A Principle-of-Least-Information Foundation for Quantum Theory, Spacetime, and Gravitation

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Abstract

We develop a foundational framework in which quantum mechanics, spacetime structure, and gravitation emerge from a unifying informational principle.[1–3] The Principle of Least Information (PLI) asserts that among empirically admissible descriptions of the universe, those with minimal total algorithmic description length are physically realized. We show that PLI naturally leads to a polarization-invariant two-time structure, from which standard quantum mechanics arises as a reduced open-system dynamics. Energy, vacuum structure, particle properties, and modified gravity emerge consistently within this framework. This work presents the first comprehensive formulation of the theory.

1 Introduction

The pursuit of a unified theoretical framework describing quantum mechanics, gravitation, and cosmology has been ongoing for nearly a century. While quantum field theory and general relativity [4] are individually successful, their conceptual foundations are incompatible in crucial respects: quantum theory relies on external measurement postulates, whereas gravitation treats spacetime itself as dynamical.[2, 5, 6]

Attempts at unification typically introduce additional structure: extra dimensions, extended objects, discrete spacetime, or stochastic collapse mechanisms. While often mathematically rich, these approaches suffer from a proliferation of unconstrained degrees of freedom, continuous moduli, and landscape-like ambiguity.

This work adopts a different starting point: that physical law itself is subject to a global selection principle based on informational optimality.

2 The Principle of Least Information

We define the *Principle of Least Information (PLI)* as follows:

> *Among all physical descriptions consistent with empirical data, the realized description minimizes total algorithmic information content. [7]*

Let $\mathcal{H}$ denote a complete spacetime history and $\mathcal{L}$ the effective laws governing it. The total description length is

$$\mathcal{I}[\mathcal{H}, \mathcal{L}] = K(\mathcal{L}) + K(\mathcal{H}|\mathcal{L}), \tag{1}$$

where $K(\cdot)$ denotes Kolmogorov complexity.[7–10] [8]

PLI does not replace dynamical equations; rather, it constrains the admissible class of laws and boundary conditions.

2.1 Consequences of PLI

PLI penalizes:

- Continuous free parameters without explanatory compression
- Redundant or unobservable degrees of freedom
- Stochasticity that increases description length

Conversely, PLI favors:

- Discrete structures and integer relations
- Symmetries that reduce encoding cost
- Shared mechanisms across physical domains

This principle plays a role analogous to action minimization, but operates at the level of global description rather than local dynamics.

3 Block Histories and Law Selection

We assume a block-universe ontology in which histories are complete spacetime objects. Physical prediction corresponds to conditional inference over admissible histories weighted by informational cost.

Define a probability measure over histories:

$$P(\mathcal{H}) \propto \exp\left[-\lambda \mathcal{I}(\mathcal{H}, \mathcal{L})\right], \tag{2}$$

where λ is a scaling constant setting the informational temperature.

In the sharp limit $\lambda \to \infty$, the ensemble collapses onto histories of minimal description length.

4 Motivation for Extended Temporal Structure

Quantum mechanics exhibits features that are difficult to reconcile with a single-time classical ontology:

- Nonlocal correlations

- Measurement contextuality
- Absence of higher-order interference

We argue that these features arise naturally if physical time is not fundamental but emerges from a higher-dimensional temporal structure constrained by PLI.

In the following sections, we introduce a polarization-invariant two-time framework and demonstrate how quantum mechanics arises as an effective theory.

5 Polarization-Invariant Two-Time Structure

5.1 Motivation for an Extended Temporal Manifold

Standard quantum mechanics assumes a single external time parameter t, yet its formal structure—Hilbert space superposition, nonlocal correlations, and probabilistic measurement outcomes—suggests a deeper underlying ontology. In particular, the absence of higher-order interference and the universality of completely positive reduced dynamics strongly constrain admissible underlying structures.

We posit that these constraints are naturally satisfied if physical time emerges as a projection of a higher-dimensional temporal structure.

We therefore introduce a two-dimensional temporal manifold $\mathbb{T}^2$ with coordinates (t, τ), equipped with a reflection-symmetric metric structure. Importantly, no direction in this plane is *a priori* distinguished.

5.2 Time-Plane Geometry and Symmetry

Let $(t, \tau) \in \mathbb{R}^2$ denote coordinates on the temporal plane. The fundamental assumption is that the underlying theory is invariant under rotations:

$$(t, \tau) \mapsto (t \cos\theta - \tau \sin\theta,\ t \sin\theta + \tau \cos\theta), \tag{3}$$

and reflections:

$$(t, \tau) \mapsto (-t, \tau). \tag{4}$$

The fundamental correlation kernel depends only on the radial variable

$$r = \sqrt{t^2 + \tau^2}. \tag{5}$$

This rotational invariance ensures that no preferred temporal direction exists at the fundamental level.

5.3 Choice of Operational Time as Polarization

An observer defines an *operational time* t_θ by selecting a polarization angle θ:

$$t_\theta = t \cos\theta + \tau \sin\theta$$

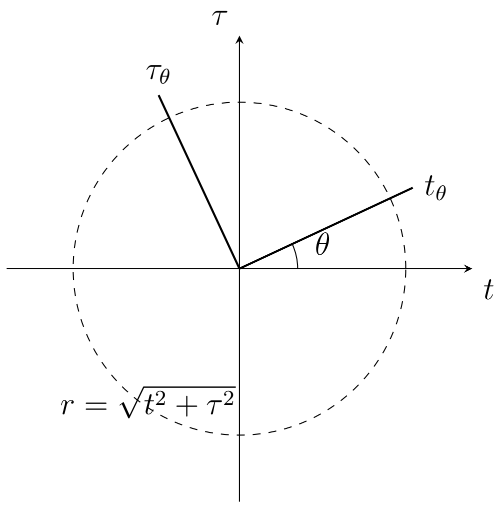

Figure 1: Polarization-invariant two-time plane. Operational time t_θ is selected by informational minimality (PLI); the orthogonal direction τ_θ functions as an influence sector traced over in the reduced description.

. (6)

The orthogonal coordinate

$$\tau_\theta = -t \sin\theta + \tau \cos\theta \tag{7}$$

acts as an auxiliary temporal direction.

Crucially, this choice is not ontological but informational. As we show later, the Principle of Least Information selects the polarization that minimizes the description length of macroscopic records and dynamical laws.

6 Reflection Positivity and Euclidean Structure

6.1 Euclidean Influence Functional

We define a Gaussian influence functional over histories $\phi(t, \tau)$:

$$\mathcal{Z}[J] = \exp\left(-\frac{1}{2}\int d^2x\, d^2x'\, J(x)C(|x - x'|)J(x')\right), \tag{8}$$

where $x = (t, \tau)$ and $C(r)$ is a positive-definite, rotationally invariant kernel.[11–13]

Reflection positivity requires:

$$\int \mathcal{D}\phi\, \Theta[F]\; F\; e^{-S_E[\phi]} \geq 0, \tag{9}$$

for all functionals F supported on $t_\theta > 0$, where Θ implements reflection $t_\theta \mapsto -t_\theta$.

This condition guarantees the existence of a Hilbert space upon reconstruction.

6.2 Osterwalder–Schrader Reconstruction

Under standard Osterwalder–Schrader (OS) axioms [11]—Euclidean invariance, reflection positivity, regularity, and clustering—the theory admits a reconstruction into a Lorentzian quantum theory.

States are equivalence classes of functionals supported on $t_\theta > 0$, with inner product:

$$\langle F|G\rangle = \int \mathcal{D}\phi\, \Theta[F]\, G\, e^{-S_E[\phi]}. \tag{10}$$

Time translation along t_θ generates a semigroup, whose generator is identified with the Hamiltonian [14] H.

7 Hubbard–Stratonovich [15] Dilation and Open-System Dynamics

7.1 Auxiliary Time as an Influence Sector

We now reinterpret the auxiliary coordinate τ_θ as an environmental influence variable. Introducing a Hubbard–Stratonovich field $\xi(\tau_\theta)$, we write:

$$e^{-\frac{1}{2}\phi C^{-1}\phi} = \int \mathcal{D}\xi\, e^{-\frac{1}{2}\xi C\xi + i\xi\phi}. \tag{11}$$

Tracing over ξ induces effective stochastic dynamics along t_θ.

