PHYSICS

Paper & Solution

Code : 55/2/N

Time : 3 Hrs. **Max. Marks : 70**

General Instruction :

(i) All questions are compulsory. There are **26** questions in all.

(ii) This question paper has **five** sections : Section A, Section B, Section C, Section D, Section E.

(iii) Section A contain **five** questions of **one** mark each. Section B contains **five** questions of **two** marks each, Section C contains **twelve** questions of **three** marks each, Section D contains **one** value based question of **four** marks and Section E contains **three** questions of **five** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all the three questions of five marks weightage. You have to attempt only one of the choices in such questions.

(v) You may use the following values of physical constants wherever necessary :

$c = 3 \times 10^8$ m/s

$h = 6.63 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1}

$\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2N^{-1}m^{-2}$

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

$m_e = 9.1 \times 10^{-31}$ kg

Mass of Neutrons = 1.675×10^{-27} kg

Mass of proton = 1.673×10^{-27} kg

Avogadro's number = 6.023×10^{23} per gram mole

Boltzmann constant = 1.38×10^{-23} JK^{-1}

SECTION-A

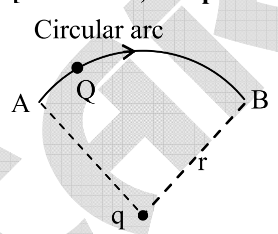
1. Why can't we see clearly through fog ? Name the phenomenon responsible for it. **[1]**

Sol. Due to scattering phenomenon, we can't see clearly through fog.

2. What is the amount of work done in moving a point charge Q around a circular arc of radius 'r' at the centre of which another point charge 'q' is located ? **[1]**

Sol. **Students may find similar question in CP exercise sheet:**

[JEE Main, Chapter : Electrostatics, Level # 1, Q.71]



A Charge partical of charge Q moving on circular arc where at the centre the charge q is located.

Work done by electrostatic force on moving charge Q

$W_{field} = -\Delta U$ (Where ΔU is change in potential energy)

$U_A = qV_A$

$U_B = qV_B$ here $V_A = V_B = \frac{kq}{r}$

$W_{field} = -(U_B - U_A)$ [So $U_A - U_B$]

$W_{field} = 0$

3. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. What are the frequencies of the side bands produced ? **[1]**

Sol. Given frequency of carrier wave $f_c = 2 \text{ MHz} = 2 \times 10^3$ kHz

Frequency of modulating signal $f_m = 5 \text{ kH}_z$

Frequency of lower side band (LSB) = $f_c - f_m$

$$\text{LSB} = (2 \times 10^3 - 5)\text{kHz}$$

$$\text{LSB} = (1995) \text{ kHz}$$

Frequency of upper side band (USB) = $f_c + f_m$

$$\text{USB} = (2 \times 10^3 + 5)\text{kHz}$$

$$\text{USB} = 2005\text{kHz}$$

4. What can be the cause of helical motion of a charged particle ? **[1]**

Sol. When charge particle is projected in a uniform magnetic field in such a way that angle between velocity and magnetic field is neither 90º nor 0 or 180º then charge particle move on helical path.

5. Define mobility of a charge carrier. What is its relation with relaxation time ? **[1]**

Sol. **Students may find similar question in CP exercise sheet:**

[JEE Main, Chapter : Current Electricity, Page No. 40, Q.26]

Mobility of a charge carriers is defined as the drift velocity of the charge carrier per unit electric field i.e.

$$\mu = \frac{v_d}{E} = \frac{e\tau}{m}$$

SECTION-B

6. Why is base band signal not transmitted directly ? Give any two reasons. **[2]**

Sol. **Students may find similar question in CP exercise sheet:**

[JEE Advance, Chapter : Communication system, Section - B, Page No. 132, Q. 4]

The needs of modulation for transmission of a signal are given below

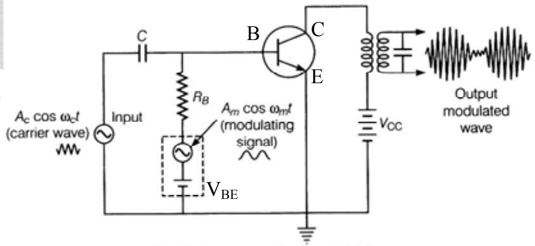
(i) The transmission of low frequency signal needs antenna of height 4-5 km which is impossible to construct. So, there is need to modulate the wave in order to reduce the height of antenna to a reasonable height.

(ii) Effective power radiated by antenna for low wavelength or high frequency wave as

$$P \propto \frac{1}{\lambda^2}$$

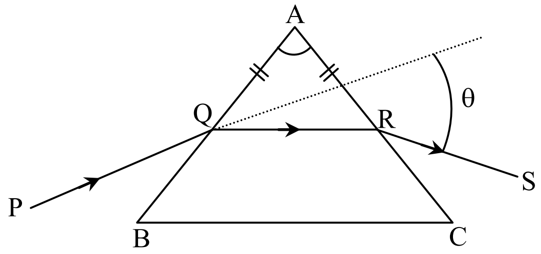
So, for effective radiation by antenna, there is need to modulate the wave.

Figure shows the amplitude modulation using a sinusoidal signal as the modulating signal.

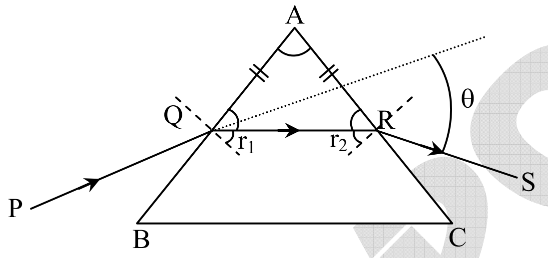


Circuit for an amplitude modulator

7. A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS such that AQ = AR. If the angle of prism A = 60º and refractive index of material of prism is $\sqrt{3}$, calculate angle θ. **[2]**



Sol.



Given AQ = AR hence in ΔAQR

$\angle Q = \angle R = \phi$(Let)

A + 2ϕ = 180º (for ΔAQR)

60 + 2ϕ = 180

$\Rightarrow \phi = 60 \Rightarrow \angle r_1 = \angle r_2 = 30^o$

This is the condition of minimum deviation

So Using relation

$$\nu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin A/2}$$

given $\nu = \sqrt{3}$ & A = 60º

$$\sqrt{3} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$\sin\left(\frac{60+\delta_{min}}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{60+\delta_{min}}{2} = 60$$

$\delta_{min} = 60^o$

8. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m/s. If the electron density in the wire is 8×10^{28} m^{-3}, calculate the resistivity of the material of wire. **[2]**

Sol. Given p.d V = 5V

$\ell = 0.1$ m

$v_d = 2.5 \times 10^{-4}$ m/s

$n = 8 \times 10^{28}$

$E = \rho J$

$$\frac{V}{\ell} = \rho\, nev_d$$

$$\rho = \frac{V}{\ell nev_d}$$

$$= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$$

$= 1.56 \times 10^{-5}$ Ω-m

9. When is H_α line in the emission spectrum of hydrogen atom obtained ? Calculate the frequency of the photon emitted during this transition. **[2]**

OR

Calculate the wavelength of radiation emitted when electron in a hydrogen atom jumps from n = ∞ to n = 1.

Sol. H_α line is obtained when electron make transition from n = 3 to n = 2.

