

5

Heat Engines

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OBJECTIVES

After studying this chapter, you will be able:

- To understand heat engines as thermal prime movers, their types and related thermal parameters
- To understand different power cycles like carnot, rankine, otto, diesel etc.

5.1 Heat Engine

5.1.1 Heat Engine as Thermal Prime Movers

Heat is defined as that transfer of energy which results from a difference in temperature, so a **heat engine** must be an engine in which a transfer of heat occurs. If heat is introduced into a system and, as the result of a cyclic process, some work appears from that system, together with some heat rejection from the system, then this is a heat engine. This is illustrated in Fig. 5.1. In practice, such an engine is the closed-circuit steam turbine plant of a power station.

On the other hand, the open-circuit internal combustion engine, such as a petrol engine, is strictly not a heat engine; fuel and air are admitted across the system boundary, combustion is internal, as the name implies, and combustion products and heat are rejected, with some work crossing the system boundary.

However, nearly all thermodynamic engines are colloquially referred to as heat engines.

5.1.2 Principle of Heat Engine

The thermodynamic engine is a device in which energy is supplied in the form of heat and some of this energy is transformed into work. It would be ideal if all the energy supplied was transformed into work.

The usual process in the engine can be followed by reference to Fig. 5.1.

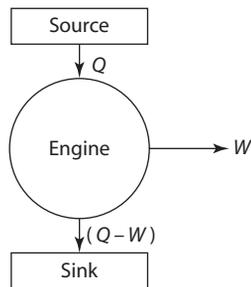


Fig. 5.1 Thermodynamic engine

With all engines there must be a source of supply of heat and, with any quantity of heat Q supplied from the source to the engine, an amount W will successfully be converted into work. This will leave a quantity of heat $(Q - W)$ to be rejected by the engine into the sink.

The ratio

$$\frac{W}{Q} = \frac{\text{Work done}}{\text{Heat received}} \quad [1]$$

is called the **thermal efficiency** and determines what fraction of the heat input has actually been successfully converted into work output. It will be evident that the object in all engines is, or should be, to make the thermal efficiency as near to unity as possible.

5.1.3 Source of Heat

The detailed list of sources of heat to get the mechanical work are:

- (a) Conventional energy sources; like coal, crude, thermal power, hydro power, nuclear power etc.
- (b) Non-conventional energy sources; which include solar energy, wind energy, tidal energy, geothermal energy and ocean wave energy, bio fuels etc.

5.1.4 Working Substance

Working substance is basically a gas or mixture of gases used for heat transfer and capable of absorbing and storing heat and released when need arise. The important thermodynamic parameters like pressure, volume, temperature and their effects will be considered for exact understanding of such process.

5.1.5 Converting Machines

Converting machines are defined as machines capable of converting heat energy of working substance into mechanical work.

5.1.6 Classification of Heat Engines

Heat engines are generally classified as follows:

- (a) Internal Combustion Engines like; liquid fuel and gas fuel engine, jet engines etc.
- (b) External Combustion Engines like; steam engines, steam turbines etc.

5.2 Heat Engine Cycles

The following survey lists and illustrates some important gas power cycles together with their date of origin and the names of the people most commonly associated with their development. All the cycles were originally developed in the nineteenth century.

The illustrations of the cycles are P - V and T - s diagrams. They are listed in date order and each is accompanied by a brief note on past and present applications.

5.2.1 Carnot Cycle

The Carnot cycle (Fig. 5.2) was conceived by Sadi Carnot in 1824. The cycle consists entirely of reversible processes.

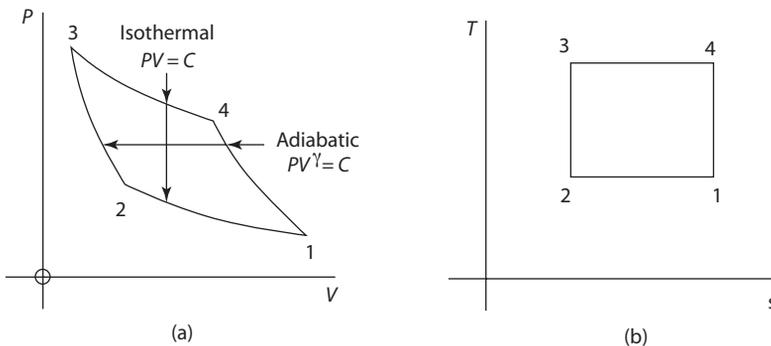


Fig. 5.2 Carnot cycle: (a) P - V diagram; (b) T - s diagram

The cycle consists of two isothermal processes, 1-2 and 3-4, together with two adiabatic processes, 2-3 and 4-1. No practical engine was ever built to run on this cycle.

It is important because it is composed of reversible processes, which are the most thermodynamically efficient processes, so its thermal efficiency establishes the maximum thermal efficiency possible within the temperature limits of the cycle.

Carnot conceived a cycle made up of thermodynamically reversible processes. By calculating the thermal efficiency of this cycle it is possible to establish the maximum possible efficiency between the temperature limits taken.

Fig. 5.3(a) illustrates the Carnot cycle on a $P-V$ diagram and Fig. 5.3(b) on a $T-s$ diagram. It consists of four reversible processes:

- **1–2 Isothermal expansion**

Pressure falls from P_1 to P_2 .

Volume increases from V_1 to V_2 .

Temperature remains constant at $T_1 = T_2$.

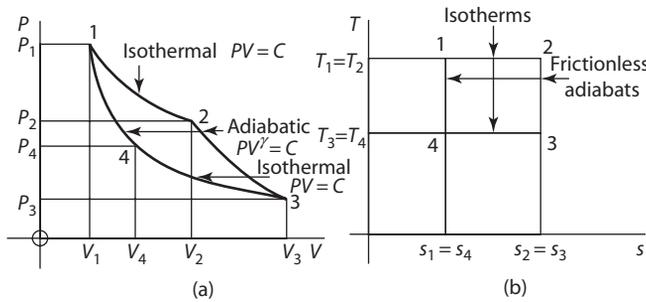


Fig. 5.3 Carnot cycle: (a) $P-V$ diagram; (b) $T-s$ diagram

$$\text{Work done} = P_1 V_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{V_2}{V_1}$$

For an isothermal process, $Q = W$

$$\therefore \text{Heat received} = mRT_1 \ln \frac{V_2}{V_1}$$

- **2–3 Adiabatic expansion**

Pressure falls from P_2 to P_3 .

Volume increases from V_2 to V_3 .

Temperature falls from T_2 to T_3 .

$$\text{Work done} = \frac{P_2 V_2 - P_3 V_3}{(\gamma - 1)} = \frac{mR(T_2 - T_3)}{(\gamma - 1)}$$

For an adiabatic process, $Q = 0$

\therefore No heat transfer during this process

- **3–4 Isothermal compression**

Pressure increases from P_3 to P_4 .

Volume reduced from V_3 to V_4 .

Temperature remains constant at $T_3 = T_4$.

$$\begin{aligned}\text{Work done} &= P_3 V_3 \ln \frac{V_4}{V_3} = -P_3 V_3 \ln \frac{V_3}{V_4} \\ &= -mRT_3 \ln \frac{V_3}{V_4}\end{aligned}$$

For an isothermal process, $Q = W$

$$\therefore \text{Heat rejected} = mRT_3 \ln \frac{V_3}{V_4}$$

- **4–1 Adiabatic compression**

Pressure increases from P_4 to P_1 .

