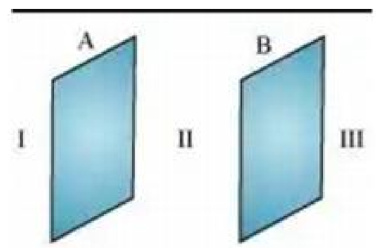
Chapter 01 and 02 [Electrostatics]

Question Number	Answers
01	A
02	D
03	C
04	D
05	A
06	A
07	B
08	A
09	A
10	D
11	A
12	A
13	B
14	C
15	A,B,C or A
16	A
17	B or A
18	C or D
19	 Surface charge density of plate A = $+17.7 \times 10^{-22}$ C/m^2 Surface charge density of plate B = -17.7×10^{-22} C/m^2 (a) In the outer region of plate I, electric field intensity E is zero. (b)Electric field intensity E in between the plates is given by relation $E = \frac{\sigma}{\epsilon_0}$ Where, ϵ_0 = Permittivity of free space = 8.85×10^{-12} N^{-1} C^2 m^{-2} $\therefore E = \frac{17.7\ x\ 10^{-22}}{8.85\ x\ 10^{-1}}$ Therefore, electric field between the plates is 2.0×10^{-10} N/C

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Net capacitance of parallel C_1 & $C_2 = C_1 + C_2$

$C_{12} = 15 + 5 = 20\,\mu F$

Net capacitance of parallel C_4 & $C_5 = C_4 + C_5$

$C_{45} = 10 + 10 = 20\,\mu F$

C_{12}, C_{45} in series, $C_{1245} = \dfrac{C_{12}C_{45}}{C_{12}+C_{45}} = \dfrac{20\times 20}{20+20} = 10\,\mu F$

C_3 in parallel with $C_{1245} = C_{1245} + C_3 = 10 + 20 = 30\,\mu F$

P.D. across $C_{1245} = 10$ V

P.D. across $C_{12} = C_{45} = 5$ V

Charge on $5\,\mu F$, $Q = CV$

$= 5 \times 10^{-6} \times 5$ C

$= \mathbf{25 \times 10^{-6}}$ **C**

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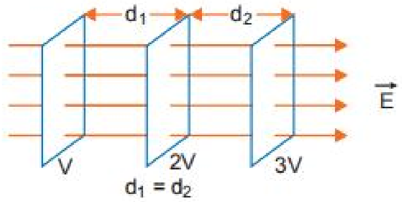
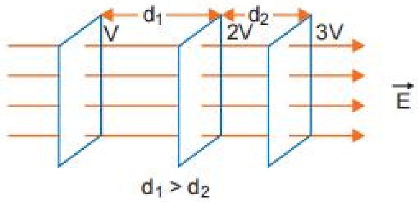
For constant electric field $\vec{E}$



For increasing electric field



Difference: For constant electric field, the equipotential surfaces are equidistant for same potential difference between these surfaces; while for increasing electric field, the separation between these surfaces decreases, in the direction of increasing field, for the same potential difference between them.

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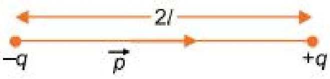
The electric dipole moment is defined as the product of either charge and the distance between the two charges. Its direction is from negative to positive charge.



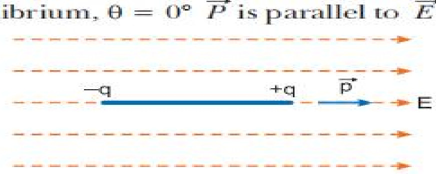
i.e., $|p| = q(2l)$

Electric dipole moment is a vector quantity.

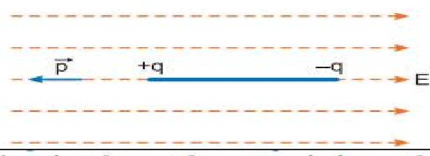
Its SI unit is coulomb-metre.

OR

(*a*) Stable equilibrium, $\theta = 0°$ $\vec{P}$ is parallel to $\vec{E}$



(*b*) Unstable equilibrium, $\theta = 180°$ $\vec{P}$ is anti parallel to $\vec{E}$



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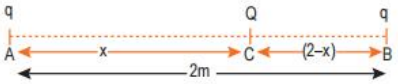
System is in equilibrium therefore net force on each charge of system will be zero.

For the total force on 'Q' to be zero

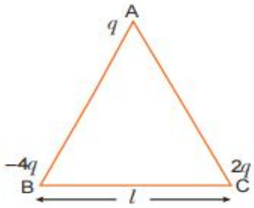
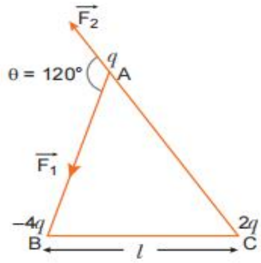
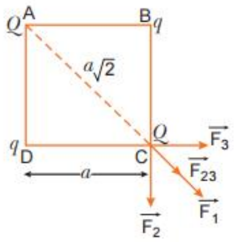


$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(2-x)^2}$$

$\Rightarrow x = 2 - x \quad \Rightarrow \quad 2x = 2$

$\Rightarrow x = \mathbf{1\ m}$

For the equilibrium of charge "q" the nature of charge Q must be opposite to the nature of charge q.

24	 Force on charge q due to the charge $-4q$ $F_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{l^2}\right)$, along AB Force on the charge q, due to the charge $2q$ $F_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{l^2}\right)$, along CA  The forces F_1 and F_2 are inclined to each other at an angle of 120° Hence, resultant electric force on charge q $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$ $= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$ $= \sqrt{F_1^2 + F_2^2 - F_1F_2}$ $= \left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\right)\sqrt{16+4-8}$ $= \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{3}q^2}{l^2}\right)$
HOTS	Let us find the force on the charge Q at the point C Force due to the other charge Q $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^2}{2a^2}\right)$ (along AC) Force due to the charge q (at B), F_2 $= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}$ along BC  Force due to the charge q (at D), F_3 $= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}$ along DC Resultant of these two equal forces $F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{qQ(\sqrt{2})}{a^2}$ (along AC) $\therefore$ Net force on charge Q (at point C) $F = F_1 + F_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2}\left[\frac{Q}{2} + \sqrt{2}q\right]$ This force is directed along AC. (For the charge Q, at the point A, the force will have the same magnitude but will be directed along CA)
25	Since two spheres are at the same potential, therefore $V_1 = V_2$ $\frac{Q_1}{4\pi\varepsilon_0 R_1} = \frac{Q_2}{4\pi\varepsilon_0 R_2}$ $\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$

	Given, $R_1 > R_2, \quad \therefore \quad Q_1 > Q_2$ ⇒ Larger sphere has more charge Now, $\sigma_1 = \frac{Q_1}{4\pi R_1^2}$ and $\sigma_2 = \frac{Q_2}{4\pi R_2^2}$ $\frac{\sigma_2}{\sigma_1} = \frac{Q_2}{Q_1} \cdot \frac{R_1^2}{R_2^2}$ $\Rightarrow \frac{\sigma_2}{\sigma_1} = \frac{R_2}{R_1} \cdot \frac{R_1^2}{R_2^2}$ [From equation (*i*)] Since $R_1 > R_2$, therefore $\sigma_2 > \sigma_1$. Charge density of smaller sphere is more than that of larger one. OR Total resistance, $R = 10\,\Omega + 20\,\Omega = 30\,\Omega$ The current, $I = \frac{V}{R} = \frac{2\text{ V}}{30\,\Omega} = \frac{1}{15}$ A Potential difference, $V = IR = \frac{1}{15} \times 10 = \frac{2}{3}$ V Charge, $q = CV = 6 \times \frac{2}{3} = \mathbf{4\ \mu C}$
26	(*i*) The capacitance of capacitor increases to K times (since $C = \frac{K\varepsilon_0 A}{d} \propto K$) (*ii*) The potential difference between the plates becomes $\frac{1}{K}$ times. **Reason:** $V = \frac{Q}{C}$; Q same, C increases to K times; $V' = \frac{V}{K}$ (*iii*) As $E = \frac{V}{d}$ and V is decreased; therefore, electric field decreases to $\frac{1}{K}$ times. (*iv*) Energy stored will be decreased. The energy becomes, $U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K}$ Thus, energy is reduced to $\frac{1}{K}$ times the initial energy.
27	(*i*) Capacitance of X, $C_X = \frac{\varepsilon_0 A}{d}$ Capacitance of Y, $C_Y = \frac{\varepsilon_r \varepsilon_0 A}{d} = 4\frac{\varepsilon_0 A}{d}$ $\therefore \quad \frac{C_Y}{C_X} = 4 \Rightarrow C_Y = 4C_X$ As X and Y are in series, so $C_{eq} = \frac{C_X C_Y}{C_X + C_Y} \Rightarrow 4\,\mu\text{F} = \frac{C_X \cdot 4C_X}{C_X + 4C_X}$ $\Rightarrow \quad C_X = \mathbf{5\ \mu F}$ and $C_Y = 4C_X = \mathbf{20\ \mu F}$ (*ii*) In series charge on each capacitor is same, so P.d. $V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$ $\therefore \quad \frac{V_X}{V_Y} = \frac{C_Y}{C_X} = 4 \Rightarrow V_X = 4V_Y$ Also $V_X + V_Y = 15$ From (*ii*) and (*iii*), $4V_Y + V_Y = 15 \Rightarrow V_Y = 3$ V $V_X = 15 - 3 = 12$ V Thus potential difference across X, $\mathbf{V_X = 12\ V}$, P.d. across Y, $\mathbf{V_Y = 3\ V}$ (*iii*) $\frac{\text{Energy stored in } X}{\text{Energy stored in } Y} = \frac{Q^2/2C_X}{Q^2/2C_Y} = \frac{C_Y}{C_X} = \frac{4}{1} \Rightarrow \frac{U_X}{U_Y} = \mathbf{\frac{4}{1}}$

