



An agent-based model of trade

Market distortions and output

February 2019

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Executive Summary

Report purpose

- This report provides an overview of an agent-based model (ABM) of domestic and international trade developed by the Centre for Economics and Business Research (Cebr), for the Institute of Economic Affairs (IEA).
- The model aims to illustrate how the domestic output of countries, and in turn global output, is affected by the presence of trade-distortions. The models capture three general distortion types: distortions limiting (i) the extent of specific cross-country trade; (ii) trade between consumers and firms (independent of their country of origin); and (iii) the information available to agents.
- The model is very much a demonstrative first-look at the issues; as such the report conceptually details how the framework can be developed and augmented going forwards.

Model overview

- The ABM has three key ingredients: the environment; agents; and behavioural rules. The structure of these ingredients is as follows:

1) Environment (countries).

The environment in the model is divided into two countries, A and B, of equal geographical size. All coordinates to the left of the origin correspond to country A, whilst all those to the right of the origin correspond to country B. This structure is then used to allocate agents to either country.

2) Agents (consumers and firms).

The model is composed of two types of agents: consumer-workers and firms.

Consumers have a reservation price that determines their maximum willingness to pay for one unit of a consumption good. The reservation price is exogenously given and normally distributed across consumers; this can be viewed as heterogeneity in consumer preferences.

Firms produce the homogenous consumption good using a production technology that exhibits economies of scale, where average cost falls with output. The production input (cost) is labour and the wages paid by firms determine consumer budgets.

The model is sufficiently general to allow for countries that differ in the number of consumers and/or firms (but in the baseline case countries are assumed identical).

3) Behavioural rules and interactions between agents.

Both consumers and firms can view a portion of their environment corresponding to a circular area around themselves, the size of which is determined by a variable radius parameter.

Consumer rule. Consumers can observe the unit-price charged by firms within their informational space: they then choose to purchase from the firm with the lowest price.

Firm rule. Firms are assumed to know the following “summary information” about consumers within their information space: they know the minimum, median and maximum reservation

prices, and the average budget. They use this information to set their price, given an *expectation* of demand at each price (minimum, median and maximum).

Trade and informational distortions within the model

- The model features three types of distortions that may inhibit the extent of trade between consumers and firms, both within a country and between countries:
 - I. **Between-country trade distortions.**
These distortions limit the extent of cross-country trade between consumers and firms; they may arise for example due to differences in domestic regulations or product standards or more direct barriers such as tariffs. When set at 100% no international trade can occur, and the model is so composed of two countries operating in autarky.
 - II. **Within-country trade distortions.**
Even if a trade between consumers and firms is mutually advantageous, the trade may not take place due to frictions/distortions that introduce uncertainty. These distortions apply both to consumers and firms from the same country, and to those from different countries.
 - III. **Technological/information distortions.**
At any given point in time, consumers are only able to observe a portion of the environment around themselves. While to some extent this is inevitable, part of the information restriction may be due to distortions that affect the amount of information that can be observed. These distortions can apply both within a country and between countries.
- The significance of I. and II above, and drives to reduce them, is well documented in the literature: see for example, OECD (2018) “Trade Facilitation and the Global Economy”.
- The within-country distortion therefore prevents transactions that would otherwise certainly take place from occurring, while the between-country distortion prevents consumers and firms accessing foreign markets, in which they may subsequently choose to trade. The technological/information distortions limit consumer visibility, meaning that potentially more optimal transactions are foregone.

Key results

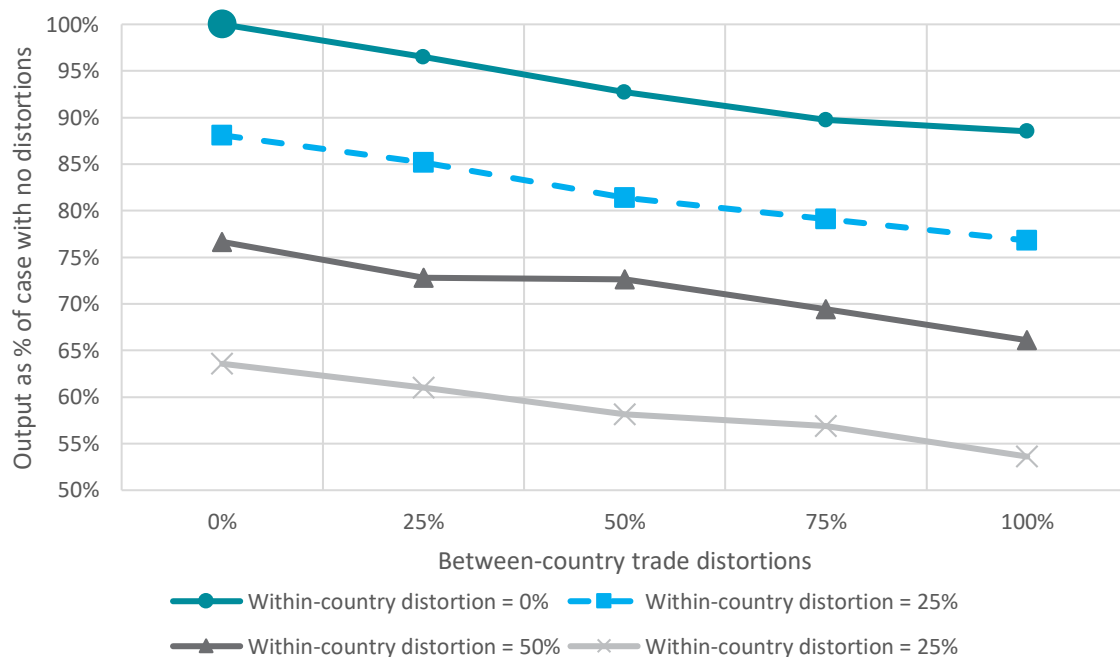
- The results in this report are expressed relative to the hypothetical **benchmark case** characterised by **zero between- or within-country trade distortions**, and modest technological distortions. The model demonstrates that between-country and within-country distortions markedly reduce output.
- *The effect of between-country trade distortions relative to benchmark*
Country and global output is unambiguously decreasing in the extent of between-country trade distortions. All else held constant, a between-country trade distortion of 25% acts to lower global output by around 4% relative to the benchmark case with no distortions. In the extreme case where no cross-country trade can occur (100% distortion) and countries operate in autarky, global output is around 11% lower than in the benchmark case with no distortions.
- *The effect of within-country trade distortions relative to benchmark*
Country and global output also falls unambiguously with the extent of within-country trade distortions. For example, a within-country trade distortion of 25% implies that one quarter of beneficial transactions between consumers and firms within a country are prohibited through

distortions, and this acts to reduce global output by approximately 14% relative to the benchmark case with no distortions.

- The joint effects of between- and within-country distortions relative to benchmark*

Within the context of the model, the magnitude of output losses in the presence of both within- and between-country distortions is broadly equivalent to the sum of the partial effects (there may be nonlinearities, but they are not marked and require further investigation alongside a less independent treatment of distortions). For example, with between- and within-country trade distortions set at 25% - such that one quarter of consumers and firms within a country cannot trade internationally while one quarter of beneficial transactions between agents within a country do not occur – global output is approximately 17% lower than the benchmark case.

Figure A: Average output over time as a proportion of output under the benchmark case (no trade distortions)



Source: See Figure 10 in main report

- The model shows that **within-country distortions have a greater impact on domestic and global output than do between-country distortions**. This arises because most economic transactions occur within a country, rather than across countries. This is likely to hold for a great many countries. **This may point to the importance of domestic liberalisation policies, as well as international trade liberalisation.**
- While technology in the model does not develop over time, the benefits of trade are realised through firms optimising their use of the current technology. One may expect that the optimisation of current technologies would catalyse the development of more advanced methods, so thus expanding the technological frontier.

Further research going forwards

- These results must be interpreted within the context of the stylistic ABM developed here. While the model captures several realistic facets of trade, it also makes several strict simplifications concerning labour markets; factor mobility; and intertemporal consumption and production decisions.

- Going forwards, a richer – though more complex – framework would relax these assumptions and investigate the extent to which the results developed here continue to hold.

1 Introduction

1.1 Report purpose

This is a report by the Centre for Economics and Business Research (Cebr) for the Institute of Economic Affairs (IEA): this report documents the development and physical output of an agent-based model (ABM) of domestic and international trade between consumers and firms. The purpose of the model is to illustrate how physical output – both at country level and globally – is affected by the presence of trade distortions. This report focuses on the implications of trade distortions for global output.

The model developed here is very much a first-look at the issues, but one that we hope presents solid foundations for a richer and more realistic framework going forwards. The premise of the model is simple: consumers will trade with firms if the price charged by a firm is weakly less than their reservation price, whilst a firm will trade with a consumer if it is profitable to produce the quantity demanded. These transactions may be limited through trade-distortions between countries, trade distortions within a country, or simply by the degree of information available to agents.