Volume reduced from V_4 to V_1 .

Temperature increases from T_4 to T_1 .

$$\begin{aligned}\text{Work done} &= \frac{P_4 V_4 - P_1 V_1}{(\gamma - 1)} = -\frac{(P_1 V_1 - P_4 V_4)}{(\gamma - 1)} \\ &= -\frac{mR(T_1 - T_4)}{(\gamma - 1)}\end{aligned}$$

For the adiabatic process, $Q = 0$

\therefore No heat transfer during this process

Note that this process returns the gas to its original state at 1.

The net work done during this cycle may be determined by summing the areas beneath the various processes, taking the expansions as positive areas and the compressions as negative areas. Thus

$$\begin{aligned}\text{Net work done/cycle} &= \oint W \\ &= \text{Area under 1–2} + \text{Area under 2–3} - \text{Area under 3–4} \\ &\quad - \text{Area under 4–1} \\ &= \text{Area 1234} \\ &= \text{Area enclosed by cycle}\end{aligned}$$

or

$$\oint W = mRT_1 \ln \frac{V_2}{V_1} + \frac{mRT(T_2 - T_3)}{(\gamma - 1)} - mRT_3 \ln \frac{V_3}{V_4} - \frac{mR(T_1 - T_4)}{(\gamma - 1)} \quad [1]$$

Now $T_1 = T_2$ and $T_3 = T_4$, from the isotherms.

$$\therefore \frac{mR(T_2 - T_3)}{(\gamma - 1)} = \frac{mR(T_1 - T_4)}{(\gamma - 1)}$$

Hence, from equation [1]

$$\oint W = mRT_1 \ln \frac{V_2}{V_1} - mRT_3 \ln \frac{V_3}{V_4} \quad [2]$$

Now for adiabatic 1-4,

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1} \right)^{(\gamma - 1)} \quad [3]$$

for adiabatic 2-3

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2} \right)^{(\gamma - 1)} \quad [4]$$

But $T_1 = T_2$ and $T_3 = T_4$

$$\therefore \frac{T_1}{T_4} = \frac{T_2}{T_3} \quad [5]$$

Hence, from equations [3] and [4]

$$\frac{V_4}{V_1} = \frac{V_3}{V_2} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad [6]$$

Substituting equation [6] in equation [2]

$$\oint W = mR \ln \frac{V_2}{V_1} (T_1 - T_3) \quad [7]$$

This is positive work done and this is always the case if the processes of a cycle proceed in a clockwise direction. Net external work can thus be obtained from such cycles.

If the processes proceed in an anticlockwise direction then the work done is negative, in which case equation [7] now becomes

$$\oint W = -mR \ln \frac{V_2}{V_1} (T_1 - T_3) \quad [8]$$

Negative work means that net external work must be put in to carry out such cycles.

Now

$$\text{Thermal } \eta = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$$

So from the analysis given above

$$\begin{aligned} \text{Thermal } \eta &= \frac{mRT_1 \ln(V_2/V_1) - mRT_3 \ln(V_3/V_4)}{mRT_1 \ln(V_2/V_1)} \\ &= \frac{mR \ln(V_2/V_1)(T_1 - T_3)}{mR \ln(V_2/V_1)T_1} \end{aligned}$$

And since $V_2/V_1 = V_3/V_4$ from equation [6]

$$\text{Thermal } \eta = \frac{T_1 - T_3}{T_1} \quad [9]$$

$$= \frac{\text{Max. abs. temp.} - \text{Min. abs. temp.}}{\text{Max. abs. temp.}} \quad [10]$$

Now from equation [9]

$$\text{Thermal } \eta = 1 - \frac{T_3}{T_1} \quad [11]$$

and from equations [3], [4] and [5]

$$\frac{T_1}{T_3} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)} = r_v^{(\gamma-1)}$$

where r_v = adiabatic compression and expansion volume ratio

So from equation [11]

$$\text{Thermal } \eta = 1 - \frac{1}{r_v^{(\gamma-1)}} \quad [12]$$

This thermal efficiency gives the maximum possible thermal efficiency obtainable between any two given temperature limits.

From the T - s diagram (Fig. 5.13(b))

$$\begin{aligned} \text{Heat received from 1 to 2} &= T_1(s_1 - s_2) \\ &= \text{Area under 1-2} \end{aligned} \quad [13]$$

$$\begin{aligned} \text{Heat rejected from 3 to 4} &= T_3(s_3 - s_4) \\ &= \text{Area under 3-4} \end{aligned} \quad [14]$$

$$\begin{aligned}
\text{Thermal efficiency} &= \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}} \\
&= \frac{T_1(s_2 - s_1) - T_3(s_3 - s_4)}{T_1(s_2 - s_1)} \\
&= \frac{(T_1 - T_3)(s_2 - s_1)}{T_1(s_2 - s_1)} \quad \text{since } (s_2 - s_1) = (s_3 - s_4) \\
&= \frac{T_1 - T_3}{T_1} \tag{15}
\end{aligned}$$

Note that this solution for the thermal efficiency of the Carnot cycle is somewhat simpler than that given in the previous work. Also

$$\begin{aligned}
\oint W &= \text{Heat received} - \text{Heat rejected} \\
&= (T_1 - T_2)(s_2 - s_1) \tag{16}
\end{aligned}$$

Now, the Carnot cycle has the maximum thermal efficiency obtainable within given temperature limits, so it is possible to suggest that if any engine working between the same temperature limits has a thermal efficiency lower than that of the Carnot cycle, then thermal efficiency improvement for the engine is theoretically possible. All practical engines have a thermal efficiency much lower than the Carnot thermal efficiency.

The ultimate aim, in practice, should be an attempt to reach an efficiency as near 100 per cent as possible. How could this be achieved?

$$\text{Carnot thermal } \eta = \frac{\text{Max. abs. temp.} - \text{Min. abs. temp.}}{\text{Max. abs. temp.}}$$

For this equation to be a maximum it must equal unity, in which case the thermal efficiency would be 100 per cent.

Consider the minimum absolute temperature as being fixed. If the maximum absolute temperature is increased, the magnitude of the right-hand quotient gets larger. Eventually, as the maximum absolute temperature $\rightarrow \infty$, the quotient $\rightarrow 1$. It is quite impossible to have an infinitely high maximum absolute temperature. In any case, contemporary engine materials will not stand up to continuous exposure at the high temperatures which are obtainable.

Now consider the maximum absolute temperature as being fixed. If the minimum absolute temperature is reduced to zero (to the absolute zero of temperature) then, once again, the quotient $\rightarrow 1$ and the Carnot efficiency $\rightarrow 100$ per cent. This too is impossible because all working substances will have liquefied and solidified before reaching this low temperature. In any case, the absolute zero of temperature is such a difficult temperature to achieve that it is out of the question as the sink temperature of an engine.

It would appear from this that a thermal efficiency of 100 per cent is impossible to achieve in practice. Hence all practical engines are inefficient. However, a guide has been given as to how the thermal

efficiency of engines delivering net work may be improved. That is, to spread the maximum and minimum temperatures as far apart as possible, consistent with the satisfactory safe operation of the engine.