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(*a*) Capacitance across C_3 & C_4

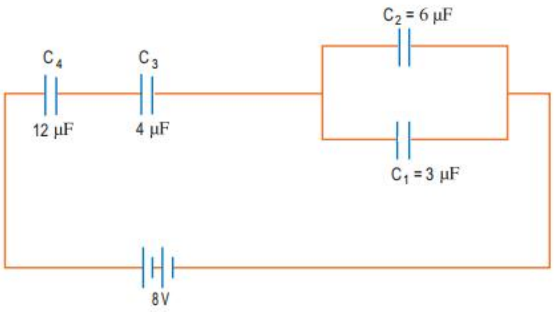
$$C_{34} = \frac{12\times 4}{16} = 3\,\mu\text{F}$$

Capacitance across C_2 & C_1

$$C_{12} = 6 + 3 = 9\,\mu\text{F}$$

Equivalent capacitance

$$C_{eq} = \frac{9\times 3}{12} = \mathbf{\frac{9\,\mu F}{4}}$$



(*b*) (*i*) $Q_1 = 6\,\mu C,\ V_1 = \frac{Q_1}{C_1}$

$$= \frac{6\times 10^{-6}}{3\times 10^{-6}} = 2\text{ V}$$

$$Q_2 = C_2V_1 = 6\times 10^{-6}\times 2 = 12\,\mu\text{C}$$

As C_3 & C_4 are in series they carry a charge of $18\,\mu$C each

(*ii*) $Q = 18\,\mu$C

$C_{43} = 3\,\mu$F

$$E_{34} = \frac{1}{2}\frac{Q^2}{C_{34}} = \frac{1}{2}\times\frac{(18\times 10^{-6})^2}{3\times 10^{-6}}$$

$$E_{34} = \mathbf{54\times 10^{-6}\,joule}$$

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Let r, q and v be the radius, charge and potential of the small drop.

The total charge on bigger drop is sum of all charge on small drops.

(*i*) $\therefore\quad Q = \boldsymbol{Nq}$ (where Q is charge on bigger drop)

(*ii*) The volume of N small drops $= N\frac{4}{3}\pi r^3$

Volume of the bigger drop $\frac{4}{3}\pi R^3$

Hence, $\quad N\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad\Rightarrow\quad R = N^{1/3}r$

Potential on bigger drop, $V = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{R}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Nq}{N^{1/3}r} = \frac{1}{4\pi\varepsilon_0}\frac{N^{2/3}.q}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.N^{2/3} = \boldsymbol{N^{2/3}v} \quad \left[\therefore v = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right]$$

(*iii*) Capacitance $= 4\pi\varepsilon_0 R$

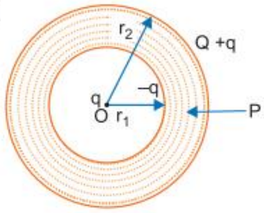
$$= 4\pi\varepsilon_0 N^{1/3} r$$

$$= N^{1/3}(4\pi\varepsilon_0 r)$$

$$= \mathbf{N^{1/3}C}$$ [where C is capacitance of the small drop]

OR

(*a*) Charge Q resides on outer surface of spherical conducting shell. Due to charge q placed at centre, charge induced on inner surface is $-q$ and on outer surface it is $+q$. So, total charge on inner surface $-q$ and on outer surface it is $Q + q$.



(*i*) Surface charge density on inner surface $= -\frac{q}{4\pi r_1^2}$

(*ii*) Surface charge density on outer surface $= \frac{Q+q}{4\pi r_2^2}$

(*b*) For external points, whole charge acts at centre, so electric field at distance $x > r_2$,

$$E(x) = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{x^2}.$$

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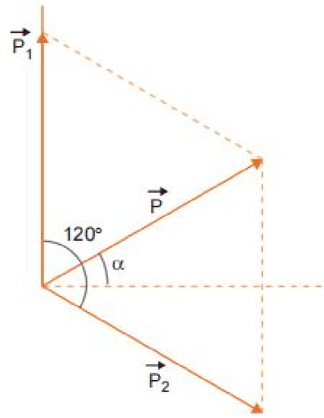
Resultant dipole moment

$$\vec{p}_r = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 120^\circ}$$

$$= \sqrt{2p^2 + 2p^2\cos 120^\circ} \quad (\because p_1 = p_2 = p)$$

$$= \sqrt{2p^2 + (2p^2)\times\left(-\frac{1}{2}\right)} = \sqrt{2p^2 - p^2} = p,$$



Using law of addition of vectors, we can see that the resultant dipole makes an angle of 60° with the y axis or 30° with x - axis.

Torque, $\vec{\tau} = \vec{p}\times\vec{E}$ ($\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$)

$$= pE\sin 30^\circ = \frac{1}{2}pE.$$

Direction of torque is into the plane of paper or along positive Z-direction.

OR

Let the charge $2q$ be placed at point P as shown. The force due to q is to the left and that due to $-3q$ is to the right.

$$\therefore \quad \frac{2q^2}{4\pi\varepsilon_0 x^2} = \frac{6q^2}{4\pi\varepsilon_0(d+x)^2} \Rightarrow (d+x)^2 = 3x^2$$

$$\therefore \quad 2x^2 - 2dx - d^2 = 0 \quad \Rightarrow \quad x = \frac{d}{2} \pm \frac{\sqrt{3}\,d}{2}$$

(–ve sign shows charge $2q$ at p would be lie between q and $-3q$ and hence is unacceptable.)

$$\Rightarrow \quad x = \frac{d}{2} + \frac{\sqrt{3}\,d}{2} = \frac{d}{2}(1+\sqrt{3}) \text{ to the left of } q.$$

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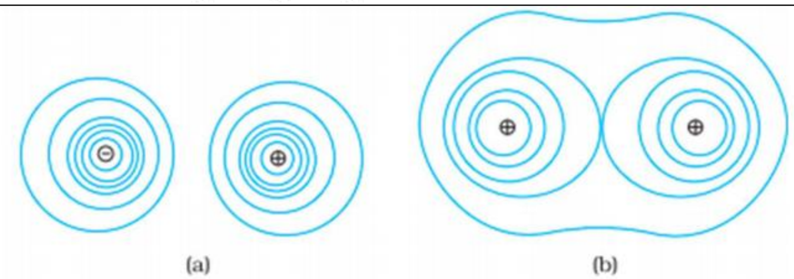

FIGURE 2.11 Some equipotential surfaces for (a) a dipole, (b) two identical positive charges.

Here, A =6 x 10^{-3} m^2, d =3mm=3x10^{-3}m
(i) Capacitance, C=ϵ_0A/d = (8.85x10^{-12} x 6x10^{-3}/3x10^{-3}) = 17.7x10^{-12} F
(ii) Charge, Q = CV =17.7 x10^{-12}x100=17.7x10^{-10}C
(iii) New charge Q'= KQ=6x17.7 x10^{-10} =1.062 x 10^{-8} C

OR

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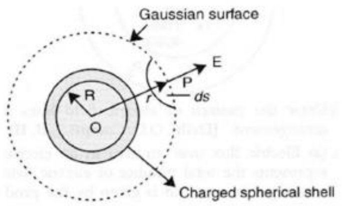
$$\frac{K(-q)Q}{x} + \frac{kQ(-q)}{x} + \frac{k(-q)(-q)}{2x} = 0$$

$$\frac{-2kqQ}{x} + \frac{kq^2}{2x} = 0 \text{ or } \frac{kq^2}{2x} = \frac{2kqQ}{x}$$

$$q = 4Q \quad \text{or} \quad \frac{Q}{q} = \frac{1}{4}$$

Electric field due to a uniformly charged thin spherical shell:



When point P lies outside the spherical shell: Suppose that we have calculate field at the point P at a distance r (r>R) from its centre. Draw Gaussian surface through point P so as to enclose the charged spherical shell. Gaussian surface is a spherical surface of radius r and centre O.

