3 Properties of Gases

CHAPTER OUTLINE

OBJECTIVES

After studying this chapter, you will be able:

■ To understand the concept of gases, their behaviour under heating, cooling, expansion, compression etc.
■ To understand gas laws and perfect gas equation
■ To understand important thermodynamic properties of gases

3.1 Introduction

This is an investigation into single–phase systems. The single–phase being considered is that phase above the critical point when a substance is called a gas.

The wide use made of gases in the field of engineering makes it necessary to investigate their behaviour when they are heated, cooled, expanded or compressed. The beginning of any investigation, such as the behaviour of gases, is usually made by conducting experiments; from the results obtained laws are determined which govern their behaviour. The first two laws in this chapter were established by experiment.

3.2 Boyle's Law

With any mass of gas it is possible to vary the pressure, volume and temperature. In i this experiment it is arranged that the temperature of a fixed mass of gas remains constant while corresponding changes in pressure and volume are observed. An apparatus suitable for conducting such an experiment is illustrated in Fig. 3.1.
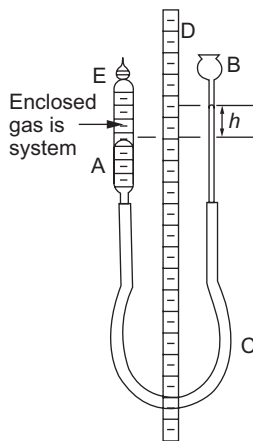

Fig. 3.1 Boyle's law apparatus

An inverted glass pipette A is connected to a glass thistle funnel B by means of a long rubber, or plastic, tube C. Both the pipette and the thistle funnel are mounted vertically such that they can be moved up or down on either side of a vertical scale D. With the tap E open, the apparatus is filled with a suitable quantity of mercury. It is possible to adjust the height of the mercury columns, and hence the volume of gas in the pipette — the system — 2 by moving the thistle funnel up or down. If the tap is then closed, a fixed mass of gas is trapped in the apparatus and modification to the height of the thistle funnel will bring about pressure changes in the gas which will be accompanied by corresponding changes in volume of the gas.

The pipette is calibrated to read the volume of gas contained in it; the vertical scale serves to establish the difference in height h of the two mercury columns. The absolute pressure of the gas will be given by the sum of the height h and the barometer reading. In order to satisfy the condition that the temperature should remain constant, a period of time is allowed to elapse after every change of condition before any new readings are taken. After a suitable number of results are obtained, the corresponding values of absolute pressure and volume are plotted on a graph; the curve is shown in Fig. 3.2.
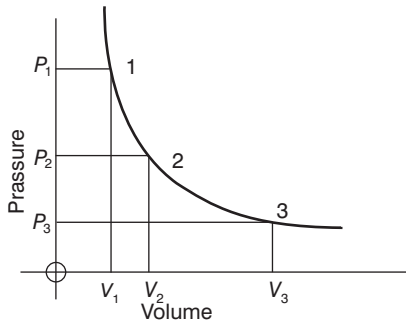
Fig. 3.2   Boyle's law graph

Taking any point on the curve, 1 say, the product of its corresponding pressure and volume *P* and *V* will equal some number, *C* say. Investigation of other points, such as 2 and 3, shows that, within the limits of experimental error, the products of their corresponding pressures and volumes also equal this same number, or

$$P_1 V_1 = P_2 V_2 = P_3 V_3 = C, \text{ a constant} \tag{1}$$

Further experiments at different fixed temperatures, with different fixed masses and with different gases yield the same result, although the constant *C* will be different with each quantity of gas, each fixed temperature and each type of gas.

From the results of this experiment, a general statement may be made:

> During a change of state of any gas in which the mass and the temperature remain constant, the volume varies inversely as the pressure.

Expressed mathematically

$$PV = C, \text{ a constant} \tag{2}$$

This is known as Boyle's law, named after its discoverer, Robert Boyle (1627–1691), an English scientist. As a point of interest, a Frenchman, Edme Mariotte, made the same discovery at about the same time while working quite independently of Boyle.

The graph of the law $PV = C$ is a rectangular hyperbola. Note also that if $PV = C$, then $P = C/V$. If *P* is plotted against $1/V$ the result will be a straight line passing through the origin and of slope *C* as shown in Fig. 3.3. This method could be used as a check of the above results; by plotting *P* against $1/V$ and obtaining a straight line passing through the origin, $PV = C$ would be proved.

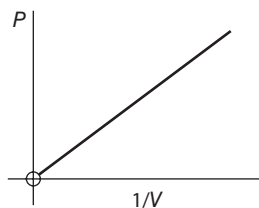The temperature is constant during a process carried out according to Boyle's law, so the process is isothermal.

Fig. 3.3   Plot of P against 1/*V*

Example 3.1   *During an experiment on Boyle's law, the original volume of air trapped in the apparatus, with the two mercury levels of the same, as 20000 mm³. The apparatus was then modified such that the volume of air became 17 000 mm³, while the temperature remained constant. If the barometer reading was 765 mm Hg, what was the new pressure exerted on the air in mm Hg? Also, what was the difference in the two mercury column levels?*

SOLUTION

Since both levels of mercury are the same at the beginning, then

P_1 = atmospheric pressure = **765 mm Hg**

Now Boyle's law states that $PV = C$, a constant, from this

$$P_1V_1 = P_2V_2$$

$$\therefore P_2 = P_1 \frac{V_1}{V_2} = 765 \times \frac{20000}{17000}$$

$$= \textbf{900 mm Hg}$$

Notice that the pressure has been left in mm Hg during this part of the solution. This can be done because there is a pressure term on both sides of the equation. As long as both terms have the same units, the equality will hold. The final pressure $P_2 = 900$ mm Hg and the atmospheric pressure = 765 mm Hg, so

Difference in height of the two mercury columns = 900 − 765

$$= \textbf{135 mm}$$

This will be the height h which was mentioned in the work on Boyle's law apparatus.

Example 3.2   *A gas whose original pressure and volume were 300 kN/m² and 0.14 m³ is expanded until its new pressure is 60 kN/m² while its temperature remains constant. What is its new volume?*

SOLUTION

The temperature remains constant, so this is an expansion according to Boyle's law.

$$\therefore P_1V_1 = P_2V_2 \text{ or } V_2 = V_1 \frac{P_1}{P_2}$$

$$\therefore V_2 = 0.14 \times \frac{300}{60}$$

$$= 0.7 \text{m}^3$$

3.3   Charle's Law

Consider now an experiment in which the pressure of a fixed mass of gas is kept constant while the volume and temperature are varied. A simple piece of apparatus on which to conduct such an experiment is illustrated in Fig. 3.4.
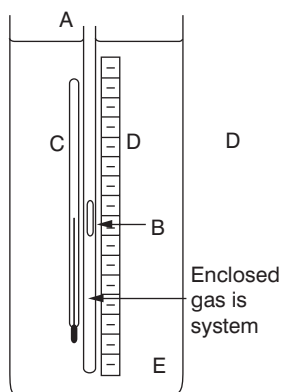
Fig. 3.4   Charles' law apparatus

A long glass tube A with one end sealed has a pellet of mercury B introduced; the pellet acts as a piston enclosing a fixed mass of gas, the system, in the end of the tube. Thermometer C and a volume scale D are attached along this tube and the assembly is immersed in a water bath E. The temperature of the water bath is then varied; this will be accompanied by changes in volume of the gas which will be registered by the mercury, pellet moving along the glass tube. The pressure of the gas in the tube will remain constant because the open end of the tube is always presented to the same external pressure conditions. The corresponding gas volumes and temperatures observed during the conduct of the experiment are recorded and plotted on a graph. The graph obtained is a straight line, as illustrated in Fig. 3.5, showing a linear relationship' between volume and temperature of a fixed mass of gas when the pressure remains constant. This takes the form:

$$V = Ct + V_0 \qquad\qquad\qquad\qquad [1]$$

where V = volume
t = temperature
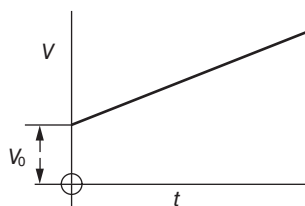C = slope
V_0 = intercept on V axis

Fig. 3.5   Plot of *V* against *t*

Further experiments at different pressures, with different masses and with different gases give similar results. An interesting point, however, is that if all the straight lines obtained are extended back to cut the temperature axis, they all cut this axis at the same point. This is illustrated in Fig. 3.6.
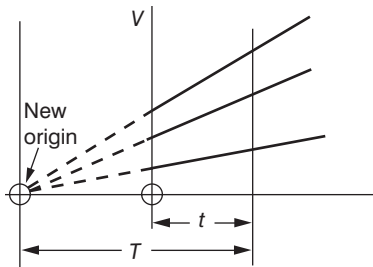
Fig. 3.6   Extended Charles' law plots

If this is the case then perhaps it would be better to use this point as a new origin and the law of the graph would then become

$$V = CT \qquad [2]$$

where T is the temperature recorded from the new origin. Equation [2] is of a better form than equation [1] because the constant V_0 is now absent. But reading temperature from this new origin has introduced a new temperature scale. Now the graph cuts the temperature axis at approximately $-273°C$. The value of T in equation [2] is therefore determined by adding 273°C to the value of t as recorded from the thermometer, or

$$T = (t + 273) = TK \qquad [3]$$

Temperature recorded in this manner is called **absolute temperature** and the new zero is called the **absolute zero of temperature.** The reason for this can be seen by referring to Fig. 3.6. It will be noted that, at the new zero, all volumes have reduced *to* zero. No further reduction seems possible because there is nothing left to reduce in temperature! The fallacy of this argument is that extension of the straight lines to determine the new zero assumes that the gas remains as a gas in the low-temperature region. This is not true in practice because all gases, on being cooled, will eventually liquefy and then finally solidify, thus losing their properties as a gas. Experiments on the problem of an absolute zero of temperature have shown, however, that approximately $-273°C$ appears to be the lowest temperature possible, and is extremely difficult to reach. From equation [2] it follows that

$$\frac{V}{T} = C \text{ , a constant} \qquad [4]$$

In words this may be stated as follows:

During the change of state of any gas in which the mass and pressure remain constant, the volume varies in proportion with the absolute temperature.