While this paper characterises distortions to trade in a very general manner (see Section 1.3), it is set against the backdrop of a growing literature that explores in detail the type of trade distortions that may arise, and in turn the measures that can be put in place to facilitate trade.¹

The literature recognises that trade distortions may arise through legitimate and important public policy objectives, but that there may be more efficient ways to achieve these objectives. For example, the removal of any redundant or duplicated controls and checks may go a long way to facilitate trade, particularly to the extent that they impose a disproportionate administrative burden on smaller firms.²

There is also a body of literature that discusses the extent of economic gains that may result from lowering trade distortions and so facilitating trade: for, example, the OECD estimates that the Trade facilitation agreement could increase world trade by 0.6%.³ Many studies conjecture that the output gains from more general reductions in distortions could be substantial.⁴

As is well documented within this report, we make several simplifying modelling assumptions that should be relaxed going forwards, in particular with regard to the intertemporal decisions of consumers, the scope of trade and the abstraction of a formal model of labour supply.

¹ See, for example, OECD (2018) “Trade Facilitation and the Global Economy”, OECD Publishing, <http://dx.doi.org/10.1787/9789264277571-2-en>

² It is important to stress that the removal of trade distortions does *not* refer to the removal or dilution of any important regulatory standards, such as those around health or safety.

³ OECD (2018) “Trade Facilitation and the Global Economy”, OECD Publishing, <http://dx.doi.org/10.1787/9789264277571-2-en>

⁴ See, for example:

- Singham, S.A. and Abbott, A.F. (2011) “Enhancing Welfare by Attacking Anticompetitive Market Distortions”, *Revue Concurrences*, No. 4
- Singham, S.A., Rangan, S. and Bradley, R. (2014) “The Effect of Anticompetitive Market Distortions (ACMDs) on Global Markets”, *Revue Concurrences*, No.4

1.2 Background on the economics of international trade

Following the seminal contribution of Krugman (1979)⁵, a significant body of economic literature has analysed the implications of economies of scale for international trade. Krugman's key insight was that trade need not be driven differing comparative advantages across countries, but rather by economies of scale. This helps explain why in reality we observe trade between countries that are very similar in terms of production, technology and endowments. These gains from scale are conceptually different to gains from specialisation, while also helping to rebut mercantilist notions of trade that focus on the accumulation of trade surpluses.

Economies of scale arise in industries where there are significant fixed costs and operational synergies associated in production, such that the average cost of production may fall over a range of output levels. The implication here is that opening up markets to international trade can allow firms to increase the scale of production and exploit economies of scale. The key theoretical result is that the volume of trade will increase, with a given good being produced in one country.

The existing economic theory largely focuses on equilibrium outcomes and assumes that both (representative) consumers and firms optimise their consumption and production decisions, respectively. Whilst the validity of these assumptions is not the specific focus of this paper, it is well-recognised that consumers may not act fully rationally and that pathways to equilibrium outcomes require greater attention. An alternative modelling approach that can help provide answers to these questions – with respect to the dynamic outcomes that can emerge – is that of Agent-Based Modelling.

1.3 An Agent-Based Model of trade

Agent-Based Models (ABMs) situate agents – such as consumers and firms – within an environment that may be physical, or network based (or both) and allow these agents to interact with each other and their environment. Such modelling frameworks have a natural application to the economics of trade, and indeed this is the focus of this paper.

We develop a simple model of within- and between-country trade that is composed of

- A physical *environment* that is partitioned into two countries (A and B);
- Two types of *agents* – consumers and firms; and
- The *behavioural rules* of consumers and firms.

The total population of consumers and firms are allocated across countries A and B. Consumers have a reservation price that determines their maximum willingness-to-pay for one unit of a homogenous consumption good that is produced by firms. Firms require labour to produce the good and face a cost function that exhibits economies of scale (as in Krugman, 1979). The costs faced by firms are distributed to the consumers as wages/remuneration for their labour supply⁶.

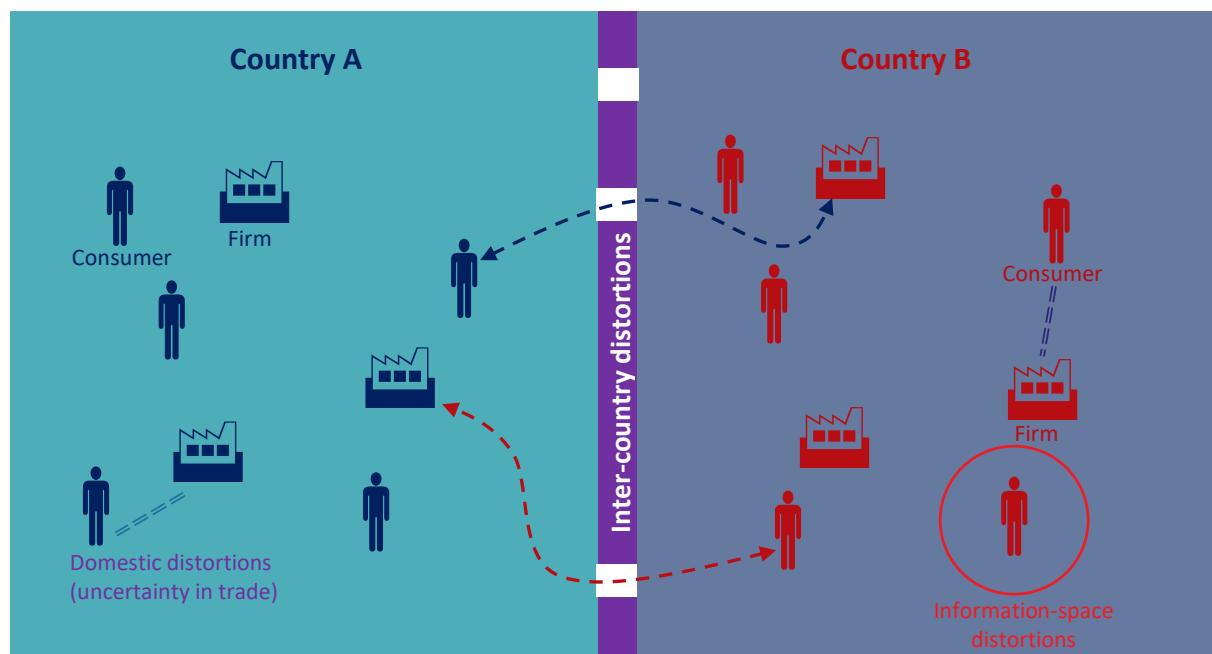
An important facet of ABMs is the information space that is available to agents. In this model both consumers and firms can observe a circular area around themselves; the size of these circular areas is determined by radius parameters. A consumer can observe the unit-price charged by firms within its sampling radius and subsequently chooses to purchase from the firm within this subset of firms that has

⁵ Krugman, P.R. (1979) "Increasing returns, monopolistic competition, and international trade", *Journal of International Economics*, 9, pp. 469-479.

⁶ There is no formal model of labour supply/choice.

the lowest price. Firms cannot observe the specific reservation prices of individual consumers but can observe a set of “summary statistics” about these consumers: they then set their price based on an expectation of the demand that they will face at a range of price levels.

Figure 1: Graphical overview of the ABM of trade



Notes: Both countries (A and B) are composed of consumers and firms. Within the model one can either assume that the number and distribution of parameters across consumers and firms are the same across both countries or can allow for underlying differences. The figure illustrates three types of distortions:

1. *Inter-country distortions*: The porous vertical line represents a barrier to cross-country trade. The greater the number of pores, the greater the number of consumers that can trade with firms from another country (the porous line is illustrative – in the model we capture the distortion through assuming that only a fraction of consumers can move across the border).
2. *Intra-country distortions*: Once consumers and firms have agreed to a trade (because it is mutually beneficial), the trade may only occur with some probability depending negatively on the extent of distortions.
3. *Information-space distortions*: Consumers and firms can only observe a fraction of their environment; this parameter can be varied to capture how information restrictions may affect trade.

The purpose of the model is to establish how total volume⁷ output – both of countries and globally – depends on the magnitude of trade distortions that exist between- and within-countries. The model features three types of trade distortions that may inhibit the extent of trade between consumers and firms, both within a country and between countries:

- *Between-country trade distortions* limit the extent of cross-country trade. These distortions operate through restricting the number of consumers from country A that can trade with firms in country B,

⁷ Throughout this report “output” related to volume output. A richer modelling environment is needed to fully explore the magnitude of the value impacts compared to the volume impacts.

and vice versa. Within the context of the model, this is equivalent to restricting the number of firms in country B that can attract consumers from country A, and vice versa.

- *Within-country trade distortions* generate uncertainty in otherwise beneficial transactions between consumers and firms. These mutually beneficial trades only take place with some probability that is decreasing in the level of the distortion.

NB. The within-country distortion prevents transactions that would otherwise certainly take place from occurring, while the between-country distortion prevents consumers and firms accessing foreign markets, in which they may subsequently choose to trade. For example, the lowering of between-country distortions does not in general guarantee that additional trade will take place, whereas lowering within-country distortions should.