Let $\vec{E}$ be the electric field at point P, then the electric flux through area element of area $\overrightarrow{ds}$ is given by

$$d\varphi = \vec{E}.\overrightarrow{ds}$$

Since $\overrightarrow{ds}$ is also along normal to the surface

$$d\varphi = E\,dS$$

∴ Total electric flux through the Gaussian surface is given by

$$\varphi = \oint E ds = E \oint ds$$

$$\text{Now, } \oint ds = 4\pi r^2 \quad ...(i)$$

$$= E x 4\pi r^2$$

Since the charge enclosed by the Gaussian surface is q, according to the Gauss's theorem,

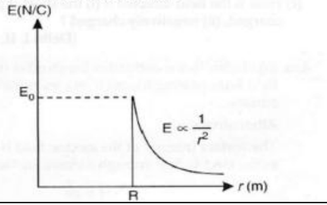
$$\varphi = \frac{q}{\epsilon_0}........(ii)$$

From equation (i) and (ii) we obtain

$$E \text{ x } 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \text{ (for r>R)}$$

A graph showing the variation of electric field as a function of *r* is shown below.



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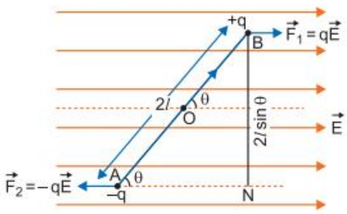
(*a*) Consider an electric dipole placed in a uniform electric field of strength E in such a way that its dipole moment $\vec{p}$ makes an angle θ with the direction of $\vec{E}$. The charges of dipole are $-q$ and $+q$ at separation $2l$ the dipole moment of electric dipole,



$$p = q2l \quad ...(i)$$

Force: The force on charge $+q$ is, $\vec{F}_1 = q\vec{E}$, along the direction of field $\vec{E}$.

The force on charge $-q$ is $\vec{F}_2 = q\vec{E}$, opposite to the direction of field $\vec{E}$.

Obviously forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude but opposite in direction; hence net force on electric dipole in uniform electric field is

$$F = F_1 - F_2 = qE - qE = 0 \text{ (zero)}$$

As net force on electric dipole is zero, so dipole does not undergo any translatory motion.

Torque: The forces $\vec{F}_1$ and $\vec{F}_2$ form a couple (or torque) which tends to rotate and align the dipole along the direction of electric field. This couple is called the torque and is denoted by τ.

$\therefore$ Torque τ = magnitude of one force × perpendicular distance between lines of action of forces

$$= qE\,(BN) = qE\,(2l \sin\theta) = (q2l)\,E \sin\theta$$

$$= pE \sin\theta \quad \text{[using (i)]} \quad ...(ii)$$

Clearly, the magnitude of torque depends on orientation (θ) of the electric dipole relative to electric field. Torque (τ) is a vector quantity whose direction is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$ given by right hand screw rule.

In vector form $\vec{\tau} = \vec{p} \times \vec{E}$ ..(*iii*)

Thus, if an electric dipole is placed in an electric field in oblique orientation, it experiences no force but experiences a torque. The torque tends to align the dipole moment along the direction of electric field.

Maximum Torque: For maximum torque sin θ should be the maximum. As the maximum value of $\sin\theta = 1$ when $\theta = 90°$

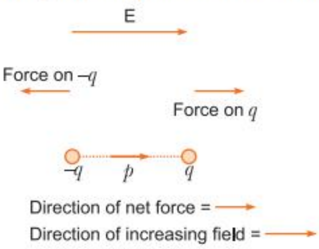
$\therefore$ Maximum torque, $\tau_{max} = pE$

When the field is non-uniform, the net force will evidently be non-zero. There will be translatory motion of the dipole.

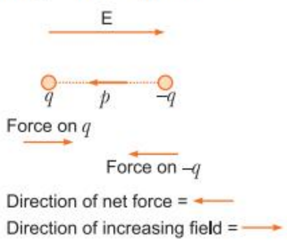
When $\vec{E}$ is parallel to $\vec{p}$, the dipole has a net force in the direction of increasing field.

When $\vec{E}$ is anti-parallel to $\vec{p}$, the net force on the dipole is in the direction of decreasing field.

In general, force depends on the orientation of $\vec{p}$ with respect to $\vec{E}$.



E parallel to p



E antiparallel to p

(*b*) Let an electric dipole be rotated in electric field from angle θ_0 to θ_1 in the direction of electric field. In this process the angle of orientation θ is changing continuously; hence the torque also changes continuously. Let at any time, the angle between dipole moment $\vec{p}$ and electric field $\vec{E}$ be θ then

Torque on dipole $\tau = pE \sin\theta$

The work done in rotating the dipole a further by small angle $d\theta$ is

$$dW = \text{Torque} \times \text{angular displacement} = pE \sin\theta \, d\theta$$

Total work done in rotating the dipole from angle θ_0 to θ_1 is given by

$$W = \int_{\theta_0}^{\theta_1} pE \sin\theta d\theta = pE[-\cos\theta]_{\theta_0}^{\theta_1}$$

$$= -pE[\cos\theta_1 - \theta_0] = pE\,(\cos\theta_0 - \cos\theta_1) \quad ..(i)$$

Special case: If electric dipole is initially in a stable equilibrium position ($\theta_0 = 0°$) and rotated through an angle $\theta(\theta_1 = \theta)$ then work done

$$W = pE[\cos 0° - \cos\theta] = pE\,(1 - \cos\theta) \quad ..(ii)$$

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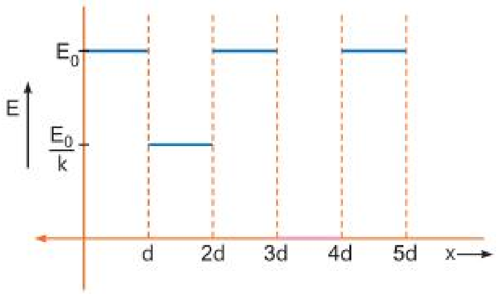
(i)

	Non-Polar (O_2)	Polar (H_2O)
(a) Absence of electric field		
Individual	No dipole moment exists	Dipole moment exists
Specimen	No dipole moment exists	Dipole are randomly oriented. Net P = 0
(b) Presence of electric field		
Individual	Dipole moment exists (molecules become polarised)	Torque acts on the molecules to align them parallel to $\vec{E}$
Specimen	Dipole moment exists	Net dipole moment exists parallel to $\vec{E}$

(ii) (a) The potential difference between the plates is given by

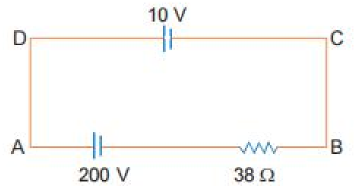
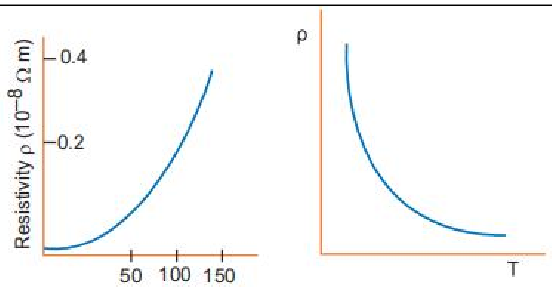
$$V = E_0 d + \frac{E_0}{K}d + E_0 d + 0 + E_0 d \quad \Rightarrow \quad V = 3E_0 d + \frac{E_0}{K}d$$

(b) E versus x graph



Chapter 03 [Current Electricity]

Question Number	Answers
01	D
02	C
03	B
04	A
05	B
06	A
07	A
08	A,B
09	D
10	B,D
11	A
12	A
13	A
14	B
15	C
16	A
17	A
18	A
19	When n resistors are connected in series, the resistance is given by $X = R + R + \dots$ upto n terms; $X = nR$. Again, when n resistors are connected in parallel, $\frac{1}{Y} = \frac{1}{R} + \frac{1}{R} + \dots$ upto n terms; $Y = \frac{R}{n}$. $\therefore \quad XY = nR \times \frac{R}{n} = R^2$
20	For balanced Wheatstone bridge, if no current flows through the galvanometer $\frac{4}{R_1} = \frac{6}{9}$ $\Rightarrow \quad R_1 = \frac{4 \times 9}{6} = 6\Omega$. For another circuit $\frac{6}{12} = \frac{R_2}{8} \Rightarrow R_2 = \frac{6 \times 8}{12} = 4\ \Omega$. $\therefore \quad \frac{R_1}{R_2} = \frac{6}{4} = \mathbf{\frac{3}{2}}$
21	(*a*) A thick copper strip offers a negligible resistance, so it does not alter the value of resistances used in the meter bridge. (*b*) If the balance point is taken in the middle, it is done to minimise the percentage error in calculating the value of unknown resistance. (*c*) Generally alloys magnin/constantan/nichrome are used in meter bridge, because these materials have low temperature coefficient of resistivity.