This is known as Charles' law.

Of historic interest, the law dealt with above is attributed to a Frenchman, Jacques A. Charles (1746–1823). It is also interesting to note that another Frenchman, Joseph-Louis Gay-Lussac (1778–1850), made the same discovery at about the same time.

The concept of the **absolute scale of temperature** has already been discussed in section 1.6.

Example 3.3   *During an experiment on Charles' law, the volume of gas trapped in the apparatus was 10 000 mm³ when the temperature was 18°C. The temperature of the gas was then raised to 85°C. Determine the new volume of gas trapped in the apparatus if the pressure exerted on the gas remained constant.*

Solution
Now according to Charles' law

$$\frac{V}{T} = C, \text{a constant}$$

From this

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad\qquad [1]$$

In order to use this equation the temperatures T_1, and T_2 must be absolute temperatures.

$$\therefore\ T_1 = 18 + 273 = \textbf{291 k}$$

and

$$T_2 = 85 + 273 = \textbf{358 k}$$

From equation [1]

$$V_2 = V_1 \frac{T_2}{V_1} \qquad\qquad [2]$$

or

$$V_2 = 10000 \times \frac{358}{291}$$
$$= \textbf{12302 mm}^3$$

Example 3.4   *A quantity of gas whose original volume and temperature are 0.2 m³ and 303°C, respectively, is cooled at constant pressure until its volume becomes 0.1 m³. What will be the final temperature of the gas?*

Solution
Again, this is a change according to Charles' law.

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = T_1 \frac{V_2}{V_1} \qquad\qquad [1]$$

The temperature is in degrees Celsius this time.

$$\therefore T_1 = 303 + 273 = 576 K$$

And from equation [1]

$$T_2 = 576 \times \frac{0.1}{0.2}$$
$$= 288 \text{ K}$$
$$\therefore\ t_2 = 288 - 273 = 15°X$$

3.4   Characteristic Equation of a Perfect Gas and Gas Constant

In the previous two experiments it was arranged that, in each case, one of the three conditions of state, pressure, volume and temperature, remained constant while a law connecting variations in the other two was established. An investigation must now be made into the more general change of state of a gas which neither pressure, volume nor temperature remains constant. Consider a gas whose original state is pressure P_1 volume V_1 and temperature T_1, and let this gas pass through a change of state such that its final state is P_2, V_2 and T_2. Inspection of Fig. 3.7 will show that there are an infinite number of paths which connect states 1 and 2 when the process is shown on a $P–V$ diagram.

   The concern at the moment, however, is not in how the state changed from 1 to 2, but in the fact that since states 1 and 2 can exist for the same mass of gas, is there any law connecting them? This being the case, a choice of path from 1 to 2 is quite arbitrary, and it is therefore reasonable to assume a path about which something is already known. Boyle's and Charles' laws supply the answer. Figure 3.8 shows that it is quite possible to move from 1 to 2 by first carrying out a Boyle's law change down to some inter-mediate state A, say, then carry out a Charles' law change to the final condition.
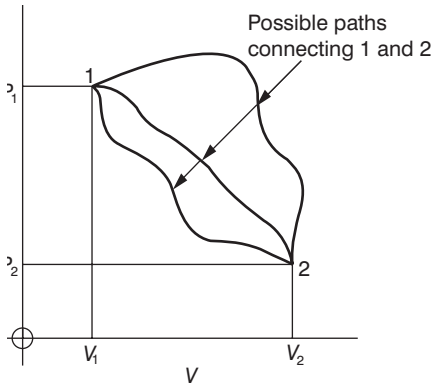

Fig. 3.7   Three of The Infinite Number of Paths From 1 to 2
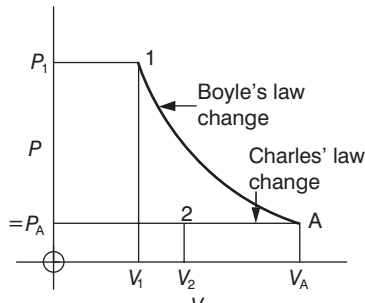

Fig. 3.8   Getting from 1 to 2 in two known stages

   Consider the Boyle's law change from 1 to A. In this case the temperature remains reactant at T_1 Also

$$P_1V_1 = P_A V_A \qquad\qquad\qquad\qquad [1]$$

All the pressure change must take place during this process because there will be no change in pressure during the Charles' law process which follows. In this case $P_A = P_2$.

Equation [1] becomes $P_1 V_1 = P_2 V_2$.

or

$$V_A = \frac{P_1 V_1}{P_2} \qquad \qquad [2]$$

Consider now the Charles' law change from A to 2. In this case the pressure remains constant at P_2. Also

$$\frac{V_A}{T_A} = \frac{V_2}{T_2} \qquad \qquad [3]$$

During the Boyle's law change from 1 to A the temperature remained constant.

$$\therefore T_A = T_1$$

from which equation [3] becomes

$$\frac{V_A}{T_1} = \frac{V_2}{T_2} \qquad \qquad [4]$$

But

$$V_A = \frac{P_1 V_1}{P_2}$$

from equation [2] and substituting this in equation [4]

$$\frac{P_1 V_1}{P_2 T_1} = \frac{V_2}{T_2}$$

from which

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Now any change of state from state 1 would produce a similar result, and hence equation [5] could be extended to read

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4} = ..., etc. \qquad \qquad [6]$$

where 3 and 4 represent other new conditions of state of the same mass of gas.

From equation [6] it follows that for any fixed mass of gas, changes of state are connected by the equation

$$\frac{PV}{T} = a \text{ constant} \qquad \qquad [7]$$

Sooner or later it will be necessary to know the actual mass of gas used during any particular process. Let V = volume of 1 kg of gas, the specific volume (see section 1.4).

Then from equation [7]

$$\frac{PV}{T} = a \text{ constant} \tag{8}$$

When 1 kg of gas is considered, this constant is written R and is called the **characteristic gas constant**, sometimes, the **specific gas constant**. So for 1 kg of gas

$$\frac{PV}{T} = R \tag{9}$$

Now consider the case when there are m kg of gas. Multiply both sides of equation [9] by m, then

$$\frac{P(mV)}{T} = mR$$

But mv is the total volume of the gas being used, V, so for m kg of gas it follows that

$$\frac{PV}{T} = mR$$

or

$$PV = mRT \tag{10}$$

This is known as the **characteristic equation of a perfect gas**.

The units of R can be obtained from equation [9]. If pressure is in N/m², specific volume is in m³/kg and temperature in K, then

$$\frac{PV}{T} = R = \frac{N}{m^2} \times \frac{m^3}{kg} \times \frac{1}{K} = \frac{Nm}{kgK} = \frac{J}{kgK}$$

For air, the value of R is usually of the order 0.287 kJ/kg K.

Actually the value of R is numerically equal to the work done when 1 kg of gas is heated at constant pressure through 1 degree rise of temperature. This can be shown as follows. Consider 1 kg of gas at original state P_1, V_1, T_1 and let it be heated at constant pressure through 1 degree. The new state will then be P_1, V_2, $(T_1 + 1)$.

Now from the characteristic equation

$$P_1 V_1 = RT_1 \tag{11}$$

and

$$P_1 V_2 = R(T_1 + 1) \tag{12}$$

Subtracting equation [11] from equation [12] gives

$$P_1(V_2 - V_1) = R \tag{13}$$

This is equal to the area under the graph of the process plotted on a P–V diagram and this has been shown to be equal to the work done (see section 1.5.1).

A further point to note about this work on the characteristic equation is that it is now possible to predict the behaviour of a gas if the volume remains constant. If this is the case then $V_1 = V_2$, and from equation [5], since $P_1 V_1/T_1 = P_2 V_2/T_2$, it follows that

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

[14]

Very accurate experiment shows that actual gases do not obey the above gas laws exactly. The deviation is very small, however, and can be sensibly neglected in all general calculations. A perfect gas may be defined as a gas which obeys the gas laws exactly.

Example 3.5 *A gas whose original pressure, volume and temperature were 140 kN/m², 0.1 m³ and 25°C, respectively, is compressed such that, its new pressure is 700 kN/m² and its new temperature is 60°C. Determine the new volume of the gas.*

SOLUTION

By the characteristic equation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \text{ and } T_1 = 25 + 273 = \textbf{298 K}$$

$$\text{also } T_2 = 60 + 273 = \textbf{333 K}$$

$$\therefore \ V_2 \ \frac{P_1}{P_2}\frac{T_2}{T_1} V_1 = \frac{140}{700} \times \frac{333}{298} \times 0.1 = \textbf{0.0223 m}^{3}$$

Example 3.6 *A quantity of gas has a pressure of 350 kN/m² when its volume is 0.03 m³ and its temperature is 35°C. If the value of R = 0.29 kJ/kg K, determine the mass of gas present. If the pressure of this gas is now increased to 1.05 MN/m² while the volume remains constant, what will be the new temperature of the gas?*

SOLUTION

By the characteristic equation

$$PV = mRT$$

$$\therefore \ m = \frac{PV}{RT} = \frac{350 \times 10^3 \times 0.3}{0.29 \times 10^3 \times 308} = \textbf{0.118 kg}$$

For the second part of the problem

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \text{ and in this case, } V_1 = V_2$$

$$\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ or } T_2 = T_1 \frac{P_2}{P_1} = 308 \times \frac{1.05 \times 10^6}{0.35 \times 10^6}$$

$$= 308 \times 3 \ (T_1 = 35 + 273 = 308 \text{ K})$$

$$= \textbf{924 K}$$

$$\therefore \ T_2 = 924 - 273 = \textbf{651}°\text{X}$$

3.5   *Internal Energy of a Gas*

The internal energy term appears in the steady-flow energy equation and the non-flow energy equation so an investigation is required into the way it applies to a gas. Joule carried out an experiment on this subject from which he concluded:

The internal energy of a gas is a function of temperature only and is independent of changes in pressure and volume.