$E_3 - E_2 = hf$ (According to Bohr)

$$-\frac{13.6}{3^2} - \left(-\frac{13.6}{2^2}\right) = \frac{hf}{1.6 \times 10^{-19}}$$

$$13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{hf}{1.6 \times 10^{-19}}$$

$$13.6\left[\frac{5}{36}\right] = \frac{6.63 \times 10^{-34} f}{1.6 \times 10^{-19}}$$

$$f = \frac{0.4558 \times 10^{-19}}{10^{-34}}$$

$\Rightarrow 0.4558 \times 10^{15}$

4.558×10^{14} Hz

OR

In hydrogen atom wavelength of radiation for transition n = ∞ to n = 1

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_L^2} - \frac{1}{n_H^2}\right] \qquad \left\{\begin{array}{c|c} \text{Here} & \text{For} \\ n_L = 1 & \text{H atom} \\ n_H = \infty & Z = 1 \end{array}\right\}$$

$$\frac{1}{\lambda} = R(1)^2\left[\frac{1}{1^2} - \frac{1}{\infty}\right]$$

$$\frac{1}{\lambda} = R \Rightarrow \lambda = \frac{1}{R}$$

$\boxed{\lambda \approx 912\text{Å}}$ $\{R = 1.09 \times 10^7\ m^{-1}\}$

10. A proton and an α particle are accelerated through the same potential difference. Which one of the two has (i) greater de-Broblie wavelength, and (ii) less kinetic energy ? Justify your answer. **[2]**

Sol. When a charge particle is accelerated through V potential difference then its kinetic energy

K.E. = qV (q = charge; V = potential difference)

Proton and α-particle accelerated through same potential difference so

because $q_\alpha > q_{proton}$ ($q_\alpha = +2e$ and $q_{proton} = +e$)

$K.E_\alpha > K.E_{proton}$

their Debroglie wavelength $\lambda = \frac{h}{\sqrt{2mqV}}$

V = same

$m_\alpha q_\alpha > m_p q_p$ $[m_\alpha = 4\ m_p]$

So $\lambda_\alpha < \lambda_p$ (λ_α = Debroglie wavelength of α-particle; λ_p = Debroglie wavelength of proton)

Ans. (1) Proton have greater Debroglie wavelength

(2) Proton have lesser kinetic energy

SECTION-C

11. State two important properties of photon which are used to write Einstein's photoelectric equation. Define
(i) stopping potential and
(ii) threshold frequency, using Einstein's equation and drawing necessary plot between relevant quantities. **[3]**

Sol. Characteristic properties of photons :

(i) Energy of photon is directly proportional to the frequency (or inversely proportional to the wavelength).

(ii) In photon-electron collision, total energy and momentum of the system of two constituents remains constant.

(iii) In the interaction of photons with the free electrons, the entire energy of photon is absorbed.

Cut off or stopping potential is that minimum value of negative potential at anode which just stops the photo electric current

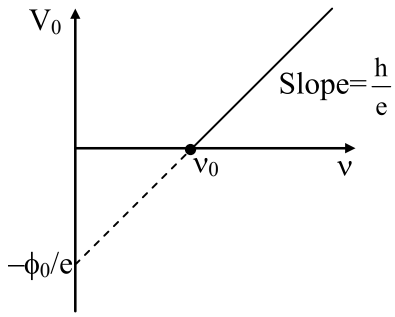
For a given material, there is a minimum frequency of light frequency is called as threshold frequency.

By Einstein's photo electric equation

$KE_{max} = \frac{hC}{\lambda} - \phi_0 = h\nu - h\nu_0$ where $\phi_0 = h\nu_0$

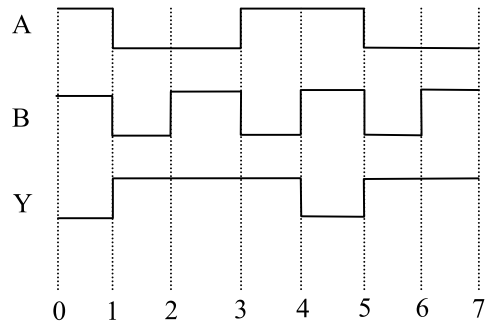
$eV_0 = h\nu - h\nu_0$

$V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0$



12. (i) Distinguish between a conductor and a semi conductor on the basis of energy band diagram.

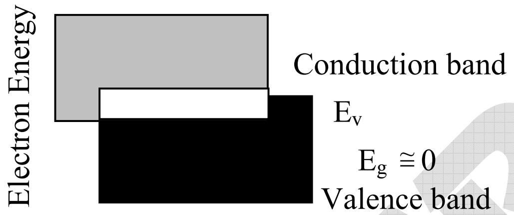
(ii) The following figure show the input waveforms (A, B) and the output waveform (Y) of a gate. Identify the gate, write its truth table and draw its logic symbol. **[3]**

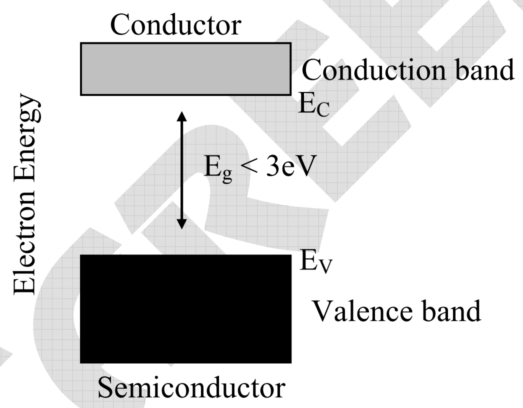


Sol. **Students may find similar question in CP exercise sheet:**

[(ii) JEE Main, Page No. 31, Q. 33]

(i)





Distinguishing features

(a) In conductors : Valence band and conduction band overlap each other.

In semiconductors : Valence band and conduction band are separated by a small energy gap.

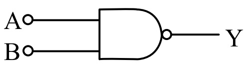
(b) In conductors : Large number of free electrons are available in conduction band.

In semiconductors : A very small number of electrons are available for electrical conduction.

(ii) **Gate** From the given output waveform, it is clear that output is zero only when both inputs are 1, so the gate is NAND gate

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Logic symbol



$$Y = \overline{A \cdot B}$$

13. (a) Derive the mathematical expression for law of radioactive decay for a sample of a radioactive nucleus.
(b) How is the mean life of a given radioactive nucleus related to the decay constant ? **[3]**

Sol. **(a) Radioactive decay Law :** The rate of decay of radioactive nuclei is directly proportional to the number of undecayed nuclei at that time.

Derivation of formula

Suppose initially the number of atoms in radioactive element is N_0 and N the number of atoms after time t. After time t, let dN be the number of atoms which disintegrate in a short interval dt, then rate of disintegration will be $\frac{dN}{dt}$, this is also called the activity of the substance/element.

According to Rutherford-Soddy law

$$\frac{dN}{dt} \propto N$$

or $$\frac{dN}{dt} = -\lambda N \qquad ....(i)$$

where λ is a constant, called decay constant or disintegration constant of the element. Its unit is s^{-1}. Negative sign shows that the rate of disintegration decreases with increase of time. For a given element/substance λ is a constant and is different for different elements. Equation (i) may be rewritten as

$$\frac{dN}{N} = -\lambda dt$$

Integrating $\log_e N = -\lambda t + C \qquad ....(ii)$

where C is a constant of integration.