22	When n resistors are in series, $I = \frac{E}{R + nR}$; When n resistors are in parallel, $\frac{E}{R + \frac{R}{n}} = 10I$ $\frac{1+n}{1+\frac{1}{n}} = 10 \Rightarrow \frac{1+n}{n+1}n = 10 \quad \Rightarrow \quad n = \mathbf{10.}$
23	The resistance of filament, $R = \frac{V}{I} = \frac{V^2}{P}$ At constant voltage V, the resistance $R \propto \frac{1}{P}$ That is the resistance of filament of 100 W bulb is greater than that of 1000 W bulb. The ratio of resistances $= \frac{R_1}{R_2} = \frac{P_2}{P_1} = \frac{1000}{100} = \frac{10}{1} = \mathbf{10:1}$
24	Applying Kirchoff's law for the loop $ABCDA$, we have $+200 - 38I - 10 = 0$ $38I = 190$ $I = \frac{190}{38} = \mathbf{5\ A}$  **Alternatively:** The two cells are in opposition. ∴ Net emf = 200 V – 10 V = 190 V Now, $I = \frac{V}{R} = \frac{190\ V}{38\ \Omega} = \mathbf{5\ A}$
25	(*i*) **Drift Velocity:** The average velocity acquired by the free electrons of a conductor in a direction opposite to the externally applied electric field is called drift velocity. The drift velocity will remain the same with lattice ions/atoms. (*ii*) **Relaxation Time:** The average time of free travel of free electrons between two successive collisions is called the **relaxation time**.
26	We know that $\rho = \frac{m}{ne^2\tau}$ Where m is mass of electron ρ = charge density, τ = relaxation time e = charge on the electron.  (*i*) In case of conductors with increase in temperature, relaxation time decreases, so resistivity increases. (*ii*) In case of semiconductors with increase in temperature number density (n) of free electrons increases, hence resistivity decreases.
27	ALREADY SOLVED

28

(*a*) (*i*) Thick copper strips are used to minimize resistance of connections which are not accounted for in the bridge formula.

(*ii*) Balance point is preferred near midpoint of bridge wire to minimize percentage error in resistance (R).

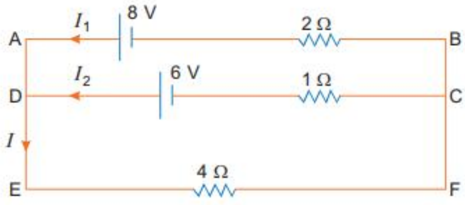
(*b*) $I = I_1 + I_2$ ...(*i*)

In loop *ABCDA*

$-8 + 2I_1 - 1 \times I_2 + 6 = 0$ ...(*ii*)

In loop *DEFCD*

$$-4I - 1 \times I_2 + 6 = 0$$
$$4I + I_2 = 6$$
$$4(I_1 + I_2) + I_2 = 6$$
$$4I_1 + 5I_2 = 6 \quad ...(iii)$$



From equations (*i*), (*ii*) and (*iii*) we get

$$I_1 = \frac{8}{7}\text{A},\ I_2 = \frac{2}{7}\text{A},\ I = \frac{10}{7}\text{A}$$

Potential difference across resistor 4Ω is:

$$V = \frac{10}{7} \times 4 = \frac{40}{7} \text{ volt}$$

29

ALREADY SOLVED

30

The acceleration, $\vec{a} = -\frac{e}{m}\vec{E}$

The average drift velocity is given by, $v_d = -\frac{eE}{m}\tau$

(τ = average time between collisions or relaxation time)

If n is the number of free electrons per unit volume, the current I is given by

$$I = neA|v_d|$$
$$= \frac{e^2A}{m}\tau n|E|$$

But $I = |j|A$ (where j= current density)

Therefore, we get

$$|j| = \frac{ne^2}{m}\tau|E|.$$

The term $\frac{ne^2}{m}\tau$ is conductivity.

$$\therefore \sigma = \frac{ne^2\tau}{m}$$

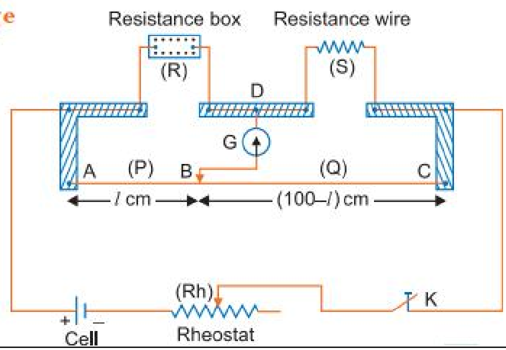
$$\Rightarrow \quad J = \sigma E$$

31

Metre Bridge: Special Case of Wheatstone Bridge

It is a practical device based on the principle of Wheatstone bridge to determine the unknown resistance of a wire.

If ratio of arms resistors in Wheatstone bridge is constant, then no current flows through the galvanometer (or bridgewire).



Construction: It consists of a uniform 1 metre long wire *AC* of constantan or manganin fixed along a scale on a wooden base (fig.) The ends *A* and *C* of wire are joined to two *L*-shaped copper strips carrying connecting screws as shown. In between these copper strips, there is a third straight copper strip having three connecting screws. The middle screw D is connected to a sensitive galvanometer. The other terminal of galvanometer is connected to a sliding jockey B. The jockey can be made to move anywhere parallel to wire AC.

Circuit: To find the unknown resistance S, the circuit is complete as shown in fig. The unknown resistance wire of resistance S is connected across the gap between points C and D and a resistance box is connected across the gap between the points A and D. A cell, a rheostat and a key (K) is connected between the points A and C by means of connecting screws. In the experiment when the sliding jockey touches the wire AC at any point, then the wire is divided into two parts. These two parts AB and BC act as the resistances P and Q of the Wheatstone bridge. In this way the resistances of arms AB, BC, AD and DC form the resistances P, Q, R and S of Wheatstone bridge. Thus the circuit of metre bridge is the same as that of Wheatstone bridge.

Method: To determine the unknown resistance, first of all key K is closed and a resistance R is taken out from resistance box in such a way that on pressing jockey B at end points A and C, the deflection in galvanometer is on both the sides. Now jockey is slided on wire at such a position that on pressing the jockey on the wire at that point, there is no deflection in the galvanometer G. In this position, the points B and D are at the same potential; therefore the bridge is balanced. The point B is called the null point. The length of both parts AB and BC of the wire are read on the scale. The condition of balance of Wheatstone bridge is

$$\frac{P}{Q} = \frac{R}{S}$$

⇒ Unknown resistance, $S = \left(\frac{Q}{P}\right)R$ ...(*i*)

To Determine Specific Resistance:

If r is the resistance per cm length of wire AC and l cm is the length of wire AB, then length of wire BC will be $(100 - l)$ cm

$\therefore P$ = resistance of wire $AB = lr$

Q = resistance of wire BC= $(100 - l)r$

Substituting these values in equation (*i*), we get

$$\text{or} \quad S = \frac{(100-l)r}{lr} \times R \quad \text{or} \quad S = \frac{100-l}{l}R \qquad ...(ii)$$

As the resistance (R) of wire (AB) is known, the resistance S may be calculated.

A number of observations are taken for different resistances taken in resistance box and S is calculated each time and the mean value of S is found.

Specific resistance $\rho = \frac{SA}{l} = \frac{S\pi r^2}{L}$

Knowing resistance S, radius r by screw gauge and length of wire L by metre scale, the value of ρ may be calculated.

If a small resistance is to be measured, all other resistances used in the circuit, including the galvanometer, should be low to ensure sensitivity of the bridge. Also the resistance of thick copper strips and connecting wires (end resistences) become comparable to the resistance to be measured. This results in large error in measurement.

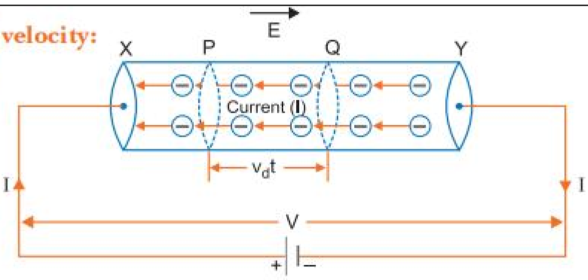
Precautions:

(*i*) In this experiment the resistances of the copper strips and connecting screws have not been taken into account. These resistances are called end-resistances. Therefore very small resistances cannot be found accurately by metre bridge. The resistance S should not be very small.

(*ii*) The current should not flow in the metre bridge wire for a long time, otherwise the wire will become hot and its resistance will be changed.

32

Relation between electric current and drift velocity:



Consider a uniform metallic wire XY of length l and cross-sectional area A. A potential difference V is applied across the ends X and Y of the wire. This causes an electric field at each point of the wire of strength

$$E = \frac{V}{l}. \qquad ...(i)$$

Due to this electric field, the electrons gain a drift velocity v_d opposite to direction of electric field. If q be the charge passing through the cross-section of wire in t seconds, then

Current in wire $I = \frac{q}{t}$ ...(ii)

The distance traversed by each electron in time t = average velocity × time = $v_d t$

If we consider two planes P and Q at a distance $v_d\, t$ in a conductor, then the total charge flowing in time t will be equal to the total charge on the electrons present within the cylinder PQ.