In section 5.1.2, the concept of work ratio for a cycle was discussed. This was defined as

$$\text{Work ratio} = \frac{\text{Net work done}}{\text{Positive work done}}$$

From equation [7] for the net positive work Carnot cycle

$$\text{Net work done} = \oint W = mR \ln \frac{V_2}{V_1} (T_1 - T_3)$$

The positive work done during the Carnot cycle occurs during process, 1–2 and 2–3 (see Fig. 5.3(a)). The area under these processes shown on the P - V diagram gives the positive work done, thus

$$\text{Positive work done} = mRT_1 \ln \frac{V_2}{V_1} + \frac{mR(T_1 - T_3)}{(\gamma - 1)} \quad [17]$$

Hence, for the net positive work Carnot cycle

$$\begin{aligned} \text{Work ratio} &= \frac{mR \ln \frac{V_2}{V_1} (T_1 - T_3)}{mRT_1 \ln \frac{V_2}{V_1} + \frac{mR(T_1 - T_3)}{(\gamma - 1)}} \\ &= \frac{\ln \frac{V_2}{V_1} (T_1 - T_3)}{T_1 \ln \frac{V_2}{V_1} + \frac{(T_1 - T_3)}{(\gamma - 1)}} \end{aligned} \quad [18]$$

It was suggested that good ideal thermal efficiency together with good work ratio is required of a cycle, if possible. It is unfortunate that the Carnot cycle has a low work ratio even though it has the highest ideal thermal efficiency. The Carnot cycle is sometimes called the **constant temperature cycle** because heat is transferred during the isothermal processes only.

Example 5.1 A Carnot cycle using a gas has temperature limits 400°C and 70°C. Determine the thermal efficiency of the cycle.

SOLUTION

We have

$$\text{Thermal } \eta = \frac{T_1 - T_3}{T_1}$$

Now $T_1 = 400 + 273 = 673 \text{ K}$

$T_3 = 70 + 273 = 343 \text{ K}$

$$\begin{aligned} \therefore \text{Thermal } \eta &= \frac{673 - 343}{673} \\ &= \frac{330}{673} \\ &= 0.49 \\ &= 49\% \end{aligned}$$

Example 5.2 The overall volume expansion ratio of a Carnot cycle is 15. The temperature limits of the cycle are 260°C and 21°C . Determine

(a) the volume ratios of the isothermal and adiabatic processes

(b) the thermal efficiency of the cycle

Take $\gamma = 1.4$.

SOLUTION

First draw a diagram (Fig. 5.4).

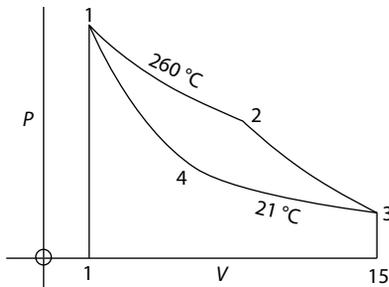


Fig. 5.4 Diagram for Example 5.2

(a)

For the adiabatic processes

$$\frac{T_1}{T_4} = \frac{T_2}{T_3} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)}$$

$$\therefore \frac{V_4}{V_1} = \frac{V_3}{V_2} = \left(\frac{T_1}{T_4}\right)^{1/(\gamma-1)} = \left(\frac{T_2}{T_3}\right)^{1/(\gamma-1)} \quad \text{and } T_1 = T_2 = 260 + 273 = 533 \text{ K}$$

$$T_3 = T_4 = 21 + 273 = 294 \text{ K}$$

$$\begin{aligned} &= \left(\frac{533}{294}\right)^{1/(1.4-1)} \\ &= 1.812^{1/0.4} \\ &= 4.426 \end{aligned}$$

The volume ratio of adiabatics r_v is 4.426

$$\begin{aligned}\text{Volume ratio of isothermals} &= \frac{V_3}{V_4} = \frac{V_3 V_1}{V_1 V_4} \\ &= \frac{15}{4.426} \\ &= \mathbf{3.39}\end{aligned}$$

(b)

$$\begin{aligned}\text{Thermal efficiency} &= \frac{T_1 - T_4}{T_1} \\ &= \frac{533 - 294}{533} \\ &= \frac{239}{533} \\ &= 0.448 \\ &= \mathbf{44.8\%}\end{aligned}$$

Alternatively

$$\begin{aligned}\text{Thermal efficiency} &= 1 - \frac{1}{r_v^{(\gamma-1)}} = 1 - \frac{1}{4.426^{(1.4-1)}} = 1 - \frac{1}{4.426^{0.4}} \\ &= 1 - \frac{1}{1.812} \\ &= 1 - 0.552 \\ &= 0.448 \\ &= \mathbf{44.8\%}\end{aligned}$$

Example 5.3 One kilogram of air is taken through a Carnot cycle. The initial pressure and temperature of the air are 1.73 MN/m^2 and 300°C , respectively. From the initial conditions, the air is expanded isothermally to three times its initial volume and then further expanded adiabatically to six times its initial volume. Isothermal compression, followed by adiabatic compression, completes the cycle. Determine

- the pressure, volume and temperature at each corner of the cycle
- the thermal efficiency of the cycle
- the work done per cycle
- the work ratio

Take $R = 0.29 \text{ kJ/kg K}$, $\gamma = 1.4$.

SOLUTION

First draw a diagram (Fig. 5.5).

(a)

For the isothermal process 1–2

Pressure $P_1 = 1.73 \text{ MN/m}^2$; Temperature $t_1 = 300^\circ\text{C}$

Now $P_1V_1 = mRT_1$ and $T_1 = 300 + 273 = 573 \text{ K}$

$$\begin{aligned} \therefore V_1 &= \frac{mRT_1}{P_1} = \frac{1 \times 0.29 \times 573}{1730} \\ &= 0.096 \text{ m}^3 \end{aligned}$$

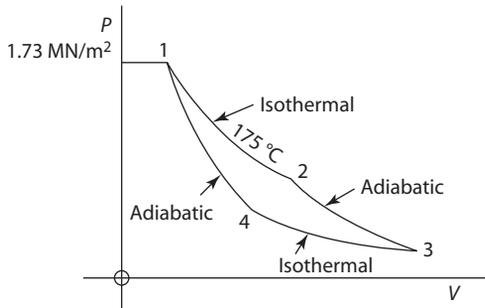


Fig. 5.5 Diagram for Example 5.3

Temperature remains constant at $t_2 = 300^\circ\text{C}$. The volume becomes three times the initial volume.

$$\therefore V_2 = 3 \times V_1 = 3 \times 0.096 = 0.288 \text{ m}^3$$

For an isothermal process, $P_1V_1 = P_2V_2$

$$\therefore P_2 = \frac{P_1V_1}{V_2} = \frac{P_1}{3} = \frac{1730}{3} = 576.7 \text{ kN/m}^2$$

For the adiabatic process 2–3

$$\text{Volume } V_3 = 6 \times V_1 = 6 \times 0.096 = 0.576 \text{ m}^3$$

For the adiabatic process, $T_1/T_3 = (V_3/V_2)^{(\gamma-1)}$

$$\begin{aligned} \therefore T_3 &= T_2 \left(\frac{V_2}{V_3} \right)^{(\gamma-1)} = 573 \times \left(\frac{0.288}{0.576} \right)^{(1.4-1)} \\ &= 573 \times \left(\frac{1}{2} \right)^{0.4} = \frac{573}{2^{0.4}} \\ &= \frac{573}{1.32} \\ &= 434 \text{ K} \end{aligned}$$

$$t_3 = 434 - 273 = 161 \text{ }^\circ\text{C}$$

$$\text{Also } P_2V_2^\gamma = P_3V_3^\gamma$$

$$\begin{aligned} \therefore P_3 &= P_2 \left(\frac{V_2}{V_3} \right)^\gamma = \frac{576.7}{2^{1.4}} \\ &= \frac{576}{2.639} \\ &= 218.53 \text{ kN/m}^2 \end{aligned}$$

