International Journal of Sciences

Research Article Volume 11 – October 2022 (10)

Using Lagrange's Multiplier Method to Solve Mathematical Contest Problem

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Dedicated to:
My Father Dr.prof. Mohammad Ali
My Mother And Teacher of all time Dr. Ruma Ali
My 2'nd Father :Engr.Ali Ahmed Jewel
My 2'nd mother and teacher of my early age Surraya Ahmed

Abstract: "Most people study mathematics to satisfy some requirements .Some Study Math to learn the tricks of nature so they may find out how to make things bigger or smaller or faster or more sensitive . But a few ,a very few ,study math because they wonder -not how things work ,but why they work .They wonder what is at the bottom of things -the very bottom ,if there is a bottom . This paper will be very useful to them ."-M.A.Rusho. Remember when you first participate any math contest or olympiad maybe national or International you see some terrible function and the question state that you have to find this function maximum or minimum value. You become frustrated thinking I have to take the derivative first then equalising it to 0 , then find the optimal value of X blah blah blah……. . now it's time to reconstruct your mind for solving this type of Mathematical problem in a suitably way . First we have to think that is the difference between School textbook math problem And Olympiad Math Problem . The core difference is problem solving . But you may think that in school mathematics we all solve problem . But this problem solving is different like you can solve it by GUIDE or may a teacher can help or this problem may directly solved by any standard formula like(a+b)^2 . But in Olympiad problem You are totally in a new place , totally difference problem you didn't see it . But you have too slove it now Many people find this difficulty .BUT IT IS not difficult if you solve it by making some useful strategies . This paper although written for High School Student's but those who will read it thoroughly they will understand it is for useful for undergraduate or beg gainer of a graduate student from engineering background . Though Calculus , linear algebra , partial differential equation is not included in Math Olympiad Syllabus . But if you are really a Math Enthusiast I believe you already learn it !!! If you didn't don't be hesitate . Learn It from any book or course . Then came here to read this thesis paper . Best Wishes to all !!! Happy Problem Solving !!

Keywords: Finding Maximum and Minimum of a Function, Real Analysis, Single Variable Problem, Partial Derivative, Topology, Complex Analysis, Lagrangian Transform

Introduction:
Here is the general procedure of myself to teach you the method

- First I will Show you the question
- Then you will see a general procedure of solving a problem
- Since every one learn math by doing not just reading , you will see a lot of example
- Then last section you will think about the problem differently in your way
- Then you learn IT!!!Apply in the contest

A rectangular Box open at the top, is to have the volume of 32 cc . Then Find the dimension of the box that requires the least material for its construction ???

General Procedure
By seeing this it may seem simple but it is not . It will be one of the difficult problem you see in this paper . So what is the general procedure . First you have to declare an auxiliary function . An auxiliary function is that function that store all the information of a question . An auxiliary function is made by the statement in the question given and some Lagrangian λ multiply with what you want the the maximum of a function . Then you will find the partial derivatives with all it's dimension and make it equal to 0 . And Find the Lagrangian multiplier λ . By finding it you will find all the dimension of this function and determine the answer ! By Reading this I hope you have learnt nothing!!!That's what a teachers do! So let's see in application :.

Define a Auxiliary function
$$f(x,y,z,\lambda)=f(x,y,z)+\lambda g(x,y,x)$$

By seeing it first it may seem something robust . But it is not . f(x,y,z) is the function of answer what you want I.e the least material for its construction . And the 2'nd part is actually equivalent to 0 . Why because g(x,y,z) is equal to $volume(V) - xyz$ *. But we will not make it to 0 . Because if we do , we will not find the Lagrangian 'λ'.*

So then f(x,y,z) is the surface area you want to find since it is open at the top the surface area will be
$f(x,y,z) = S = xy + 2yz + zx$
**N.P=Here x is length , y is breadth and z is height . And the volume is*
$V(x,y,z) = xyz$

Then the real valued function is :-
$f(x,y,z,\lambda) = xy + 2yz + zx + \lambda * (xyz - 32)$

Then find the partial derivative of this function by it 's dimension and equalising it to 0.