The volume of this cylinder = cross sectional area × height

$$= A\, v_d\, t$$

If n is the number of free electrons in the wire per unit volume, then the number of free electrons in the cylinder = $n\,(Av_d\, t)$

If charge on each electron is $-e$ ($e = 1.6 \times 10^{-19}$C), then the total charge flowing through a cross-section of the wire

$$q = (nAv_d t)(-e) = -neAv_d t \quad ..(iii)$$

∴ Current flowing in the wire,

$$I = \frac{q}{t} = \frac{-neAv_d\, t}{t}$$

i.e., current $I = -neAv_d$ ...(iv)

This is the relation between electric current and drift velocity. Negative sign shows that the direction of current is opposite to the drift velocity.

Numerically $I = -neAv_d$ ...(v)

∴ Current density, $J = \frac{I}{A} = nev_d$

⇒ $J \propto v_d$.

That is, current density of a metallic conductor is directly proportional to the drift velocity.

33

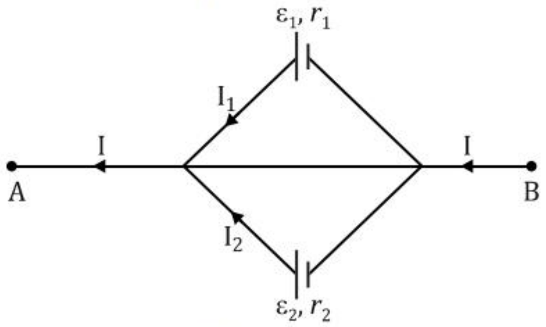
Drift velocity: It is the average velocity acquired by the free electrons superimposed over the random motion in the direction opposite to electric field and along the length of the metallic conductor.
Derivation I = ne A V_d

Here, $I = I_1 + I_2$ ...(*i*)

Let V = Potential difference between A and B.

For cell ε_1

Then, $V = \varepsilon_1 - I_1 r_1 \Rightarrow I_1 = \frac{\varepsilon_1 - V}{r_1}$



Similarly, for cell ε_2 $I_2 = \frac{\varepsilon_2 - V}{r_2}$

Putting these values in equation (*i*)

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

or $$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or $$V = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \quad ...(ii)$$

Comparting the above equation with the equivalent circuit of emf 'ε_{eq}' and internal resistance 'r_{eq}' then,

$$V = \varepsilon_{eq} - I r_{eq} \quad ...(iii)$$

Then

(*i*) $\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$ (*ii*) $r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$

(*iii*) The potential difference between A and B

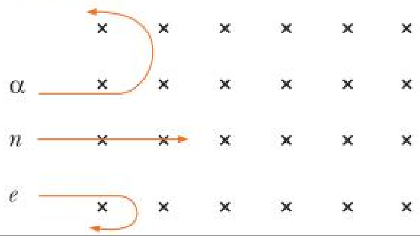
$$V = \varepsilon_{eq} - I r_{eq}$$

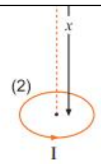
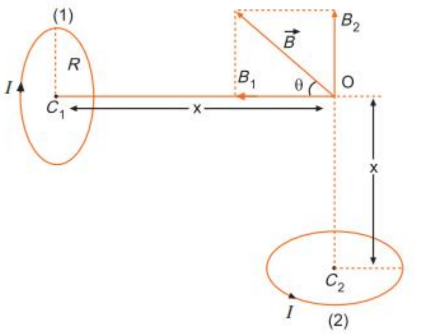
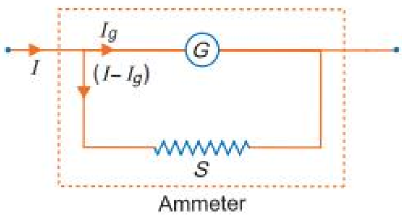
OR

Junction rule: At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction

Loop rule: The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero
Derivation

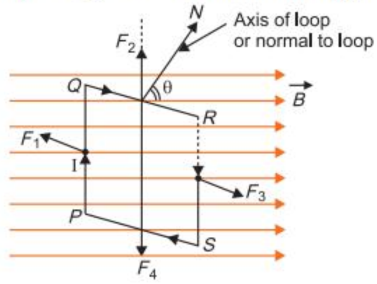
Chapter 04 and 05 [Magnetostatics]

Question Number	Answers
01	C
02	A
03	D
04	D
05	C
06	A
07	C
08	C
09	B
10	B
11	A,B
12	C
13	B
14	A
15	B
16	C
17	D
18	A
19	ALREADY SOLVED
20	Here, $B_H = B$ and $\delta = 60°$ We know that $B_H = B_E \cos\delta$ $B = B_E \cos 60° \quad \Rightarrow B_E = 2B$ At equator $\delta = 0°$ $\therefore \quad B_H = 2B\cos 0° = 2B$
21	ALREADY SOLVED
22	(i) $\vec{F} = q(\vec{v} \times \vec{B})$ (ii) Force on alpha particle and electron are opposite to each other, magnitude of mass per charge ratio of alpha particle is more than electron (*i.e.*, $r \propto \frac{m}{q}$) hence radius of alpha particle is more than radius of electron. 
23	Charge on deutron (q_d) = charge on proton (q_p) $q_d = q_p$ Radius of circular path $(r) = \frac{P}{Bq} \quad \left(\because qvB = \frac{mv^2}{r}\right)$ $r \propto \frac{1}{q}$ [for constant momentum (P)] So, $\frac{r_p}{r_d} = \frac{q_d}{q_p} = \frac{q_p}{q_p} = 1$ Hence, $r_p : r_d = 1 : 1$

24	A - diamagnetic B- paramagnetic The magnetic susceptibility of A is small negative and that of B is small positive.
25	A diamagnetic specimen would move towards the weaker region of the field while a paramagnetic specimen would move towards the stronger region.
26	Diagram Derivation The ampere is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, and placed one metre apart in vacuum, would exert on each of these conductors a force equal to 2×10^{-7}newtons per metre of length.
27	Magnetic field due to coil 1 at point O $\vec{B}_1 = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$ along $\overrightarrow{OC_1}$  Magnetic field due to coil 2 at point O $\vec{B}_2 = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$ along $\overrightarrow{C_2O}$ Both $\vec{B}_1$ and $\vec{B}_2$ are mutually perpendicular, so the net magnetic field at O is $B = \sqrt{B_1^2+B_2^2} = \sqrt{2}B_1 \text{(as } B_1 = B_2)$ $= \sqrt{2}\frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$ As $R << x$ $B = \frac{\sqrt{2}\mu_0 I R^2}{2.x^3} = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}.I(\pi R^2)}{x^3}$ $= \frac{\mu_0}{4\pi}\frac{2\sqrt{2}.IA}{x^3}$  where $A = \pi R^2$ is area of loop. $\tan\theta = \frac{B_2}{B_1} \Rightarrow \tan\theta = 1 \quad (\because B_2 = B_1)$ $\Rightarrow \quad \theta = \frac{\pi}{4}$ $\therefore$ $\vec{B}$ is directed at an angle $\frac{\pi}{4}$ with the direction of magnetic field $\vec{B}_1$.
28	ALREADY SOLVED
29	(*a*) By connecting a small resistance called shunt (S) in parallel to coil of the galvanometer. The value of S is related to the maximum current (I) to be measured as $S = \frac{I_g G}{I - I_g}$. (*b*) Given, $G = 15\,\Omega$ $I_g = 4\times10^{-3}$A $I = 6$ A $\because$ $I_g G = (I - I_g)S$ $S = \frac{I_g G}{I - I_g} = \frac{4\times10^{-3}\times15}{6-4\times10^{-3}}$ $= 0.01\,\Omega$  The galvanometer can be converted into ammeter of given range by connecting a shunt resistance of 0.01 Ω in parallel.
30	ALREADY SOLVED

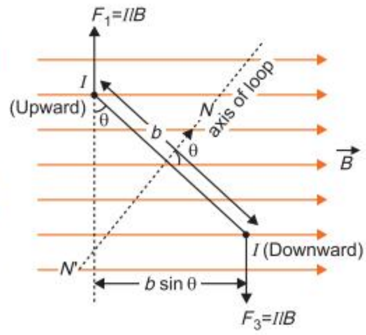
31

Torque on a current carrying loop: Consider a rectangular loop $PQRS$ of length l, breadth b suspended in a uniform magnetic field $\vec{B}$. The length of loop = $PQ = RS = l$ and breadth $QR = SP = b$. Let at any instant the normal to the plane of loop make an angle θ with the direction of magnetic field $\vec{B}$ and I be the current in the loop. We know that a force acts on a current carrying wire placed in a magnetic field. Therefore, each side of the loop will experience a force. The net force and torque acting on the loop will be determined by the forces acting on all sides of the loop. Suppose that the forces on sides PQ, QR, RS and SP are $\vec{F}_1, \vec{F}_2, \vec{F}_3$ and $\vec{F}_4$ respectively. The sides QR and SP make angle $(90° - \theta)$ with the direction of magnetic field. Therefore each of the forces $\vec{F}_2$ and $\vec{F}_4$ acting on these sides has same magnitude $F' = Blb \sin(90° - \theta) = Blb \cos\theta$. According to Fleming's left hand rule the forces F_2 and F_4 are equal and opposite but their line of action is same. Therefore these forces cancel each other *i.e.*, the resultant of $\vec{F}_2$ and $\vec{F}_4$ is zero.