For the isothermal process 3–4

$$t_3 = t_4 = 161^\circ\text{C}$$

Now for both adiabatic processes, the temperature ratio is the same because they both have the same end temperatures.

$$\therefore \frac{T_1}{T_4} = \frac{T_2}{T_3} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)}$$

$$\therefore \frac{V_4}{V_1} = \frac{V_3}{V_2} = 2$$

$$\therefore V_4 = 2V_1 = 2 \times 0.096 = \mathbf{0.192 \text{ m}^3}$$

For the isothermal process, $P_3V_3 = P_4V_4$

$$\therefore P_4 = P_3 \frac{V_3}{V_4} = 218.53 \times \frac{0.576}{0.192} = 218.3 \times 3 = \mathbf{655.6 \text{ kN/m}^2}$$

Alternatively, $P_1V_1^\gamma = P_4V_4^\gamma$

$$\begin{aligned} \therefore P_4 &= P_1 \left(\frac{V_1}{V_4}\right)^\gamma = 1730 \times \left(\frac{1}{2}\right)^{1.4} \\ &= \frac{1730}{2^{1.4}} = \frac{1730}{2.639} \\ &= \mathbf{655.6 \text{ kN/m}^2} \end{aligned}$$

(b)

$$\begin{aligned} \text{Thermal efficiency} &= \frac{T_1 - T_3}{T_1} \\ &= \frac{573 - 464}{573} \\ &= \frac{109}{573} \\ &= 0.19 \\ &= \mathbf{19\%} \end{aligned}$$

(c)

$$\begin{aligned} \oint W &= mR \ln \frac{V_2}{V_1} (T_1 - T_3) \\ &= 1 \times 0.29 \times \ln 3 \times (573 - 464) \\ &= 1 \times 0.29 \times 1.099 \times 109 \\ &= \mathbf{34.7 \text{ kJ}} \end{aligned}$$

Alternatively

$$\begin{aligned} \oint W &= \text{Heat received} \times \text{Thermal efficiency} \\ &= mRT_1 \ln \frac{V_2}{V_1} \times 0.19 \\ &= 1 \times 0.29 \times 573 \times 1.099 \times 0.19 \\ &= \mathbf{34.7 \text{ kJ}} \end{aligned}$$

(d)

$$\begin{aligned} \text{Work ratio} &= \frac{\ln \frac{V_2}{V_1} (T_1 - T_3)}{T_1 \ln \frac{V_2}{V_1} + \frac{(T_1 - T_3)}{(y-1)}} \\ &= \frac{1.099 \times 109}{(573 \times 1.099) + \frac{109}{0.4}} \\ &= \frac{119.79}{629.73 + 272.5} \\ &= \frac{119.79}{902.23} \\ &= \mathbf{0.133} \end{aligned}$$

Note that this is a very low work ratio.

5.2.2 Carnot Cycle and Steam Plant

It was suggested that the Carnot cycle had the greatest efficiency possible between any two given limits of temperature. It is important, therefore, to see whether the Carnot cycle can be successfully applied to a steam plant. From this will be developed the **Rankine cycle** and the concepts of **reheat** and **feed heat**.

To understand the Carnot cycle in a steam plant, consider the P - V diagram in Fig. 5.6(a).

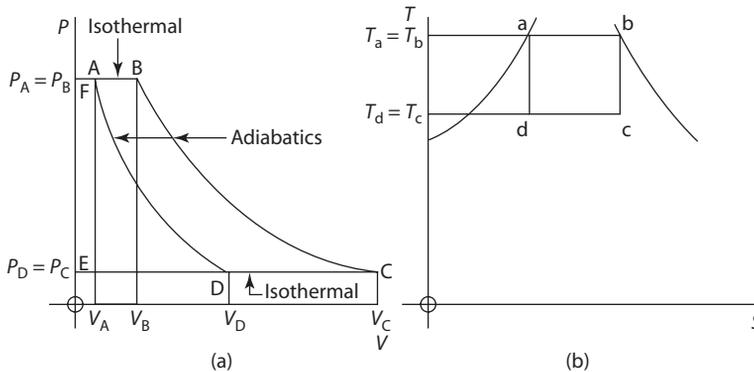


Fig 5.6 Carnot cycle for steam plant: (a) P - V diagram; (b) T - s diagram

- **AB** Water at boiler pressure P_B and volume V_A is fed from the feed pump into the boiler. This is shown as process AF. In the boiler, the water is converted into steam at pressure P_B . The volume of the steam produced is V_B . This volume of steam V_B is then fed from the boiler into the engine or turbine. This is shown as process FB.

The conversion of water into steam at constant pressure takes place at constant temperature, the saturation temperature T_B , as long as the steam does not enter the superheat phase. If the steam produced is either wet or dry saturated, the process is isothermal.

- **BC** The steam is expanded frictionlessly and adiabatically in the engine or turbine.
- **CD** The steam, after expansion, is passed from the engine or turbine into a condenser. This is shown as process CE. In the condenser the volume of the steam is reduced from V_C to V_D . This process takes place at constant condenser pressure P_C and at constant condenser saturation temperature T_C . This process is therefore isothermal.
- **DA** The partially condensed steam at pressure P_C and volume V_D is fed from the condenser into the feed pump. This is shown as process ED. In the feed pump the steam is compressed frictionlessly and adiabatically to boiler pressure P_B . This is shown as process DA. The compression converts the wet steam at condenser pressure into water at boiler pressure. This water is fed into the boiler, shown as process AF, and the cycle is repeated.

Now the P - V diagram is really composed of two diagrams. There is the engine or turbine diagram FBCE, whose area will give work output; there is also the feed pump diagram EDAF, whose area will give the required work input to run the feed pump.

The net work output from the plant, therefore, will be the net area of these two diagrams. This is the area ABCD.

Area ABCD is enclosed by two isothermal processes and two adiabatic processes, so this is a Carnot cycle. Its thermal efficiency will be given by $(T_B - T_C)/T_B$, the maximum efficiency possible between these temperature limits.

The T - s diagram of the cycle is shown in Fig. 5.6(b).

- **ab** represents the constant temperature formation of the steam in the boiler.
- **bc** represents the frictionless adiabatic (isentropic) expansion of the steam in the engine or turbine.
- **cd** represents the condensation of the steam in the condenser.
- **da** represents the frictionless adiabatic (isentropic) compression of the steam in the feed pump back to water at boiler pressure at point a.

Now this cycle for operation in a steam plant is practical up to a point. The isothermal expansion of the steam in the boiler and the adiabatic expansion of the steam in the engine or turbine (especially in turbines) is reasonable.

The impractical part is in the handling of the steam in the condenser and feed pump. In the condenser, the steam is only partially condensed and condensation must be stopped at point d. Also the feed pump must be capable of handling both wet steam and water.

A slight modification to this cycle, however, will produce a cycle which is more practical, although it will have a reduced thermal efficiency. This cycle is the Rankine cycle and is usually accepted as the appropriate ideal cycle for steam plant.

5.2.3 Rankine Cycle

The modification made to the Carnot cycle to produce the Rankine cycle is that, instead of stopping the condensation in the condenser at some intermediate condition, the condensation is completed. This is shown in Fig. 5.7(a). On the T - s diagram, Fig. 5.7(b), the Carnot cycle would be $abcg$. For the Rankine cycle, however, condensation is continued until it is complete at point d , all water. This water can be successfully dealt with in a feed pump in which its pressure can be raised feeding it back into the boiler. The cycle thus becomes more practical.