At $t = 0, N = N_0$

$\therefore$ $\log_e N_0 = 0 + C \Rightarrow C = \log_e N_0$

$\therefore$ Equation (ii) gives $\log_e N = -\lambda t + \log_e N_0$

or $\log_e N - \log_e N_0 = -\lambda t$

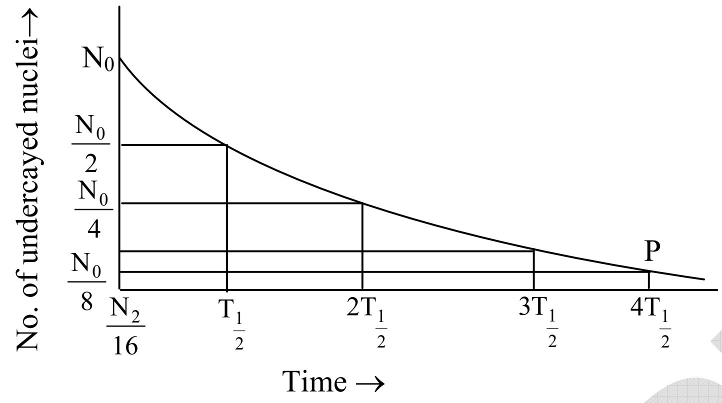
or $$\log_e \frac{N}{N_0} = -\lambda t$$

or $$\frac{N}{N_0} = e^{-\lambda t}$$

$\therefore$ $$N = N_0 e^{-\lambda t} \qquad ...(iii)$$

According to this equation, the number of undecayed atoms/nuclei of a given radioactive element decreases exponentially with time (i.e., more rapidly at first and slowly afterwards.)

Mark of $N = \frac{N_0}{16}$ in terms of $T_{1/2}$ is shown in figure.

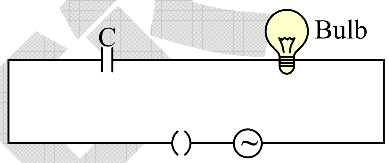


(b) Relation between mean life and half life

Mean life $\tau = \frac{1}{\lambda}$ where $\lambda \to$ decay constant

14. (i) When an AC source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero.

(ii) A bulb is connected in series with a variable capacitor and an A.C. source as shown. What happens to the brightness of the bulb when the key is plugged in and capacitance of the capacitor is gradually reduced **[3]**



Sol. **Students may find similar question in CP exercise sheet:**

[(ii) JEE Main, Chapter : Alternating Current, Page No. 27, Q.1]

(i) Power dissipation in AC circuit is $P = V_{rms} I_{rms} \cos\phi$ where $\cos\phi = \frac{R}{Z}$

for an ideal capacitor R = 0

So $\cos \phi = 0$, So $P = 0$

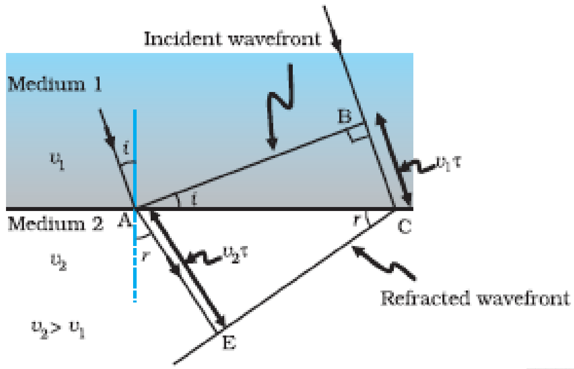
Hence power dissipated is minimum.

(ii) When AC source is connected, the capacitor offers capacitive reactance $X_C = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C, X_C increases. Therefore, glow of the bulb reduces.

15. (i) Derive Snell's law on the basis of Huygen's wave theory when light is travelling from a denser to a rarer medium

(ii) Draw the sketches to differentiate between plane wavefront and spherical wavefront. **[3]**

Sol. (i)



Let PP′ represent the surface separating medium 1 and medium 2, as shown in Fig. Let v_1 and v_2 represent the speed of light in medium 1 and medium 2, respectively. We assume a plane wavefront AB propagating in the direction A′A incident on the interface at an angle i as shown in the figure. Let τ be the time taken by the wavefront to travel the distance BC. Thus,

$$BC = v_1 \tau$$

Thus we obtain

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

Now, if c represents the speed of light in vacuum, then,

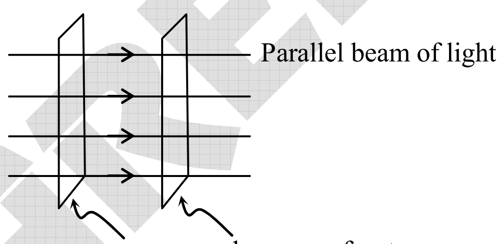
$$n_1 = \frac{c}{v_1} \text{ and } n_2 = \frac{c}{v_2}$$

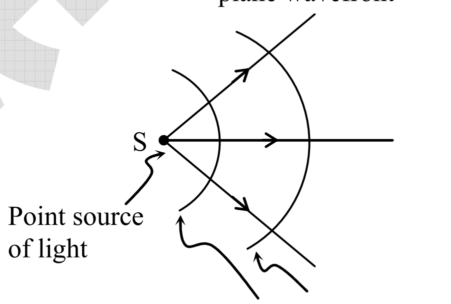
are known as the refractive indices of medium 1 and medium 2, respectively. In terms of the refractive indices, Eq. can be written as

$$n_1 \sin i = n_2 \sin r$$

This is the *Snell's law of refraction*.

(ii)





16. (i) A screen is placed at a distance of 100 cm from an object. The image of the object is formed on the screen by a convex lens for two different location of the lens separated by 20 cm. Calculate the focal length of the lens used.

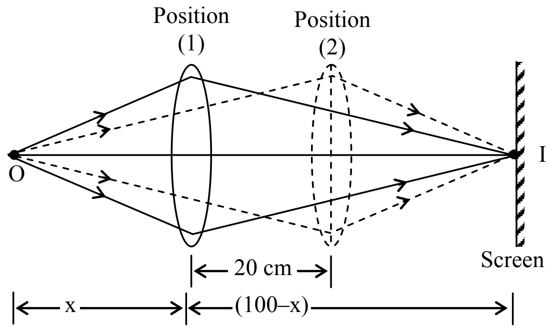
(ii) A converging lens is kept coaxially in contact with a diverging lens-both the lenses being of equal focal length. What is the focal length of the combination ? **[3]**

Sol. **Students may find similar question in CP exercise sheet:**

[(i) JEE Advance, Chapter : Refraction at curved surface, Page No. 24, Q.22]

[(ii) JEE Advance, Chapter : Refraction at curved surface, Page No. 27, Q.1]

(i)



Lens at position (1)

Object distance, u = –x

Image distance, v = (100 – x)

Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{100 - x} + \frac{1}{x} \qquad \ldots(1)$$

Lens at position (2)

Object distance, u = – (x + 20)

Image distance, v = (80 – x)

Using lens formula

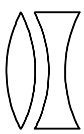
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{(80 - x)} + \frac{1}{x + 20} \qquad \ldots(2)$$

Solving equation (1) & (2)

f = 24 cm.

(ii)



(both are of equal focal length = f)

equivalent power of combination

$P_{eqL} = P_1 + P_2$

$$P_{eqL} = \left(\frac{1}{+f} + \frac{1}{-f}\right)$$

$$P_{eqL} = 0$$

$$\frac{1}{f_{eqL}} = 0 \Rightarrow \boxed{f_{eqL} = \infty}$$

focal length of the combination is infinite.

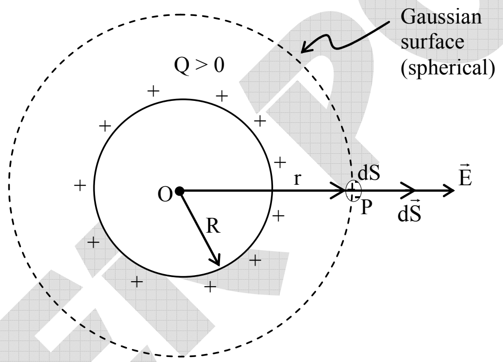
17. Find the electric field intensity due to a uniformly charged spherical shell at a point (i) outside the shell and (ii) inside the shell. Plot the graph of electric field with distance from the centre of the shell. **[3]**

Sol. **Students may find similar question in CP exercise sheet:**
[JEE Main, Chapter : Electrostatics, Page No. 34, Q.9]

Object : Here we have to find electric field intensity due to a uniformly charged spherical conducting shell at outside point.