The sides PQ and RS of current loop are perpendicular to the magnetic field, therefore the magnitude of each of forces $\vec{F}_1$ and $\vec{F}_3$ is $F = IlB \sin 90° = IlB$

According to Fleming's left hand rule the forces $\vec{F}_1$ and $\vec{F}_3$ acting on sides PQ and RS are equal and opposite, but their lines of action are different; therefore the resultant force of $\vec{F}_1$ and $\vec{F}_3$ is zero, but they form a couple called the ***deflecting couple***. When the normal to plane of loop makes an angle with the direction of magnetic field the perpendicular distance between F_1 and F_3 is $b \sin\theta$.



∴ Moment of couple or Torque,

$$\tau = (\text{Magnitude of one force F}) \times \text{perpendicular distance} = (BIl).(b \sin\theta) = I(lb) B \sin\theta$$

But lb = area of loop $= A$ (say)

∴ Torque, $\tau = IAB \sin\theta$

If the loop contains N-turns, then $\tau = NIAB \sin\theta$

In vector form $\vec{\tau} = NI\vec{A} \times \vec{B}$

The magnetic dipole moment of rectangular current loop $= M = NIA$

$$\therefore \quad \vec{\tau} = \vec{M} \times \vec{B}$$

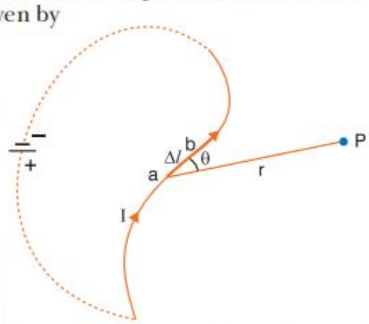
Direction of torque is perpendicular to direction of area of loop as well as the direction of magnetic field *i.e.*, along $IA \times B$.

The current loop would be in stable equilibrium, if magnetic dipole moment is in the direction of the magnetic field $(\vec{B})$.

32

Biot-Savart law: Suppose the current I is flowing in a conductor and there is a small current element '*ab*' of length Δl. According to Biot-Savart the magnetic field (ΔB) produced due to this current element at a point P distant r from the element is given by

$$\Delta B \propto \frac{I\Delta l\sin\theta}{r^2} \text{ or } \Delta B = \frac{\mu}{4\pi}\frac{I\,\Delta l\sin\theta}{r^2} \quad ...(i)$$



where $\frac{\mu}{4\pi}$ is a constant of proportionality. It depends on the medium between the current element and point of observation (P). μ is called the permeability of medium. Equation (i) is called Biot-Savart law. The product of current (I) and length element (Δl) (*i.e.*, $I\ \Delta l$) is called the **current element**. Current element is a vector quantity, its direction is along the direction of current. If the conductor be placed in vacuum (or air), then μ is replaced by μ_0; where μ_0 is called the permeability of free space (or air). In S.I. system $\mu_0 = 4\pi \times 10^{-7}$ weber/ampere-metre (or newton/ampere^2).

Thus $\frac{\mu_0}{4\pi} = 10^{-7}$ weber/ampere × metre

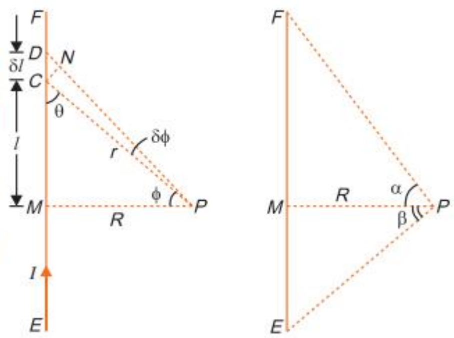
As in most cases the medium surrounding the conductor is air, therefore, in general, Biot-Savart law is written as

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\Delta l\sin\theta}{r^2}$$

The direction of magnetic field is perpendicular to the plane containing current element and the line joining point of observation to current element. So in vector form the expression for magnetic field takes the form

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\vec{r}}{r^3}$$

Derivation of formula for magnetic field due to a current carrying wire using Biot-Savart law: Consider a wire EF carrying current I in upward direction. The point of observation is P at a finite distance R from the wire. If PM is perpendicular dropped from P on wire; then $PM = R$. The wire may be supposed to be formed of a large number of small current elements. Consider a small element CD of length δl at a distance l from M.



(a) (b)

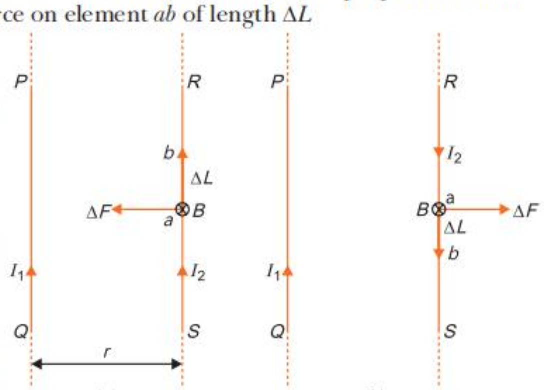
Let $\angle CPM = \phi$

and $\angle CPD = \delta\phi$, $\angle PDM = \theta$

The length δl is very small, so that $\angle PCM$ may also be taken equal to θ.

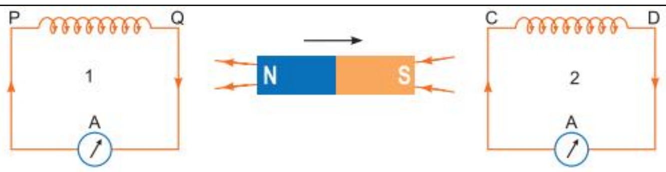
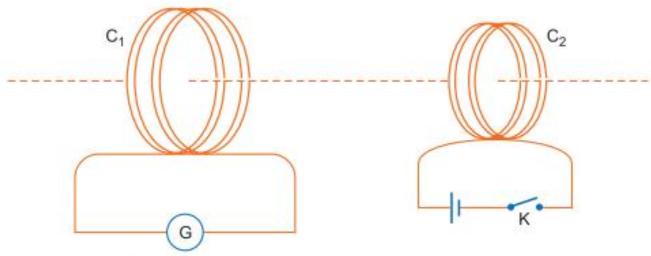
The perpendicular dropped from C on PD is CN. The angle formed between element $I\,\vec{\delta l}$ and $\vec{r}\,(=\vec{CP})$ is $(\pi - \theta)$. Therefore according to Biot-Savart law, the magnetic field due to current element $I\vec{\delta l}$ at P is

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\,\delta l\sin(\pi-\theta)}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\delta l\sin\theta}{r^2} \quad ...(i)$$

	But in Δ *CND*, $\sin\theta = \sin(\angle CDN) = \frac{CN}{CD} = \frac{r\,\delta\phi}{\delta l}$ or $\delta l \sin\theta = r\,\delta\phi$ $\therefore$ From equation (*i*) $\delta B = \frac{\mu_0}{4\pi}\frac{I r\,\delta\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\delta\phi}{r}$ ...(*ii*) Again from fig. $\cos\phi = \frac{R}{r} \Rightarrow r = \frac{R}{\cos\phi}$ From equation (*ii*) $\delta B = \frac{\mu_0}{4\pi}\frac{I\cos\phi\,\delta\phi}{R}$ ...(*iii*) If the wire is of finite length and its ends make angles α and β with line *MP*, then net magnetic field (*B*) at *P* is obtained by summing over magnetic fields due to all current elements, *i.e.*, $B = \int_{-\beta}^{\alpha}\frac{\mu_0}{4\pi}\frac{I\cos\phi\,d\phi}{R} = \frac{\mu_0 I}{4\pi R}\int_{-\beta}^{\alpha}\cos\phi\,d\phi$ $\frac{\mu_0 I}{4\pi R}[\sin\phi]_{-\beta}^{\alpha} = \frac{\mu_0 I}{4\pi R}[\sin\alpha - \sin(-\beta)]$ *i.e.*, $B = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$ This is expression for magnetic field due to current carrying wire of finite length. **If the wire is of infinite length** (or very long), then $\alpha = \beta \Rightarrow \pi/2$ $\therefore$ $B = \frac{\mu_0 I}{4\pi R}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = \frac{\mu_0 I}{4\pi R}[1+1]$ or $B = \frac{\mu_0 I}{2\pi R}$
33	Suppose two long thin straight conductors (or wires) *PQ* and *RS* are placed parallel to each other in vacuum (or air) carrying currents I_1 and I_2 respectively. It has been observed experimentally that when the currents in the wire are in the same direction, they experience an attractive force (fig. *a*) and when they carry currents in opposite directions, they experience a repulsive force (fig. *b*). Let the conductors *PQ* and *RS* carry currents I_1 and I_2 in same direction and placed at separation *r*. Consider a current–element '*ab*' of length ΔL of wire *RS*. The magnetic field produced by current-carrying conductor *PQ* at the location of other wire *RS* $B_1 = \frac{\mu_0 I_1}{2\pi r}$ ...(*i*) According to Maxwell's right hand rule or right hand palm rule number 1, the direction of B_1 will be perpendicular to the plane of paper and directed downward. Due to this magnetic field, each element of other wire experiences a force. The direction of current element is perpendicular to the magnetic field; therefore the magnetic force on element *ab* of length ΔL $\Delta F = B_1 I_2 \Delta L \sin 90^o = \frac{\mu_0 I_1}{2\pi r} I_2 \Delta L$ $\therefore$ The total force on conductor of length *L* will be $F = \frac{\mu_0 I_1 I_2}{2\pi r}\Sigma\Delta L = \frac{\mu_0 I_1 I_2}{2\pi r}L$ $\therefore$ Force acting per unit length of conductor $f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ N/m ...(*ii*) According to Fleming's left hand rule, the direction of magnetic force will be towards *PQ* *i.e.*, the force will be attractive.  (*a*) (*b*)