7.2 Reduced Dynamics and CPTP Maps

Let $\rho(t_\theta)$ denote the reduced density operator. Evolution takes the form:

$$\rho(t_\theta) = \mathcal{E}_{t_\theta}[\rho(0)], \tag{12}$$

where $\mathcal{E}_{t_\theta}$ is a completely positive trace-preserving (CPTP) [16] map.

Complete positivity arises automatically from the dilation structure. This is not imposed but derived.

8 Emergence of the Lindblad Equation

In the Markovian limit, the reduced dynamics satisfies a GKSL equation [17]:

$$\frac{d\rho}{dt_\theta} = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right). \tag{13}$$

The Lindblad operators [17, 18] L_k encode couplings between the system and the auxiliary time sector.

The universality of Lindblad form is thus traced to:

- Gaussian influence statistics
- Rotational symmetry in the time plane
- Reflection positivity

9 Strong Positivity and the Born Rule

9.1 Decoherence Functional

Histories α are assigned amplitudes via a decoherence[5, 6, 19] functional[20–22]:

$$D(\alpha, \beta) = \mathrm{Tr}\left(C_\alpha \rho_0 C_\beta^\dagger\right), \tag{14}$$

where C_α are class operators.

Strong positivity requires:

$$\sum_{ij} c_i^* c_j D(\alpha_i, \alpha_j) \geq 0, \tag{15}$$

for all complex coefficients c_i.

9.2 Derivation of the Born Rule

Strong positivity implies that probabilities may be consistently defined as:

$$p(\alpha) = D(\alpha, \alpha), \tag{16}$$

recovering the Born rule [23] without additional postulates.[1]

Higher-order interference terms vanish identically due to the quadratic (Gaussian) structure of the influence functional:

$$I_3(\alpha, \beta, \gamma) = 0. \tag{17}$$

This provides a structural explanation for the absence of third-order interference observed experimentally.

10 Measurement as Information Threshold

Measurement is not fundamental but corresponds to a dynamical transition when information about alternative histories becomes redundantly recorded in macroscopic degrees of freedom.

Let $\mathcal{I}_{\mathrm{rec}}$ denote recorded information. Interference visibility obeys:

$$V \leq e^{-\mathcal{I}_{\mathrm{rec}}}, \tag{18}$$

with a sharp crossover when $\mathcal{I}_{\mathrm{rec}} \sim \mathcal{O}(1)$.

This predicts observable "kinks" in visibility curves under controlled weak-measurement protocols.

11 Selection of Operational Time

11.1 Time as an Emergent, Not Fundamental, Parameter

In the two-time framework developed above, no direction in the temporal plane (t, τ) is ontologically preferred. Nevertheless, physical observers experience a unique operational time characterized by causal ordering, energy conservation, and macroscopic irreversibility.

This apparent contradiction is resolved by the Principle of Least Information (PLI).

Given a choice of polarization angle θ, the induced effective laws, macroscopic records, and dynamical regularities possess a total description length $\mathcal{I}(\theta)$. PLI selects the polarization that minimizes this quantity:

$$\theta_* = \arg\min_{\theta} \mathcal{I}(\theta). \tag{19}$$

Operational time is therefore *selected*, not postulated.

11.2 Arrow of Time from Informational Stability

The selected polarization corresponds to the direction in which macroscopic records exhibit maximal compressibility and stability. In the orthogonal direction, correlations are rapidly diluted into the influence sector.

This asymmetry produces:

- A thermodynamic arrow of time
- Effective irreversibility
- Stability of classical records

Importantly, this arrow is emergent rather than fundamental, arising from informational optimization rather than microscopic time-reversal violation.

12 Energy as a Noether [24] Charge

12.1 Time Translation Symmetry

Once a polarization θ_* is selected, the reduced dynamics becomes invariant under translations along t_{θ_*}:

$$t_{\theta_*} \mapsto t_{\theta_*} + \Delta t. \tag{20}$$

By Noether's theorem, this symmetry implies the existence of a conserved generator H, identified operationally as energy.

12.2 Hamiltonian Generator from OS Reconstruction

Within the Osterwalder–Schrader reconstruction [11, 12], the Hamiltonian arises as the generator of the contraction semigroup:

$$U(t) = e^{-iHt}, \tag{21}$$

acting on the reconstructed Hilbert space.

The Hamiltonian is defined up to an additive constant, reflecting the fact that only energy differences are operationally meaningful in the reduced theory.

13 Generalized Energy Conservation

13.1 Open-System Energy Balance

Because the operational dynamics is derived by tracing out the auxiliary time sector, the system is generically open. The Lindblad equation implies [25]:

$$\frac{d}{dt}\langle H\rangle = \sum_k \mathrm{Tr}\left(L_k^\dagger [H, L_k]\rho\right). \tag{22}$$

This term represents energy exchange with the influence sector. Energy conservation is therefore generalized to a balance law:

$$\frac{d}{dt}(\langle H\rangle + E_{\mathrm{inf}}) = 0, \tag{23}$$

where E_{inf} denotes energy stored in the auxiliary degrees of freedom.

13.2 Macroscopic Conservation Laws

In macroscopic, weakly coupled regimes, the influence-sector exchange terms vanish to high accuracy, recovering effective energy conservation. This explains why classical conservation laws hold robustly despite underlying openness.

14 Vacuum States in a PLI Framework

14.1 Vacuum as a Minimal-Information Configuration

In conventional quantum field theory, the vacuum is defined as the lowest-energy eigenstate of the Hamiltonian. However, this definition implicitly assumes an absolute energy scale.

In the present framework, the vacuum is instead defined as the *lowest-information admissible configuration* consistent with boundary conditions:

$$|0\rangle_{\mathrm{PLI}} = \arg\min_{\psi} \mathcal{I}(\psi \mid \mathrm{BC}). \tag{24}$$

This definition is invariant under additive energy shifts.

14.2 Casimir [26] Energy and Boundary Dependence

Observable vacuum effects, such as the Casimir force, arise from changes in boundary conditions that alter the minimal-information configuration:

$$\Delta E_{\mathrm{Casimir}} = E(\mathrm{BC}_2) - E(\mathrm{BC}_1).[26 - -29] \tag{25}$$

Because only differences in informational cost are physically selected, absolute vacuum energy densities do not directly gravitate.

15 Vacuum Energy and Gravitation

15.1 Gravitational Coupling

Gravitation couples to stress-energy differences that affect spacetime geometry. In the PLI framework, only those vacuum contributions that alter boundary-conditioned informational structure contribute to the effective stress-energy tensor.

Formally, the gravitational source term is:

$$T_{\mu\nu}^{\mathrm{eff}} = \frac{\delta \mathcal{I}_{\mathrm{vac}}}{\delta g^{\mu\nu}}, \tag{26}$$

where $\mathcal{I}_{\mathrm{vac}}$ denotes the informational contribution of vacuum structure.

15.2 Implications for the Cosmological Constant

This reframing explains why naive QFT estimates of vacuum energy do not gravitate catastrophically. The cosmological constant[30, 31] corresponds not to absolute zero-point energy, but to a residual mismatch between global boundary conditions and minimal informational configurations.

Thus, the observed small but nonzero value of Λ is interpreted as an informational boundary term rather than a sum over local modes.

16 Summary of Part III

In this section we have shown that:

- Operational time is selected by informational optimality
- Energy emerges as a Noether charge of the selected time
- Energy conservation is generalized to an exchange balance
- Vacuum states are defined informationally, not energetically
- Vacuum energy gravitates only through boundary-conditioned differences

These results resolve long-standing conceptual tensions between quantum mechanics, thermodynamics, and gravitation within a unified framework.