- *Informational distortions/restrictions* limit the information that is available to firms, as captured through the extent to which they can observe their environment. The results generated by the model can therefore be conditioned on the scale of informational distortions.

The model suggests that between- and within-country trade distortions have a substantial effect on country and global levels of output. For example, a between-country trade distortion of 25% acts to lower global output relative to the benchmark case with no distortions by around 4%. A within-country trade distortion of 25% acts to lower global output relative to the benchmark case with no distortions by around 14%. When both types of distortion are present, output is approximately 17% lower than the benchmark case with no distortions. While further investigation is required, the joint effect of the two types of distortions would appear to be “broadly” linear (i.e. the sum of the two partial effects of each distortion).

Within the context of the model, **within-country distortions have a greater impact on domestic and global output than do between-country distortions**. This arises because most economic transactions occur within a country, rather than across countries. This is likely to hold for a great many countries. This may point to the importance of **domestic liberalisation policies**, as well as **international trade liberalisation**.

It is important to note that the ABM developed here is a highly stylised representation of trade. It makes several strict assumptions that should be relaxed going forwards, in order to establish the extent to which the results continue to hold in a richer – but ultimately more complex – model.

- First, the model is silent on the distributional implications of trade within and between countries. As there is no underlying model of labour supply/choice, it assumes that all consumers receive the same income (a fraction of the total labour costs faced by firms). This implies that all consumers are employed and either (i) all command the same wage and work for the same amount of time, or (ii) command different wages but differences in labour supply mean everyone ultimately earns the same.
- Second, consumer behaviour is myopic: consumers will attempt to exhaust their budget constraint on the consumption good in each period. The model should be generalised going forwards to include intertemporal consumption decisions and so saving.
- Thirdly, the current of the model does not explicitly tackle the trade-related issues of labour, capital and other factor mobility. For instance, **the ability to reach economies of scale in cost may be contingent on countries also having access to a common pool of labour and other pooled resources that would require more than the liberalisation of trade in a narrow sense**.

- Lastly, the current model covers the gains from trade associated with scale, as has already been mentioned. **We have not looked at the gains from specialisation that come through comparative advantage.** Indeed, an extension of the framework to cover trade through specialisation would provide an enormously valuable addition to the current work, especially in terms of how scale interacts with scope.

1.4 Report structure

The remainder of this report is structured as follows. Section 2 provides a detailed overview of the ABM developed in this paper: it describes the behaviour of consumers and firms; the rules that govern their interaction; and how barriers to international trade are modelled. Section 3 then proceeds to discuss the key results obtained. Section 4 then presents the concluding remarks. The Appendix discusses some of the key model equations.

2 Methodology: an overview of the ABM

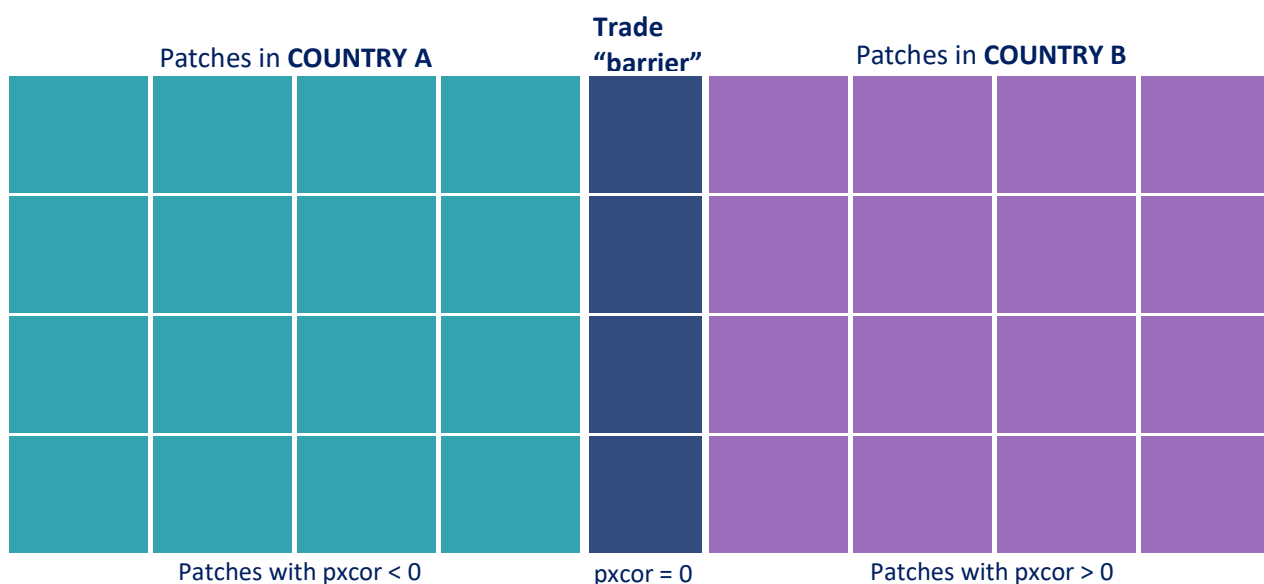
This section provides a detailed overview of the ABM of within- and between-country trade developed in this report, focusing on the heuristics and decision rules of the two types of agent in the model: consumers and firms.

Section 2.1 describes the structure of the model environment; sections 2.2 and 2.3 then proceed to discuss the decision-making process of consumers and firms, and how they interact with each other. Following this, Section 2.4 provides detail on the cost function that firms face, and how this exhibits economies of scale in production (a more technical treatment of costs can be found in the appendix).

2.1 The modelling environment

The physical environment is a square grid of “patches” that are divided into two types: country A and country B. Each patch is labelled as belonging to country A, country B or neither (trade “barrier”). The appeal of this approach is that we can then allocate agents (consumers and firms) to a given country through instructing them to be setup on patches with a given country label.

Figure 2: The modelling environment



Notes. “ $pxcor$ ” denotes position of the patch on horizontal (x) axis. Country A is defined by all patches with negative x -coordinates, while Country B is defined by all patches with positive x -coordinates. The trade barrier is composed of all patches at the origin. The figure is illustrative; the number of patches drawn here are not representative of those in the model.
Source: Cebr analysis.

The environment is constructed such that each country has the same geographical size, though the concentration of consumers and firms within a given country can be varied. The default setting is to have an equal distribution of consumers and firms across each country. Our concern in this framework is largely with the effect of trade distortions on global output, as opposed to any asymmetric/distributional consequences for individual countries.

2.2 Consumers

The model is endowed with a total number of consumers; these consumers are then distributed across country A and country B, respectively. Both the total number of consumers and the country allocations (shares) of these consumers can be varied by the user.

Consumers have the following key attributes:

- “*country*” – this variable denotes whether a given consumer belongs to country A or B.
- “*reservation-price*” – the maximum amount that a consumer would be willing to pay for one unit of the commodity.
- “*intl?*” – a Boolean variable (true/false) that determines whether a consumer can purchase the commodity from firms in a different country. I.e. if a consumer in country A (B) has *intl?*=true they can purchase the commodity from a firm in country B (A).
- “*budget*” – the total resources available to a consumer in a given period. The budget is cumulative, such that any resources not spent in the previous period are carried forward into the current period.

Reservation prices are distributed across consumers, leading to heterogeneity in preferences. We assume a standard normal distribution, as characterised by mean and standard deviation (s.d.) parameters. Formally, if we let R denote the reservation price:

$$R \sim \mathcal{N}(\text{mean}, s. d.)$$

Both the distribution parameters and the type of distribution function itself can be adjusted by the user.

Consumer perspective

Figure 3 provides an overview of the information set available to consumers. Consumers can observe the prices charged by firms that are sufficiently close to themselves: they can observe the prices charged by firms within a circular area around themselves, the size of which is determined by the radius parameter “consumer-sampling”.

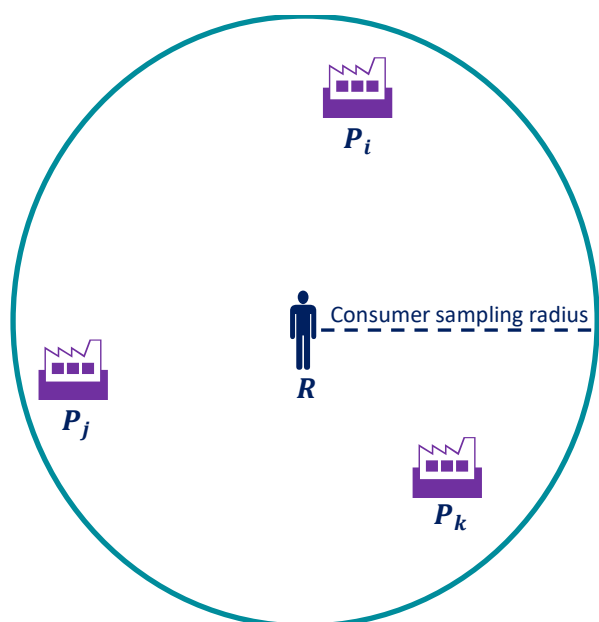
Consumers thus have local information on prices, as opposed to a complete picture of all the possible prices that are available in the economy. Note that this radius parameter is the same for all consumers in the model; as such it is defined as a global variable and not included in the list above. While it may be interesting to examine the implications of differences in information space across consumers, this would really require a framework that examines more closely the distributional implications of trade across consumers within a given country.