	On the other hand if the currents I_1 and I_2 in wires are in opposite directions, the force will be repulsive. The magnitude of force in each case remains the same. **Definition of SI unit of Current (ampere):** In SI system of fundamental unit of current 'ampere' is defined assuming the force between the two current carrying wires as standard. The force between two parallel current carrying conductors of separation r is $f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ N/m If $I_1 = I_2 = 1$ A, $r = 1$ m, then $f = \frac{\mu_0}{2\pi} = \mathbf{2 \times 10^{-7}\, N/m}$ Thus 1 ampere is the current which when flowing in each of parallel conductors placed at separation 1 m in vacuum exert a force of 2×10^{-7} on 1 m length of either wire.

Chapter 06 and 07 [E.M.I and A.C.]

Question Number	Answers
01	B
02	C
03	B
04	A
05	B
06	B
07	C
08	B
09	B
10	D
11	A
12	A
13	B
14	B
15	C
16	C
17	A
18	B
19	ALREADY SOLVED
20	 In figure, N-pole is receding away coil (PQ), so in coil (PQ), the nearer faces will act as S-pole and in coil (CD) the nearer face will also act as S-pole to oppose the approach of magnet towards coil (CD), so currents in coils will flow clockwise as seen from the side of magnet. The direction of current will be from P to Q in coil (PQ) and from C to D in coil (CD).
21	Loop *abc* is entering the magnetic field, so magnetic flux linked with it begins to increase. According to Lenz's law, the current induced opposes the increases in magnetic flux, so current induced will be **anticlockwise** which tends to decrease the magnetic field. Loop *defg* is leaving the magnetic field; so flux linked with it tends to decrease, the induced current will be **clockwise** to produce magnetic field downward to oppose the decrease in magnetic flux.
22	 (*a*) The deflection in galvanometer may be made large by (*i*) moving coil C_2 towards C_1 with high speed. (*ii*) by placing a soft iron laminated core at the centre of coil C_1. (*b*) The induced current can be demonstrated by connecting a torch bulb (in place of galvanometer) in coil C_1. Due to induced current the bulb begins to glow.

23

Given, $L = 1.0$ H; $C = 20\ \mu F = 20 \times 10^{-6}$ F

$R = 300\ \Omega$; $V_{rms} = 50$ V; $\nu = \frac{50}{\pi}$ Hz

Inductive reactance $X_L = \omega L = 2\pi\nu L = 2 \times \pi \times \frac{50}{\pi} \times 1 = 100\ \Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times \pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} = 500\ \Omega$

Impedance of circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(300)^2 + (500-100)^2} = \sqrt{90000 + 160000} = \sqrt{250000} = 500\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{50}{500} = \mathbf{0.1\ A}$$

24

In given *ac*, there are identical positive and negative half cycles, so the mean value of current is zero; but the rms value is not zero.

$$(I^2)_{mean} = \frac{\int_0^T I^2\,dt}{\int_0^T dt} = \frac{\int_0^{T/2} (2)^2\,dt + \int_{T/2}^{T} (-2)^2\,dt}{T} = \frac{\int_0^T 4\,dt}{T} = 4$$

$$I_{rms} = \sqrt{4} = \mathbf{2\ A}$$

25

ALREADY SOLVED

26

Area of the circular loop $= \pi r^2$

$$= 3.14 \times (0.12)^2\ m^2 = 4.5 \times 10^{-2}\ m^2$$

$$E = -\frac{d\varphi}{dt} = -\frac{d}{dt}(BA) = -A\,\frac{dB}{dt} = -A\,.\,\frac{B_2 - B_1}{t_2 - t_1}$$

For $0 < t < 2$s

$$E_1 = -4.5 \times 10^{-2} \times \left\{\frac{1-0}{2-0}\right\} = -2.25 \times 10^{-2}\ V$$

$$\therefore\quad I_1 = \frac{E_1}{R} = \frac{-2.25 \times 10^{-2}}{8.5}\ A = -2.6 \times 10^{-3}\ A = -2.6\ mA$$

For $2s < t < 4s$,

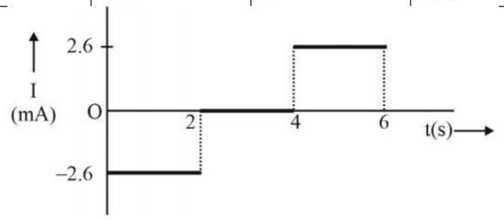
$$E_2 = -4.5 \times 10^{-2} \times \left\{\frac{1-1}{4-2}\right\} = 0$$

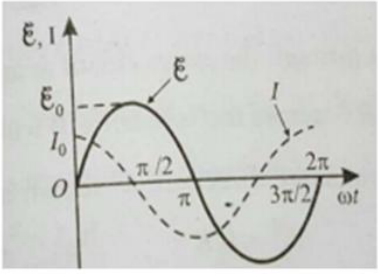
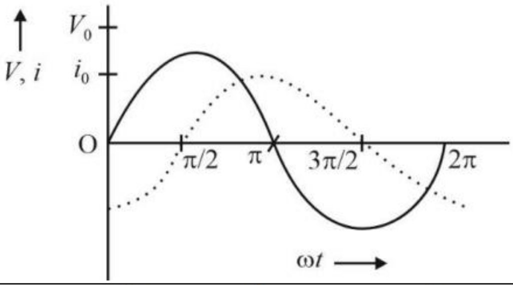
$$\therefore\quad I_2 = \frac{E_2}{R} = 0$$

For $4s < t < 6s$,

$$I_3 = -\frac{4.5 \times 10^{-2}}{8.5} \times \left\{\frac{0-1}{6-4}\right\} A = 2.6\ mA$$

	0<t<2s	2<t<4s	4<t<6s
E(V)	-0.023	0	+0.023
I(mA)	-2.6	0	+2.6



27	Derivation  **OR** Derivation 
28	(*a*) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 80\times 10^{-6}}} = \frac{1}{\sqrt{400\times 10^{-6}}}$ $\omega = \frac{1000}{20} = 50 \text{ rad/s} \Rightarrow f = \frac{\omega}{2\pi} = \frac{50}{2\pi} = \mathbf{\frac{25}{\pi}}$ **Hz** (*b*) At resonance, $Z = R =$ **40 Ω** $I_{max} = \frac{230\sqrt{2}}{R} = \frac{230\sqrt{2}}{40} =$ **8.1 A** (*c*) $V_C = I_{max}X_C = \frac{230\sqrt{2}}{40}\times\frac{1}{50\times 80\times 10^{-6}} = 2025 \text{ V}$ $\quad [\because X_C = \frac{1}{\omega C}]$ $V_L = I_{max}X_L = \frac{230\sqrt{2}}{40}\times 50\times 5 = 2025 \text{ V}$ $\quad [\because X_L = \omega L]$ $V_C - V_L = 0$
29	(*a*) *A* (*b*) Zero (*c*) *L* or *C* or *LC* Series combination of *L* and *C*

30

Self inductance – Using formula $\phi = LI$, if $I = 1$ Ampere then $L = \phi$

Self inductance of the coil is equal to the magnitude of the magnetic flux linked with the coil, when a unit current flows through it.

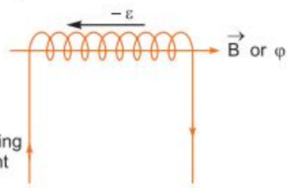
Alternatively

Using formula $|-\varepsilon| = L\frac{dI}{dt}$

If $\frac{dI}{dt} = 1$ A/s then $L = |-\varepsilon|$

Self inductance of the coil is equal to the magnitude of induced emf produced in the coil itself, when the current varies at rate 1 A/s.

