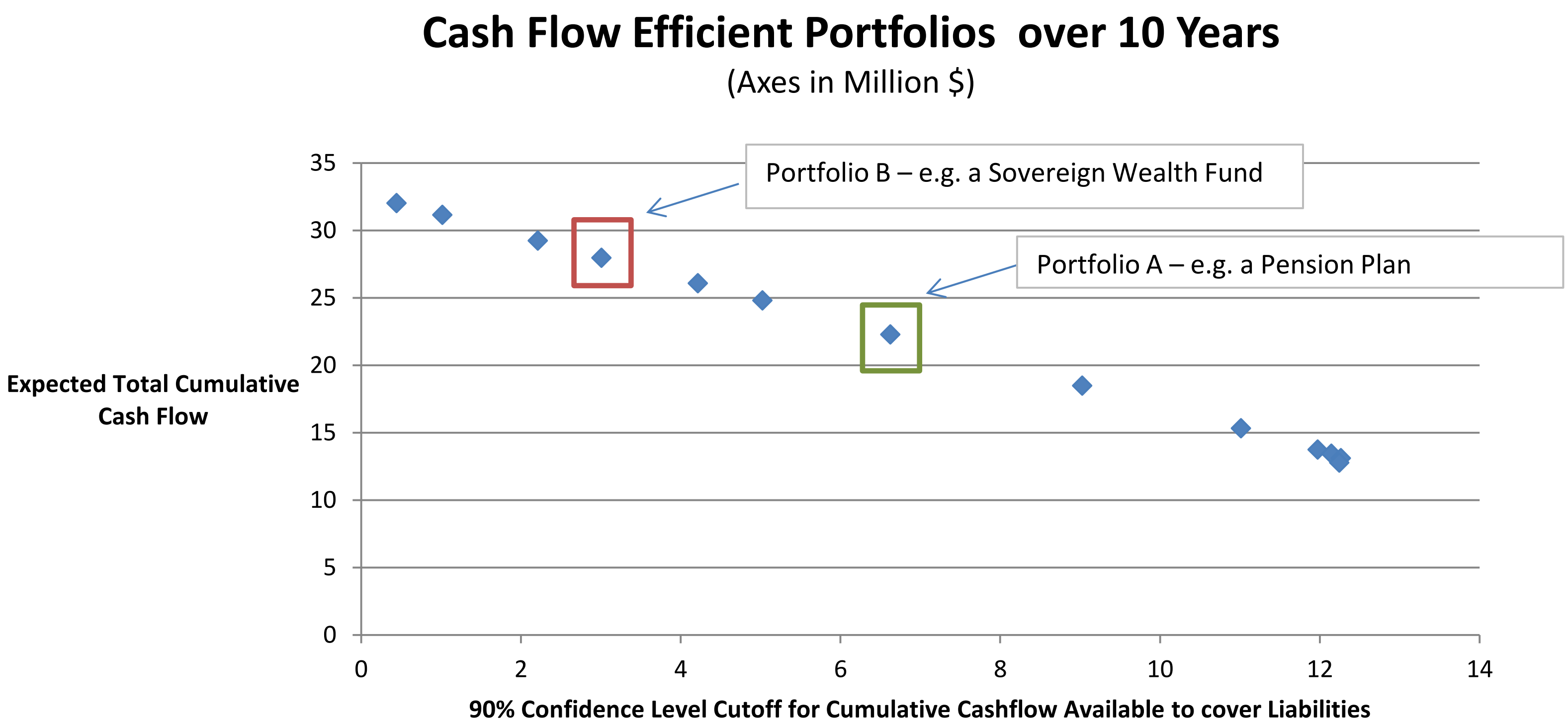
Application: Liability Driven Optimization (LDO)

- We can then formulate the optimization problem at each period over the investment horizon, taking into account the interplay between periodic and cumulative performance of liquid and “non-tradeable” assets.
- What we care about is the performance of the liquid public assets over each horizon, as well as the cash flows produced by the illiquid asset over the same period, as well as the reinvestment value of those cashflows to a later period.

Feasible Set of Efficient Portfolios



Efficient Portfolios with different Liquidity Levels



Efficient Portfolios with different Liquidity Levels

Portfolio A:
Provides $ 6.3 Mill.
with 90% confidence

Asset Class	Portfolio Weight
Bonds	52%
PE	34%
Stocks	14%

Portfolio B:
Provides $ 2.4 Mill.
with 90% confidence

Asset Class	Portfolio Weight
Bonds	35%
PE	46%
Stocks	19%

NOTE: *The examples herein are for illustration purposes only and do not represent investment advice or recommendations. Investor and investment circumstances vary widely including but not limited to number of asset class definitions, asset class underlying investment cash flow capacity and dispersion, investment horizons, etc. and each set of circumstances should be evaluated on a case by case basis.*

Key Points About LDO

- LDO has a deceptive resemblance to MVO. Mean-variance trade-off is not the end objective of LDO, as there are multi period constraints and dependencies that are factored in the LDO analysis.

- Applicable to private asset content beyond Private Equity. Private Debt, Real Assets, and Resource Assets also weave into LDO well using granular cash flow and risk models as the ones made available by Northfield and Aspequity.

- It may make sense in the early periods to offset redemption liabilities with borrowing in order to have a more aggressive profile that will bring higher expected results with high liquidity confidence in later periods… *Or not*. Cannot be determined without first performing the type of rigorous analysis outlined here

- If the Private Assets to which the investor has access are strong performers, they can get high weights in the optimal mix. Otherwise public equities compete well on the growth dimension. There is no one-size fits-all prescribed allocation, the investor and investment details are important.

Final Thought on Diversification and Higher Moments

- It can be shown that a moment of order *j* has the following first derivative with respect to a position $\mathbf{x_s}$:

$$E[(V - E[V])^j]' = j \sum_{k=1}^{N\ outcomes} p_k \left(\sum_{i=1}^{M\ positions} x_i V_{i,k} - E[V] \right)^{j-1} * (V_s - E[V_s])$$

- This expression can be thought of a "covariance" of the Value of a particular position S with performance of the overall portfolio to a certain power. Due to arguments similar to those made earlier in this presentation it can be shown that if a position diversifies away a particular central moment in a single period, the effect is magnified over several periods.

Summary

- Cash flow forecasting models for private asset classes are a key ingredient in being able to build a correlation estimation framework for a cross-asset class portfolio.

- Robust cumulative cash flow projection is important. Simulations that derive the specific horizon asset class risk models are accounting for the very complex dynamics of the cash flow accumulation and incorporating higher distributional moments

- Risk model that span asset classes is important. The approach also puts emphasis on the fact that multi-period asset class correlations are distinct and different form a single-period correlations, and handles the that transformation starting with a baseline risk model with broad asset class coverage and ends with a multi-period representation

- The overall inference is that multi-periodicity promotes diversification of a multi-asset class portfolio from a number of perspectives. The extent of these dynamics are specific to portfolios, assets, and investor circumstances.

Question and Answer Session

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