This is known as **Joule's law.**

A sketch of Joule's apparatus is shown in Fig. 3.9. Two copper vessels A and B were connected together as shown and were isolated from each other by means of a gas tap C. Vessel A was filled with compressed air to a pressure of about 21 atmospheres (about 2.1 MN/m^2) and vessel B was exhausted to a condition of vacuum. This assembly was immersed in a water bath D and temperature recordings were made by means of a thermometer E. After leaving the apparatus for some time, in order to let the temperature conditions become steady, the gas tap C was opened and some air from vessel A expanded into vessel B while the pressure dropped, eventually to stabilise at some new common pressure. The volume was then equal to the total volume of vessels A and B. During this process no change in temperature was observed.
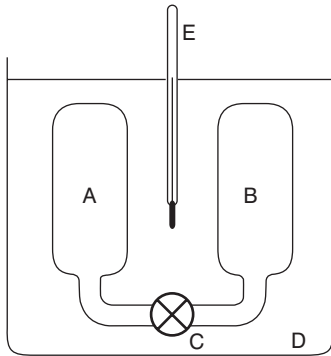

Fig. 3.9   Joule's law apparatus

No work transfer occurred because this was a free expansion into a vacuum, so $W = 0$.
And no heat was transferred during the expansion, so $Q = 0$.
Applying this to the non-flow energy equation for the expansion of a gas

$$Q = \Delta U + W \tag{1}$$

it follows that for this experiment

$$0 = \Delta U + 0 \tag{2}$$

or

$$\Delta U + 0 \tag{3}$$

Both the pressure and the volume changed during the experiment. But the temperature did not change, nor did the internal energy, so it seemed reasonable to assume that the internal energy of a gas was a function of temperature only.

Later experiments, carried out rather more accurately by Joule in conjunction with William Thomson (Lord Kelvin), showed that there was a very small change in temperature during such an expansion of a gas, but this temperature change is small enough to be neglected. Joule's law is assumed to hold in all normal practical cases.

The problem now is to develop an expression which will give the magnitude of the change in internal energy of a gas and, by Joule's law, this expression must be a function of temperature only. Before this can be accomplished, it is necessary to discuss the specific heat capacities of a gas.

3.6   Specific Heat Capacities of a Gas

The specific heat capacity of a substance may be defined as the amount of heat transfer required to raise unit mass of a substance through 1 degree difference in temperature (see section 1.7.1). Apply this statement to a gas and, at first sight, this definition may seem reasonable. But consider Fig. 3.10.
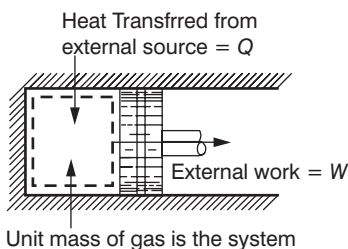

Fig. 3.10   Using a piston to determine specific heat capacity of a gas

The figure shows a piston enclosing unit mass of gas, the system, in a cylinder. This unit mass of gas could'' be heated from some outside source such that the temperature of the gas is raised by 1 degree. The amount of heat transfer to accomplish this 1 degree rise in temperature will depend upon what happens to the piston. For example, the piston could be fixed, then the gas would be heated at constant volume and a certain quantity of heat would bring about the 1 degree rise in temperature. Or the gas in the cylinder could be allowed to expand, moving the piston and doing external work W. The extent to which the piston is allowed to move is one of an infinite number of possible arrangements. The amount of heat transfer will depend upon the piston movement, so there are infinitely many heat supply quantities, each able to produce a 1 degree rise in temperature. It appears there are an infinite number of possible specific heat capacities for a gas. If the specific heat capacity of a gas is quoted, therefore, it is necessary to define the conditions under which the specific heat capacity was measured. Two important cases are called the **principal specific heat capacities.**

- *The specific heat capacity at constant volume* This is defined as the amount of heat which transfers to or from unit mass of gas while the temperature changes by 1 degree and the volume remains constant. It is written C_v.

- *The specific heat capacity at constant pressure* This is defined as the amount of heat which transfers to or from unit mass of gas while the temperature changes by 1 degree and the pressure remains constant. It is written C_p.

The specific heat capacities at constant volume and constant pressure rise in value with temperature. For calculations, it is usual to assume an average value of specific heat capacity within the temperature range being considered.

Table 3.1 Table of average specific heat capacities of gases

Gas	C_p(kJ/kg K)	C_v(kJ/kg K)
Air	1.006	0.718
Carbon dioxide	0.87	0.67
Carbon monoxide	1.04	0.74
Hydrogen	14.4	10.2
Nitrogen	1.04	0.74
Oxygen	0.92	0.66
Methane	2.29	1.74
Sulphur dioxide	0.65	0.52

3.7 Specific Heat Capacity at Constant Volume

Let a mass of gas m be heated at constant volume such that its temperature rises *from T*1 to *T*2 and its pressure rises from *P*1 to *P2,* then

Heat received by the gas = mass × specific heat capacity at constant volume

$$\times \text{ rise in temperature}$$
$$= mC_v(T_2 - T_1) \quad [1]$$

Constant volume heating is a particular case of a non-flow process carried out on a gas. Consider the non-flow energy equation applied to constant volume heating.

$$Q = \Delta U + W \quad [2]$$

No external work is done during constant volume heating. This can be seen by inspecting Fig. 3.11, in which pressure is plotted against volume. The process appears as a vertical straight line. There is no area beneath this line, so no external work is done.
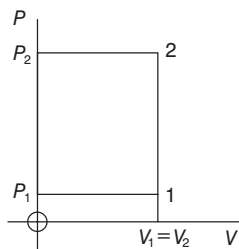
$$\therefore \ W = 0$$

Fig. 3.11 Constant volume process on P-V diagram

Hence, equation [2] becomes

$Q = \Delta U$

or

$mC_v(T_2 - T_1) = U_2 - U_1$

which can be written

$$mC_v\,\Delta T = \Delta U$$

It follows, therefore, that all the heat added during constant volume heating goes completely into increasing the stock of internal energy of the gas. Conversely, if a gas is cooled at constant volume, the heat rejected will be at the expense of the stock of internal energy of the gas. If the new pressure is required, it may be found by the application of the characteristic equation of a perfect gas.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \qquad\qquad\qquad [5]$$

and for this case, $V_1 = V_2$.

$$\therefore\ \ \frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad\qquad\qquad [6]$$

or

$$P_2 = P_1 \frac{T_2}{T_1} \qquad\qquad\qquad [7]$$

If unit mass of gas is considered, equation [3] can be written

$$C_V\,(T_2 - T_1 = u_2 - u_1) \qquad\qquad\qquad [8]$$

from which

$$C_v = \frac{u_2 - u_1}{T_2 - T_1} = \frac{\Delta u}{\Delta T} \qquad\qquad\qquad [9]$$

At any particular absolute temperature T, therefore, and as $\Delta T \to 0$, equation [9] can be written

$$C_v = \left(\frac{du}{dT} \right)_v$$

The notation $(du/dT)_v$ means that the process is being considered while holding the volume constant.

Equation [10] is often used mathematically to define the specific heat capacity of a gas at constant volume.

From equation [10]

$$\mathrm{d}u = C_v dT \qquad\qquad\qquad [11]$$

A further very important point arises out of equation [3]. Remember how Joule's law states that the internal energy of a gas is a function of temperature only. An inspection of equation [3] will show that this expression does, in fact, give the change of internal energy as a function of temperature only since m is constant and C_v is assumed constant, being given an average value within the temperature range T_1 to T_2. The expression $U_2 - U_1 = mC_v(T_2 - T_1)$ will therefore give the change of internal energy dur-

ing any process where the temperature changes from original temperature T_1 to final temperature T_2. It will be noted that, from this expression for the change of internal energy, at any absolute temperature T, it appears that the total internal energy of a gas, reckoned from the absolute zero of temperature, is given by $U = mC_v T$. This seems to indicate that, at the absolute zero of temperature, a gas assesses no internal energy, a point which is probably true. But the actual value of the internal energy, $U = mC_v T$, is probably not true, since this expression assumes that the gas remains as a gas all the way down to the absolute zero of temperature. This is actually not the case because a gas will liquefy and eventually solidify before absolute zero is reached, thus its gaseous properties will be lost. But in general, only the change of internal energy is required, so there is no need to enquire into its actual value for a gas at any absolute temperature T; the expression for the change of internal energy, $U_2 - U_1 = mC_v(T_2 - T_1)$, will give all that is required.

A further point about the internal energy of a gas concerns its graphical representation. It has been suggested that the change of internal energy is all that is generally required. Hence any graphical representation need only plot its change, not its absolute value. In order to plot values of the change of internal energy, a common origin must be chosen from which to reckon all changes. It is usual to choose the origin as 0°C at which temperature the internal energy is suggested as being zero. This *is* really not true, but the overall change of internal energy from one temperature to another is not affected by the choice of an origin for the graph, an arbitrary choice, in any case. But with this choice of origin, all values of internal energy at temperatures below 0°C will appear as negative.