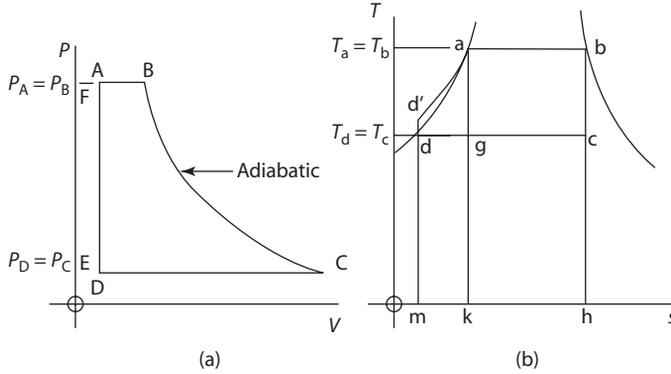


Fig. 5.7 Rankine cycle: (a) P - V diagram; (b) T - s diagram

This is shown exaggerated as process dd' on the T - s diagram. In the boiler the temperature of the water is raised at boiler pressure, shown as process $d'a$, thus the cycle is completed. The complete Rankine cycle is therefore $abcd'd'a$.

On the P - V diagram, Fig. 5.7(a), there are two cycles.

The work done in the engine or turbine is represented by the area $FBCE$. There is also the feed pump work, represented by the area $EDAF$. The feed pump work is negative because work must be put into the pump.

Hence

$$\text{Work done/cycle} = \text{Area } ABCD \tag{1}$$

Using the steady-flow equation, neglecting changes in potential and kinetic energy, and putting $Q = 0$ for an adiabatic expansion, the energy equation becomes

$$h_1 = h_2 + W$$

or

$$\text{Specific work} = h_1 - h_2 \tag{2}$$

Using symbols as in Fig. 10.14, then

$$\begin{aligned} \text{Specific work} &= h_b - h_c \\ &= \text{Area } FBCE \text{ on } P\text{-}V \text{ diagram} \end{aligned} \tag{3}$$

$$\begin{aligned} \text{The feed pump work/unit mass} &= \text{Area } EDAF \\ &= (P_B - P_C)v_D \end{aligned} \tag{4}$$

Hence

$$\text{Net work done/cycle} = (h_b - h_c) - (P_B - P_C)v_D \tag{5}$$

Now the heat transfer required in the boiler to convert the water at d' into steam at b is

$$h_b - h_{d'} \quad [6]$$

But the total energy of the water entering the boiler at d' is

Liquid enthalpy at d + Feedpump work

or

$$h_{d'} = h_d + (P_B - P_C)v_D \quad [7]$$

Substituting equation [7] in equation [6] gives the heat transfer required in the boiler as

$$h_b - [h_d + (P_B - P_C)v_D] = (h_b - h_d) - (P_B - P_C)v_D \quad [8]$$

The thermal efficiency of the cycle is

$$\frac{\text{Work done/cycle}}{\text{Heat received/cycle}}$$

Hence, from equations [5] and [8], the thermal efficiency of the Rankine cycle is

$$\frac{(h_b - h_c) - (P_B - P_C)v_D}{(h_b - h_d) - (P_B - P_C)v_D} \quad [9]$$

But the feed pump term $(P_B - P_C)v_D$, is small compared with the other energy quantities, so it can be neglected.

Thus, equation [9] gives the Rankine efficiency as

$$\frac{(h_b - h_c)}{(h_b - h_d)} \quad [10]$$

The cycle is named after William John Rankine (1820–1872), a Glasgow university professor.

If the work done by the feed pump is neglected and if the steam expansion can be expressed in the form $PV^n = \text{constant}$, the P - V diagram for the Rankine cycle is as shown in Fig. 5.8. From this diagram

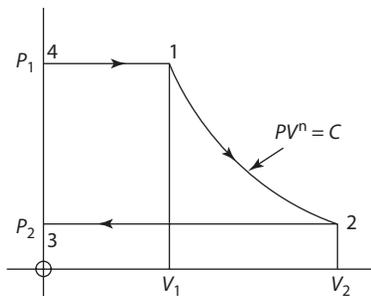


Fig. 5.8 Rankine cycle: P - V diagram

$$\begin{aligned}
 \text{Work done} &= \text{Area under } 4-1 + \text{Area under } 1-2 \\
 &\quad - \text{Area under } 2-3 \\
 &= P_1 V_1 + \frac{(P_1 V_1 - P_2 V_2)}{n-1} - P_2 V_2 \\
 &= (P_1 V_1 - P_2 V_2) + \frac{(P_1 V_1 - P_2 V_2)}{n-1} \\
 &= (P_1 V_1 - P_2 V_2) \left(1 + \frac{1}{n-1} \right) \\
 &= (P_1 V_1 - P_2 V_2) \frac{(n-1) + 1}{n-1}
 \end{aligned}$$

$$\text{Work done} = \frac{n}{n-1} (P_1 V_1 - P_2 V_2) \quad [11]$$

If superheated steam is used in the Rankine cycle then the appearance of the cycle on the T - s diagram is as shown in Fig. 5.9. The difference between this diagram and Fig. 5.7(b) is the inclusion of superheat line bc . The complete cycle is now $abcdee'$.

The thermal efficiency has the same form as before. Using the lettering of Fig. 5.9

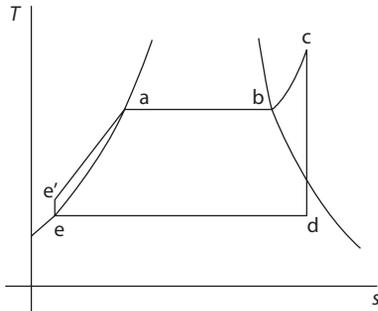


Fig. 5.9 Rankine cycle using superheated steam

$$\text{Rankine } \eta = \frac{h_c - h_d}{h_c - h_e} \quad [12]$$

There is little gain in thermal efficiency as a result of using superheated steam instead of using saturated steam. The chief advantages of using superheated steam are as follows:

- There is little or no condensation loss in transmission.
- There is greater potential for enthalpy drop and hence for work done.

And by using superheated steam, there is a further departure from the Carnot cycle because the final temperature of the steam is above the constant saturation temperature of the boiler. The Rankine cycle will have a high work ratio ($\rightarrow 1$) because the net work done per cycle is very close to the positive work done per cycle; the feed pump work is very low by comparison. The Rankine cycle will have a higher work ratio than the Carnot vapour cycle.

Example 5.4 A steam turbine plant operates on the Rankine cycle. Steam is delivered from the boiler to the turbine at a pressure of 3.5 MN/m^2 and with a temperature of 350°C . Steam from the turbine exhausts into a condenser at a pressure of 10 kN/m^2 . Condensate from the condenser is returned to the boiler using a feed pump. Neglecting losses, determine

- the energy supplied in the boiler per kilogram of steam generated
- the dryness fraction of the steam entering the condenser
- the Rankine efficiency

SOLUTION

First draw a diagram of the Rankine cycle (Fig. 5.10).