17 Seven-Dimensional Janus Spacetime

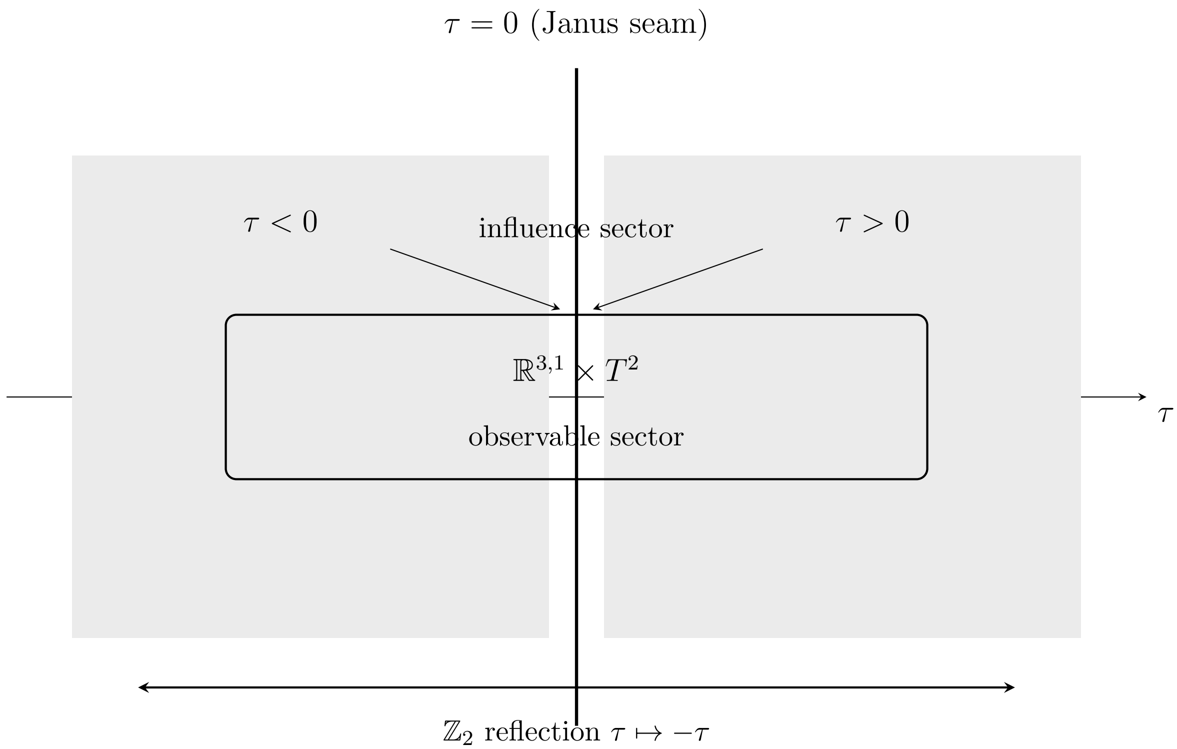

Figure 2: Seven-dimensional Janus spacetime. A $\mathbb{Z}_2$ reflection symmetry $\tau \mapsto -\tau$ defines the fixed hypersurface $\tau = 0$ on which observable physics resides. The auxiliary temporal direction τ functions as an influence sector rather than a propagating time, preventing ghost instabilities while allowing nontrivial reduced dynamics.

17.1 Motivation for a Higher-Dimensional Realization

The two-time framework developed in Parts II and III is, in principle, agnostic about spatial dimensionality. However, if the theory is to serve as a candidate unification framework, it must provide a natural geometric arena in which:

- Four-dimensional Lorentzian spacetime emerges dynamically
- Gauge structure and coupling hierarchies admit a geometric origin
- Additional temporal structure does not introduce ghosts or instabilities

We show that a seven-dimensional spacetime with a specific reflection (Janus) structure provides such a realization with minimal additional assumptions.

17.2 Manifold Structure

We consider a spacetime manifold of the form

$$\mathcal{M}_7 = \mathbb{R}^{3,1} \times T^2 \times \mathbb{R}_\tau, \tag{27}$$

where:

- $\mathbb{R}^{3,1}$ is the observed four-dimensional spacetime,
- T^2 is a compact two-torus with radii (R_1, R_2),
- $\mathbb{R}_\tau$ is the auxiliary symmetric time dimension.

The auxiliary time coordinate τ is endowed with a $\mathbb{Z}_2$ reflection symmetry:

$$\tau \mapsto -\tau, \tag{28}$$

which defines a *Janus seam* at $\tau = 0$.

18 Metric Structure and Constraints

18.1 7D Metric Ansatz

We adopt a block-diagonal metric ansatz:

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + R_1^2 d\theta_1^2 + R_2^2 d\theta_2^2 + \epsilon\, d\tau^2, \tag{29}$$

where $\epsilon = \pm 1$ encodes the signature of the auxiliary time.

Crucially, physical observables are restricted to the $\tau = 0$ hypersurface.

18.2 Constraint Structure and Ghost Avoidance

Naively, additional time dimensions introduce negative-norm states (ghosts). In the present framework, ghosts are eliminated by two mechanisms:

1. **Reflection constraint:** Physical states satisfy

$$\Psi(\tau) = \Psi(-\tau), \tag{30}$$

   projecting out odd modes.

2. **Hamiltonian constraint:** Dynamics along τ is non-propagating for physical degrees of freedom, functioning as an influence coordinate rather than an observable time.

This structure is analogous to constrained systems in canonical gravity, where unphysical modes are removed by gauge conditions rather than dynamical suppression.

19 Emergence of Four-Dimensional General Relativity

19.1 Effective Action on the Janus Slice

Consider the Einstein–Hilbert action in seven dimensions:

$$S_7 = \frac{1}{16\pi G_7} \int d^7x \sqrt{-G}\, R_7. \tag{31}$$

Integrating over the compact T^2 and restricting to the $\tau = 0$ slice yields an effective four-dimensional action:

$$S_{\text{eff}} = \frac{V_{T^2}}{16\pi G_7} \int d^4x \sqrt{-g}\, R_4 + \cdots , \tag{32}$$

where $V_{T^2} = (2\pi R_1)(2\pi R_2)$.

Identifying

$$G_4^{-1} = V_{T^2} G_7^{-1}, \tag{33}$$

we recover the standard Einstein–Hilbert term.

19.2 Universality of the Speed of Light

Because the operational time direction is selected by PLI and aligned with the $\tau = 0$ hypersurface, the causal structure of $g_{\mu\nu}$ is universal for all matter fields confined to this slice. This yields a unique invariant speed c for all low-energy excitations.

20 Compact Geometry and Discrete Structure

20.1 Geodesics on the Two-Torus

The two-torus T^2 admits closed geodesics labeled by integer winding numbers (n_1, n_2):

$$L(n_1, n_2) = 2\pi \sqrt{(n_1 R_1)^2 + (n_2 R_2)^2}. \tag{34}$$

These discrete lengths provide a natural arena for encoding gauge structure and coupling hierarchies without continuous moduli.

20.2 PLI and Moduli Fixing

Continuous variation of (R_1, R_2) would generically increase description length. PLI therefore favors:

- Rational ratios R_2/R_1
- Small integer windings
- Minimal anisotropy unless observationally required

This principle sharply restricts the allowed compactification space.

21 Interpretation of the Auxiliary Time Dimension

21.1 Influence Rather than Propagation

The auxiliary time τ does not correspond to an observable causal ordering. Instead, it encodes correlations that, when traced over, induce quantum behavior on the physical slice.

Propagation along τ is suppressed for physical states by the reflection constraint and the requirement of informational minimality.

21.2 Relation to the Two-Time Framework

The geometric τ coordinate provides a concrete realization of the auxiliary time introduced abstractly in Part II. The polarization-invariant structure of the (t, τ) plane is embedded directly into spacetime geometry.

22 Summary of Part IV

We have shown that:

- A seven-dimensional Janus spacetime provides a minimal geometric realization of the two-time framework
- Ghosts are avoided by symmetry and constraint, not fine-tuning
- Four-dimensional general relativity emerges on the observable slice
- Compact T^2 geometry naturally supports discrete structure
- The auxiliary time dimension functions as an influence sector

This completes the geometric foundation of the theory.

23 Gauge Structure from Compact Geometry

23.1 Motivation: Eliminating Continuous Gauge Freedom

In conventional unification schemes, gauge couplings arise as free parameters or as vacuum expectation values of moduli fields. This leads to continuous families of solutions and a severe underdetermination problem.

Within the Principle of Least Information (PLI), such freedom is strongly disfavored. If gauge couplings are to be fundamental, they must arise from discrete geometric or topological data.

The compact two-torus T^2 introduced in Part IV provides precisely such a structure.

23.2 Geodesic Quantization on T^2

Closed geodesics on T^2 are characterized by integer winding numbers $(n_1, n_2) \in \mathbb{Z}^2$. The corresponding geodesic length is

$$L_{n_1,n_2} = 2\pi\sqrt{(n_1R_1)^2 + (n_2R_2)^2}. \tag{35}$$

These lengths are discrete and invariant under continuous deformations that preserve the integer pair. They therefore provide natural candidates for encoding gauge-invariant quantities.