Consider the equation for the change of internal energy

$$U_2 - U_1 = mC_v(T_2 = T_1)$$

If the origin is chosen as suggested, $U_1 = 0$ when $T_1 = 273.15$ K; substituting in the above equation gives

$$U = mC_v(T - 273.15)$$

where U (written U_2 above) = the value of the internal energy of the gas at temperature T (written T_2 above)

This equation allows determination of the internal energy of a gas, from which a graph may be plotted. It is also possible to produce a set of tables for the internal energy of a gas by using the same method.

Example 3.7   *2 kg of gas, occupying 0.7 m³, had an original temperature of 15°C. It was then heated at constant volume until its temperature became 135°C. Determine the heat transferred to the gas and its final pressure. Take $C_v = 0.72\ kJ/kg\ K$ and $R = 0.29\ kJ/kg\ K$.*

SOLUTION

Heat transferred at constant volume $= mC_v(T_2 = T_1)$
$$= 2 \times 0.72 \times (135 - 15)$$
$$= 2 \times 0.72 \times 120$$
$$= \mathbf{172.8\ kJ}$$

Now $P_1V_1 = mRT_1$ and $T_1 = (15 + 273)$ K $= \mathbf{288\ K}$

$$\therefore P_1 = \frac{mRT_1}{V_1} = \frac{2 \times 0.29 \times 288}{0.7} = \frac{167.04}{0.7} = \textbf{288.6 kN/m}^2$$

Since the volume remains constant, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \therefore \quad P_2 = P_1 \frac{T_2}{T_1} \text{ and } T_2 = (135 + 273) \text{ K} = \textbf{408 K}$$

$$= 238.6 \times \frac{408}{288} = \textbf{338.02 KN/m}^2$$

3.8   Specific Heat Capacity at Constant Pressure

Let a mass of gas m be heated at constant pressure such that its temperature rises from T_1 to T_2 and its volume increases from V_1 to V_2 then

Heat received by the gas = mass × specific heat capacity at constant pressure
$$\times \text{ rise in temperature}$$
$$= m C_p (T_2 - T_1) \qquad [1]$$

Constant pressure heating is another case of a non–flow process carried out on a gas. Consider the non–flow energy equation applied to constant pressure heating

$$Q = \Delta U + W \qquad [2]$$
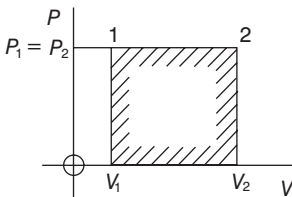
Fig. 3.12   Constant pressure process on P–V diagram

In this case external work is done by the gas. Figure 3.12 shows a graph of a constant pressure process plotted on a *P–V* diagram. This graph has a definite area beneath the constant pressure line, which gives the work done, $P(V_2 - V_1)$, where P is the constant pressure ($P = P_1 = P_2$). In this constant pressure case, equation [2] becomes

$$mC_p(T_2 - T_1) = (U_2 - U_1) + P(V_2 - V_1) \qquad [3]$$
$$= (U_2 + PV_2) - U_1 + PV_1)$$

or

$$m C_p (T_2 - T_1) = H_2 - H_1 \qquad [4]$$

i.e.

Heat transferred at constant pressure = Change of enthalpy

This was shown to be the case when dealing with the constant pressure formation of steam. Equation [4] could be written

$$m\,C_p\,\Delta T \;=\; \Delta H \tag{5}$$

Also, if unit mass of gas is considered, equation [4] becomes

$$C_p(T_2 - T_1) = h_2 - h_1 \tag{6}$$

from which

$$C_P = \frac{h_2 - h_1}{T_2 - T_2} = \frac{\Delta h}{\Delta T} \tag{7}$$

At any particular absolute temperature *T,* therefore, and as $\Delta T \rightarrow 0$, equation [7] can be written

$$C_P = \left(\frac{dh}{dT}\right)_P \tag{8}$$

The notation $(\;)_P$ means that the process is being considered while holding the pressure constant. From equation [8]

$$dh = C_p dT \tag{9}$$

From equation [3]

$$U_2 - U_1 = mC_p(T_2 - T_1) - P(V_2 - V_1)$$

or

$$U_2 - U_1 = mC_p(T_2 - T_1) - mR(T_2 - T_1) \tag{10}$$

since

$$PV = mRT$$

This again gives an expression for the change of internal energy of a gas in terms of temperature only, so this can also be used as a method for the determination of the change of internal energy during any process when a temperature change from T_1 *to* T_2 occurs. The expression determined during the constant volume analysis, $U_2 - U_1 = mC_v(T_2 - T_1)$, is of a simpler form and is usually used instead of equation [10].

It should be noted that enthalpy tables could be made up for a gas, using equation [4], in much the same way as indicated for internal energy tables discussed during the constant volume analysis.

If the new volume is required after a constant pressure process, this too may be obtained by using the characteristic equation of a perfect gas.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{11}$$

and for this case,

$$P_1 = P_2 \qquad\qquad\qquad\qquad\qquad\qquad [12]$$

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

or

$$V_2 = V_1 \frac{T_2}{T_1}$$

Example 3.8   *A gas whose pressure, volume and temperature are 275 kN/m², 0.09 m³ and 185°C, respectively, has its state changed at constant pressure until its temperature becomes 15°C. Determine the heat transferred from the gas and the work done on the gas during the process. Take R = 0.29 kJ/kg K, Cₚ = 1.005 kJ/kg K.*

SOLUTION

First determine the mass of gas used.

Now $P_1 V_1 = mRT_1$ and $T_1 = (185 + 273)$ **K = 458 K**

$$\therefore \quad m = \frac{P_1 V_1}{RT_1} = \frac{275 \times 10^3 \times 0.09}{0.29 \times 10^3 \times 458} = \mathbf{0.186\ kg}$$

Heat transferred $mc_p(T_2 = T_1)$ and $T_2 = (15 + 273)$ K

$$= \mathbf{288\,K}$$

Heat transferred $= 0.186 \times 1.005 \times (288 - 458)$

$$= 0.186 \times 1.005 \times = -31.78 \text{kJ} (-170)$$

Notice the negative sign, indicating that the heat has been transferred from the gas Since the pressure remains constant, then

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\therefore \ V_2 = V_1 \frac{T_2}{T_1} = 0.09 \times \frac{288}{458} \quad 0.056\ 6\ \text{m}^3$$

Work done $= P\,(V_2 - V_1)$

$$= 275 \times (0.56\ 6 - 0.009)$$
$$= 275 \times (-0.0334)$$
$$= \mathbf{-\ 9.19\ kJ}$$

3.9   Relation Between Specific Heat Capacities of a Gas

It has been shown that if a mass of gas *m* has its temperature changed from T_1 to T_2 then the change of internal energy can be. determined by the expressions

$$U_2 - U_1 = m C_v (T_2 - T_1) \qquad\qquad\qquad\qquad [1]$$

and

$$U_2 - U_1 = m\,C_p(T_1 - T_1) - m\,R\,(T_2 - T_1) \tag{2}$$

If the temperature change is the same for both expressions then it follows that equation [1] equals equation [2] because the change of internal energy is a function of temperature only, by Joule's law.

$$\therefore\ mC_v(T_2 - T_1) = m\,C_p(T_2 - T_1) - m\,R\,(T_2 - T_1)$$

from which

$$C_p = C_v - R$$

since $m(T_2 - T_1)$ is common throughout.

$$\therefore\ C_p = C_v - R \tag{3}$$

$$= \frac{PV}{T}\left(\text{Since }\ \frac{PV}{T} = R\right) \tag{4}$$

3.10   Polytropic Process and a Gas

Section 1.5.2 discussed the general concept of the polytropic process. A gas is no exception to this concept; if a mass of gas is expanded or compressed, the general law of expansion or compression has the polytropic form

$$PV^n = C \tag{1}$$

For two state points 1 and 2

$$P_1V_1^n = P_2V_2^n \tag{2}$$

Furthermore

$$\text{work done } = \frac{P_1V_1 - P_2V_2}{n-1} \tag{3}$$

This was shown in section 1.5.2.
By the characteristic equation

$$PV = mRT \tag{4}$$

Substituting [4] in [3]

$$\text{work done } = mR\frac{(T_1 - T_2)}{n-1} \tag{5}$$

Apply the non–flow energy equation

$$Q = \Delta U \ + \ W$$
$$= (U_2 - U_1) \ + \ W$$

$$= mC_v \left(T_2 - T_1 \right) + \frac{P_1 V_1 - P_2 V_2}{n-1} \tag{6}$$

$$= mC_v \left(T_2 - T_1 \right) + \frac{mR \left(T_1 - T_2 \right)}{n-1} \tag{7}$$

3.11 Combination of Polytropic Law with Perfect Gas Characteristic Equation

The law $PV^n = C$ will enable calculations to be made of the changes in pressure and volume which occur during a polytropic process. Combining this with the characteristic equation of a perfect gas will enable variations in temperature to be determined.

Consider a polytropic process in which the state of a gas changes from P_1, V_1, T_1 to P_2, T_2. By the polytropic law

$$P_1 V_1{}^n \ = \ P_2 V^n{}_2 \tag{1}$$

By the characteristic equationΔ

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{2}$$

From equation [2]

$$\frac{T_1}{T_2} = \frac{P_1 V_1}{P_2 V_2} \tag{3}$$

From equation [1]

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^n \tag{4}$$

Substituting equation [4] in equation [3]

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^n \frac{V_2}{V_1} = \left(\frac{V_2}{V_1} \right)^n \left(\frac{V_2}{V_1} \right)^{-1}$$

or

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{(n-1)} \tag{5}$$

Also, from equation [4]

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2} \right)^{1/n} \text{ or } \frac{V_1}{V_2} = \left(\frac{P_2}{P_1} \right)^{1/n} \tag{6}$$

Substituting equation [6] in equation [3]

$$\frac{T_1}{T_2} = \frac{P_1}{P_2}\left(\frac{P_2}{P_1}\right)^{1/n} = \frac{P_1}{P_2}\left(\frac{P_1}{P_2}\right)^{-1/n}$$

or

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n} \tag{7}$$

Combining equations [5] and [7]

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n} = \left(\frac{V_2}{V_1}\right)^{(n-1)} \tag{8}$$

or in other words

$$\text{Ratio of absolute temperatures} = (\text{Ratio of pressures})^{(n-1)/n}$$
$$= (\text{Inverse ratio of volumes})^{(n-1)}$$

This expression gives the relationship between pressure, volume and temperature when a gas state changes according to the law $PV^n = C$.