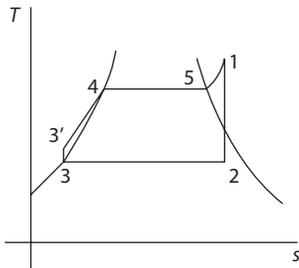


Fig. 5.10 Diagram for Example 10.6

(a)

Energy supplied in boiler/kg steam = $(h_1 - h_3)$.

At 3.5 MN/m^2 and 350°C

$$\begin{aligned} h_1 &= 3139 - \frac{1.5}{2}(3139 - 3095) \\ &= 3139 - \left(\frac{1.5}{2} \times 44\right) \\ &= 3139 - 33 \\ &= \mathbf{3106 \text{ kJ/kg}} \end{aligned}$$

At 10 kN/m^2 , $h_3 = 191.8 \text{ kJ/kg}$

$$\begin{aligned} \therefore h_1 - h_3 &= 3106 - 191.8 \\ &= \mathbf{2914.2 \text{ kJ/kg}} \end{aligned}$$

This is the energy supplied in the boiler.

(b)

Expansion through the turbine is theoretically isentropic, so $s_1 = s_2$.

$$\begin{aligned} s_1 &= 6.960 - \frac{1.5}{2}(6.960 - 6.587) \\ &= 6.960 - \left(\frac{1.5}{2} \times 0.373\right) \\ &= 6.960 - 0.28 \\ &= \mathbf{6.680 \text{ kJ/kg K}} \end{aligned}$$

$$\begin{aligned}
 s_2 &= s_{f2} + x_2(s_{g2} - s_{f2}) \\
 &= 0.649 + x_2(8.151 - 0.649) \\
 &= 6.680
 \end{aligned}$$

$$\begin{aligned}
 \therefore x_2 &= \frac{6.680 - 0.649}{8.151 - 0.649} \\
 &= \frac{6.031}{7.502} \\
 &= \mathbf{0.804}
 \end{aligned}$$

This is the dryness fraction of the steam entering the condenser.

(c)

$$\text{Rankine } \eta = \frac{(h_1 - h_2)}{(h_1 - h_3)}$$

$$\begin{aligned}
 h_2 &= h_{f2} + x_2 h_{g2} = 191.8 + (0.803 \times 2392.9) \\
 &= 191.8 + 1921.5 \\
 &= \mathbf{2113.3 \text{ kJ / kg}}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Rankine } \eta &= \frac{3106 - 2113.3}{3106 - 191.8} \\
 &= \frac{992.7}{2914.2} \\
 &= 0.341 \\
 &= \mathbf{34.1\%}
 \end{aligned}$$

Example 5.5 A steam plant operates on the Rankine cycle. Steam is supplied at a pressure of 1 MN/m^2 and with a dryness fraction of 0.97 . The steam exhausts into a condenser at a pressure of 15 kN/m^2 . Determine the Rankine efficiency. If the expansion of the steam is assumed to follow the law $PV^{1.135} = C$, estimate the specific work done and compare this with that obtained when determining the Rankine efficiency.

SOLUTION

First draw a diagram, of the Rankine cycle (Fig. 5.11).

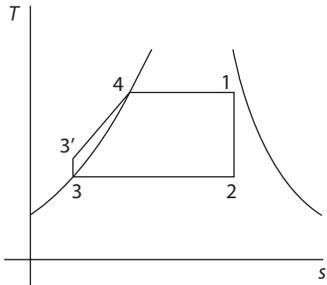


Fig. 5.11 Diagram for Example 5.5

$$\begin{aligned} h_1 &= 762.6 + (0.97 \times 2013.6) \\ &= 762.6 + 1953.2 \\ &= \mathbf{2715.8 \text{ kJ/kg K}} \end{aligned}$$

$$\begin{aligned} s_1 &= 2.138 + 0.97 \times (6.583 - 2.138) \\ &= 2.138 + (0.97 \times 4.445) = 2.138 + 4.332 \\ &= \mathbf{6.470 \text{ kJ/kg K}} \end{aligned}$$

Now $s_1 = s_2$

$$\begin{aligned} \therefore 6.470 &= 0.755 + x_2(8.009 - 0.755) \\ x_2 &= \frac{6.470 - 0.755}{8.009 - 0.755} = \frac{5.715}{7.254} \\ &= \mathbf{0.788} \end{aligned}$$

$$\begin{aligned} h_2 &= 226.0 + (0.788 \times 2373.2) \\ &= 226.0 + 1870.1 \\ &= \mathbf{2096.1 \text{ kJ / kg K}} \end{aligned}$$

(a)

$$\begin{aligned} \text{Rankine } \eta &= \frac{h_1 - h_2}{h_1 - h_3} = \frac{2715.8 - 2096.1}{2715.8 - 226.0} = \frac{619.7}{2489.8} = \mathbf{0.249} \\ &= 0.249 \times 100 \\ &= \mathbf{24.9\%} \end{aligned}$$

(b)

Specific work done = $\frac{n}{n-1}(P_1V_1 - P_2V_2)$ from equation (11), pages 305

$$\begin{aligned} V_1 &= x_1V_{g1} = 0.97 \times 0.1943 = 0.188 \text{ m}^3/\text{kg} \\ V_2 &= x_2V_{g2} = 0.788 \times 10.02 = \mathbf{7.896 \text{ m}^3 / \text{kg}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Specific work done} &= \frac{1.135}{1.135-1} \left[(1 \times 10^3 \times 0.188) - (15 \times 7.896) \right] \\ &= \frac{1.135}{0.135} (188 - 118.44) = \frac{1.135}{0.135} \times 69.56 \\ &= \mathbf{584.8 \text{ kJ / kg}} \end{aligned}$$

$$\begin{aligned} \text{Specific work done (from Rankine } \eta) &= h_1 - h_2 = 2715.8 - 2096.1 \\ &= \mathbf{619.7 \text{ kJ / kg}} \end{aligned}$$

Example 5.6 Steam is supplied from a boiler to a steam engine at a pressure of 1.1 MN/m^2 and at a temperature of 250°C . It is expanded isentropically to a release pressure of 0.28 MN/m^2 . Its pressure then falls at constant volume to 35 kN/m^2 , when it is exhausted to a condenser. In the condenser the steam is condensed to water with

no undercooling and this water is then pumped back into the boiler. Determine (a) the Rankine efficiency (b) the specific steam consumption in kg/kWh (c) the Carnot efficiency for the same temperature limits of the cycle

SOLUTION

First draw a diagram (Fig. 5.12).