Given : Here it is given that radius of shell is R and a positive charge Q distributed uniformly on the surface of a thin spherical shell of radius R.

Procedure : To determine electric field intensity at a point P outside the sphere such that OP = r ; (r > R)



We construct a sphere of radius r as a Gaussian surface as shown in figure and take a small area elements ds on it and direction of $d\vec{s}$ will be radially outward
Total electric flux through Gaussian Surface
Now from Gauss's law

$$\Phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{\Sigma q}{\varepsilon_0}$$

Where Σq = net charged enclosed in Gaussian Surface

$\because$ at point P direction of electric field $\vec{E}$ and $d\vec{s}$ is same i.e. $\theta = 0^\circ$ and $\Sigma q = Q$

$$\Rightarrow \oint E ds \cos 0^\circ = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \oint E ds = \frac{Q}{\varepsilon_0}$$

As electric field (magnitude) is constant at every point on Gaussian Surface

$\because$ $|\vec{E}|$ = constant

$$E \oint ds = \frac{Q}{\varepsilon_0}$$

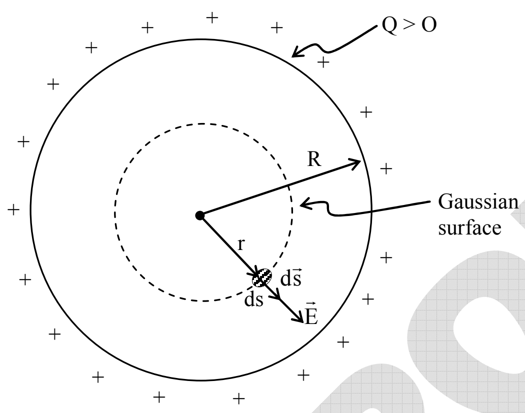
$\oint ds$ = Surface area of Gaussian Surface = $4\pi r^2$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ (magnitude)}$$

or $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ (vector form)

(ii) At a point inside the shell
From Gauss's law



$$\Phi_E = \oint \vec{E}\cdot d\vec{S} = \frac{\Sigma q}{\varepsilon_0}$$

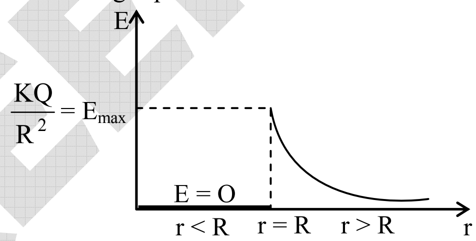
$\because$ no charge is enclosed by the Gaussian surface

$\Sigma q = 0$

$\Rightarrow \oint \vec{E}\cdot d\vec{S} = O$

$\Rightarrow E = O$

So, there is no electric field inside a charge spherical shell.



18. What is space wave propagation ? State the factors which limit its range of propagation. Derive an expression for the maximum line of sight distance between two antennas for space wave propagation. **[3]**

Sol. **Students may find similar question in CP exercise sheet:**

[JEE Advance, Chapter : Communication system, Page No. 130, Q.27]

&

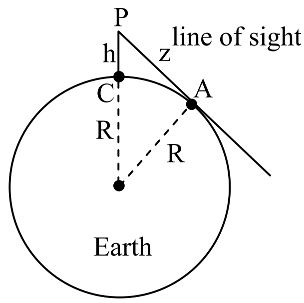
[JEE Advance, Chapter : Communication system, Page No. 130, Q.31(i)]

When a wave propagates in a straight line, from the transmitting antenna to the receiving antenna, its mode of propagation is called space wave communication.

Frequency range : Above 40 MHz.

Space waves are used for the Line of Sight (LOS) communication. Space wave communication involves the transmission from transmitter, traveling along a straight line in space, reaches to receiving antenna.

The range of their frequencies is 40 MHz and above. The range of space wave propagation is limited by line of sight distance between transmission to receiver /repeater antenna.



h <<< R

h = height of antenna

r = Radius of earth

Range = cA

As h is very small CA ≈ PA

using plythagores theorem

$PA^2 + R^2 = (R + h)^2$

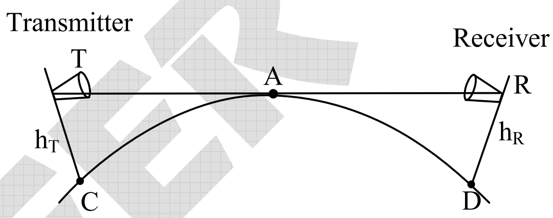
$PA^2 + R^2 = R^2 + 2Rh + h^2$

(neglecting h^2)

PA = $\sqrt{2Rh}$

Range ⇒ $\sqrt{2Rh}$

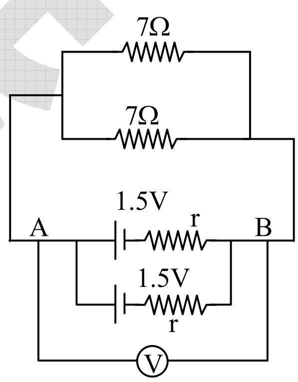
on ground surface



Total range on ground = CA + AD = $\sqrt{2Rh_T} + \sqrt{2Rh_R}$

19. Two identical cells of emf 1.5 V each joined in parallel supply energy to an external circuit consisting of two resistances of 7Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V. Calculate the internal resistance of each cell. **[3]**

Sol.



E_{eq} of cells across AB $E_{eq} = \dfrac{E_1r_2 + E_2r_1}{r_1 + r_2}$

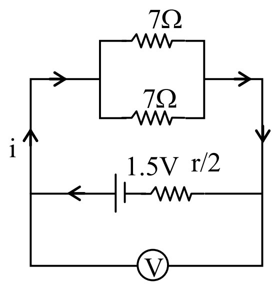
$E_1 = E_2 = 1.5V$

$E_{eq} = \dfrac{1.5r + 1.5r}{2r}$

$E_{eq} = 1.5$ volt

equivalent resistance $r_{eq} = \dfrac{r_1\ r_2}{r_1 + r_2} \Rightarrow r_{eq} = \dfrac{r^2}{2r} \Rightarrow \boxed{r_{eq} = \dfrac{r}{2}}$

equivalent circuit



$i = \dfrac{1.5}{\dfrac{7}{2} + \dfrac{r}{2}} \Rightarrow i = \dfrac{3}{7 + r}$

Reading of voltameter

$V = iR$

$1.4 = i\left(\dfrac{7}{2}\right)$

$\left(\dfrac{3}{7 + r}\right)\dfrac{7}{2} = 1.4 \Rightarrow 7 + r = 7.5;\quad r = 0.5\ \Omega$

20. (i) Name two important processes that occur during the formation of an pn junction.