From equation [8], by raising each term to the power $n/(n-1)$ it follows that

$$\left(\frac{T_1}{T_2}\right)^{n/(n-1)} = \left(\frac{P_1}{P_2}\right)^{(n-1)n-n/(n-1)} = \left(\frac{V_2}{V_1}\right)^{(n-1)-n/(n-1)}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{n} = \left(\frac{T_1}{T_2}\right)^{n/(n-1)} \tag{9}$$

Also, by raising each term in equation [8] to the power $1/(n-1)$ it follows that

$$\left(\frac{T_1}{T_2}\right)^{1/(n-1)} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n-1(n-1)} = \left(\frac{V_2}{V_1}\right)^{(n-1)-1(n-1)}$$

or

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{1/(n-1)} = \left(\frac{P_1}{P_2}\right)^{1/n} \tag{10}$$

Having developed these expressions, it might be useful to note that, initially, it may be difficult to know which one to use for the solution of a particular problem. There is no hard and fast rule, but notice that

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

requires five conditions of state to be known before solving for the sixth. If five conditions are not known, another expression may be more appropriate, such as

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \ \text{ or } \ \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n}$$

Example 3.9   *A gas whose original pressure and temperature were 300 kN/m² and 25°C, respectively, is compressed according to the law P V¹·⁴ = C until its temperature becomes 180 ºC. Determine the new pressure of the gas.*

SOLUTION

It has been shown that for a polytropic compression, the relation ship between pressure and temperature is

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n}$$

From this

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{n/(n-1)}$$

$$\therefore \ \ P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{n/(n-1)}$$

Now

$$T_1 = (25 + 273) \text{ K} = \textbf{298 K}$$

and

$$T_2 = (180 + 273)\text{K} = \textbf{453 K}$$

Hence

$$P_2 = 300 \times \left(\frac{453}{298}\right)^{1.4/0.4}$$
$$= 300 \times 1.52^{3.5}$$
$$= 300 \times 4.33$$
$$= \textbf{1299 kN/m}^2 \quad \text{or} \quad \textbf{1.299 MN/m}^2$$

Example 3.10   *A gas whose original volume and temperature were 0.015 m³ and 285°C. respectively, is expanded according to the law P V¹·³⁵ = C until its volume is 0.09 m³ Determine the new temperature of the gas.*

SOLUTION

The relationship between volume and temperature during a polytropic expansion of a gas is

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{(n-1)} \text{ and } T_1 = (285 + 273) \textbf{ K = 558 K}$$

$$\therefore \ \ T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{(n-1)} = 558 \times \left(\frac{0.015}{0.09}\right)^{(1.35-1)}$$

$$= \frac{558}{6^{0.35}}$$

$$= \frac{558}{1.87}$$

$$= \textbf{298.4 K}$$

$$t_2 = 298.4 - 273 = \textbf{25.4°C}$$

Example 3.11   *0.675 kg of gas at 1.4 MN/m² and 280°C is expanded to four times the original volume according to the law PV$^{1.3}$ = C. Determine*
(a) the original and final volume of the gas
(b) the final pressure of the gas
(c) the final temperature of the gas
Take R = 0.287 *kj/kg k.*

(a)
Now $P_1 V^n_1 = mRT_1$ and $T_1 = (280 + 273)\text{K} = \textbf{553 K}$

$$\therefore \; V_1 = \frac{mRT_1}{P_1} = \frac{0.675 \times 0.287 \times 10^3 \times 553}{1.4 \times 10^6}$$

$$= \textbf{0.076 m}^3$$

The original volume is 0.076 5 m³.
Since the gas is expanded to four times its original volume, then

$$V_2 = 4V_1 = 4 \times 0.076\ 5 = \textbf{0.306 m}^3$$

The final volume is 0.306 m³

(b)

$$P_1 V^n_1 = P_2\, V^n_2$$

$$\therefore \; P_2 = P_1 \left(\frac{V_1}{V_2} \right)^n = 1.4 \left(\frac{1}{4} \right)^{1.3}$$

$$= \frac{1.4}{4^{1.3}}$$

$$= \frac{1.4}{6.06}$$

$$= \textbf{0.231 MN/m}^2$$

$$= \textbf{231 kN/m}^2$$

The final pressure is 231 kN/m².

(c)

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore \; T_2 = \frac{P_2}{P_1}\frac{V_2}{V_1}T_1 = \frac{0.231}{1.4}\times 4\times 553 \;\; = 365 \text{ K}$$

$$T_2 = 365 - 373 = \textbf{92ºC}$$

The final temperature is 92ºC .

Example 3.12   *0.25 kg of air at a pressure of 140 kN/m² occupies 0.15 m³ and from this condition it is compressed to 1.4 MN/m² according to the law PV^{1.25} = C. Determine*
(a) The change of internal energy of the air
(b) the work done on or by the air
(c) the heat received or rejected by the air
Take C_p = 1.005 U/kg K, C_v = 0.718 kJ/kg

(a)

Now $C_p - C_v = R$

$\therefore$ R = 1.005 − 0.718 = **0.287 kj/kg K**

Also $P_1V_1 = mRT_1$

$$\therefore \; T_1 = \frac{P_1V_1}{mR} = \frac{140\times 10^3 \times 0.15}{0.25\times 0.287\times 10^3} = \textbf{292.7 K}$$

Also $\dfrac{T_1}{T_2} = \left(\dfrac{P_1}{P_2}\right)^{(n-1)/n}$

$$\therefore \qquad T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n-1)/n} = 292.7 \times \left(\frac{1.4\times 10^6}{140\times 10^3}\right)^{0.25/1.25}$$

$$= 292.7\times 10^{1/5} = 292.7 \; \times {}^5\sqrt{10}$$
$$= 292.7 \times 1.58$$
$$= \textbf{463.9 K}$$

Change of internal energy

$$\Delta U = U_2 - U_1 = mC_v\,(T_2 - T_1)$$
$$= 0.25 \times 0.718 \times (463.9 - 292.7)$$
$$= 0.25 \times 0.718 \times 171.2$$
$$= \textbf{30.73 kJ}$$

This is positive, so it is a gain of internal energy to the air.

(b)

$$\text{Work done, } W = \frac{mR\left(T_1 - T_2\right)}{n-1} = \frac{0.25\times 0.287\times \left(292.7 - 463.9\right)}{1.25 - 1}$$

$$= \frac{0.25\times 0.287\times \left(-171.2\right)}{0.25}$$

$$= \textbf{−49.1 kJ}$$

This is negative, so the work is done on the air.

(c)

$$Q = \Delta U + W$$
$$\therefore \quad Q = 30.73 - 49.1 = -\textbf{18.37 kJ}$$

This is negative, so the heat is rejected by the air.

3.12 Adiabatic Process and a Gas

When dealing with the general case of a polytropic expansion or compression, it was stated that this process followed a law of the form $PV^n = C$. Now the adiabatic process can be a particular case of the polytropic process in which no heat is allowed to enter or leave during the progress of the process. From this it appears there should be a particular value of the index *n* which will satisfy this condition. An investigation is therefore necessary to see if this is the case.

Consider an adiabatic expansion or compression in which a change of state occurs from P_1, V_1, T_i to P_2, V_2, T_2. Then

$$\text{Change of internal energy} = mC_v(T_2 - T_1) \tag{1}$$

Also

$$\text{Work done during the process} = \frac{P_1V_1 - P_2V_2}{(\gamma - 1)} \tag{2}$$

$$= \frac{mR(T_1 - T_2)}{(\gamma - 1)} \tag{3}$$

Where γ (gamma) is the particular index which will satisfy the case of an adiabatic process (sometimes the adiabatic index is written *k*).

From the polytropic law, if γ is the adiabatic index

$$P_1 V^\gamma_1 = P_2 V^\gamma_2 \tag{4}$$

Also from the polytropic law

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma} = \left(\frac{V_2}{V_1}\right)^{(\gamma-1)} \tag{5}$$

and by the characteristic equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \tag{6}$$

Applying the non–flow energy equation

$$Q = \Delta U + W$$

For an adiabatic process $Q = 0$

$$\therefore \; 0 = \Delta U + W$$

or

$$W = -\Delta U \tag{7}$$

Work is done at the expense of internal energy during an adiabatic expansion Internal energy increases at the expense of work during an adiabatic compression. Substituting equations [1] and [3] in equation [7]

$$\frac{mR(T_1 - T_2)}{(\gamma - 1)} = -mc_V(T_2 - T_1)$$

$$\therefore \frac{mR(T_1 - T_2)}{(\gamma - 1)} = mc(T_1 - T_2)$$

from which

$$\frac{R}{(\gamma - 1)} = C_v \tag{8}$$

since $m(T_1 - T_2)$ is a common term on both sides. From this

$$\frac{R}{C_v} = (\gamma - 1) \tag{9}$$

or

$$\gamma = \frac{R}{C_v} + 1 = \frac{R + C_v}{C_v} \tag{10}$$

Now $R = C_p - C_v$; substituting in equation [8]

$$\gamma = \frac{C_p - C_v + C_v}{C_v}$$

or

$$\gamma = \frac{C_p}{C_v} \tag{11}$$

From this, then, the law for an adiabatic expansion or compression of a gas is $pv^\gamma = C$, where $\gamma = C_p/C_v$, the ratio of the specific heat capacities at constant pressure and constant volume. The theoretical adiabatic process is sometimes said to be a **frictionless** adiabatic process. The reason for this is perhaps best understood by attempting to suggest a practical way of carrying out an adiabatic process. If a piece of apparatus for carrying out an expansion or compression could be constructed of a perfect heat insulating material then an adiabatic process would be quite possible. But no perfect heat insulator exists, so perhaps the nearest approach to an adiabatic process is to complete the process very rapidly, in which case there is very little time for heat exchange between the gas and its surroundings.