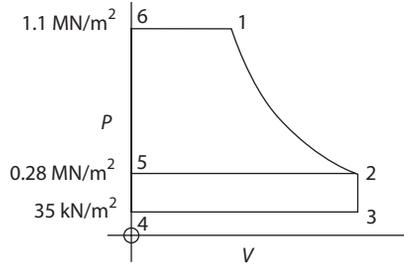


Fig. 5.12 Diagram for Example 5.6

(a)

At 1.1 MN/m² and 250°C

$$\begin{aligned} h_1 &= 2943 - 0.1 \times (2943 - 2902) \\ &= 2943 - (0.1 \times 41) \\ &= 2943 - 4.1 \\ &= \mathbf{2938.9 \text{ kJ/kg}} \end{aligned}$$

For the isentropic 1-2, $s_1 = s_2$

$$\begin{aligned} s_1 &= 6.926 - 0.1 \times (6.926 - 6.545) \\ &= 6.926 - (0.1 \times 0.381) \\ &= 6.926 - 0.038 \\ &= \mathbf{6.888 \text{ kJ/kg K}} \end{aligned}$$

$$\begin{aligned} s_2 &= s_{f2} + x_2(s_{g2} - s_{f2}) = 1.647 + x_2(7.014 - 1.647) \\ \therefore 6.888 &= 1.647 + x_2(7.014 - 1.647) \end{aligned}$$

$$x_2 = \frac{6.888 - 1.647}{7.014 - 1.647} = \frac{5.241}{5.367} = \mathbf{0.977}$$

$$\begin{aligned} h_2 &= 551.4 + (0.977 \times 2170.1) \\ &= 551.4 + 2120.2 \\ &= \mathbf{2671.6 \text{ kJ/kg}} \end{aligned}$$

$$\begin{aligned} v_2 &= x_2 v_{g2} = 0.977 \times 0.646 \\ &= \mathbf{0.631 \text{ m}^3/\text{kg}} \end{aligned}$$

Work done = Area 61234 = Area 6125 + Area 5234

$$\begin{aligned} \text{Area 6125} &= h_1 - h_2 = 2938.9 - 2671.6 \\ &= \mathbf{267.3 \text{ kJ/kg}} \end{aligned}$$

$$\begin{aligned} \text{Area } 5234 &= v_2(P_2 - P_3) = 0.631 \times (280 - 35) \\ &= 0.631 \times 245 \\ &= \mathbf{154.6 \text{ kJ / kg}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Work done} &= 267.3 + 154.6 \\ &= \mathbf{421.9 \text{ kJ / kg}} \end{aligned}$$

$$\begin{aligned} \text{Energy received} &= \text{Specific enthalpy of steam supplied} - \text{Specific enthalpy of water} \\ &\quad \text{at condenser pressure} \\ &= 2938.9 - 304.3 \\ &= \mathbf{2634.6 \text{ kJ / kg}} \end{aligned}$$

$$\therefore \text{Rankine } \eta = \frac{421.9}{2634.6} = 0.16 = \mathbf{16\%}$$

(b)

$$1 \text{ kWh} = 10^3 \times 3600 \text{ kJ}$$

$$\therefore \text{Specific steam consumption} = \frac{3600}{421.9} = \mathbf{8.53 \text{ kg / kWh}}$$

(c)

$$\text{Carnot } \eta = (T_1 - T_3)/T_1$$

$$T_1 = 250 + 273 = 523 \text{ K}$$

$$t_3 = 72.7 \text{ }^\circ\text{C}$$

$$T_3 = 72.7 + 273 = 345.7 \text{ K}$$

$$\therefore \text{Carnot } \eta = \frac{523 - 345.7}{523} = \frac{177.3}{523} = 0.339 = \mathbf{33.9\%}$$

5.2.4 Otto Cycle

The constant volume cycle (Fig. 5.13) consists of two adiabatic processes, 1–2 and 3–4, together with two constant volume processes, 2–3 and 4–1.

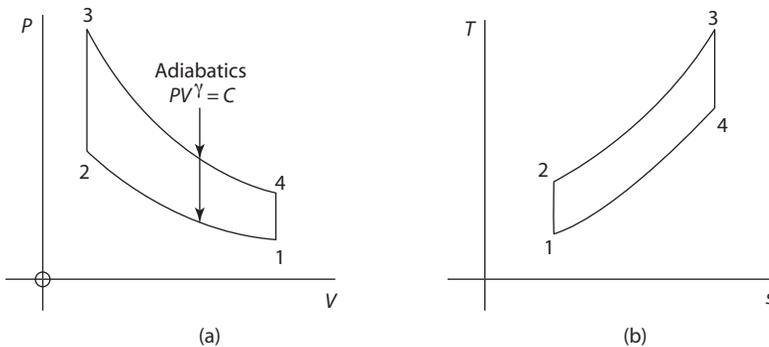


Fig. 5.13 Constant volume cycle: (a) P - V diagram; (b) T - s diagram

The cycle was originally conceived by a Frenchman named Beau de Rochas in 1862. And in 1867 it was successfully applied to an internal combustion engine by Dr Nikolaus August Otto, a German engineer, so it is often called the Otto cycle.

The cycle is of importance because it is the theoretical cycle on which the modern petrol engine is based.

Fig. 5.14(a) is a P - V diagram of the constant volume cycle; Fig. 5.14(b) is the corresponding T - s diagram. The cycle is arranged as follows:

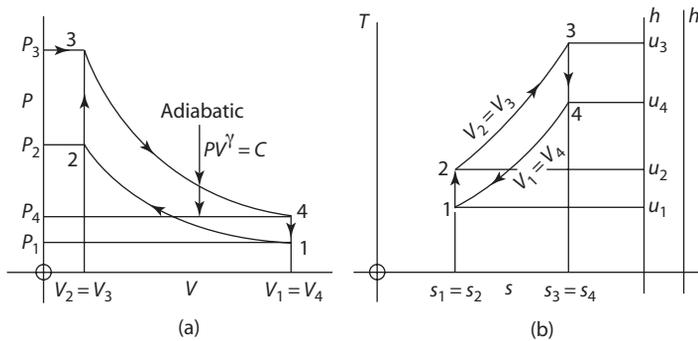


Fig. 5.14 Constant volume cycle: (a) P - V diagram; (b) T - s diagram

- **1-2 Adiabatic compression of the gas according to the law $PV^\gamma = C$**
 Pressure increases from P_1 to P_2 .
 Volume decreases from V_1 to V_2 .
 Temperature increases from T_1 to T_2 .
 Entropy remains constant at $s_1 = s_2$.
- **2-3 Constant volume heat addition**
 Volume remains constant at $V_2 = V_3$.
 Pressure increases from P_2 to P_3 .
 Temperature increases from T_2 to T_3 .
 Entropy increases from s_2 to s_3 .
- **3-4 Adiabatic expansion of the gas according to the law $PV^\gamma = C$**
 Pressure decreases from P_3 to P_4 .
 Volume increases from V_3 to V_4 .
 Temperature decreases from T_3 to T_4 .
 Entropy remains constant at $s_3 = s_4$.
- **4-1 Constant volume heat rejection**
 Volume remains constant at $V_4 = V_1$.
 Pressure decreases from P_4 to P_1 .
 Temperature decreases from T_4 to T_1 .
 Entropy decreases from s_4 to s_1 .
 This process completes the cycle and returns the gas to its original state at 1.

An analysis of the properties P , V and T at state points, 1, 2, 3 and 4 will now be made and it will be assumed that P_1 , V_1 and T_1 are known.

- P_1 , V_1 , T_1 .
- Assume that the volume ratio V_1/V_2 is known. Recall, that the constant volume cycle is the theoretical cycle for a petrol or gas engine. The ratio V_1/V_2 for a petrol or gas engine is often called the compression ratio of the engine. In fact, the ratio V_1/V_2 is a volume ratio. It is sometimes called the geometric compression ratio.

$$T_1/T_2 = (V_2/V_1)^{(\gamma-1)}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma-1)} = T_1 r_v^{(\gamma-1)}$$

where $r_v = \begin{cases} V_1/V_2 = \text{adiabatic compression volume ratio} \\ V_4/V_3 = \text{adiabatic expansion volume ratio} \end{cases}$

$$\text{Also } P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1 r_v^\gamma$$

- $V_3 = V_2$ because the volume remains constant.

$$P_3/T_3 = P_2/T_2$$

$$\therefore T_3 = T_2 \left(\frac{P_3}{P_2} \right) = \frac{P_3}{P_2} T_1 r_v^{(\gamma-1)}$$

- $T_3/T_4 = (V_4/V_3)^{(\gamma-1)} = r_v^{(\gamma-1)}$
- $$\therefore T_4 = \frac{T_3}{r_v^{(\gamma-1)}} = \frac{P_3}{P_2} T_1 \frac{r_v^{(\gamma-1)}}{r_v^{(\gamma-1)}} = \frac{P_3}{P_2} T_1$$

$$\text{Also } P_4 V_4^\gamma = P_3 V_3^\gamma$$

$$\therefore P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma = \frac{P_3}{r_v^\gamma}$$

Also, from the constant volume process 4–1, $P_4/T_4 = P_1/T_1$

$$\therefore T_4 = \frac{P_4}{P_1} T_1 = \frac{P_3}{P_2} T_1$$

From this it follows that $P_4/P_1 = P_3/P_2$.