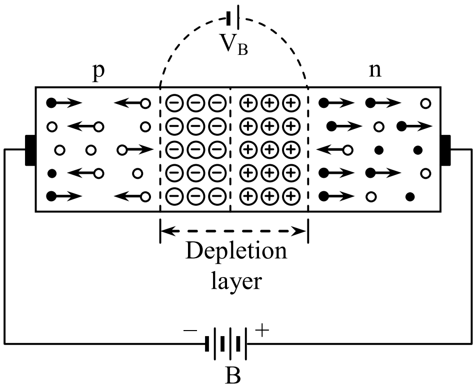
(ii) Draw the circuit diagram of a full wave rectifier along with the input and output waveforms. Briefly explain how the output voltage/current is unidirectional. **[3]**

Sol. Students may find similar question in CP exercise sheet:

[(i) JEE Advance, Chapter : Semiconductor Devices, Page No. 33, Q.64]

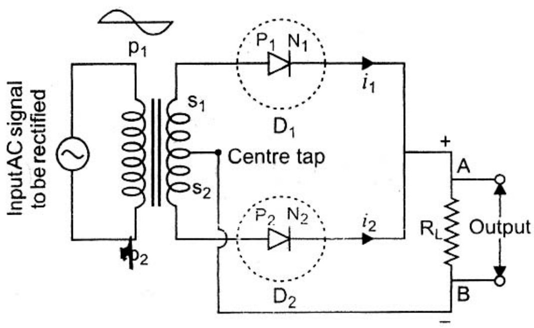
[(ii) JEE Advance, Chapter : Semiconductor Devices, Page No. 33, Q.66]

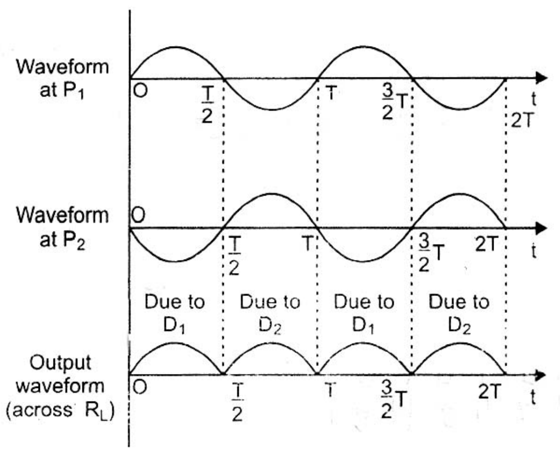
(i)



During formation of p-n junction, diffusion and drift of charge takes place.

(ii) **Rectification:** Rectification means conversion of ac into dc. A p-n diode acts as a rectifier because an ac changes polarity periodically and a p-n diode conducts only when it is forward biased; it does not conduct when reverse biased.





Working: The AC input voltage across secondary s_1 and s_2 change polarity after each half cycle. Suppose during the first half cycle of input AC signal, the terminal s_1 is positive relative to centre tap O and s_2 is negative relative to O. Then diode D_1 is forward biased and diode D_2 is reverse biased. Therefore, diode D_1 conducts while diode D_2 does not. The direction of current (i_1) due to diode D_1 in load resistance R_L is directed from A to B. In next half cycle, the terminal s_1 is negative and s_2 is positive relative to centre tap O. The diode D_1 is reverse biased and diode D_2 is forward biased. Therefore, diode D_2 conducts while D_1 does not. The direction of current (i_2) due to diode D_2 in load resistance R_L is still from A to B. Thus the current in load resistance R_L is in the same direction for both half cycle of input AC voltage. Thus for input AC signal the output current is a continuous series of unidirectional pulses.

In a full wave rectifier, if input frequency is f hertz, then output frequency will be 2f hertz because for each cycle of input, two positive half cycles of output are obtained.

21. State Ampere's circuital law. Use this law to find magnetic field due to straight infinite current carrying wire. How are the magnetic field lines different from the electrostatic field lines ?

OR

State the principle of a cyclotron. Show that the time period of revolution of particles in a cyclotron is independent of their speeds. Why is this property necessary for the operation of a cyclotron ? **[3]**

Sol. **Students may find similar question in CP exercise sheet:**
[JEE Main, Chapter : Magnetic effect of current, Page No. 34, Q.2]

OR

[JEE Main, Chapter : Magnetic effect of current, Page No. 36, Q.10(a)]

Ampere's circuital law : The line integral of magnetic field over a closed loop is μ_0 times the total current passing through the loop.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I$$

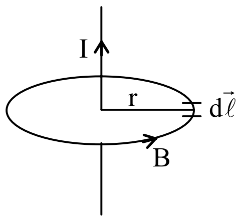
Consider an amperian loop of radius r and applying ampere's circuital law

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I$$

$$B\,(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_o I}{2\pi r}$$

$$B \propto \frac{1}{r} \quad (r > a)$$



The magnetic field lines form continuous closed loops, where as electrostatic field lines never forms closed loop.

OR

Principle. The frequency of revolution of the charged particle in magnetic field is independent of its energy.

If particle is performing circular motion due to magnetic force then centripetal force = Magnetic force

$$\Rightarrow \frac{mv^2}{r} = q\,v\,B\sin 90°$$

$$\Rightarrow r = \frac{mv}{qB}$$

$$\therefore \text{Time period} = \frac{2\pi r}{v}$$

$$T = \frac{2\pi}{v}\,.\,\frac{mv}{qB} = \frac{2\pi m}{qB}$$

The time period of revolution of particles in a cyclotron is independent of their speeds

This property is necessary for the operation of a cyclotron to attain the resonance condition ($\nu_a = \nu_c$) where ν_a is the frequency of the applied voltage and ν_c is cyclotron frequency.

22. How are electromagnetic waves produce ? What is the source of energy of these waves ? Write mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis. Write any two important properties of electromagnetic waves. **[3]**

Sol. **Students may find similar question in CP exercise sheet:**
[JEE Advance, Chapter : EMW, Page No. 93, Q.48]

Electromagnetic waves are produced by accelerated or oscillating charges and do not need any medium for propagation.

The source of energy of these waves are energy of oscillating charge particle.

Mathematical expressions for electric and magnetic fields of an electromagnetic wave propagating along the z-axis – $E_x = E_0 \sin [kz - \omega t]$

$B_y = B_0 \sin [kz - \omega t]$

Properties of electromagnetic waves

1. The direction of E, B and direction of propagation of waves are mutually perpendicular to one another.
2. The amplitude ratio of E and B gives velocity of light.
3. Electromagnetic waves are not deflected by electric and magnetic fields.

SECTION-D

23. Seema's uncle was advised by his doctor to have an MRI (Magnetic Resonance Imaging) scan of his brain. Her uncle felt it to be expensive and wanted to postpone it.

When Seema learnt about this, she took the help of her family and also approached the doctor, who also offered a substantial discount. She then convinced her uncle to undergo the test to enable the doctor to know the condition of his brain. The information thus obtained greatly helped the doctor to treat him properly.

Based on the above paragraph, answer the following questions:

(a) What according to you are the values displayed by Seema, her family and the doctor ?

(b) What could be the possible reason for MRI test to be so expensive ?

(c) Assuming that MRI test was performed using a magnetic field of 0.1 T., find the minimum and maximum values of the force that the magnetic field could exert on a proton (charge = 1.6×10^{-19} C) moving with a speed of 10^4 m/s. **[4]**

Sol. (a) Depend on student what he / she think about value of Seema

(b) Availability of MRI test is very less. Only in big Hospital it is available so due to monopoly of big hospital they are just charging huge amount.

(c) Force experienced by proton $\vec{F} = q\ (\vec{V} \times \vec{B})$

$|\vec{F}| = q\ vB\sin\theta$

maximum force $|\vec{F}|_{max} = qvB$

$\Rightarrow |\vec{F}|_{max} = 1.6 \times 10^{-19} \times 10^4 \times 0.1$

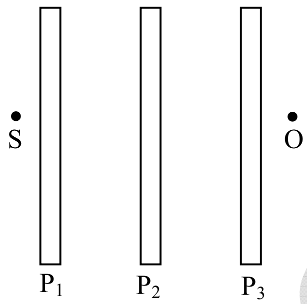
$|\vec{F}|_{max} = 1.6 \times 10^{-16}$ N

Minimum force $|\vec{F}|_{min}$ = Zero

SECTION-E

24. (a) Why does unpolarised light from a source show a variation in intensity when viewed through a polaroid which is rotated ? Show with the help of a diagram, how unpolarised light from sun gets linearly polarised by scattering.