Expression for magnetic energy



When a time varying current flows through the coil, back *emf* $(-\varepsilon)$ produces, which opposes the growth of the current flow. It means some work needs to be done against induced emf in establishing a current I. This work done will be stored as magnetic potential energy.

For the current I at any instant, the rate of work done is

$$\frac{dW}{dt} = (-\varepsilon) I$$

Only for inductive effect of the coil $|-\varepsilon| = L\frac{dI}{dt}$

$$\therefore \quad \frac{dW}{dt} = L\left(\frac{dI}{dt}\right)I \quad \Rightarrow \quad dW = LI\,dI$$

From work-energy theorem

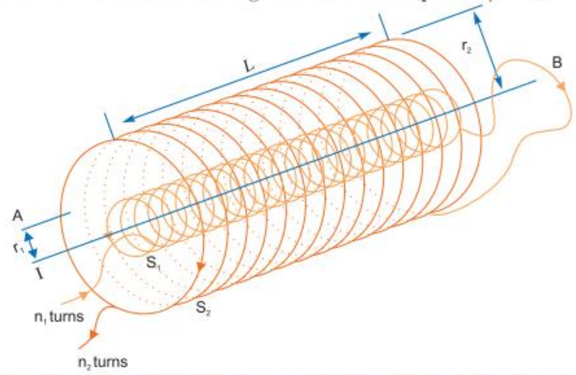
$$dU = LI\,dI$$

$$\therefore \quad U = \int_0^I LI dI = \frac{1}{2}LI^2$$

31

(*a*) When current flowing in one of two nearby coils is changed, the magnetic flux linked with the other coil changes; due to which an emf is induced in it (other coil). This phenomenon of electromagnetic induction is called the mutual induction. The coil, in which current is changed is called the primary coil and the coil in which emf is induced is called the secondary coil. **The SI unit of mutual inductance is henry.**

(*b*) Mutual inductance is numerically equal to the magnetic flux linked with one coil (secondary coil) when unit current flows through the other coil (primary coil).



Consider two long co-axial solenoids, each of length L. Let n_1 be the number of turns per unit length of the inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of the outer solenoid S_2 of radius r_2.

Imagine a time varying current I_2 through S_2 which sets up a time varying magnetic flux ϕ_1 through S_1.

$$\therefore \quad \phi_1 = M_{12}(I_2) \quad \quad ...(i)$$

where, M_{12} = Coefficient of mutual inductance of solenoid S_1 with respect to solenoid S_2

Magnetic field due to the current I_2 in S_2 is

$$B_2 = \mu_0 n_2 I_2$$

$\therefore$ Magnetic flux through S_1 is

$$\phi_1 = B_2 A_1 N_1$$

where, $N_1 = n_1 L$ and L = length of the solenoid

$$\phi_1 = (\mu_0 n_2 I_2)(\pi r_1^2)(n_1 L)$$

$$\phi_1 = \mu_0 n_1 n_2 \pi r_1^2 L I_2 \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 L \quad ...(iii)$$

Let us consider the reverse case.

A time varying current I_1 through S_1 develops a flux ϕ_2 through S_2.

$$\therefore \quad \phi_2 = M_{21}(I_1) \quad ...(iv)$$

where, M_{21} = Coefficient of mutual inductance of solenoid S_2 with respect to solenoid S_1

Magnetic flux due to I_1 in S_1 is confined solely inside S_1 as the solenoids are assumed to be very long.

There is no magnetic field outside S_1 due to current I_1 in S_1.

The magnetic flux linked with S_2 is

$$\therefore \quad \phi_2 = B_1 A_1 N_2 = (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 L)$$

$$\phi_2 = \mu_0 n_1 n_2 \pi r_1^2 L I_1 \quad ...(v)$$

From equations (*iv*) and (*v*), we get

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 \quad ...(vi)$$

From equations (*iii*) and (*vi*), we get

$$M_{12} = M_{21} = M = \mu_0 n_1 n_2 \pi r_1^2 L$$

We can write the above equation as

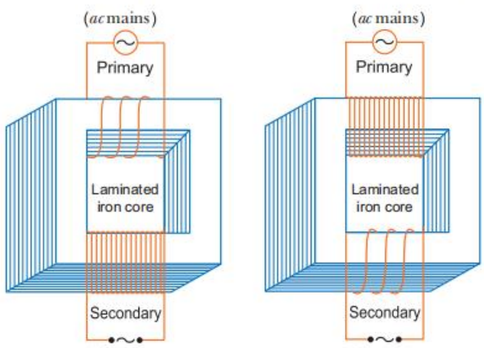
$$M = \mu_0 \left(\frac{N_1}{L}\right)\left(\frac{N_2}{L}\right)\pi r^2 \times L$$

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{L}$$

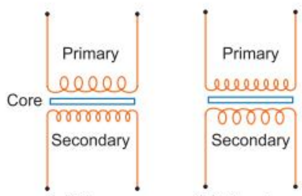
32

(*a*) **Transformer:** A transformer converts low voltage into high voltage *ac* and vice-versa.

Construction: It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is called *primary coil* and other is called the *secondary coil*. The terminals of primary coils are connected to *ac* mains and the terminals of the secondary coil are connected to external circuit in which alternating current of desired voltage is required. Transformers are of two types:



(a) Step up Transformer (b) Step down Transformer



(a) Step up (b) Step down

1. **Step up Transformer:** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil (*i.e.*, $N_S > N_P$).
2. **Step down Transformer:** It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (*i.e.*, $N_S < N_P$).

Working: When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let N_P be the number of turns in primary coil, N_S the number of turns in secondary coil and ϕ the magnetic flux linked with each turn. ***We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same.*** According to Faraday's laws the emf induced in the primary coil

$$\varepsilon_P = -N_P \frac{\Delta\phi}{\Delta t} \quad ...(i)$$

and emf induced in the secondary coil

$$\varepsilon_S = -N_S \frac{\Delta\phi}{\Delta t} \quad ...(ii)$$

From (*i*) and (*ii*)

$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} \quad ...(iii)$$

If the resistance of primary coil is negligible, the emf (ε_P) induced in the primary coil, will be equal to the applied potential difference (V_P) across its ends. Similarly if the secondary circuit is open, then the potential difference V_S across its ends will be equal to the emf (ε_S) induced in it; therefore

$$\frac{V_S}{V_P} = \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} = r(\text{say}) \quad ...(iv)$$

where $r = \frac{N_S}{N_P}$ is called the transformation ratio. If i_P and i_S are the instantaneous currents in primary and secondary coils and there is no loss of energy; then

For about 100% efficiency, Power in primary = Power in secondary

$$V_P\, i_P = V_S\, i_S$$

$$\frac{i_S}{i_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{1}{r} \quad ...(v)$$

In step up transformer, $N_S > N_P \rightarrow r > 1$;

So $V_S > V_P$ and $i_S < i_P$

i.e., step up transformer increases the voltage, but decreases the current.

In step down transformer, $N_S < N_P \rightarrow r < 1$

so $V_S < V_P$ and $i_S > i_P$

i.e., step down transformer decreases the voltage, but increases the current.

(*ii*) Given, $V_P = 2200$ V

$N_P = 3000$ turns

$V_S = 220$ V

We have, $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$$N_S = \frac{V_S}{V_P} \times N_P$$

$$= \frac{220}{2200} \times 3000$$

N_S = **300 turns**

33

(*a*) The device '*X*' is a capacitor.

(*b*) Curve B : Voltage

Curve C : Current

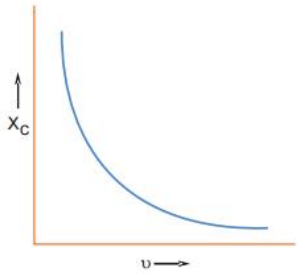
Curve A : Power consumed in the circuit

Reason : This is because current leads the voltage in phase by $\frac{\pi}{2}$ for a capacitor.

(*c*) Impedance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

$$\Rightarrow \qquad X_C \propto \frac{1}{\nu}$$

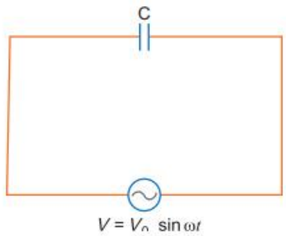


(*d*) Voltage applied to the circuit is

$$V = V_0 \sin \omega t$$

Due to this voltage, a charge will be produced which will charge the plates of the capacitor with positive and negative charges.

$$V = \frac{Q}{C} \qquad \Rightarrow \qquad Q = CV$$



Therefore, the instantaneous value of the current in the circuit is

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$$

$$\therefore \qquad I = \omega C V_0 \cos \omega t = \frac{V_0}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where, $I_0 = \frac{V_0}{\frac{1}{\omega C}}$ = Peak value of current

Hence, current leads the voltage in phase by $\frac{\pi}{2}$.