But, when such a process is carried out, it is found that with both the compression and the expansion, the final temperature is slightly higher than the calculated value. Now since the process is very rapid, the heat transfer required to increase the temperature above the adiabatic temperature could not have transferred from the outside. The answer to this is friction, turbulence and shock within the gas itself. Energy is required to overcome these effects and it will appear as a slightly increased temperature of the gas above its theoretical value. If these effects are neglected then the adiabatic process is said to be **frictionless**.

The average value of γ, the adiabatic index, for air is of the order of 1.4.

Example 3.13 *A gas expands adiabatically from a pressure and volume of 700 kN/m² and 0.015 m³ respectively, to a pressure of 140 kN/m². Determine the final volume and the work done be the gas. Determine, also, the change of internal energy in this case. Take, $C_p = 1.046 \ kJ/kg \ K, \ C_v = 0.152 \ kJ/kgK$.*

SOLUTION

Adiabatic index $= \gamma = C_p/C_v = 1.046/0.752 = $**1.39**

For an adiabatic expansion

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\therefore \left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{P_1}{P_2} \ \text{ or } \ \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$$

from which

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/\gamma} = 0.015 \times \left(\frac{700}{140}\right)^{1.39}$$

$$= 0.015 \times 5^{1/1.39} = 0.015 \times {}^{1.39}\sqrt{5}$$
$$= 0.015 \times 3.18$$
$$= \textbf{0.04m}^3$$

The final volume is 0.048 m³.

$$\text{Work done } = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$= \frac{(700 \times 0.015) - (140 \times 0.048)}{1.39 - 1}$$

$$= \frac{10.5 - 6.72}{0.39} = \frac{3.78}{0.39}$$

$$= \textbf{9.69 kJ}$$

For an adiabatic process

$$W = -\Delta U$$

or

$$\Delta U = -W$$

∴ Change of internal energy = **−9.69 kJ**

This is a loss of internal energy from the gas.

3.13   Isothermal Process and a Gas

An isothermal process is defined as a process carried out such that the temperature remains constant throughout the process. This is evidently the same as a process carried out according to Boyle's law. The law for an isothermal expansion or compression of a gas is therefore

$PV = C$, a constant   [1]

Thus, for a change of state from 1 to 2

$P_1 V_1 = P_2 V_2$ [2]

$T_1 = T_2 = T =$ constant temperature   [3]

Now the law $PV = C$ is that of a rectangular hyperbola. In section 1.22 it was shown that

Work done $= PV \ln \dfrac{V_2}{V_1}$ [4]

This, therefore, is the expression which will give the work done during an isothermal process on a gas.

From the characteristic equation

$PV = mRT$ [5]

Substituting equation [5] in [4]

Work done $= mRT \ln \dfrac{V_2}{V_1}$ [6]

Applying the non–flow energy equation

$Q = \Delta U + W.$

For an isothermal process $T =$ constant, and by Joule's law, the internal energy of a gas is a function of temperature only, so if $T =$ constant, there is no change of internal energy.

Hence, for an isothermal process

$\Delta U = 0$

So the energy equation becomes

$Q = W$ [7]

$= PV \ln \dfrac{V_2}{V_1} = mRT \ln \dfrac{V_2}{V_1}$ [8]

It follows that, during an isothermal expansion, all the heat transferred is converted into external work. Conversely, during an isothermal compression, all the work done on the gas is rejected by the gas

as heat transfer. This sometimes seems a little odd at first, but remember the temperature must remain constant throughout so the internal energy before the process will be the same as the internal energy after the process. For an expansion, external work is performed by the gas. During an isothermal process the internal energy content of the gas must remain constant, so it appears that any heat transferred to the gas must immediately be dissipated in carrying out the external work. A similar analysis holds for the case of an isothermal compression. The energy input is in the form of work done on the gas and is immediately rejected as heat transfer.

Example 3.14   *A quantity of gas occupies a volume of 0.4 m³ at a pressure of 100 kN/m² and a temperature of 20°C. The gas is compressed isothermally to a pressure of 450 kN/m² then expanded adiabatically to its initial volume. For this quantity of gas determine*
(a) the heat transferred during the compression
(b) the change of internal energy during the expansion
(c) the mass of gas
Assume that, for the gas, γ = 1.4, C_p = 1.0 kJ/kg K.

(a)
For the isothermal compression

$$P_1 V_1 = P_2 V_2$$

$$\therefore V_2 = V_1 \frac{P_1}{P_2} = 0.4 \times \frac{100}{450} = \textbf{0.089 m}^3$$

Now $Q = \Delta U + W$ and for an isothermal process on a gas $\Delta U = 0$

$$\therefore \ Q = W = PV \ln r = PV \ln P_1/P_2$$

$$= 100 \times 0.4 \times \ln \frac{100}{450}$$

$$= -100 \times 0.4 \times \ln \frac{450}{100}$$

$$= -40 \times \ln 4.5 = -40 \times 1.5$$
$$= -\textbf{600 kJ}$$

This is heat rejected.

(b)
For the adiabatic expansion

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\therefore P_3 = P_2 \left(\frac{V_2}{V_3} \right)^{\gamma} = 450 \times \left(\frac{0.089}{0.4} \right)^{1.4}$$

$$= \frac{450}{4.5^{1.4}}$$

$$= \frac{450}{8.21}$$

$$= \textbf{54.8 kN/m}^2$$

Now $Q = \Delta U + W$ and for an adiabatic process $Q = 0$

$$\therefore \ 0 = \Delta U + W$$

or

$$\Delta U = -W = \frac{-\left(P_2 V_2 - P_3 V_3\right)}{\gamma - 1}$$

$$= \frac{-\left(450 \times 0.089 - 54.8 \times 0.4\right)}{1.4 - 1}$$

$$= \frac{-\left(40.05 - 21.9\right)}{0.4}$$

$$= \frac{-18.15}{0.4}$$

$$= -43.4 \text{ kJ}$$

This is a loss of internal energy.

(c)

$$C_p - C_v = R \quad \text{and} \quad C_p / C_v = \gamma \quad \text{so} \quad C_v = C_p / \gamma$$

$$\therefore \left(C_p - C_p / \gamma\right) = R = C_p \left(1 - \frac{1}{\gamma}\right)$$

$$\therefore R = 1.0 \times \left(1 - \frac{1}{1.4}\right) = 1.0 \times \left(1 - 0.714\right) \ = \textbf{0.286 kJ/kg K}$$

$$P_1 V_1 = mRT_1 \quad \text{and} \quad T_1 = (20 + 273) \text{ K} = \textbf{293 K}$$

$$\therefore \ m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 10^3 \times 0.4}{0.286 \times 10^3 \times 293} \ = 0.477 \text{ kg}$$

3.14   Non-flow Energy Equation

Consider the expansion or compression of a gas according to the law $PV^n = C$ in which the state changes from $P_1 \ V_1, \ T_1,$ to $P_2, \ V_2, \ T_2$. It has been shown that the change of internal energy is

$$\Delta U = mC_v(T_2 - T_1) \tag{1}$$

Also, the work done during the change is

$$W = \frac{mR\left(T_1 - T_2\right)}{n-1} \qquad\qquad [2]$$

Substituting equations [1] and [2] into the non-flow energy equation

   $Q = \Delta U + W$

Then

$$Q = mC_v\left(T_2 - T_1\right) + \frac{mR\left(T_1 - T_2\right)}{n-1} \qquad\qquad [3]$$

Now

   $C_p - C_v = R \qquad\qquad [4]$

and

$$\frac{C_p}{C_v} = \gamma \qquad\qquad [5]$$

From equation [5] $C_p = \gamma C_v$; substituting into equation [4] gives

   $\gamma\, C_v - C_v = R \text{ or } C_v\,(\gamma - 1) = R \qquad\qquad [6]$

$$\therefore C_v = \frac{R}{\left(\gamma - 1\right)} \qquad\qquad [7]$$

Substituting equation [7] into equation [3]

$$Q = m\frac{R}{\left(\gamma-1\right)}\left(T_2 - T_1\right) + m\frac{R}{\left(n-1\right)}\left(T_1 - T_2\right)$$

$$= m\frac{R}{\left(\gamma-1\right)}\left(T_1 - T_2\right) - m\frac{R}{\left(\gamma-1\right)}\left(T_1 - T_2\right)$$

$$= \left[\frac{1}{\left(n-1\right)} - \frac{1}{\left(\gamma-1\right)}\right] mRT\left(T_1 - T_2\right)$$

$$= \left[\frac{\left(\gamma-1\right)-\left(n-1\right)}{\left(n-1\right)\left(\gamma-1\right)}\right] mRT\left(T_1 - T_2\right)$$

$$= \frac{\left(\gamma - n\right)}{\left(\gamma - n\right)}\frac{mR\left(T_1 - T_2\right)}{\left(n-1\right)} \qquad\qquad [8]$$

Or

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{ Polytropic work} \qquad\qquad [9]$$

From this equation it is possible to examine what happens to the heat received or rejected during an expansion or compression of a gas if the value of the index *n* is varied. For a compression the work done is negative. In this case

If $n > \gamma$ then $\dfrac{\gamma - n}{\gamma - n}$ is negative, so Q is positive, i.e. heat is received.