(b) Three identical polaroid sheets P_1, P_2 and P_3 are oriented so that the pass axis of P_2 and P_3 are inclined at angles of 60º and 90º respectively with the pass axis of P_1. A monochromatic source S of unpolarized light of intensity I_0 is kept in front of the polaroid sheet P_1 as shown in the figure. Determine the intensities of light as observed by the observer at O, when polaroid P_3 is rotated with respect to P_2 at angles θ = 30º and 60º. **[5]**



OR

(c) Derive an expression for path difference in Young's double slit experiment and obtain the conditions for constructive and destructive interference at a point on the screen.

(b) The intensity at the central maxima in Young's double slit experiment is I_0. Find out the intensity at a point where the path difference is $\frac{\lambda}{6}, \frac{\lambda}{4}$ and $\frac{\lambda}{3}$.

Sol. **Students may find similar question in CP exercise sheet:**

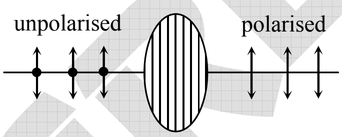
[(a) JEE Advance, Chapter : Polarization, Page No. 250, Q.3]

OR

[(a) JEE Advance, Chapter : Wave nature of light, Page No. 20, Q.2]

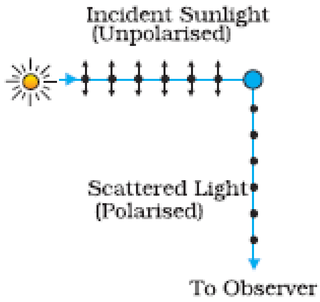
[(b) JEE Advance, Chapter : Wave nature of light, Page No. 19, Q.26(ii)]

(a) When vibrations of light wave are confined to only one direction then light is called linearly polarised.

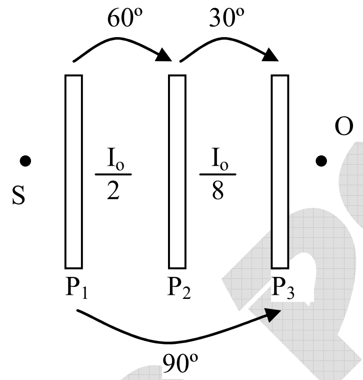


When sunlight passes through polaroid then components parallel to axis passes unaffected and components perpendicular to axis are absorbed so transmitted light is polarised.

As Fig. shows, the incident sunlight is unpolarised. The dots stand for polarisation perpendicular to the plane of the figure. The double arrows show polarisation in the plane of the figure. (There is no phase relation between these two in unpolarised light). Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions. We have drawn an observer looking at 90° to the direction of the sun. Clearly, charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore represented by dots. It is polarised perpendicular to the plane of the figure. This explains the polarisation of scattered light from the sky.



(b) On rotating the polaroid, intensity remains unchanged as half of the incident intensity.



Intensity of unpolarized light on P_1 is I_0

After passing through P_1 the intensity is $\left(\frac{I_0}{2}\right)$

After passing through P_2 polaroid

$I' = I\cos^2\theta$ where $\left(I = \frac{I_0}{2}\right)$

$I' = \frac{I_0}{2}\cos^2 60^o$

$I' = \frac{I_0}{8}$

Case I[st]

P_3 rotated $\theta = 30^o$ towards pass axis of P_2

Now P_3 and P_2 have parallel pass axis

So, $I'' = I'\cos^2\theta$ (here $\theta = 0^o$)

$I'' = I'$

$I'' = \frac{I_0}{8}$

Case II[nd]

P_3 rotated away ($\theta = 30^o$) from pass axis of P_2.

Now angle between pass axis of P_2 & P_3 will be

$\phi = 60^o$

$I'' = I' \cos^2 60^o$

So $I'' = \frac{I_0}{8}\left(\frac{1}{2}\right)^2$

$I'' = \frac{I_0}{32}$

Case IIIrd :

P_3 rotated towards pass axis of P_2 by $\theta = 60^o$

Now again angle between pass axis of P_2 & P_3 will be

$\phi = 30^o$

So $I'' = I' \cos^2 30^o$

So $I'' = \frac{I_0}{8}\left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow$ $I'' = \frac{3I_0}{32}$

Case IVrd

P_3 rotated away from pass axis of P by $\theta = 60$

Now angle between pass axis of P_2 & P_3 will be

$\phi = 90^o$

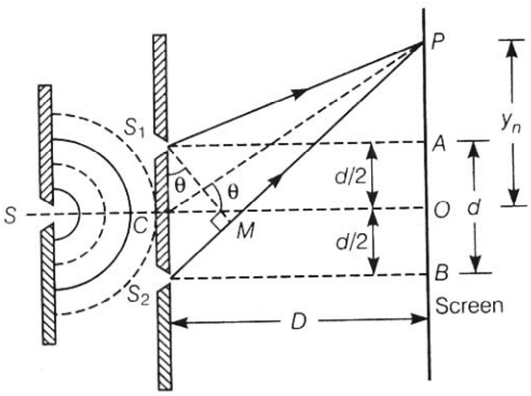
So $I'' = I' \cos^2 90^o$

So $I'' = 0$

OR

(a) To observe interference fringe pattern, there is need to have coherent sources of light which can produce light of constant phase difference.

Let two coherent sources of light, S_1 and S_2 (narrow slits) are derived from a source S. The two slits, S_1 and S_2 are equidistant from source, S. Now suppose S_1 and S_2 are separated by distance d. The slits and screen are distance D apart.



Considering any arbitrary point P on the screen at a distance y_n from the centre O.

The path difference between interfering waves is given by $S_2P - S_1P$

i.e., Path difference = $S_2P - S_1P$

i.e., Path difference = $S_2P - S_1P = S_2M$

$= d \sin\theta$

where, $S_1M \perp S_2P$

[$\because \angle S_2S_1M = \angle OCP$ (By geometry)

$\Rightarrow S_1P = PM \Rightarrow S_2P = S_2M$]

If θ is small, then $\sin\theta \approx \theta \approx \tan\theta$

$\therefore$ Path difference,

$S_2P - S_1P = S_2M = d \sin\theta \approx d \tan\theta$

Path difference $= d\left(\frac{y_n}{D}\right)$ ...(i)

[$\because$ In ΔPCO, $\tan\theta = \frac{OP}{CO} = \frac{y_n}{D}$]

For constructive interference

Path difference = $n\lambda$, where, n = 0, 1, 2, …

$\frac{dy_n}{D} = n\lambda$ [From Eq. (i)]

$\Rightarrow \quad y_n = \frac{Dn\lambda}{d}$

$\Rightarrow \quad y_{n+1} = \frac{D(n+1)\lambda}{d}$

$\because$ Fringe width of dark fringe = $y_{n+1} - y_n$

[$\because$ dark fringe exist between two bright fringes]

$\beta = \frac{D\lambda}{d}(n+1) - \frac{Dn\lambda}{d} = \frac{d\lambda}{d}(n+1-n) = \frac{D\lambda}{d}$

Fringe width of dark fringe, $\beta = \frac{D\lambda}{d}$ ...(ii)

For destructive interference

Path difference $= (2n-1)\frac{\lambda}{2}$, where n = 1, 2, 3, …

$\Rightarrow \quad \frac{y_n' d}{D} = (2n-1)\frac{\lambda}{2}$ [From Eq. (i)]

$\Rightarrow \quad y_n' = \frac{(2n-1)D\lambda}{2d}$

where, y_n' is the separation of nth order dark fringe from central fringe.