Given this local information on prices, a consumer then chooses to purchase the commodity from the firm that offers the cheapest unit price. For simplicity, we assume that a consumer will request to purchase the quantity that exhausts their budget constraint. We therefore abstract from any intertemporal consumption decisions, i.e. decisions to save and borrow. Consumer behaviour can therefore be described as myopic.⁸

⁸ As will be discussed in the concluding remarks, this assumption will need to be relaxed in future developments of the model.

Consumer movement

Figure 3: Consumer perspective



Notes. The consumer has a reservation price of R , which is the maximum amount that they would be willing to pay for a unit of a given product. The consumer can search for the lowest price offered by firms: the search region is a circular space whose area is determined by the radius parameter “firm-sampling”. Within the figure the prices charged by firms i, j and k are denoted by P_i, P_j and P_k , respectively. The consumer will identify the minimum of these prices and only purchase if $R \geq \text{MIN} \{P_i, P_j, P_k\}$.

Source: Cebr analysis

In order to encounter firms with different prices, consumers move around the environment. For simplicity, we impose a “mechanical” consumer movement procedure: each period a consumer will move a given distance forward in a given direction. The movement of consumers (effectively information gathering) is assumed to be costless.

However, a consumer may be restricted in their movement by between-country trade distortions. If a consumer faces these trade distortions, they can only move within their country: whenever they reach the trade-barrier they will reverse their heading. Alternatively, a consumer who is unrestricted because they do not face trade-distortions can move freely between countries.

This prescriptive movement process is a strong simplification; a richer framework going forwards would analyse optimal movement decisions whereby consumers will move in the direction of firms with the optimal price-quantity combination given their budget. Furthermore, there may be periods in which it is suboptimal for consumers to move because the most affordable prices are already available to them.

Whilst we capture between-country trade-distortions through restrictions to consumer movement, this can be readily interpreted as the ability of firms to access consumers in foreign markets. For example, if a consumer in country A can traverse to country B and purchase the commodity off a firm in country B, we can interpret this as that firm in country B not facing any export restrictions.

2.3 Firms

The model is endowed with an initial number of firms; with these firms distributed across country A and country B. Both the initial number of firms and the country allocations can be varied.

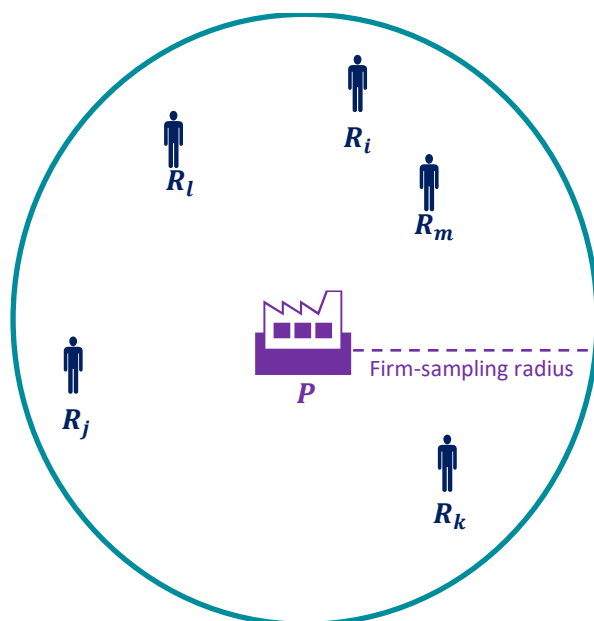
Firms have the following key attributes:

- “country” – as with consumers, this variable denotes whether a firm belongs to country A or B.
- “price” – this is a placeholder variable for the price charged in a given period by a firm. This is a choice variable of the firm.
- “quantity” – the quantity of output produced by a firm in a given period.
- “deal?” – a variable that records whether a firm agrees to a given trade with a consumer.

Firm perspective

A firm can observe the number of consumers within a circular area around itself, with the size of this area determined by the radius parameter “firm-sampling”. In terms of informational asymmetries, a firm cannot observe the specific reservation price of a consumer (for this is private information), but it can observe a set of “summary statistics”. A firm can observe the *minimum, median and max* reservation price of these consumers. It can also observe the average budget across these consumers.

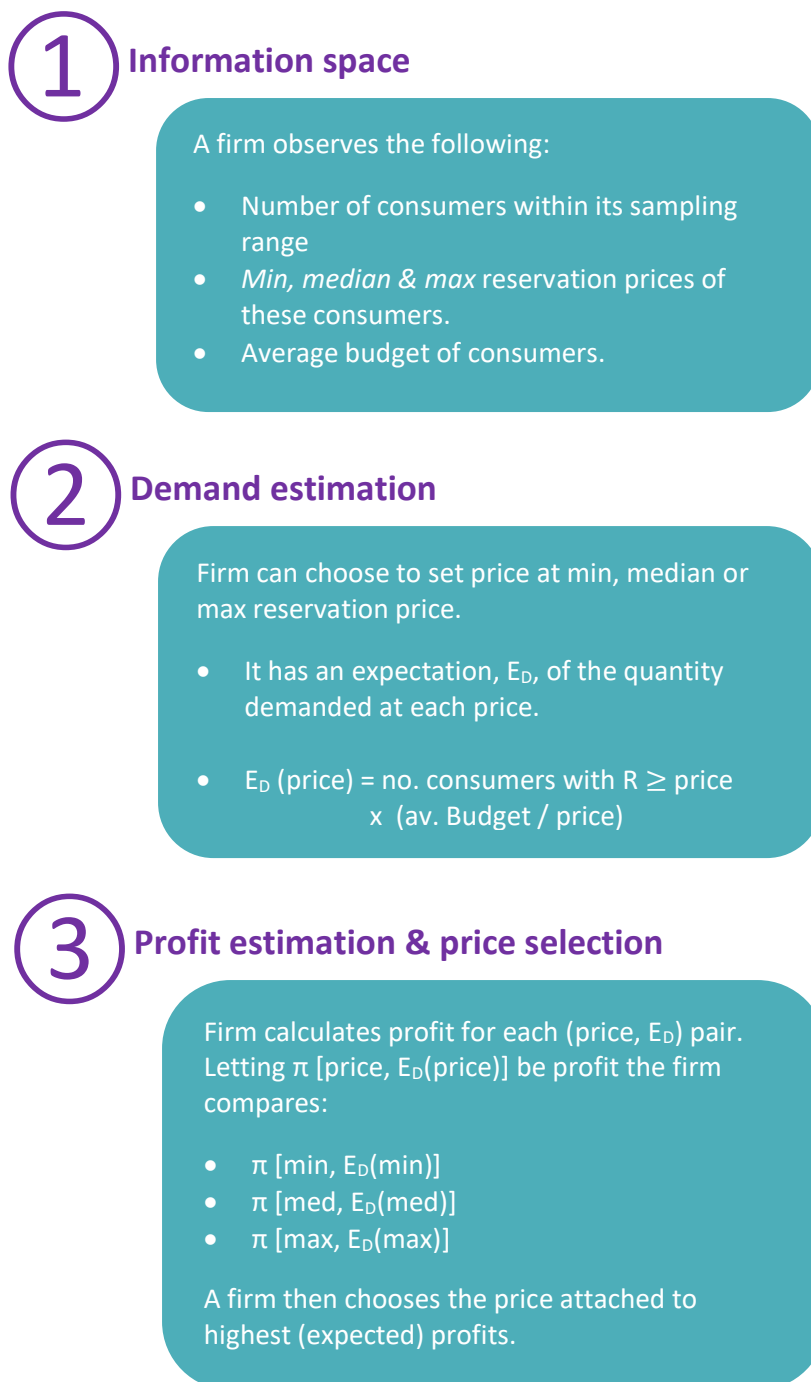
Figure 4: Firm perspective



Notes. The firm must choose what price-per-unit to charge for its product. This will be a function of the demand that it perceives, and so its perception of the reservation prices that consumers have. We assume that any given firm is unable to observe the specific reservation prices of consumers, but only able to observe a set of summary statistics concerning the reservation prices and budget of consumers in a circular area around it. The size of this circular area is determined by the radius parameter “firm-sampling”.