If $n < \gamma$ then $\dfrac{\gamma - n}{\gamma - n}$ is positive, so Q is negative, i.e. heat is rejected.

For an expansion the work done is positive. In this case

If $n > \gamma$ then $\dfrac{\gamma - n}{\gamma - n}$ is negative, so Q is negative, i.e. heat is rejected

If $n < \gamma$ then $\dfrac{\gamma - n}{\gamma - n}$ is positive, so Q is positive, i.e. heat is received.

Note that

If $n = \gamma$ then $\dfrac{\gamma - n}{\gamma - n} = 0$, so $Q = 0$, i.e. this is the adiabatic case.

If $n = $ I then $\dfrac{\gamma - n}{\gamma - n} = 1$ so $Q = $ work done, i.e. this is the isothermal case.

Note that this analysis has shown how to control the value of the index *n*. The control of the index *n* is obtained by the extent to which heat is allowed, or not allowed, to pass out of, or into, the gas during the compression or expansion. Substituting equation [6] in equation [8]

$$Q = \frac{(\gamma - n)}{(\gamma - 1)} m C_v (\gamma - 1) \frac{(T_1 - T_2)}{(n - 1)}$$

$$= m C_v \frac{(\gamma - n)}{(\gamma - 1)}(T_1 - T_2) \qquad [11]$$

$$= m C_n (T_1 - T_2)$$

where

$$C_n = C_v \frac{(\gamma - n)}{(n - 1)} \qquad [12]$$

C_n is called the **polytropic specific heat capacity.**

Example 3.15   *A gas expands according to the law $PV^{1.3} = C$ from a pressure of 1 MN/m² and a volume 0.003 m³ to a pressure of 0.1 MN/m². Determine the heat received or rejected by the gas during this process. Determine the polytropic specific heat capacity. Take $\gamma = 1.4$, $C_v = 0.718$ kJ/kg K.*

SOLUTION

Now $P_1V_1 = P_2V_2$

$$\therefore V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/n} == 0.003\times\left(\frac{1}{0.1}\right)^{1/3}$$

$$= 0.003\times 101^{/1.3}$$
$$= 0.003\times 5.88$$
$$= \textbf{0.0 176 m}$$

Heat received or rejected is

$$Q = \frac{(\gamma-n)}{(\gamma-1)}\times\text{work done}$$

$$= \frac{(\gamma-n)}{\gamma-1}\times\frac{(P_1V_1 - P_2V_2)}{n-1}$$

$$= \frac{(1.4-1.3)}{(1.4-1)}\times\frac{(1\times0.003 - 0.1\times0.001}{1.3-1}$$

$$= \frac{0.1}{0.4}\times\frac{0.003-0.00176}{0.3}$$

$$= \frac{1}{4}\times\frac{0.00124}{0.3}$$

$$= \frac{0.00124}{1.2}$$
$$= 0.00103 \text{ MJ}$$
$$= \textbf{1.03 kJ}$$

This is positive, so heat is received by the gas.

$$C_n - C_v\frac{(\gamma-n)}{(n-1)} = 0.718\times\frac{(1.4-1.3)}{(1.3-1)}$$

$$= 0.718\times\frac{0.1}{0.3}$$
$$= \textbf{0.239 KJ/kg K}$$

Summary

Boyle's Law

During a change of state of any gas in which the mass and the temperature remain constant, the volume varies inversely as the pressure.

Mathematically $PV = C$, a constant

Charle's Law

During the change of state of any gas in which the mass and pressure remain constant, the volume varies in proportion with the absolute temperature.

Mathematically

$$\frac{V}{T} = C, \text{ a constant}$$

Characteristic Equation of a Perfect Gas and Gas Constant

$$PV = mRT$$

The units of R can be obtained from equation [9]. If pressure is in N/m2, specific volume is in m3/kg and temperature in K, then

$$\frac{Pv}{T} = R = \frac{N}{m^2} \times \frac{m^3}{kg} \times \frac{1}{K} = \frac{Nm}{kgK} = \frac{J}{kgK}$$

For air, the value of R is usually of the order 0.287 kJ/kg K.

Internal Energy of a Gas (Joule's Law)

The internal energy of a gas is a function of temperature only and is independent of changes in pressure and volume. This is known as Joule's law.

Specific Heat Capacities of a Gas

The specific heat capacity at constant volume is defined as the amount of heat which trans=fers to or from unit mass of gas while the temperature changes by 1 degree and the volume remains constant. It is written as C_v.

The specific heat capacity at constant pressure is defined as the amount of heat which transfers to or from unit mass of gas while the temperature changes by 1 degree and the pressure remains constant. It is written as C_p.

$$mc_r \, \Delta T = \Delta U$$

$$mc_v \, \Delta T = \Delta H$$

$$c_p = c_v - R$$

Polytropic Process and a Gas

$$P_1 V_1^n = P_2 V_2^n$$

$$\text{work done} = mR\frac{(T_1 - T_2)}{n-1}$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n} = \left(\frac{V_2}{V_1}\right)^{(n-1)}$$

Adiabatic Process and a Gas

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\text{Work done} = \frac{P_1V_1 - P_2P_2}{g-1}$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(r-1)/r} = \left(\frac{V_2}{V_1}\right)^{(r-1)}$$

$$Y = \frac{c_p}{c_v}$$

Isothermal Process and a Gas

$PV = C$, a constant

$$\text{Work done} = mRT \ \ln \frac{V_2}{V_1}$$

$$P_1V_1 = P_2V_2$$

Non-flow Energy Equation

$$Q = \frac{(y-n)}{(y-1)} \times \text{work done}$$

$$c_n = c_v \frac{(y-n)}{(y-1)}$$

Questions

1. A quantity of gas has an initial pressure of 140 kN/m^2 and volume 0.14 m^3. It is then compressed to a pressure of 700 kN/m^2 while the temperature remains constant Determine the final volume of the gas.

   [0.028 m^3]

2. A quantity of gas has an initial volume of 0.06 m^3 and a temperature of 15°C. It is expanded to a volume of 0.12 m^3 while the pressure remains constant. Determine the final temperature of the gas.

   [303 ºC]

3. A mass of gas has an initial pressure of 1 bar. and a temperature of 20°C. The temperature of the gas is now in reased to 550°C while the volume remains constant Determine the final pressure of the gas.

   [2.81 bar]

4. A mass of air has an initial pressure of 1.3 MN/m^2, volume 0.014 m^3 and temperature 135°C. It is expanded until its final pressure is 275 kN/m^2 and its volume become 0.056 m^3. Determine

   (a) the mass of air
   (b) the final temperature
   Take $R = 0.287$ kJ/kg K.

   [(a) 0.155 kg; (b)**72 ºC**]

5. A quantity of gas has an initial pressure and volume of 0.1 MN/m^2 and 0**.**1 m^3 respectively. It is compressed to a final pressure of 1.4 MN/m2 according to the law PV^{l26} = constant. Determine the final volume of the gas.

   [0.0123 m^3 (= 12.35 liters]

6. A quantity of gas has an initial volume and temperature of 1.2 litres and 15 ºC respectively. It is expanded to a volume of 3.6 litres according to the law $PV^{1.4}$ = constant. Determine the final volume of the gas.

[0ºC].

7. A mass of gas has an initial pressure and temperature of 0.11 MN/m² and 15ºC respectively. It is compressed according to the law $PV^{1.3}$ = constant unit the temperature becomes 90°C. Determine the final pressure of the gas.

[0.299 MN/m²]

8. 0.23 **kg** of air has an initial pressure of 1.7 MN/m² and a temperature of 200°C. It is expanded to a pressure of 0.34 MN/m² according to the law $PV^{1.35}$ = constant. Determine the work transferred during the expansion. Take R = 0.29 kJ/kg K.

[30.72 kJ]

9. 0.1 kg of gas is heated by means of an electric heater for a period of 10 min, during which time the pressure of the gas remains constant. The temperature of the gas is increased from 16°C to 78°C. The power used by the heater is 20 watts. Assuming no losses, determine

   (a) the specific heat capacity of the gas at constant pressure
   (b) the specific heat capacity of the gas at constant volume
   (c) the characteristic gas constant
   (d) the density of the gas at a temperature of 16°C and with a pressure of 0.12 MN/m² For the gas, take γ = 1.38.

[(a) 1.935 kJ/kg K; (b) 1.402 kJ/kg K; (c) 0.533 kJ/kg K; (d) 0.78 kg/m³]

10. An engine has a swept volume of 15 litres and a volume ratio of compression of 14:1. The air in the engine at the beginning of compression has a temperature and pressure of 30**°C** and 95 kN/m², respectively. The air is compressed according to the law $PV^{1.34}$ = C. At the end of the compression the air is heated at constant volume through a pressure ratio of 1.6:1. Determine

    (a) the temperature and pressure of the air at the end of the compression
    (b) the temperature and pressure of the air at the end of the constant volume process
    (c) the heat transfer required to carry out the constant volume process
    For the air, take C_p = 1.005 kJ/kg K, R = 0.24 kJ/kg K.

[(a) 469.4°C, 3262.3 kN/m²; (b) 915°C, 5219.7 kN/m²; (c) 7.16 kJ]

11. One kilogram of gas at an initial pressure of 0.11 MN/m² and a temperature of 15°C. It is compressed isothermally until the volume becomes 0.1 m³. Determine

    (a) the final pressure
    (b) the final temperature
    (c) the heat transfer
    If the compression had been adiabatic, determine
    (d) the finalpressure
    (e) the final temperature
    (f) the work transfer
     For the gas, take C_p = 0.92 kJ/kg K, C_v = 0.66 kJ/kg K.