$\therefore \quad y_{n+1}' = (2n+1)\frac{D\lambda}{2d}$

$\therefore$ Fringe width of bright fringe

= Separation between (n + 1)th and nth order dark fringe from centred fringe,

$\Rightarrow \beta = y_{n+1}' - y_n'$

or $\beta = \frac{(2n+1)D\lambda}{2d} - \frac{(2n-1)D\lambda}{2d}$

$= \frac{D\lambda}{2d}[2n + 1 - 2n + 1] = \frac{D\lambda}{2d}[2]$

Fringe width of bright fringe.

$\beta = \frac{D\lambda}{d}$ …(iii)

From Eqs. (ii) and (iii), we can see that,
fringe width of dark fringe = fringe width of bright fringe

$\beta = \frac{D\lambda}{d}$

The fringe width decrease as wavelength gets reduced when interference set up is taken from air to water.

(b) Intensity at central maxima is given = I_0

$\Rightarrow \quad I_{max} = I_0$

Intensity of resultant wave when the light waves interfere at $\Delta\phi$ phase difference is

$I_{res} = I_{max} \cos^2\left(\frac{\Delta\phi}{2}\right)$

$I_{res} = I_o \cos^2\left(\frac{\Delta\phi}{2}\right)$ ($\Delta\phi$ = phase difference)

(1) Path difference

$\Delta x = \frac{\lambda}{6}$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{6}\right) \Rightarrow \Delta\phi = \frac{\pi}{3}$

$I_{res} = I_o \cos^2\left(\frac{\frac{\pi}{3}}{2}\right)$

$I_{res} = I_o \cos^2\left(\frac{\pi}{6}\right)$

$I_{res} = I_o\left(\frac{\sqrt{3}}{2}\right)^2$

$I_{res} = \frac{3I_o}{4}$

(2) Path difference $\Delta x = \frac{\lambda}{4}$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) \Rightarrow \Delta\phi = \frac{\pi}{2}$

$I_{res} = I_o \cos^2\left(\frac{\pi/2}{2}\right)$

$$I_{res} = I_o \cos^2\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad I_{res} = I_o\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad I_{res} = \frac{I_0}{2}$$

(3) Path difference $\Delta x = \dfrac{\lambda}{3}$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x) \Rightarrow \Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{3}$$

$$\Delta\phi = \frac{2\pi}{3}$$

$$I_{res} = I_o \cos^2\left(\frac{\frac{2\pi}{3}}{2}\right)$$

$$I_{res} = I_0 \cos^2\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad I_{res} = I_o\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad I_{res} = \frac{I_0}{4}$$

25. (a) Distinguish, with the help of a suitable diagram, the difference in the behaviour of a conductor and a dielectric placed in an external electric field. How does polarised dielectric modify the original external field ?

(b) A capacitor of capacitance C is charged fully by connecting it to a battery of emf E. It is then disconnected from the battery. If the separation between the plates of the capacitor is now doubled, how will the following change ? **[5]**

(i) charge stored by the capacitor.

(ii) field strength between the plates.

(iii) energy stored by the capacitor.

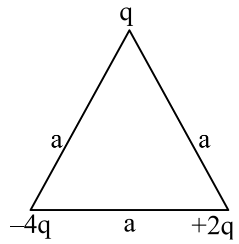
Justify your answer in each case.

OR

(a) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at that point.

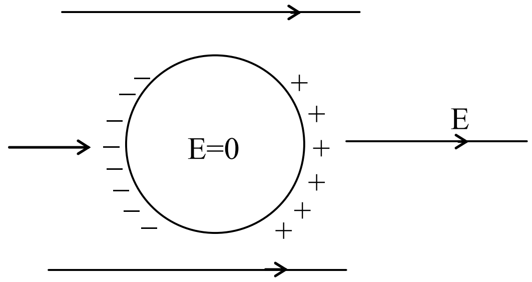
Draw a sketch of equipotential surfaces due to a single charge (–q), depicting the electric field lines due to the charge

(b) Obtained an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.

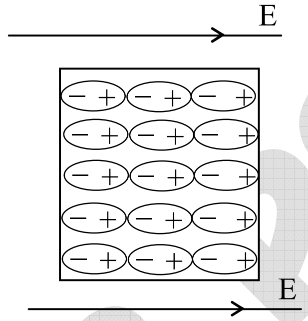


Sol. Students may find similar question in CP exercise sheet:
[(b) JEE Main, Chapter : Capacitance, Page No.26, Q.24]
OR
[(b) JEE Main, Chapter : Electrostatic, Page No. 28, Q.41]

Sol. (a) When conductor is placed in an external electric field then induction phenomena occur, due to which induced charge get develop on the conductor surface.



When dielectric is placed in an external electric field then polarization phenomena will occur.



(b) After disconnected from battery and doubling the separation between two plates

(i) charge on capacitor remains same.

i.e., $CV = C' V'$

$\Rightarrow CV = \left(\frac{C}{2}\right)V'$

$\Rightarrow V' = 2V$

(ii) $\because$ Electric field between the plates

$E' = \frac{V'}{d'} = \frac{2V}{2d}$

$E' = \frac{V}{d} = E$

$\Rightarrow$ Electric field between the two plates remains same.

(iii) Energy stored in capacitor when connected from battery

$$U_1 = \frac{q^2}{2C}$$

Now, energy stored in capacitor after disconnection from battery

$$U_2 = \frac{q^2}{2(C')} = \frac{q^2}{2\times\left(\frac{C}{2}\right)} = \frac{q^2}{C}$$

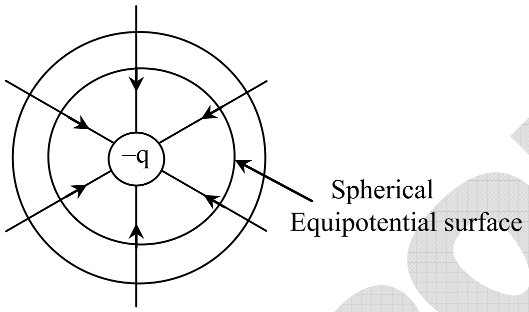
$$\Rightarrow U_2 = 2\left(\frac{q^2}{2C}\right) = 2U_1$$

$U_2 = 2\ U_1$

Energy stored in capacitor gets doubled to its initial value.

OR

(a) If the field were not normal to the equipotential surface, it would have non-zero component along the surface. To move a unit test charge against the direction of the component of the field, work would have to be done. But this is in contradiction to the definition of an equipotential surface: there is no potential difference between any two points on the surface and no work is required to move a test charge on the surface.



(b) Total electrostatic potential energy of system

$$U = U_{12} + U_{23} + U_{31}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q(-4q)}{a} + \frac{(-4q)(2q)}{a} + \frac{q(2q)}{a}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{10q^2}{a}$$

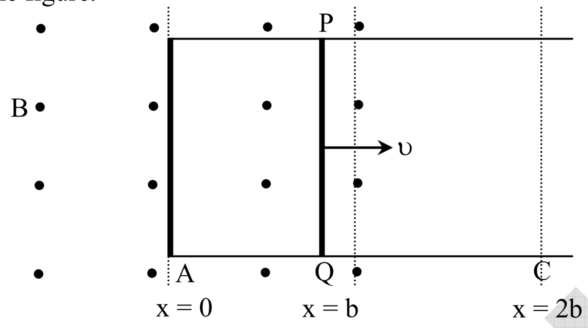
$\therefore$ Work done to dissociate the system

$$W = -U$$

$$= \frac{1}{4\pi\varepsilon_0}\ \frac{10q^2}{a}$$

26. (a) When a bar magnet is pushed towards (or away) from the coil connected to a galvanometer, the pointer in the galvanometer deflects. Identify the phenomenon causing this deflection and write the factors on which the amount and direction of the deflection depends. State the laws describing this phenomenon.