Source: Cebr analysis

Figure 5: Firm price setting



Source: Cebr analysis

Firm price setting rule

Firms aim to maximise their profits, given limited information on the preferences (reservation prices) of individual consumers. The price-setting process is depicted in Figure 5. As stated above, a firm is in general unable to observe the specific reservation prices of consumers but can instead observe a set of summary statistics about these consumers: namely the minimum, median and maximum reservation prices, in addition to the average budget of these consumers. Given this information, a firm can form an expectation of the quantity of their product that would be demanded at a given price. Formally, the demand a firm would expect to face at a given price is

$$E_D(\text{price}) = (\text{no. of consumers with } R \geq \text{price}) \times \left(\frac{\text{av. budget}}{\text{price}} \right)$$

For simplicity, a firm can set its price at one of three levels: the *min*, *median*, or *max* reservation price. If it sets its price at the minimum reservation price then it would expect to have demand from all consumers within its sampling range, with the total quantity demanded being the product of (i) the total number of consumers and (ii) the ratio of average budget to price. Conversely, if a firm sets its price at the maximum reservation price it can only expect to have demand from those consumers with the highest reservation price. Its expected demand is then the product of (i) only those consumers with the highest reservation price and (ii) the ratio of average budget to price.

A firm will compare the expected profits across the three price options (min, median and max) and choose that which yields the highest expected profit.

We are here implicitly assuming that a given firm is unaware of the price charged by other firms in the market: it effectively assumes that all the consumers near it will choose to purchase from it whenever the price that it charges falls below the reservation price of consumers. However, a firm may be inadvertently “undercut” by other firms that charge a lower price and attract away some of these consumers. Going forwards, a richer version of the model would allow for strategic interactions between firms when setting their price.

2.4 The cost function and economies of scale

Analogous to Krugman (1979)⁹, each firm faces a cost function, $c(q)$, of the form

$$c(q) = FC + MVC \times q$$

where q denotes the quantity produced, FC is a fixed cost that is incurred independent of the quantity of output produced; and MVC is a constant marginal variable cost (such that variable costs are linearly proportional to the quantity of output produced.)¹⁰

Economies of scale arise when the average cost of production falls with output; it can be readily demonstrated that this arises with the functional form adopted here. The average cost per unit of production is

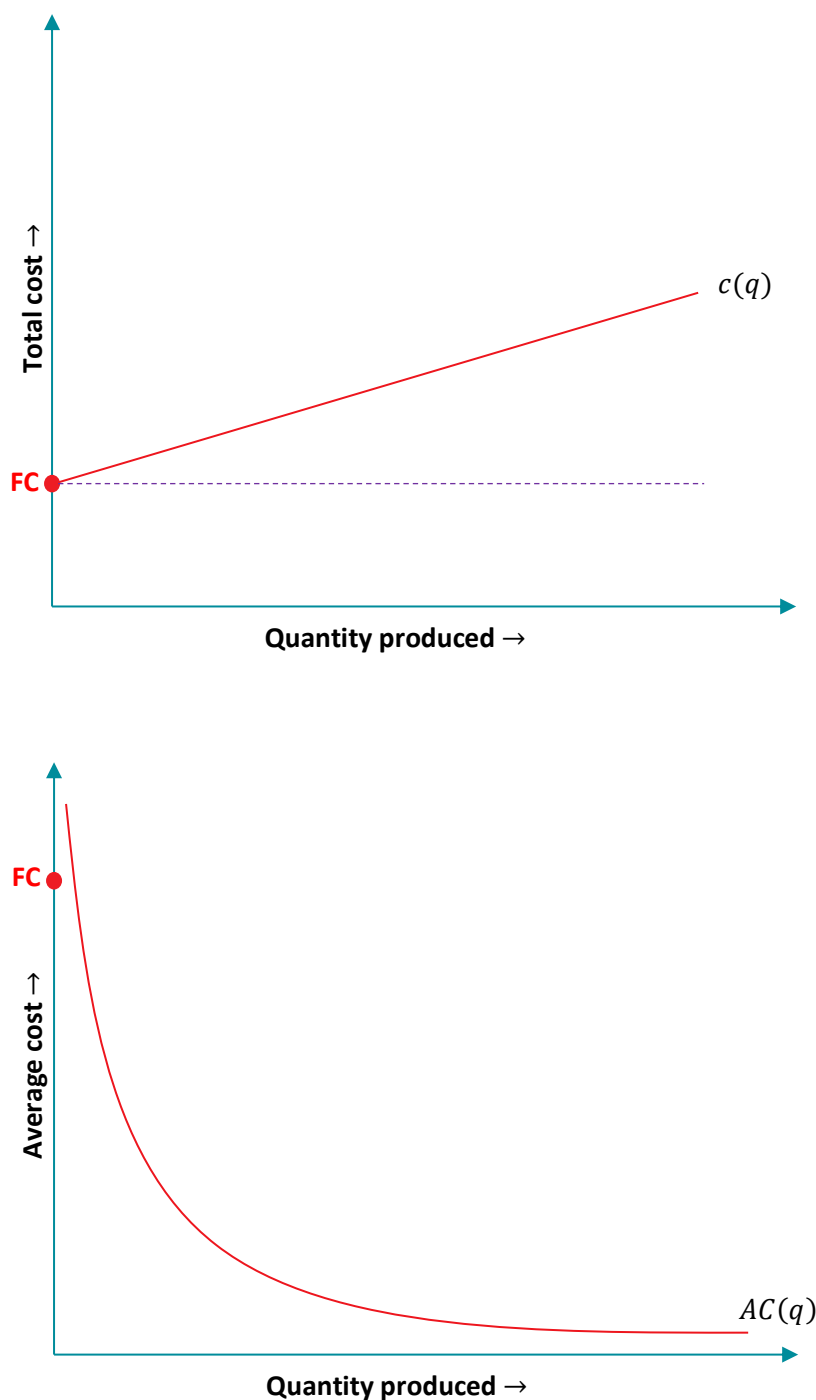
$$AC(q) = \frac{c(q)}{q} = \frac{FC}{q} + MVC$$

Given the constant marginal variable costs, average cost falls unambiguously with the quantity produced.

⁹ Krugman, P.R. (1979) “Increasing returns, monopolistic competition, and international trade”, *Journal of International Economics*, 9, pp. 469-479.

¹⁰ The constant marginal variable cost is a simplification. A more general cost function with fixed and variable costs would take the form $c(q) = k(FC, VC(q))$; where the function k is increasing in both the fixed and variable costs, but where the variable costs need not be linearly increasing in the quantity of output produced.

Figure 6: The cost function



Notes: FC denotes A “fixed cost” of production (a cost that is incurred independent of the quantity produced), whilst VC denotes a variable cost that is proportional (increasing in) the quantity of output produced. As depicted, an implication of the fixed cost and constant marginal variable cost is that average costs fall monotonically with the quantity produced. Note that average cost falls monotonically precisely because the marginal variable cost is constant; were the marginal variable costs instead increasing in the quantity produced then average cost would only fall over a range of quantities.
Source: Cebr analysis

The properties of the cost function are depicted graphically in Figure 6.

Capturing average cost within the ABM

Within a dynamic ABM, one must think carefully about how average cost of production is measured and recorded. As economies of scale will arise when a firm increases its output and the average cost falls, it is important that average costs are recorded at firm level and subsequently averaged across all firms within a country or globally. We therefore consider the following measures of average cost.

- *Average cost conditioned by country and time.*
This measure takes the mean value of the average cost faced by firms in a given country at a given point in time.
- *Average cost conditioned by time.*
This measure takes the average value across countries, of the average cost conditioned by country and time. It is therefore conditional on time.
- *Average cost conditioned by country.*
This measure takes the average value over time, of the average cost conditioned by country and time. It is therefore conditional on time.
- *Unconditional Average cost.*
This is the mean value of the average cost faced by all firms in the model, averaged across time.

3 Discussion of the results

This section presents the key outputs from the ABM. Results are generated through systematically varying the extent of one of the trade distortions, all else held constant.

To facilitate discussion, we throughout compare results with the following benchmark case:

Benchmark case: *The model outcomes that arise when “between-country trade distortions” = 0%; “within-country trade distortions” = 0%; and “consumer-vision” = 2 patches.*

The analysis is structured as follows: sections 3.1 and 3.2 assess the effects of between-country and within-country trade distortions; section 3.3 illustrates the joint effects of these distortions (i.e. when both occur); and finally section 3.4 discusses some of the implications of information distortions.