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Published at: http://www.ijsciences.com/pub/issue/2022-10/
DOI: 10.18483/ijSci.2622; Online ISSN: 2305-3925; Print ISSN: 2410-4477

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$Fx = y + 2z + \lambda * yz$=0———(1)
Fy=$x + 2z + \lambda xz$=0—————-(2)
$fz = 2y + 2x + \lambda(xy) = 0$————-(3)
Then finding the value of λ from this 3 equation is :-
$1/z + 2/y = 1/z + 2/x = 2/x + 2/y = -\lambda$

Then the dimension become
$x = y = 2z$

Then if we substitute in volume equation you will probably find this
y=4 ; x=4;z=2;

Now our answer become after substituting in our real vaulted function is s=xy+2yz+2zx or s=16+16+16 or 39 cm^2.(ans)

Now Let's solve another 2 more problem . You will generally see this type of problem in IMO Olympiad or IYMC online challenge . Please check the previous question if you don't fully understand . Because I will do this question very quickly .

Find the maximum value of $x^m y^n z^p$,where x+y+z=a
Here
$f(x, y, z) = x^m y^n z^p$ *and*
$g(x, y, z) = x + y + z - a$
Now Let's define auxiliary function :-
$f(x, y, z, \lambda) = x^m y^n z^p + \lambda(x + y + z - a)$

Then taking partial derivative of this function we find :->
Fx=$mx^(m - 1)y^n z^p + \lambda$*=0*
$fy = x^m ny^(n - 1)z^p + \lambda$*=0*
Fz =$x^m y^n pz^p - 1 + \lambda = 0$
Now ->
$\lambda = -mx^(m - 1^) y^n z^p = -x^m n^(n - 1)z^p$
$= -x^m y^n pz^p - 1$

Value of y and z in terms of x is
$y = (n/m)x$ *and z =*$(p/n)y$

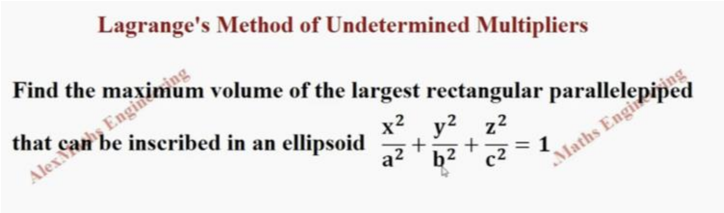
Not put the values in x+y+z=a equation we find again the stationary point is
$x = am/(m + n + P)$
Y =$an/m + n + p$
Z=ap/m+n+p
Then The maxima Function is
f(x,y,z)=$a^(m + n + p) * m^m * n^n * z^p/(m + n + p)$
(Ans)

Last But Not least Question :
If the perimeter of a triangle is constant then the maximum area of a triangle is when it is Equilateral There perimeter of this triangle is a+b+c=k

Then g(x)=a+b+c-k
Let , The area is A
Then $A^2 = K/2 * (k/2 - a) * (k/2 - b) * (k/2 - c)$
The Auxiliary function is
f(x,yx,z,λ)=$K/2 * (k/2 - a) * (k/2 - b) * (k/2 - c) + \lambda * (a + b + c - k)$——-

Now it 's your Work ! I'm Tired . The next procedure is similar like other 2 question . If you really understand it then solve it and send me in email ; rusho.ali17@gmail.com

Also You can do this question:
If you are very enthusiastic in math, you must



participate in Math Olympiad Both online and online . Here is some list of Mathematical Contest
https://molympiad.wordpress.com/list-of-mathematics-competitions/
https://www.idtech.com/blog/best-math-competitions-for-kids-teens
https://iymc.info/en/
Scholarship Contest
https://mathchallenge.in.th/online/

This article is written mainly for Olympiad Candidates to help them in becoming a math Olympiad expert. Though I Am Not A Math Olympiad expert !!! But I will try my best to help you . Always Remember For becoming a successful mathematical you must not be good in Math Olympiad internationally . Many Successful mathematician who got fields medal lex. : 2021 fields medal June HUH,Maryna Viazovska , Hugo Duminil-Copin was not good at math Olympiad . But they always like to do math more and more . So Stay hungry , Stay foolish .

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