(b) Sketch the change in flux, emf and force when a conducting rod PQ of resistance R and length ℓ moves freely to and fro between A and C with speed υ on a rectangular conductor placed in uniform magnetic field as shown in the figure. **[5]**



OR

In a series LCR circuit connected to an a.c. source of voltage $v = v_m \sin\omega t$, use phasor diagram to derive an expression for the current in the circuit. hence obtained the expression for the power dissipated in the circuit. Show that power dissipated at resonance is maximum.

Sol. **Students may find similar question in CP exercise sheet:**
[(b) JEE Main, Chapter : EMI, Page No. 32, Q.47]
OR
[JEE Main, Chapter : AC, Page No. 31, Q.10]

(a) When a bar magnet pushed towards or away from coil, magnetic flux passing through coil change with time and cause induced emf hence induced current according to faraday's law of induction. Induced emf in the coil is given as

$$\boxed{\varepsilon = -\frac{Nd\phi}{dt}}$$

Induced emf and hence current depends on
(i) no. of turns in the coil (ii) motion of magnet
Direction of current depends on the motion of magnet whether moving towards coil or away from the coil.

Faraday's Laws of Electromagnetic Induction

(i) Whenever there is a change in magnetic flux linked with a coil, an emf is induced in the coil. The induced emf is proportional to the rate of change of magnetic flux linked with the coil

i.e., $\varepsilon \propto \frac{\Delta\phi}{\Delta t}$

(ii) emf induced in the coil opposes the change in flux, i.e.,

$$\varepsilon \propto -\frac{\Delta\phi}{\Delta t} \Rightarrow \varepsilon = -k\frac{\Delta\phi}{\Delta t}$$

where k is a constant of proportionality

Negative sign represents opposition to change in flux.

In SI system ϕ is in weber, t in second, ε in volt, when k = 1, $\varepsilon = -\frac{\Delta\phi}{\Delta t}$

If the coil has N-turns, then $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$

(b) Case I When PQ moves forward.

(i) For $0 \le x < b$

Magnetic field, B exists in the region.

$\therefore$ Area of loop PQRS = lx

$\therefore$ Magnetic flux linked with loop PQRS,

$\phi = BA = Blx$

$\phi = Blx$ …(i) $[b > x \ge 0]$

(ii) For, $2b \ge x \ge b$

$B = 0$

$\therefore$ Flux linked with loop PQRS is uniform and given by

$\phi' = Blb$ $(x = b)$

Forward journey

Thus, for $b > x \ge 0$

flux $\phi = Blx$

$\Rightarrow$ $\phi \propto x$

For $2b \ge x \ge b$

Flux, $\phi = Bbl$ [Constant]

Return journey

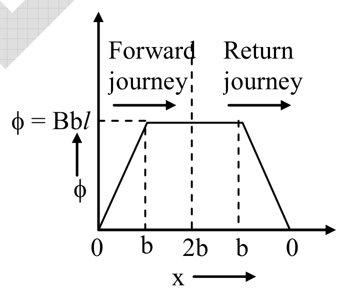
For $b \le x \le 2b$,

ϕ = constant = Bbl

For $0 \le x \le b$,

$\phi = Blx$ [Decreasing]

Graphical representation



Case II For $b > x \ge 0$

As, $\phi = Blx$

$$\Rightarrow \frac{d\phi}{dt} = Bl\frac{dx}{dt} = Bvl \left[\because v = \frac{dx}{dt}\right]$$

Induced emf $e = -\dfrac{d\phi}{dt} = -vBl$

For $2b \ge x \ge b$,

As, $\phi' = Bbl$

$\frac{d\phi'}{dt} = 0$

$\Rightarrow \quad e = 0$

Forward journey

For $b > x \ge 0$

$e = -vBl$

For $2b \ge x \ge b$

$e = 0$

Backward journey

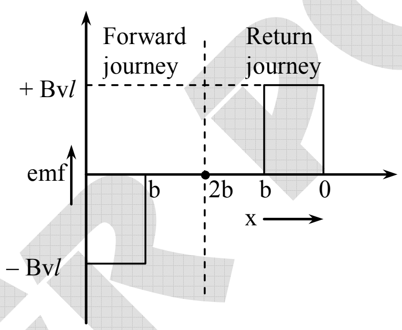
For $b > x \ge 0$

$e = vBl$

For $2b \ge x \ge b$,

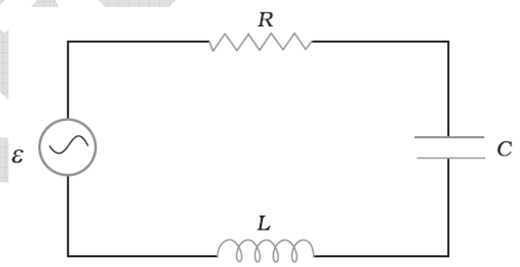
$e = 0$

Variation of induced emf.



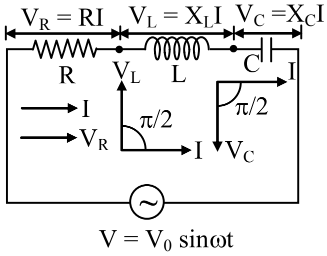
OR

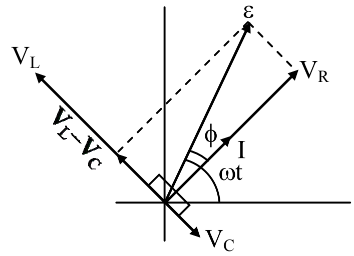
(a)



A series LCR circuit connected to an ac source

Figure shows a series LCR circuit connected to an ac source ε. As usual, we take the voltage of the source to be $v = v_m \sin \omega t$.





Assuming, $X_L > X_C$

$\Rightarrow V_L > V_C$

$\because$ Net voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

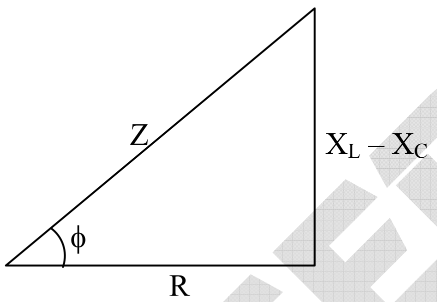
where, V_L, V_C and V_R are PD across L, C and R respectively.

But, $V_R = IR$, $V_L = IX_L$, $V_C = IX_C$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance, $Z = \dfrac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$



$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

So $I = I_m \sin(\omega t + \phi)$ where ϕ is the phase difference between voltage and current source.

Power dissipated in AC circuit :

We have seen that a voltage $v = v_m \sin\omega t$ applied to a series RLC circuit drives a current in the circuit given by $i = i_m \sin(\omega t + \phi)$ where

$$i_m = \frac{v_m}{Z} \quad \text{and } \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Therefore, the instantaneous power p supplied by the soruce is

$$p = vi = (v_m \sin \omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2}[\cos\phi - \cos[2\omega t + \phi]$$

The average power over a cycle is given by the average of the two terms in R.H.S of above equation. It is only the second term which is time-dependent. Its average is zero (the positive half of the cosine cancels the negative half). Therefore,

$$P = \frac{v_m i_m}{2}\cos\phi = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$= VI\cos\phi$$

This can also be written as

$$P = I^2 Z \cos\phi$$

Power dissipated at resonance in LCR circuit : At resonance $X_L - X_C = 0$. Therefore, $\cos\phi = 1$ and $P = I^2Z = I^2$. That is, maximum power is dissipated in a circuit (through R) at resonance.