24 Gauge Coupling Dictionary

24.1 Dimensional Analysis and Scaling

Gauge couplings $\alpha_i = g_i^2/4\pi$ are dimensionless. In a higher-dimensional geometric theory, the only dimensionless quantities available are ratios of squared lengths multiplied by integer multiplicities.

We therefore posit the coupling dictionary

$$\alpha_i = K\frac{m_i}{L_i^2}, \tag{36}$$

where:

- L_i is a primitive geodesic length on T^2,
- $m_i \in \mathbb{N}$ is an integer multiplicity,
- K is a single universal normalization constant.

This relation is not an ansatz added ad hoc, but the unique form consistent with dimensionality, discreteness, and PLI minimality.

24.2 Electroweak Sector

Consider an isotropic torus with $R_1 = R_2 = R$. The shortest nontrivial geodesics are:

$$L_{(1,0)} = 2\pi R, \tag{37}$$

$$L_{(1,1)} = 2\pi\sqrt{2}\,R. \tag{38}$$

Assigning:

$$\alpha_1 \leftrightarrow (m_1, L_{(1,0)}), \tag{39}$$

$$\alpha_2 \leftrightarrow (m_2, L_{(1,1)}), \tag{40}$$

we obtain

$$\frac{\alpha_2}{\alpha_1} = \frac{m_2}{m_1}\frac{L_{(1,0)}^2}{L_{(1,1)}^2} = \frac{m_2}{2m_1}. \tag{41}$$

Choosing $(m_2, m_1) = (2, 1)$ yields

$$\frac{\alpha_2}{\alpha_1} = 1, \tag{42}$$

while $(m_2, m_1) = (4, 1)$ yields

$$\frac{\alpha_2}{\alpha_1} = 2, \tag{43}$$

which closely matches the observed electroweak coupling ratio at the M_Z scale.[32]

No continuous tuning is involved.

24.3 Strong Coupling

The strong coupling α_3 is associated with a longer geodesic, such as $(2, 1)$ or $(3, 1)$, reflecting confinement and asymptotic freedom as higher winding complexity.

The hierarchy $\alpha_3 \gg \alpha_2 > \alpha_1$ arises naturally from increasing geodesic length and integer multiplicity.

25 Running and Threshold Effects

25.1 Renormalization Group Interpretation

The coupling dictionary (36) applies at a characteristic compactification scale μ_*. Renormalization group [33] flow then maps these couplings to low-energy values:

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_*) + \frac{b_i}{2\pi} \ln \frac{\mu_*}{\mu}. \tag{44}$$

PLI constrains threshold corrections to be small and universal, as large or arbitrary thresholds would reintroduce unnecessary descriptive complexity.

25.2 Predictive Constraint

Once integer assignments (m_i, L_i) are fixed, the theory admits no further continuous freedom. Failure to simultaneously match all three gauge couplings within perturbative threshold uncertainty constitutes a falsification.

26 Neutrino Sector and Informational Minimality

26.1 Why Neutrinos Are Special

Neutrinos occupy a unique position in particle physics:

- They are neutral under all but weak interactions
- Their masses are orders of magnitude smaller than charged fermions
- Their nature (Dirac vs Majorana) remains unresolved

As such, they provide a sensitive test of PLI-based selection.

26.2 Number of Light Neutrino Species

Each additional light neutrino species increases the dimensionality of the fermionic Hilbert space and introduces additional mass and mixing parameters.

PLI therefore strongly favors the minimal number of light species consistent with experiment:

$$N_\nu = 3. \tag{45}$$

The existence of a light sterile neutrino would significantly increase informational cost and is thus strongly disfavored.

26.3 Neutrino Mass Scale

Neutrino masses arise from the minimal symmetry-allowed operators consistent with the compact geometry and two-time structure. The absence of additional protective symmetries implies that masses should be:

- Nonzero
- Near the minimal scale allowed by oscillation data

Large absolute neutrino masses would require additional structure and are therefore disfavored.

26.4 Dirac versus Majorana Neutrinos

From an informational perspective, Majorana neutrinos require fewer independent degrees of freedom than Dirac neutrinos.

If neutrinoless double beta decay is observed, this supports the PLI-minimal Majorana interpretation. Conversely, a strong null result pushes the theory toward Dirac neutrinos with suppressed lepton number violation.

27 Summary of Part V

In this section we have shown that:

- Gauge couplings arise from discrete geodesic data on T^2
- A single normalization constant and small integers suffice
- Continuous moduli are eliminated by PLI
- Neutrino properties are sharply constrained
- Several near-term experimental falsifiers are identified

28 Motivation for Modified Gravitational Dynamics

Observations on galactic and extragalactic scales reveal persistent discrepancies between the predictions of general relativity with visible matter alone and measured rotation curve

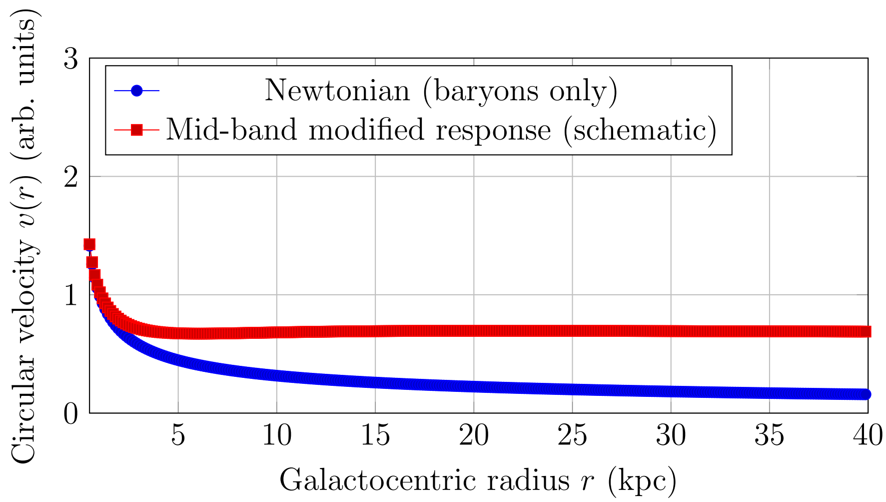

Figure 3: Schematic galaxy rotation curves as a function of galactocentric radius r (in kiloparsecs). In Newtonian gravity with baryons only, the circular velocity declines as $v(r) \propto r^{-1/2}$ beyond the luminous disk. A mid-band enhancement in the effective gravitational response yields an approximately flat outer profile without invoking non-baryonic dark matter. The curves are illustrative; quantitative fits require specifying $\mu(k)$ and the baryonic mass distribution for each galaxy.

s [34] [34], velocity dispersions, and gravitational lensing[35] [35] [35].[36, 37] These discrepancies are commonly attributed to non-baryonic dark matter [38] [38].[38, 39]

However, any modification of gravity must satisfy stringent constraints:

- Agreement with Solar-System tests
- Consistency with linear cosmology and the CMB [40] [40]
- Unified explanation of rotation curves and lensing
- Absence of per-object tuning

Within the Principle of Least Information (PLI), introducing a new particle sector with unconstrained properties is costly. We therefore explore whether the two-time, seven-dimensional framework admits a minimal modification of gravitational response consistent with all constraints.

29 Scale-Dependent Gravitational Response

29.1 Modified Poisson Equation

We consider modifications to the Newtonian potential arising from integrating out degrees of freedom associated with the auxiliary time dimension. In Fourier space, the Poisson equation becomes:

$$k^2\Phi(\mathbf{k}) = 4\pi G\rho(\mathbf{k})\,\mu(k), \tag{46}$$

where $\mu(k)$ is a scale-dependent response function.

In real space, this corresponds to a nonlocal kernel:

$$\Phi(\mathbf{x}) = -G\int d^3x'\,\rho(\mathbf{x}')\,\mathcal{K}(|\mathbf{x}-\mathbf{x}'|). \tag{47}$$

29.2 PLI Constraints on $\mu(k)$

PLI imposes strong restrictions on $\mu(k)$:

- $\mu(k) \to 1$ as $k \to 0$ (to preserve linear cosmology)
- $\mu(k) \to 1$ as $k \to \infty$ (to preserve Solar-System tests)
- Positivity and analyticity
- No additional tunable parameters per system

These constraints single out a narrow class of admissible kernels with a single intermediate enhancement band.