[(a) 0.748 MN/m²; (b) 15°C; (c) −143.8 kJ; (d) 1.591 MN/m²;
(e) 339.9°C; (f) −214.4 kJ]

12. A gas has an initial pressure, volume and temperature of 140 kN/m², 0.012 m³ and 100°C, respectively. The gas is compressed to a final pressure of 2.8 MN/m² and volume of 0.001 2 m². Determine

    (a) the index of compression if the compression is assumed to follow the law PVn = C

(b) the final temperature of the gas

(c) the work transfer

(d) the change of internal energy of the gas

For the gas, take $R = 0.287$ kJ/kg K, $C_v = 0.717$ kJ/kg K.

[(a) $n = 1.3$; (b) 471°C; (c) −5.6 kJ; (d) 4.2 kJ]

13. A gas has a density of 0.09 kg/m^3 at a temperature of 0°C and a pressure of 1.013 bar. Determine

   (a) the characteristic gas constant

   (b) the specific volume of the gas at a temperature of 70°C and a pressure of 2.07 bar

   If a volume of 5.6 m of the gas at an initial pressure of 1.02 bar and temperature

   0°C is heated at constant presure to a final temperature of 50°C, determine

   (c) the heat transfer

   (d) the change of internal energy of the gas

   (e) the work transfer

   For the gas, take $C_v = 10.08$ kJ/kg K.

[(a) 4.13 kJ/kg K; (b) 6.84 m^3/kg; (c) 360.2 kJ; (d) 255.5 kJ; (e) 104.7 kJ]

14. A gas has an initial pressure, volume and temperature of 95 kN/m^2, 14 litres and 100°C, respectively. The gas is compressed according to the law $PV^{1.3} = C$ through a volume ratio of 14:1. Determine

   (a) the work transfer

   (b) the change of internal energy

   (c) the heat transfer

   For. the gas, take $R = 0.29$ kJ/kg K, $C_v = 0.72$ kJ/kg K.

[(a) −5.35 kJ; (b) 4.015 kJ; (c) −1.335 kJ]

15. A gas at an initial pressure of 690 KN/m^2 and temperature of 185°C has a mass of 0.45 kg. The gas is expanded adiabatically to a final pressure of 138 KN/m^2 with a fall of temperature of 165°C. The work transfer during the expansion is 53 kJ. For the gas, determine

   (a) the specific heat capacity at constant volume

   (b) the adiabatic index

   (c) the specific heat capacity at constant pressure

[(a) 0.714 kJ/kg K; (b) 1.385; (c) 0.989 kJ/kg K]

16. An air receiver has a volume of 4.25 m^3 and contains air at a pressure of 650 kN/m^2 and a temperature of 120°C. The air is cooled to a temperature of 40°C. Determine

   (a) the final pressure of the air

   (b) the change of internal energy of the air

   For the air, take $R = 0.29$ kJ/kg K, $C_v = 0.717$ kJ/kg K.

[(a) 517.7 kN/m^2; (b) −1390 kJ]

17. An internal combustion engine has a cylinder bore of 165 mm and a piston stroke of 300 mm. The volume ratio of compression is 8:1. At the commencement of the working stroke the pressure of the gas in the clearance volume is 4.5 MN/m^2 and the temperature is 400°C. The gas expands at constant pressure while the piston moves a distance of 45 mm down the cylinder. Determine

   (a) the temperature of the gas at the end of the piston movement

   (b) the work transfer during the piston movement

   (c) the heat transfer during the piston movement

   For the gas, take $R = 0.29$ kJ/kg K, $C_p = 1.005$ kJ/kg K.

[(a) 1102°C; (b) 4.32 kJ; (c) 14.32 kJ]

18. An engine has a volume ratio of compression of 12:1. At the beginning of compression the gas in the cylinder has a pressure, volume and temperature of 110 kN/m^2, 0.28 m^3 and 80°C, respectively. The gas is compressed according to the law $PV^{1.28} = C$. Determine

  (a) the pressure of the gas after compression

  (b) the temperature of the gas after compression

  (c) the work transfer during compression

  (d) the heat transfer during compression

For the gas, take C_p = 1.0 kJ/kg K, C_v = 0.71 kJ/kg K.

[(a) 2.647 MN/m²; (b) 433°C; (c) –107.5 kJ; (d) –34.9 kJ]

19. A quantity 01 gas has an initial pressure, volume and temperature of 1.4 MN/m² , 0.14 m³ and 300°C, respectively. The gas is expanded adiabatically to a pressure of 280 kN/m². Determine

  (a) the mass of gas

  (b) the temperature of the gas after the expansion

  (c) the work transfer

  (d) the change of internal energy of the gas

For the gas, take C_p = 1.04 kJ/kg K, C_v = 0.74 kJ/kg K.

[(a) 1.14 kg; (b) 87.4°C; (c) 179.5 kJ; (d) -179.5 kJ]

20. 0.45 kg of gas is expanded adiabatically until the pressure is halved and the temperature of the gas falls from 220°C to 130°C. During the expansion there is a work transfer from 27 kJ. Determine (a) the adiabatic index of the gas (b) the characteristic gas constant

[(a) 1.408; (b) 0.272 kJ/kg K]

21. A quantity of gas has a mass of 0.2 kg and an initial temperature of 15°C. It is compressed adiabatically through a volume ratio of 4:1. The final temperature after compression is 237°C. The work transfer during compression is 33 kJ. For the gas, determine

  (a) the specific heat capacity at constant volume

  (b) the adiabatic index

  (c) the specific heat capacity at constant pressure

  (d) the characteristic gas constant

[(a) 0.743 kJ/kg K; (b) 1.412; (c) 1.049 kJ/kg K; (d) 0.306 kJ/kg K]

22. The cylinder of an engine has a stroke of 300 mm and a bore of 250 mm. The volume ratio of compression is 14:1. Air in the cylinder at the beginning of compression has a pressure of 96 kN/m² and a temperature of 93°C. The air is compressed for the full stroke according to the law $PV^{1.3} = C.$ Determine

  (a) he mass of air

  (b) the work transfer

  (c) the heat transfer

For the air, take γ = 1.4, C_p = 1.006 kJ/kg K.

[(a) 0.0145 kg; (b) – 6.15 kJ; (c) –1.54 kJ]

23. A quantity of air has a pressure, volume and temperature of 104 kN/m², 30 litres and 38°C, respectively. The temperature of the air is raised (i) by heating while the volume remains constant until the pressure becomes 208 kN/m²; and (ii) by adiabatic compression to a volume of 6 litres. For both cases, determine

  (a) the final temperature

  (b) the work transfer

  (c) the change of internal energy

  (d) the heat transfer

For the air, take R = 0.29 kJ/kg K, γ – 1.4.

[(i) (a) 349°C; (b) 0; (c) 7.8 kJ/kg; (d) 7.8 kJ

(ii) (a) 319°C; (b) –7.05 kJ; (c) 7.05 kJ; (d) 0]

24. A mass of gas has a pressure, volume and temperature of 100 kN/m² , 0.56 m³ and 20°C, respectively. It is compressed to a volume of 0.15 m³ according to the law $PV^{1.36} = C.$ The gas is then cooled at constant pressure until the volume becomes 0.1 m³. Determine

(a)   the final pressure, the final temperature and the work transfer for the compression

(b)   the final temperature and the heat transfer for the constant pressure process

for the gas, take C_p = 1.006 kJ/kg K, R = 0.287 kJ/kg K.

[(a) 600 kN/m², 197.6°C, –94.4 kJ; (b) 40.7°C,–105.1 kJ]

25. An air main is connected to a cylinder through a valve. A piston slides in the cylinder. The air in the main is maiained at a constant pressure and temperature of 1 MN/m² and 40°C, respectively. The initial pressure and volume of air in the cylinder are 140 kN/m² and 3 litres, respectively. The valve is opened, 0.11 kg of air enters the cylinder then the valve is closed. As a result of this mass transfer, the pressure in the cylinder becomes 700 kN/m² and the volume becomes 15 litres. Asuming the process to be adiabatic, determine the work done on the piston.

For air, take C_p = 1.006 kJ/kg K, C_v = 0.717 kJ/kg K.

[7.8 kJ]

Previous Years' GTU Examination Questions

1. A cylinder contains 0.6 m³ of gas at a pressure of 1.0 bar and 900°C. The gas is compressed to a volume of 0.18 m³ according to law PV^n = C. The final pressure is 5.0 bar. Assuming R = 0.287 kJ/kg K and γ = 1.4, calculate:
    (i) The mass of gas
    (ii) The value of index 'n' for compression
    (iii) The change of internal energy of gas

[Dec '08]

2. Derive characteristic equation of a perfect gas.

[Dec '08]

3. An ideal gas is heated from 25ºC to 145ºC. The mass of gas is 2 kg.
    Determine:
    (i) Specific heats
    (ii) Change in internal energy
    (iii) Change in enthalpy. Assume R = 267 J/kg K and γ =1.4 for the gas.

[Mar '09]

4. Prove that the relation between C_p and C_v is $C_p – C_v$ = R.

[Mar '09]

5. Write about combined gas law, gas constant and non-flow process.

[Mar '09]

6. A gas, whose pressure, volume, and temperatures are 2.75 bar, 0.09m3 and 185°C respectively, has the state changed at constant pressure until its temperature becomes 15°C. Calculate
    (i) Heat transferred
    (ii) Work done during the process
    Take R= 0.29 kJ/kg K and C_p = 1.005 kJ/kg K

[Jun '09]

7. With usual notations prove that $C_p – C_v$ = R.

[Jun '09]

8. What is isothermal process? Derive an expression for the work done during the isothermal process.

[Jun '10]