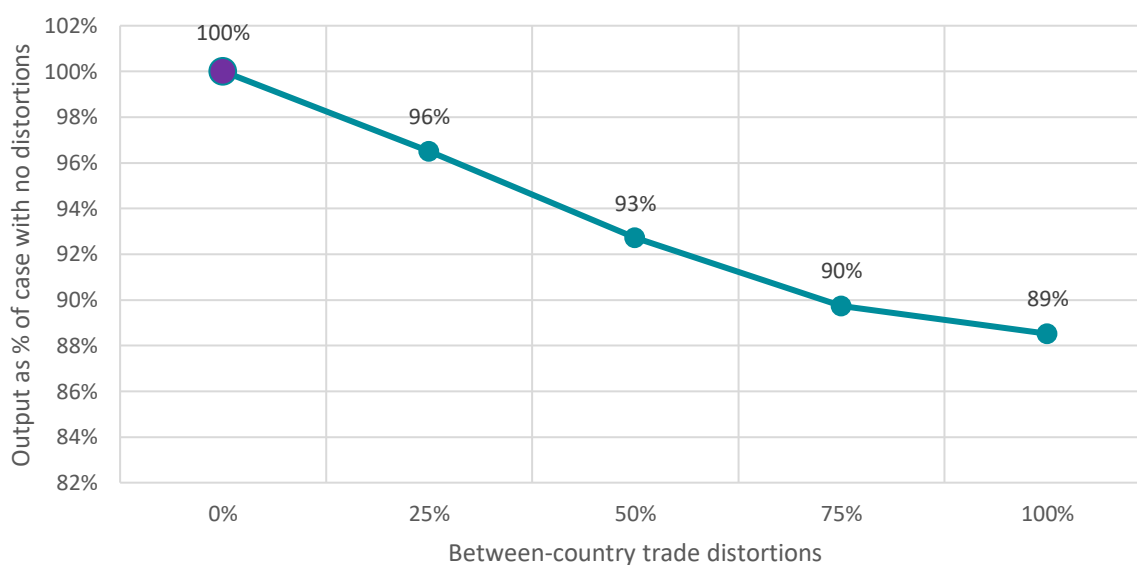
3.1 Between-country trade distortions

In all that follows we establish how output and average cost are affected by the between-country trade distortions.

Output

Figure 7 expresses the global output that arises with between-country trade distortions as a percentage of global output in the benchmark case. The direction of travel is clear: global output (averaged over time) is falling with the degree of between-country trade distortion. Output is 4% lower when 25% of consumers are unable to access foreign markets; 7% lower when half of consumers are unable to access foreign markets; and 10% lower when 75% of consumers are unable to access foreign markets.

Figure 7: The effect of between-country trade distortions on global output



Notes: This figure presents global output as a percentage of the global output achieved when there are no trade distortions (the larger marker).

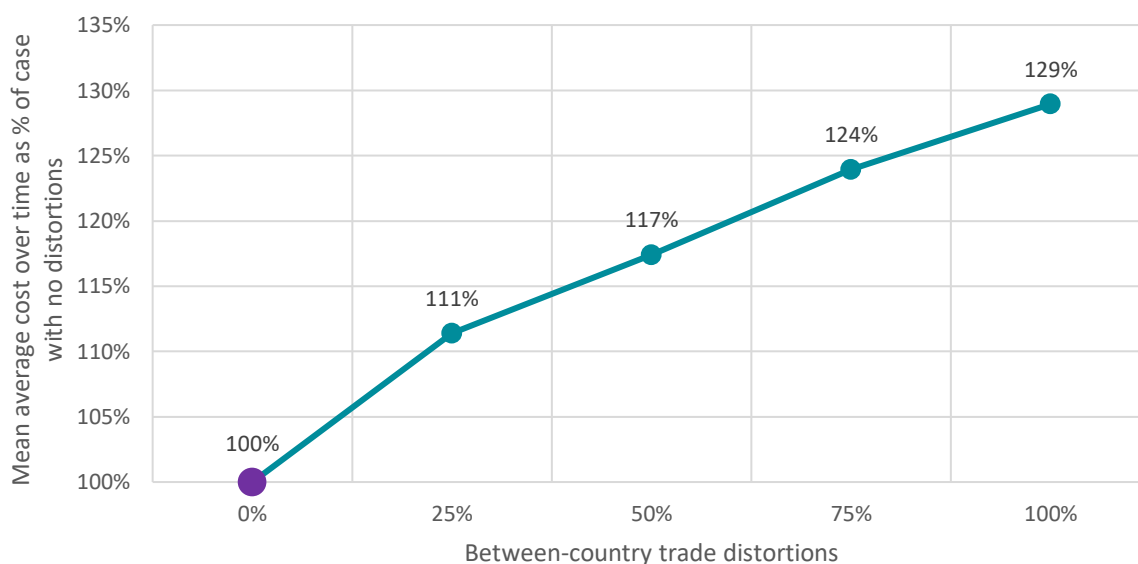
Source: Cebr analysis

Note that these percentage falls in output also hold for changes in output in both country A and B, respectively. This is unsurprising given that the distribution of consumers and firms is assumed to be identical in both countries. As stated, our focus is with the effects on global output, rather than any distributional implications across countries.

Mean average cost over time

Given the increase in average global output over time, one may expect to see a fall in the average cost of production across firms that produce. We capture this through examining how the unconditional average cost – i.e. the mean average cost across firms, averaged over time – varies with the degree of between-country trade distortion. Figure 8 expresses the unconditional average cost that arises with distortions as a percentage of the unconditional average cost with no distortions (the benchmark case). Average cost is 11% higher when 25% of consumers are unable to access foreign markets; 17% higher when half of consumers are unable to access foreign markets; 24% higher when 75% of consumers are unable to access foreign markets; and finally, 29% higher when consumer-firm trade is entirely “autarkic”.

Figure 8: The effect of between-country trade distortions on average cost



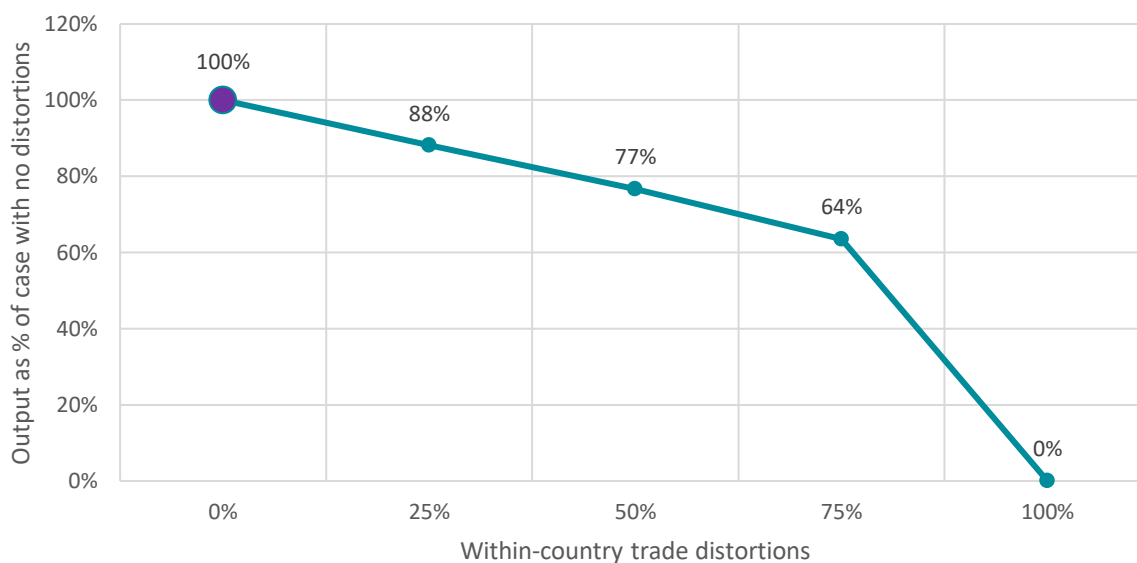
Notes: This figure presents how the average cost of firms (the average cost of a firm being averaged across firms and over time) varies with the degree of between-country trade distortion.

Source: Cebr analysis

3.2 Within-country trade distortions

Figure 9 expresses the global output that arises with within-country trade distortions as a percentage of global output in the benchmark case. Unsurprisingly, global output is falling with these distortions because they prevent mutually beneficial trades from taking place. Output is 12% lower when approximately 25% of transactions between consumers and firms that would otherwise go ahead do not due to distortions. Output largely falls linearly with the distortion, ultimately falling to zero (an extreme case) when no transactions can take place.

Figure 9: The effect of within-country trade distortions on global output



Source: Cebr analysis

While a comparison between the results in Figure 7 and Figure 9 has to be treated with some caution given the difference in how the distortions are modelled, it is interesting to note that the within-country distortion has a much more marked affect average output over time than does the between-country distortion.

To some extent, however, the intuition is clear: the within-country distortion prevents transactions that would otherwise *certainly* take place from occurring, while the between-country distortion prevents consumers and firms accessing foreign markets, in which they *may subsequently* choose to trade. For example, the lowering of between-country distortions does not in general guarantee that additional trade will take place, whereas lowering within-country distortions should.