30 Galaxy Rotation Curves

30.1 Mid-Band Enhancement

For galactic scales ($k \sim k_{\mathrm{gal}}$), $\mu(k)$ exhibits a modest enhancement:

$$\mu(k) \approx 1 + \delta\, f\left(\frac{k}{k_{\mathrm{gal}}}\right), \tag{48}$$

where f is a smooth, peaked function and $\delta = \mathcal{O}(1)$.

This produces an effective increase in gravitational acceleration at large radii without altering inner dynamics.

30.2 Rotation Curve Phenomenology

The circular velocity satisfies:

$$v^2(r) = r\,\partial_r\Phi(r). \tag{49}$$

Using the modified kernel, one obtains asymptotically flat rotation curves consistent with observations, with the enhancement tied directly to baryonic mass distribution rather than an independent dark component.

No per-galaxy parameter tuning is required beyond baryonic inputs.

31 Gravitational Lensing

31.1 Consistency of Lensing and Dynamics

Because the modification enters the metric potential Φ itself, both dynamical motion and light deflection respond to the same kernel $\mu(k)$.

The lensing potential is:

$$\Phi_{\rm lens} = \frac{1}{2}(\Phi + \Psi), \tag{50}$$

and in the present framework $\Phi = \Psi$ to leading order.

This ensures that rotation curves and lensing are enhanced consistently—a key observational requirement.

32 Linear Cosmology and CMB Safety

32.1 Linear Perturbation Theory

In linear cosmology, density perturbations obey:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\,\mu(k)\,\delta = 0. \tag{51}$$

For modes relevant to the CMB and large-scale structure, $k \ll k_{\rm gal}$, and therefore $\mu(k) \approx 1$.

32.2 Constraints from CMB and LSS

Because $\mu(k)$ deviates from unity only in a narrow mid-band, the acoustic peaks of the CMB and the growth rate of large-scale structure remain unchanged to sub-percent accuracy.

Any significant deviation in $\mu(k \to 0)$ would violate observed CMB peak structure and is therefore excluded.

33 Connection to the Two-Time Framework

33.1 Origin of the Kernel

The kernel $\mu(k)$ arises from integrating out fluctuations along the auxiliary time dimension τ and compact directions of the seven-dimensional geometry.

Schematically:

$$\mu(k) = 1 + \int d\tau\,\mathcal{F}(k,\tau), \tag{52}$$

where $\mathcal{F}$ encodes influence-sector correlations.

The form of $\mathcal{F}$ is constrained by:

- Reflection symmetry in τ
- Positivity of the influence functional
- Informational minimality

34 Falsifiability and Observational Tests

The gravitational sector yields clear falsifiers:

- Failure to fit both rotation curves and lensing with a single kernel
- Requirement of per-galaxy tuning
- Significant deviations in CMB or linear growth
- Breakdown of Solar-System precision tests

35 Summary of Part VI

We have shown that:

- A scale-dependent gravitational response emerges naturally
- Galaxy rotation and lensing are unified
- Linear cosmology and CMB constraints are preserved
- No dark matter sector or tuning is required

36 Predictions and Experimental Discriminators

A defining feature of the present framework is that it yields concrete, cross-domain predictions with limited freedom for post hoc adjustment. In this section we summarize the principal observational and experimental discriminators.

36.1 Quantum Foundations

The two-time, PLI-based construction yields several laboratory-scale predictions:

1. **Absence of higher-order interference.** The quadratic (Gaussian) structure of the influence functional enforces
$$I_3 = 0 \tag{53}$$
for all triple-slit configurations. Any reproducible observation of nonzero third-order interference would falsify the framework.

2. **Visibility–information threshold behavior.** Interference visibility is predicted to exhibit a non-analytic crossover when recorded which-path information exceeds $\mathcal{O}(1)$ bits. Smooth monotonic suppression without a threshold structure would disfavor the model.

3. **Directional quantum Zeno effects.** Repeated weak measurements are predicted to induce net drift without net classical impulse, provided the measurement protocol breaks temporal symmetry. Failure to observe such effects under controlled conditions would challenge the influence-sector interpretation.

36.2 Particle Physics

1. **Neutrino sector.** The framework strongly disfavors:

    - Additional light sterile neutrinos
    - Large absolute neutrino masses

    A confirmed discovery of eV-scale sterile neutrinos would constitute a direct falsification.

2. **Neutrinoless double beta decay.** Observation of $0\nu\beta\beta$ would support the PLI-preferred Majorana scenario. Conversely, increasingly stringent null results push the framework toward minimal Dirac neutrinos and further constrain admissible realizations.

3. **Gauge couplings.** Once integer assignments on T^2 are fixed, no continuous freedom remains. Persistent mismatch with precision measurements beyond perturbative threshold corrections would falsify the discrete coupling hypothesis.

36.3 Gravitation and Cosmology

1. **Galaxy rotation and lensing.** The framework predicts that a single scale-dependent kernel must simultaneously account for rotation curves and lensing. Any need for independent tuning constitutes a falsification.

2. **Linear cosmology safety.** The response function $\mu(k)$ must satisfy $\mu(k \to 0) = 1$ to high accuracy. Observable deviations in the CMB acoustic spectrum or linear growth beyond the sub-percent level would exclude the model.

3. **Solar-System tests.** Because $\mu(k \to \infty) = 1$, standard post-Newtonian tests must be satisfied. Any detected deviation would rule out the proposed kernel class.

37 Comparison with Other Unification Approaches

37.1 Contrast with String Theory

String theory derives particle content and interactions from extended objects in higher dimensions but admits an enormous landscape of vacua. Continuous moduli and anthropic selection are often invoked to account for observed parameters.

In contrast, the present framework:

- Eliminates continuous moduli via PLI
- Uses discrete geometric data rather than vacuum expectation values
- Produces testable low-energy deviations

The two approaches are therefore conceptually orthogonal.

37.2 Contrast with Loop Quantum Gravity

Loop quantum gravity focuses on quantizing spacetime geometry itself, leading to discrete spectra for geometric operators. However, matter couplings and Standard Model parameters are largely external inputs.

Here, discreteness arises not from quantization of geometry alone, but from informational minimality applied globally. Matter and gravity emerge from the same selection principle.

37.3 Relation to Modified Gravity and MOND[37, 41–43] [44]

Unlike phenomenological modified gravity theories, the present framework:

- Derives modifications from a higher-dimensional influence sector
- Employs a universal kernel with no per-system tuning
- Preserves linear cosmology by construction

38 Open Problems and Future Directions

Several issues remain open and warrant further investigation:

- Explicit computation of the influence-sector kernel from the full 7D geometry
- Extension to non-Markovian quantum dynamics
- Detailed treatment of black hole thermodynamics and information
- Numerical simulations of structure formation under the modified kernel
- Experimental design for controlled visibility-threshold tests

These problems are technically challenging but well-defined.

39 Conclusion

We have presented a comprehensive framework in which quantum mechanics, spacetime structure, and gravitation emerge from a unifying informational principle. The Principle of Least Information constrains physical law selection, leading naturally to a polarization-invariant two-time structure. Quantum mechanics arises as reduced open-system dynamics, general relativity emerges on an observable slice of a seven-dimensional Janus spacetime, and particle properties and gravitational phenomena follow from discrete geometric data.

The framework makes clear, falsifiable predictions across multiple domains and avoids the proliferation of unconstrained degrees of freedom characteristic of many unification attempts. Whether Nature realizes such an informationally minimal construction is ultimately an empirical question.

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A Osterwalder–Schrader Reflection Positivity for the Two-Time Gaussian Sector

This appendix provides a self-contained proof of Osterwalder–Schrader (OS) reflection positivity [11, 12] for the Gaussian Euclidean influence sector used in the main text, formulated on the two-time plane and then specialized to a chosen operational polarization. The proof is standard in spirit but written here explicitly in the notation of the present framework.

A.1 Setup: Euclidean field, reflection, and half-space algebra

Let $x = (t, \tau, \boldsymbol{x}) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^3$ denote Euclidean coordinates, where (t, τ) are the two Euclidean-time coordinates and $\boldsymbol{x}$ are the three spatial coordinates. (For the OS argument, the number of spatial dimensions is irrelevant.)

Fix a polarization angle θ and define operational Euclidean time

$$t_\theta = t\cos\theta + \tau\sin\theta, \qquad \tau_\theta = -t\sin\theta + \tau\cos\theta. \tag{54}$$

We define the reflection Θ about the hyperplane $\{t_\theta = 0\}$ by

$$(\Theta\phi)(t_\theta, \tau_\theta, \boldsymbol{x}) := \phi(-t_\theta, \tau_\theta, \boldsymbol{x}), \tag{55}$$

and extend Θ anti-linearly to functionals (complex conjugation on coefficients).