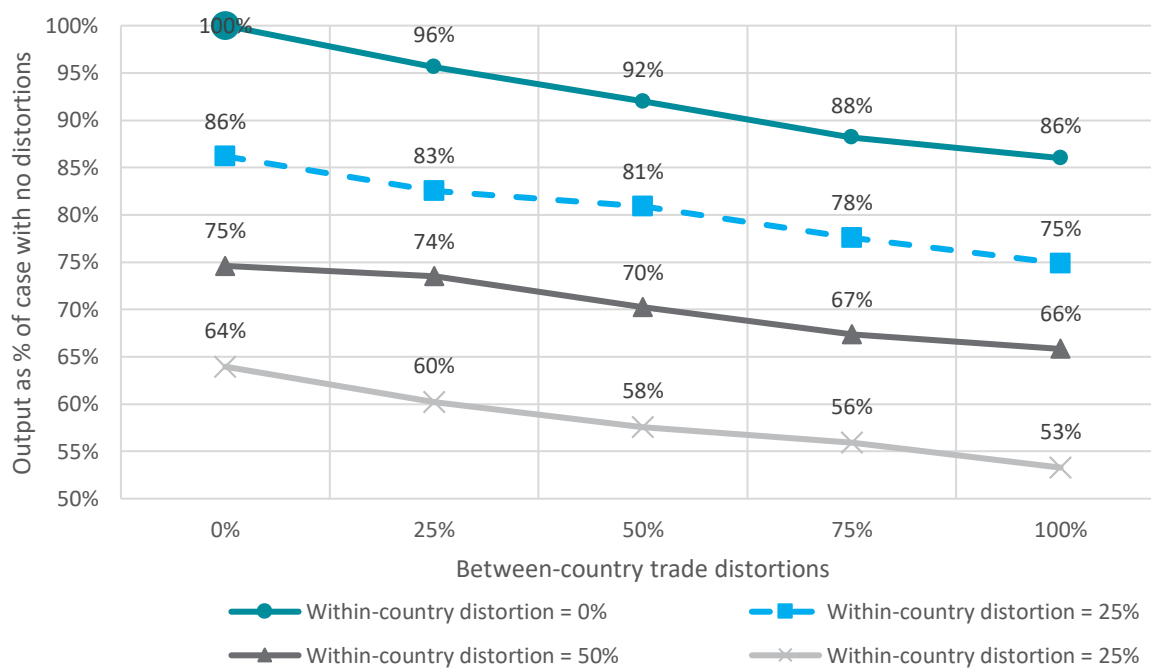
However, the impact of this distortion is not just linearly due to the prevented number of transactions. As with the trade distortion case, there are scale transmissions which come through in instances where a firm may observe multiple potential customers – but trade cannot take place with all those counterparts, thus lowering the achievable scale for each firm.

3.3 Joint effects of between- and within-country distortions

Our examination has thus far been “partial”, exploring the implications of between- or within-country distortions relative to the benchmark case, all else held constant. It is, however, interesting to explore how output is affected in the presence of both types of distortion. It is not a priori clear whether the joint effects will simply be the linear sum of the two partial effects, or whether there will be any multiplier effects whereby the overall effect is more than the sum of the two effects.

The results are presented in Figure 10, below. There are four curves, with each generated for a different level of within-country trade distortions. Movements along a given curve provide the partial effects of a between-country trade distortions, while movement across curves (for a given level of between-country distortion) provide the partial effect of within-country distortions.

Figure 10: The joint effects of between- and within-country trade distortions



Source: Cebr analysis

The key results are as follows:

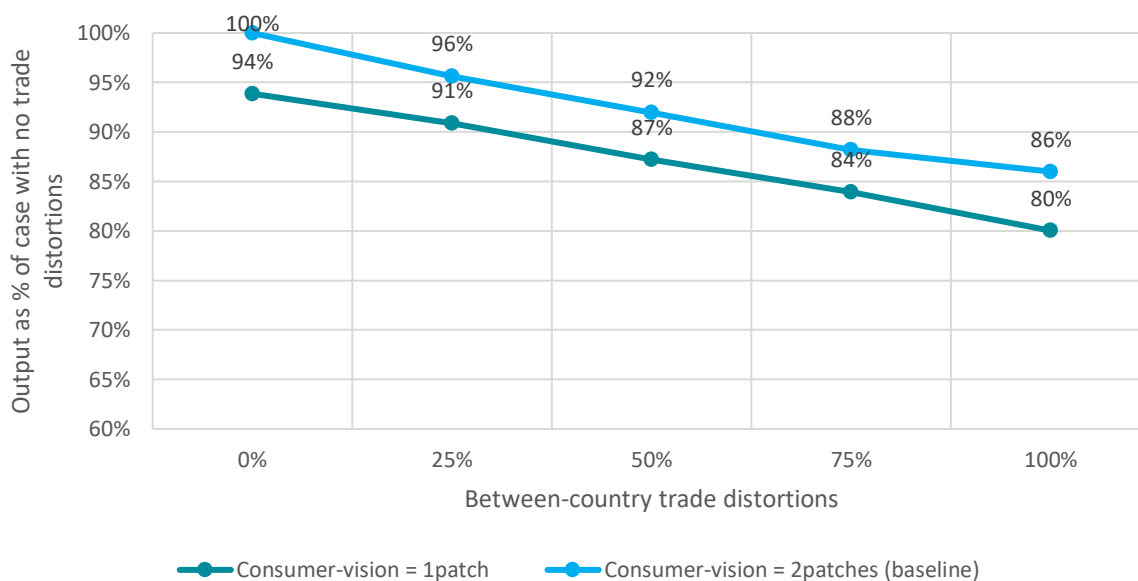
- When between- and within-country distortions are both set at 25%, global output is approximately 17% lower than in the benchmark case. The partial contribution of between-country trade distortions is 4%, while the partial contribution of within-country trade distortions is 14% (the sum of the two partial effects is therefore 18%).
- When between- and within-country distortions are both set at 50%, global output is approximately 30% lower than the benchmark case with no distortions. The partial contribution of between-country trade distortions is 8%, while the partial contribution of within-country trade distortions is 25% (the sum of the two partial effects is therefore 33%).
- When between- and within-country trade distortions are both set at 75%, global output is approximately 44% lower than in the benchmark case with no distortions. The partial contribution of between-country trade distortions is 12%, while the partial contribution of within-country trade distortions is 36% (the sum of the two partial effects is therefore 48%).

Further investigation is required to understand whether there are any additional effects at work that mean the joint effects of distortions are more than the sum of the two partial effects. The most we can say now is that that joint effects appear to be “broadly” in line with the sum of the two partial effects.

3.4 Informational distortions

This section briefly explores the implications of the extent of consumer vision on the level of global output. Figure 11 below illustrates how global output is affected when the vision of consumers is effectively halved, from 2 patches to 1 patch. Intuitively, this acts to reduce the menu of firms that a consumer can choose to purchase from in a given period; all else held constant this may mean that the consumer pays more for a given good and purchases less (though this requires further investigation).

Figure 11: The effect of informational distortions on consumers on output



The key results are as follows:

- A reduction in the extent of consumer vision from 2 patches to 1 patch reduces global output by 6% relative to the benchmark case.
- The *joint effect* of (i) a reduction in consumer vision from 2 patches to 1 patch; and (ii) a between-country trade distortion of 25% is to reduce global output by approximately 9% relative to the benchmark case.

While efforts to reduce informational distortions are likely to play an important role in facilitating trade, the analysis of these distortions has only been afforded a modest amount of attention in this report; with focus largely placed on between- and within-country distortions. The results presented here provide a specific example whereby a reduction in informational distortion unambiguously increases output. However, further investigation is required to understand the extent to which this holds universally for producers and consumers.¹¹

¹¹ Early inspection suggests there may be some non-monotonicity in the effect of increasing consumer vision on overall output.

4 Concluding remarks

4.1 Summary

This report has developed a stylised Agent-based model (ABM) of within-country and cross-country trade, coupled with information distortions. The model situates consumers and firms within one of two countries (though the framework could be readily generalised to more countries), where consumers may purchase from domestic firms in their own country or those from the other country; analogously, firms may sell to domestic consumers or to consumers abroad. Crucially, the extent to which trade can take place depends on any barriers or distortions that inhibit trade.

The trading process operates as follows. Each consumer has an (exogenously) given reservation price that determines their maximum willingness to pay for a unit of the homogenous commodity that is produced by firms; reservation prices are normally distributed across the population. Consumers can only observe a subset of the prices charged by firms – with this subset determined by a circular area around themselves.

Consumers will choose to purchase the commodity from the firm within their vision that charges the cheapest price. Firms can also only observe a subset of their environment and can observe a set of “summary statistics” about the consumers within their vision: these summary statistics include the minimum, median and maximum reservation price, in addition to the average budget of the consumers. This information allows the firm to form an expectation of its demand and to choose the quantity that maximises its profits.

The model incorporates three types of distortion that can limit the extent of trade:

- **Between-country trade distortions:** These distortions limit the extent of trade that can take place between countries. A between-country trade distortion of $x\%$ means that $x\%$ of consumers within a given country are unable to trade with foreign firms (this is, within the model, equivalent to assuming that $x\%$ of domestic firms are unable to attract foreign customers).
- **Within-country trade distortions:** These distortions limit the extent of trade than can occur within a given country. They operate as follows: when a consumer and firm have identified that they wish to trade with each they attempt to initiate the transaction, but trade-distortions ultimately render the mutually advantageous transaction unsuccessful. An $x\%$ within-country trade distortion means that, *each* mutually advantageous transaction has an $(100-x\%)$ chance of being successful. Note that these probabilities are independent, such that each transaction faces the same probability of success.
- **Informational distortions:** These distortions correspond to the extent to which consumers and firms can observe their environment, and in turn the amount of information they can garner. For example, an increase in the vision of consumers allows them to observe a greater range of prices charged by firms. Similarly, an increase in the vision of firms allows them to observe a (weakly) wider range of reservation prices; for increasing the range can never act to increase the minimum reservation price or to lower the maximum reservation price.