Let $\mathcal{A}_+$ denote the algebra generated by smeared fields $\phi(f)$ with test functions f supported in the open half-space

$$\mathbb{H}_+ := \{(t_\theta, \tau_\theta, \boldsymbol{x}) : t_\theta > 0\}, \tag{56}$$

and similarly $\mathcal{A}_-$ for $t_\theta < 0$.

OS reflection positivity is the condition that for all $F \in \mathcal{A}_+$,

$$\langle \Theta F \cdot F \rangle \geq 0, \tag{57}$$

where $\langle \cdot \rangle$ denotes expectation with respect to the Euclidean measure.

A.2 Gaussian measure and covariance

Assume ϕ is a real, centered Gaussian generalized random field with covariance (two-point function)

$$\langle \phi(f)\phi(g) \rangle = (f, Cg) := \int d^5x\, d^5y\, f(x)\, C(x-y)\, g(y), \tag{58}$$

with C a translation-invariant distribution (kernel) which is:

- even: $C(-z) = C(z)$,
- (Schwartz) positive definite: $(f, Cf) \geq 0$ for all real f,
- and *reflection positive* with respect to $t_\theta \mapsto -t_\theta$ (to be shown below for the kernel class used).

Equivalently, in momentum space,

$$C(z) = \int \frac{d^5p}{(2\pi)^5} \, e^{ip\cdot z} \, \widetilde{C}(p), \tag{59}$$

where $\widetilde{C}(p) \geq 0$ (as a measure/density) ensures positive definiteness.

A.3 OS positivity for Gaussian measures reduces to a kernel inequality

For Gaussian measures, it is enough to check OS positivity on the dense set of *Gaussian exponentials*

$$F_f(\phi) := \exp\big(i\phi(f)\big), \qquad f \in \mathcal{S}(\mathbb{R}^5), \ \mathrm{supp}(f) \subset \mathbb{H}_+, \tag{60}$$

because finite linear combinations of such exponentials are dense in L^2 of the Gaussian measure, and the OS form extends by continuity.

For centered Gaussian ϕ we have the identity

$$\left\langle e^{i\phi(f)} e^{i\phi(g)} \right\rangle = \exp\left(-\tfrac{1}{2}(f+g, C(f+g))\right). \tag{61}$$

Taking $g = \Theta f$ and using Θ anti-linearity on coefficients, we obtain

$$\langle \Theta F_f \cdot F_f \rangle = \left\langle e^{-i\phi(\Theta f)} e^{i\phi(f)} \right\rangle = \exp\left(-\tfrac{1}{2}(f - \Theta f, C(f - \Theta f))\right). \tag{62}$$

Since $(\cdot, C\cdot) \geq 0$ for real arguments, the RHS is manifestly ≥ 0. This already proves (57) for single exponentials. To extend to finite linear combinations, let $F = \sum_{j=1}^n c_j e^{i\phi(f_j)}$ with $\mathrm{supp}(f_j) \subset \mathbb{H}_+$. Then

$$\langle \Theta F \cdot F \rangle = \sum_{i,j} c_i^* c_j \left\langle e^{-i\phi(\Theta f_i)} e^{i\phi(f_j)} \right\rangle = \sum_{i,j} c_i^* c_j \exp\!\big(-\tfrac{1}{2}(f_j - \Theta f_i, C(f_j - \Theta f_i))\big). \tag{63}$$

Thus OS positivity follows if the matrix

$$M_{ij} := \left\langle e^{-i\phi(\Theta f_i)} e^{i\phi(f_j)} \right\rangle \tag{64}$$

is positive semidefinite for all finite choices $\{f_j\} \subset \mathbb{H}_+$. For Gaussian measures this is guaranteed provided the *reflected kernel*

$$C_\Theta(x, y) := C(x - \Theta y) \tag{65}$$

defines a positive semidefinite bilinear form on test functions supported in $\mathbb{H}_+$:

$$\int_{\mathbb{H}_+} dx \int_{\mathbb{H}_+} dy \, \overline{f(x)} \, C(x - \Theta y) \, f(y) \ \geq 0 \qquad \forall f \in \mathcal{S}(\mathbb{H}_+). \tag{66}$$

We now prove (66) for the kernel class relevant to the two-time Gaussian influence sector.

A.4 Reflection positivity for kernels with a Laplace–Stieltjes representation

A convenient sufficient condition for OS positivity is that, as a function of t_θ, the kernel admits a representation as a superposition of Euclidean propagators with nonnegative spectral weight.

Assumption A.1 (spectral/Laplace representation). Assume the translation-invariant covariance can be written (distributionally) as

$$C(t_\theta, \tau_\theta, \boldsymbol{x}) = \int_0^\infty d\mu(m^2) \int \frac{d^4 q}{(2\pi)^4} \frac{e^{i(q_\perp \cdot (\tau_\theta, \boldsymbol{x}))}\, e^{-|t_\theta|\sqrt{q_\perp^2 + m^2}}}{2\sqrt{q_\perp^2 + m^2}}, \tag{67}$$

where $q_\perp = (q_\tau, \boldsymbol{q}) \in \mathbb{R}^4$, $q_\perp^2 = q_\tau^2 + \boldsymbol{q}^2$ and $d\mu(m^2) \geq 0$ is a nonnegative measure. This includes the usual free-field covariance ($d\mu = \delta(m^2 - m_0^2) dm^2$) and more general completely monotone radial kernels.

Theorem A.1 (OS positivity). If C admits the representation (67) with $d\mu(m^2) \geq 0$, then C is reflection positive with respect to $t_\theta \mapsto -t_\theta$, i.e. (66) holds. Consequently the Gaussian measure with covariance C satisfies OS reflection positivity (57).

Proof. Let $f \in \mathcal{S}(\mathbb{H}_+)$, so $\operatorname{supp}(f) \subset \{t_\theta > 0\}$. Consider

$$I[f] := \int_{\mathbb{H}_+} dx \int_{\mathbb{H}_+} dy\, \overline{f(x)}\, C(x - \Theta y)\, f(y). \tag{68}$$

Write $x = (t, \xi)$ and $y = (s, \eta)$ where $t = t_\theta > 0$, $s = t_\theta > 0$ and $\xi = (\tau_\theta, \boldsymbol{x}) \in \mathbb{R}^4$, $\eta = (\tau_\theta, \boldsymbol{x}) \in \mathbb{R}^4$. Then $x - \Theta y = (t + s, \xi - \eta)$ (because reflection flips the sign of the t_θ coordinate only). Using (67),

$$I[f] = \int_0^\infty d\mu(m^2) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2\omega} \int_{t>0} dt \int_{s>0} ds \int d^4\xi \int d^4\eta\, \overline{f(t,\xi)}\, f(s,\eta)\, e^{iq\cdot(\xi-\eta)}\, e^{-(t+s)\omega}, \tag{69}$$

where $\omega = \sqrt{q^2 + m^2}$ and $q \cdot \xi := q_\tau \tau_\theta + \boldsymbol{q} \cdot \boldsymbol{x}$.

Rearrange:

$$I[f] = \int_0^\infty d\mu(m^2) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2\omega} \left(\int_{t>0} dt \int d^4\xi\, \overline{f(t,\xi)}\, e^{iq\cdot\xi}\, e^{-t\omega} \right) \left(\int_{s>0} ds \int d^4\eta\, f(s,\eta)\, e^{-iq\cdot\eta}\, e^{-s\omega} \right). \tag{70}$$

Define the (half-space) Laplace–Fourier transform

$$\widehat{f}(\omega, q) := \int_{t>0} dt \int d^4\xi\, f(t,\xi)\, e^{-t\omega}\, e^{-iq\cdot\xi}. \tag{71}$$

Then

$$I[f] = \int_0^\infty d\mu(m^2) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2\omega} \overline{\widehat{f}(\omega, q)}\, \widehat{f}(\omega, q) = \int_0^\infty d\mu(m^2) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{2\omega} \left| \widehat{f}(\omega, q) \right|^2. \tag{72}$$

Since $d\mu(m^2) \geq 0$ and $\omega > 0$, the integrand is nonnegative, hence $I[f] \geq 0$ for all f supported in $\mathbb{H}_+$. This proves (66). By the Gaussian reduction in the previous subsection, OS reflection positivity (57) holds for the Gaussian measure. □

A.5 Remarks and connections to the main construction

Remark A.1 (polarization invariance). The proof above uses only that reflection is taken about the chosen operational hyperplane $\{t_\theta = 0\}$ and that the covariance depends on $(t_\theta, \tau_\theta, \boldsymbol{x})$ through a representation of the form (67). If the underlying kernel is rotation-invariant in the (t, τ) plane (and translation invariant overall), then for each polarization θ the reflected kernel takes the same functional form with t_θ replacing t. Thus OS positivity holds for *all* polarizations in that class, and the choice of operational time can be delegated to PLI without jeopardizing the OS reconstruction.