The model suggests that between- and within-country trade distortions have a substantial effect on country and global levels of output. For example, a between-country trade distortion of 25% acts to lower global output relative to the benchmark case with no distortions by around 4%. A within-country trade distortion of 25% acts to lower global output relative to the benchmark case with no distortions by around 14%. When both types of distortion are present, output is approximately 17% lower than the benchmark case with no distortions. While further investigation is required, the joint effect of the two types of distortions would appear to be “broadly” linear (i.e. the sum of the two partial effects of each

distortion) – but this insight needs further development, especially with more nuanced and overlapping distortions.

The results must, of course, be viewed within the context of the limitations of the model. We turn to discuss this below.

4.2 Developing the framework

The ABM developed here makes several strict assumptions that, going forwards, would need to be relaxed. We discuss each of these in turn.¹²

Modelling labour supply: the consumption-leisure trade-off

The current framework does not formally model labour supply. Individuals care only about consumption and the model is silent on any disutility of labour. Several (implicit) assumptions are made around labour supply:

- *Full employment.*
In any given time period the total production costs of firms (salaries paid to workers) are distributed equally across consumers in the population. This implies that all consumers are employed and either (i) all command the same wage and work for the same amount of time, or alternatively (ii) command different wages but differences in labour supply mean everyone ultimately earns the same.
- *Unbounded labour supply.*
Workers will work as much or as little as is required by firms to meet the production requirements (demand) of these firms.

Going forwards, a richer - but more complex - model would allow consumers to choose how much labour to supply each period, given a trade-off between consumption (earnings) and leisure. This model may either assume that all individuals have the same productivity or allow for differences in ability and therefore the wage that individuals command. In such a model one would need to carefully consider how consumer preferences are modelled; the simplistic reservation price approach adopted in the current framework would likely need to be made richer. A more sophisticated labour supply framework would also allow for greater modelling depth to cover issues such as international labour mobility alongside trade in goods.

Factor mobility

A richer model would also allow for greater depth on factor mobility. This is to recognise that the scale economies of firms may be contingent or augmentable by greater pooling of inputs into production. Models that more adequately cover this would provide valuable analysis on the gains from narrow trade, versus those from deeper trading arrangements such as a single market or sophisticated trade agreements with labour mobility and investment provisions.

Expanding scale to specialisation

The interaction of scale and specialisation is an important avenue that one might want to address. For instance, it would help to provide much deeper analysis covering asymmetric liberalisation, where

¹² This is a non-exhaustive list of assumptions that would need to be relaxed going forwards.

countries liberalising trade unilaterally might suffer a loss of domestic scale – but could potentially benefit from more efficient specialisation in areas of comparative advantage.

Intertemporal consumption-saving decisions

Fully myopic consumers

A significant simplification of the current model is that consumers will - if possible - exhaust their budget in every period through consuming the maximum amount possible of the consumption good. This myopic behaviour is unrealistic in general; a richer model would therefore need to explore the issues of intertemporal decisions, saving for the future and consumption smoothing. For example, the amount that consumers save may in part depend on their expectation of the demand by firms for labour in the next period.

4.3 Case study: UK policy implications

The work undertaken by Cebr focuses on technical economic transmissions and how trade helps to facilitate these in a way that boosts global output. From a policy point of view, this has several important substantive implications that go beyond the formal political debates surrounding the UK's EU withdrawal process:

- The modelling demonstrates that key gains from trade come through scale, while the current UK-EU Brexit negotiations are disproportionately focussed on the narrow issue of trade surpluses versus deficits. The benefits of trade linked to economies of scale are not getting enough attention, especially insofar as this affects consumer welfare and the viability of industries that rely on internationally consolidated markets. While our research is politically agonistic regarding the official nature of the UK's future relationship with the EU after withdrawal, the modelling above shows that a loss of trade will hurt both parties by lowering joint output compared to what it would have otherwise been. Equally though, both the UK and the EU may seek to offset these economic losses by increasing their links with other geographies and pursuing greater domestic liberalisation.
- Cebr's work also suggests that, for the full benefits of scale to be realised, trade liberalisation (domestic and international) may require more substantial institutional elements to be implemented in areas such as product standards, labour mobility, capital mobility and dispute resolution. Without these features the ability of firms to fully harness technological scale may be limited by worker productivity constraints, capital availability and mutually exclusive product standards. Insofar as post-Brexit trade policy is concerned, this point holds true for both EU and non-EU trade. Although the Cebr research is focused on these institutional pillars as outcomes rather than processes, there are several ways to achieve this. One way for both the UK and the EU to achieve deeper levels of global integration may be through mutual recognition of standards and regulatory equivalence arrangements.
- Domestic distortions are shown to be of greater magnitude than international trade distortions for economies that trade more internally than they do externally. As demonstrated though, both impacts have significant relative magnitudes – leading us to conclude that (within the scope of the model developed) it appears to be **more economically advantageous for policy to focus on domestic liberalisation alongside efforts to lower international barriers**. This kind of approach will therefore require a commitment to multilateral liberalisation in between countries and within countries such that **overall barriers are simultaneously lowered**. While such a policy is

technically recommended, in many cases this may be difficult to achieve due to the multitude of domestic policy goals, protectionist trade policies, variances in negotiating leverage between different countries as well as political red lines that limit or prevent the benefits of international value exchange from taking place. It is also important to note that the primacy of domestic liberalisation may change as economies become increasingly linked to external markets versus internal activity.

- If the UK ends up with an independent trade policy after Brexit, the research does point to what this should seek to achieve. While the ability of the UK to achieve multilateral intra and inter-country liberalisation may be curtailed by the previously noted political and practical constraints; outside of the EU customs union this would seem to be an economically rational approach. Although our research is focused on outcomes rather than processes, one way for the UK to pursue this kind of liberalisation may be to participate in deep trade agreements that simultaneously lower both kinds of distortions.

Appendix

Deriving aggregate and average costs within the model

Consider a global economy composed of I countries, with N_i firms in each country.

Aggregate costs

Letting $q_{n,i,t}$ denote the quantity of output produced by firm n in country i at time t , the cost of production of a given firm is

$$c_{n,i,t} = FC + MVC \times q_{n,i,t}$$

Where FC denotes a fixed cost of production and MVC is the marginal variable cost associated with producing an additional unit of output.

We can readily establish that the aggregate costs of production in country i at time period t are:

$$\begin{aligned} C_{i,t} \equiv C(Q_{i,t}) &= \sum_{n=1}^{N_i} \{FC + MVC \times q_{n,i,t}\} \\ &= N_i \times FC + MVC \times \sum_{n=1}^{N_i} q_{n,i,t} \\ &= N_i \times FC + MVC \times Q_{i,t} \end{aligned}$$

Where $Q_{i,t}$ is the aggregate output in country i at time t .

It immediately follows that aggregate costs across all countries at time t are given by

$$C_t \equiv C(Q_t) = \sum_{i=1}^I C_{i,t}$$

Average costs

The average cost of production for firm n in country i at time t is

$$AC_{n,i,t} = c_{n,i,t}/q_{n,i,t}$$

It follows that the average cost of production in a given country is given through aggregating and averaging the average cost of production across the individual firms. It is therefore:

$$AC_{i,t} = \left(\frac{1}{N_i}\right) \sum_{n=1}^{N_i} (c_{n,i,t}/q_{n,i,t})$$

The average cost of production in country i across time is therefore

$$AC_i = \left(\frac{1}{T}\right) \sum_{t=1}^T AC_{i,t}$$

$$= \left(\frac{1}{T}\right) \sum_{t=1}^T \left\{ \left(\frac{1}{N_i}\right) \sum_n^N (c_{n,i,t}/q_{n,i,t}) \right\}$$

The average cost of production across all countries at time t is

$$\begin{aligned} AC_t &= \left(\frac{1}{I}\right) \sum_{i=1}^I AC_{i,t} \\ &= \left(\frac{1}{I}\right) \left\{ \left(\frac{1}{N}\right) \sum_n^N (c_{n,i,t}/q_{n,i,t}) \right\} \end{aligned}$$

Finally, the average cost across both countries and time is

$$\begin{aligned} AC &= \left(\frac{1}{T}\right) \sum_{t=1}^T \left\{ \left(\frac{1}{I}\right) \sum_{i=1}^I AC_{i,t} \right\} \\ &= \left(\frac{1}{T}\right) \sum_{t=1}^T \left\{ \left(\frac{1}{I}\right) \sum_{i=1}^I \left(\left(\frac{1}{N}\right) \sum_{n=1}^N c_{n,i,t}/q_{n,i,t} \right) \right\} \end{aligned}$$