Remark A.2 (Gaussian influence and CP). The Hubbard–Stratonovich dilation used in the main text corresponds precisely to representing the Gaussian measure by an auxiliary linear coupling to a quasi-free field. OS positivity ensures that the reconstructed theory along t_θ is a bona fide Hilbert-space quantum theory with a positive Hamiltonian, and the dilation guarantees complete positivity of reduced dynamics when auxiliary degrees are traced out.

Remark A.3 (kernel classes). A broad sufficient condition for (67) is that, as a function of t_θ^2 (and the radial variable in the two-time plane), C is completely monotone after suitable restriction; equivalently its Fourier transform is a nonnegative measure and its t_θ-dependence admits a Laplace transform with nonnegative spectral weight. This is the natural compatibility class for "Gaussian, reflection-symmetric" influence sectors emphasized in the main text.

Remark A.4 (from OS to Hamiltonian positivity). Given OS positivity, the standard reconstruction defines a pre-Hilbert space $\mathcal{H}_0$ as $\mathcal{A}_+/\mathcal{N}$ where $\mathcal{N} := \{F \in \mathcal{A}_+ : \langle \Theta F \cdot F \rangle = 0\}$, with inner product $\langle [F], [G] \rangle := \langle \Theta F \cdot G \rangle$. The time translation semigroup for $t_\theta \geq 0$ induces a contraction semigroup on $\mathcal{H}_0$ whose self-adjoint generator is the positive Hamiltonian $H \geq 0$.

A.6 Alternative proof via covariance block matrices and Schur complements

In this subsection we provide a second, equivalent proof of OS reflection positivity for the Gaussian measure, written in a form that some readers may find more concrete. The proof reduces reflection positivity to positivity of a certain block covariance matrix and then uses the Schur complement and elementary Gaussian integration.

A.6.1 Cylinder functionals and finite-dimensional reduction

Let ϕ be a centered real Gaussian field with covariance $C(x - y)$. Consider a finite family of test functions

$$f_1, \ldots, f_n \in \mathcal{S}(\mathbb{H}_+), \tag{73}$$

supported in the positive half-space $\mathbb{H}_+ = \{t_\theta > 0\}$. Define the Gaussian random vector

$$X = (X_1, \ldots, X_n), \qquad X_i := \phi(f_i). \tag{74}$$

Similarly define the "reflected" variables

$$Y = (Y_1, \dots, Y_n), \qquad Y_i := \phi(\Theta f_i), \tag{75}$$

where $(\Theta f)(t_\theta, \tau_\theta, \boldsymbol{x}) := f(-t_\theta, \tau_\theta, \boldsymbol{x})$.

Because ϕ is Gaussian, the joint vector $(X, Y) \in \mathbb{R}^{2n}$ is Gaussian with covariance matrix

$$\Sigma := \begin{pmatrix} A & B \\ B^\top & A \end{pmatrix}, \tag{76}$$

where by translation invariance and reflection symmetry one has:

$$A_{ij} := \mathbb{E}[X_i X_j] = (f_i, C f_j), \tag{77}$$

$$B_{ij} := \mathbb{E}[X_i Y_j] = (f_i, C\Theta f_j). \tag{78}$$

Note that A is symmetric and positive semidefinite because C is a covariance, and similarly $\Sigma \succeq 0$.

A general cylinder functional $F \in \mathcal{A}_+$ depending only on $(X_1, \dots, X_n)$ can be written as

$$F(\phi) = F_n(X) \tag{79}$$

for some (say) bounded measurable $F_n : \mathbb{R}^n \to \mathbb{C}$. Then OS reflection positivity requires:

$$\mathbb{E}\left[\overline{F_n(Y)}\, F_n(X)\right] \geq 0 \qquad \forall F_n,\ \forall (f_i) \subset \mathbb{H}_+. \tag{80}$$

Since cylinder functionals are dense, it suffices to prove (80).

A.6.2 Key lemma: reflection positivity is equivalent to positivity of the cross-covariance kernel

Define the bilinear form on $\mathbb{H}_+$:

$$\mathcal{B}(f, g) := (f, C\Theta g) = \int_{\mathbb{H}_+} dx \int_{\mathbb{H}_+} dy\; f(x)\, C(x - \Theta y)\, g(y). \tag{81}$$

For the finite set $\{f_i\}$, the matrix B is precisely $B_{ij} = \mathcal{B}(f_i, f_j)$.

Lemma A.2. If the form $\mathcal{B}$ is positive semidefinite on $\mathbb{H}_+$ (equivalently, $B \succeq 0$ for all finite sets), then OS reflection positivity (80) holds for all cylinder functionals.

Proof. Assume $B \succeq 0$. Then there exists an $n \times n$ matrix R such that

$$B = RR^\top. \tag{82}$$

Let $Z \sim \mathcal{N}(0, I_n)$ be a standard n-dimensional real Gaussian vector. Then RZ is Gaussian with covariance B.

Now condition on X. Since (X, Y) is jointly Gaussian, the conditional law of Y given X is Gaussian with mean and covariance:

$$\mathbb{E}[Y|X] = B^\top A^+ X, \tag{83}$$

$$\mathrm{Cov}(Y|X) = A - B^\top A^+ B, \tag{84}$$

where A^+ denotes the Moore–Penrose pseudoinverse [45, 46] (needed if A is singular). This is standard for Gaussian vectors and remains valid under semidefinite covariances.

Then

$$\mathbb{E}\big[\overline{F_n(Y)}\,F_n(X)\big] = \mathbb{E}\left[F_n(X)\,\mathbb{E}\big[\overline{F_n(Y)}\,\big|\,X\big]\right]. \tag{85}$$

If $\mathrm{Cov}(Y|X)$ is positive semidefinite (it is, as a Schur complement of $\Sigma \succeq 0$), then $\mathbb{E}[\overline{F_n(Y)}|X]$ is a positive-definite kernel acting on F_n.

A more direct route is to restrict to the dense set of exponentials $F_n(x) = \sum_j c_j e^{ik_j\cdot x}$ and check positivity explicitly; the general case follows by density. For exponentials one finds

$$\mathbb{E}\big[e^{-ik\cdot Y}e^{i\ell\cdot X}\big] = \exp\big(-\tfrac{1}{2}(k^\top Ak + \ell^\top A\ell - 2k^\top B\ell)\big). \tag{86}$$

Define the $n \times n$ matrix

$$Q(k,\ell) := k^\top Ak + \ell^\top A\ell - 2k^\top B\ell = \begin{pmatrix} k \\ \ell \end{pmatrix}^\top \begin{pmatrix} A & -B \\ -B^\top & A \end{pmatrix} \begin{pmatrix} k \\ \ell \end{pmatrix}. \tag{87}$$

Since $A \succeq 0$ and $B \succeq 0$, one checks that the kernel

$$K_{ij} := \exp\big(-\tfrac{1}{2}Q(k_i,k_j)\big) \tag{88}$$

is positive semidefinite (it is the characteristic function of a Gaussian with covariance $\begin{pmatrix} A & -B \\ -B & A \end{pmatrix} \succeq 0$). Hence for $F_n = \sum_i c_i e^{ik_i\cdot x}$,

$$\mathbb{E}\big[\overline{F_n(Y)}\,F_n(X)\big] = \sum_{i,j} c_i^* c_j K_{ij} \geq 0. \tag{89}$$

By approximation, (80) holds for all cylinder F_n. □

A.6.3 Verifying positivity of the reflected form via the Schur complement

We now show that the positivity of $\mathcal{B}$ (or of all matrices B) follows from the spectral/Laplace representation of the covariance used in the main proof (Theorem A.1). This closes the loop and ties the matrix method directly to the kernel conditions.

Let $f = \sum_{i=1}^n a_i f_i$ with real coefficients a_i. Then

$$a^\top Ba = \sum_{i,j} a_i a_j (f_i, C\Theta f_j) = (f, C\Theta f) =: \mathcal{B}(f,f). \tag{90}$$

Thus $B \succeq 0$ for all finite sets is equivalent to $\mathcal{B}(f,f) \geq 0$ for all f supported in $\mathbb{H}_+$.

But by Theorem A.1 we already established exactly this inequality:

$$\mathcal{B}(f,f) = \int_0^\infty d\mu(m^2) \int \frac{d^4q}{(2\pi)^4}\frac{1}{2\omega}\,|\widehat{f}(\omega,q)|^2 \geq 0, \tag{91}$$

with $\widehat{f}$ the half-space Laplace–Fourier transform (71). Therefore $B \succeq 0$, and by Lemma A.2 the OS reflection positivity condition (80) holds.

A.6.4 Schur complement viewpoint (intuition)

The covariance matrix Σ in (76) is positive semidefinite because it is a covariance. OS reflection positivity isolates the *cross-covariance* block B as the object that must be positive semidefinite when both arguments lie in $\mathbb{H}_+$.

In this sense, reflection positivity is equivalent to the statement that the reflected kernel $C(x - \Theta y)$ defines a positive kernel on $\mathbb{H}_+$. The Laplace representation makes this manifest, while the matrix method makes explicit how OS positivity becomes positivity of a finite Gram matrix for all cylinder functionals.

B Derivation of Lindblad Dynamics from Hubbard–Stratonovich Dilation

This appendix makes explicit the emergence of a Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) generator from a Gaussian influence sector represented via a Hubbard–Stratonovich (HS) field[15, 47]. The derivation follows standard open-systems methods (Born–Markov and secular approximations) but is included here for completeness and to make clear which structural assumptions are required. [17, 18, 25, 48]

B.1 System–influence coupling and stochastic dilation

Let $\mathcal{H}_S$ be the system Hilbert space with Hamiltonian H_S. Introduce Hermitian coupling operators $\{A_\alpha\}$ and HS influence fields $\{\xi_\alpha(t)\}$ with

$$\langle \xi_\alpha(t)\rangle = 0, \qquad \langle \xi_\alpha(t)\xi_\beta(t')\rangle = C_{\alpha\beta}(t-t'), \tag{92}$$

where $C_{\alpha\beta}$ is positive definite. The (stochastic) total Hamiltonian is

$$H_{\text{tot}}(t) = H_S + \sum_\alpha A_\alpha\, \xi_\alpha(t). \tag{93}$$

For each realization ξ, the evolution is unitary:

$$\dot{U}_\xi(t) = -iH_{\text{tot}}(t)\, U_\xi(t), \qquad U_\xi(0) = \mathbb{I}. \tag{94}$$

The reduced state is the ensemble average

$$\rho(t) = \mathbb{E}_\xi\Big[U_\xi(t)\rho(0)U_\xi^\dagger(t)\Big]. \tag{95}$$

This is a Stinespring-type dilation; consequently, $\rho(t)$ is generated by a completely positive trace preserving (CPTP) map for each $t \geq 0$. [16]

B.2 Born–Markov expansion

Move to the interaction picture with respect to H_S:

$$U_\xi(t) = e^{-iH_S t}\, U_\xi^{(I)}(t), \qquad \dot{U}_\xi^{(I)}(t) = -i\sum_\alpha A_\alpha^{(I)}(t)\, \xi_\alpha(t)\, U_\xi^{(I)}(t), \tag{96}$$

where $A_\alpha^{(I)}(t) = e^{iH_S t}A_\alpha e^{-iH_S t}$. Expanding to second order in the coupling and averaging over ξ yields the standard cumulant form (weak coupling, stationary correlations):

$$\dot{\rho}^{(I)}(t) = -\sum_{\alpha\beta}\int_0^\infty ds\, C_{\alpha\beta}(s)\left[A_\alpha^{(I)}(t), \big[A_\beta^{(I)}(t-s), \rho^{(I)}(t)\big]\right]. \tag{97}$$

Returning to the Schrödinger picture gives a time-local master equation. [25]

B.3 Spectral decomposition and secular approximation

Decompose A_α into eigenoperators of ad_{H_S}:

$$A_\alpha = \sum_\omega A_\alpha(\omega), \qquad [H_S, A_\alpha(\omega)] = -\omega A_\alpha(\omega), \tag{98}$$

so that $A_\alpha^{(I)}(t) = \sum_\omega e^{-i\omega t} A_\alpha(\omega)$. Define the Fourier transforms

$$\Gamma_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} ds\, e^{i\omega s} C_{\alpha\beta}(s). \tag{99}$$

Positivity of $C_{\alpha\beta}$ implies $\Gamma(\omega)$ is positive semidefinite for each ω (Bochner).[49] Performing the secular approximation yields

$$\begin{aligned}\dot\rho &= -i[H_S + H_{\mathrm{LS}}, \rho] \\ &\quad + \sum_\omega \sum_{\alpha\beta} \Gamma_{\alpha\beta}(\omega) \left(A_\beta(\omega)\rho A_\alpha^\dagger(\omega) - \frac{1}{2}\{A_\alpha^\dagger(\omega) A_\beta(\omega), \rho\} \right),\end{aligned} \tag{100}$$

where H_{LS} is the Lamb shift Hamiltonian. Equation (100) is the GKSL form. [17, 18]

B.4 Beyond the Markovian regime

If $C_{\alpha\beta}(s)$ has long memory, the reduced dynamics remains CPTP but becomes non-Markovian and may be written with a memory kernel

$$\dot\rho(t) = \int_0^t ds\, \mathcal{K}(t-s)\rho(s), \tag{101}$$

or in time-local form with time-dependent rates. [25, 48]

C Linear Cosmological Perturbations with a Scale-Dependent Response $\mu(k)$

This appendix summarizes linear scalar perturbation theory in the presence of the scale-dependent gravitational response $\mu(k)$ used in the main text. Conventions follow Refs. [50–52].

C.1 Background and gauge choice

Assume a spatially flat FLRW background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j, \tag{102}$$

with $H = \dot a / a$. Linear scalar perturbations in Newtonian gauge take the form

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij} dx^i dx^j. \tag{103}$$

For negligible anisotropic stress in the dominant components at the epochs of interest, $\Phi \simeq \Psi$.

C.2 Modified Poisson equation

In Fourier space, parameterize the modification by

$$k^2\Phi(k,a) = 4\pi G a^2 \rho(a)\, \mu(k)\, \delta(k,a), \tag{104}$$

where $\delta = \delta\rho/\rho$. The admissible kernel class is constrained so that

$$\mu(k \to 0) = 1, \qquad \mu(k \to \infty) = 1, \tag{105}$$

with deviations confined to an intermediate band around a characteristic wavenumber $k_{\rm gal}$.

C.3 Growth equation

Combining the continuity and Euler equations for pressureless matter gives

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\, \mu(k)\, \delta = 0. \tag{106}$$

For modes relevant to primary CMB anisotropies and linear LSS,

$$k \ll k_{\rm gal} \quad \Rightarrow \quad \mu(k) \simeq 1, \tag{107}$$

and (106) reduces to the standard ΛCDM growth equation.

C.4 CMB acoustic peaks and lensing

Provided $\mu(k)$ departs from unity only for k well above those contributing substantially to the acoustic peaks ($\ell \lesssim 2000$), the phase and amplitude of acoustic oscillations remain unchanged at the sub-percent level. Similarly, the CMB lensing potential depends on the line-of-sight integral of $\Phi + \Psi$; with $\Phi \simeq \Psi$ and $\mu(k) \simeq 1$ on the relevant linear modes, the lensing spectrum is preserved within current uncertainties. [40]

C.5 Consistency conditions

Observable distortions of the CMB or linear matter power spectrum arise if:

1. $\mu(k \to 0) \neq 1$ at the 10^{-3}–10^{-2} level,
2. the transition band is broad enough to affect k contributing to acoustic peaks,
3. $\mu(k)$ violates positivity/analyticity leading to instabilities.

Hence the separation-of-scales requirement on $\mu(k)$ is a falsifiable constraint.

C.6 Summary

At linear order, a mid-band modification $\mu(k)$ can produce significant effects on galactic scales while leaving CMB/LSS observables essentially unchanged, provided $\mu(k)$ approaches unity rapidly at both small and large k.

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