The work done during the cycle may be obtained as follows:

- Process 3–4 is an expansion which gives positive work done.
- Process 1–2 is a compression which gives negative work done.
- The net work done is the sum of the work done by these two processes.

$$\begin{aligned} \therefore \text{Net work done} &= \oint W = \text{Area under } 3-4 - \text{Area under } 1-2 \\ &= \text{Area } 1234 \\ &= \text{Area of diagram} \end{aligned}$$

$$\begin{aligned} \therefore \oint W &= \frac{(P_3V_3 - P_4V_4)}{(\gamma-1)} - \frac{(P_2V_2 - P_1V_1)}{(\gamma-1)} \\ &= \frac{(P_3V_3 - P_4V_4) - (P_2V_2 - P_1V_1)}{(\gamma-1)} \end{aligned} \quad [1]$$

$$\begin{aligned} &= \frac{mR}{(\gamma-1)} [(T_3 - T_4) - (T_2 - T_1)] \\ &\quad (\text{since } PV = mRT) \end{aligned} \quad [2]$$

Alternatively, the cycle work done can be determined using

$$\oint W = \oint Q$$

or

$$\text{Cycle work done} = \oint W = \text{Heat received} - \text{Heat rejected} \quad [3]$$

Equation [3] will hold good for any cycle.

Now in this cycle heat is received and rejected only during constant volume processes. Hence the name, constant volume cycle.

There are also two adiabatic processes, but during an adiabatic process no heat is received or rejected. Hence the adiabatic processes do not appear when the discussion is on heat received or rejected. This will be true in any other cycle where adiabatic processes appear.

In this cycle

$$\text{Heat is received from } 2-3 = mc_v(T_3 - T_2) \quad [4]$$

$$\text{Heat is rejected from } 4-1 = mc_v(T_4 - T_1) \quad [5]$$

$$\begin{aligned} \therefore \oint W &= mc_v(T_3 - T_2) - mc_v(T_4 - T_1) \\ &= mc_v [(T_3 - T_2) - (T_4 - T_1)] \end{aligned} \quad [6]$$

The thermal efficiency of the cycle may be determined from

$$\text{Thermal efficiency} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}} \quad [7]$$

$$= 1 - \frac{\text{Heat rejected}}{\text{Heat received}} \quad [8]$$

$$= 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)}$$

and assuming that c_v remains constant

$$\text{Thermal } \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad [9]$$

Equation [9] gives the thermal efficiency in terms of temperatures. Substituting temperatures in terms of T_1 in equation [9]

$$\begin{aligned} \text{Thermal } \eta &= 1 - \frac{(P_3/P_2) T_1 - T_1}{(P_3/P_2) T_1 r_v^{(\gamma-1)} - T_1 r_v^{(\gamma-1)}} \\ &= 1 - \frac{T_1(P_3/P_2 - 1)}{T_1 r_v^{(\gamma-1)} (P_3/P_2 - 1)} \\ &= 1 - \frac{1}{r_v^{(\gamma-1)}} \end{aligned} \quad [10]$$

Now consider the adiabatic processes 1–2 and 3–4.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{(\gamma-1)} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3} \right)^{(\gamma-1)}$$

But $V_1/V_2 = V_4/V_3 = r_v$

$$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_4} = r_v^{(\gamma-1)} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{T_4}{T_3} = \left(\frac{1}{r_v} \right)^{(\gamma-1)}$$

So from equation [10]

$$\text{Thermal } \eta = 1 - \frac{T_1}{T_2} \quad [11]$$

$$= 1 - \frac{T_3}{T_4} \quad [12]$$

Note that from equation [7]

$$\text{Heat received} - \text{Heat rejected} = \text{Heat received} \times \text{Thermal } \eta \quad [13]$$

Substituting equation [3] into equation [13]

$$\oint W = \text{Heat received} \times \text{Thermal } \eta \quad [14]$$

This is another way in which the work done may be determined.

In the case of this cycle,

$$\oint W = mc_v(T_3 - T_2) \times \text{Thermal } \eta \quad [15]$$

The cycle can also be analysed by using the temperature–entropy chart.

By tracing the points of the cycle round the chart, the various properties P, V, T, u, h at state points 1, 2, 3 and 4 can be found. Now for the constant volume process, it has been shown that

Heat transferred = Change of internal energy

Hence

$$\text{Heat received from 2-3} = (u_3 - u_2) \quad [16]$$

$$\text{Heat rejected from 4-1} = (u_4 - u_1) \quad [17]$$

From equations [3], [16] and [17]

$$\oint W = (u_3 - u_2) - (u_4 - u_1) \quad [18]$$

From equations [8], [16] and [17]

$$\text{Thermal } \eta = 1 - \frac{(u_4 - u_1)}{(u_3 - u_2)} \quad [19]$$

Remember that the chart is made out for unit mass of gas, so equations [16], [17] and [18] will give values per unit mass per cycle.

On a practical note, $r_v = V_1/V_2$ must increase for the thermal efficiency to increase (i.e. the so-called compression ratio must increase). In a practical engine, the ability to increase r_v is limited because of high material loading, high temperatures and fuel combustion problems.

In the case of the petrol engine, the addition of tetraethyl lead to the fuel has helped to prevent knocking or pinking, thus enabling the use of higher compression ratios. The use of tetraethyl lead is now in disfavour because of atmospheric pollution and possible damage to health. Research and development into fuel combustion techniques are making it possible to use lead-free petrol as well as to increase compression ratios.

Example 5.7 An engine works on the constant volume cycle. It has a bore of 80 mm and a stroke of 85 mm. The clearance volume of the engine is 0.06 litre. The actual thermal efficiency of the engine is 22 per cent. Determine the relative efficiency of the engine. Take $\gamma = 1.4$.

SOLUTION

Suppose the engine has bore d and the stroke l , then

$$\begin{aligned} \text{Stroke volume} &= V_1 - V_2 \\ &= \frac{\pi d^2}{4} \times l \\ &= \frac{\pi \times 80^2}{4} \times 85 \\ &= 427.257 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \text{Clearance volume} &= V_1 = 0.06 \times 10^6 \\ &= 60\,000 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} \text{Total volume} &= V_1 = (V_1 - V_2) + V_2 \\ &= 427\,257 + 60\,000 \\ &= 487\,257 \text{ mm}^3 \end{aligned}$$

$$\text{Ideal thermal } \eta = 1 - \frac{1}{r_v^{(\gamma-1)}}$$

$$\text{and } r_v = \frac{V_1}{V_2} = \frac{487\,257}{60\,000} = 8.12$$

$$\begin{aligned} \therefore \text{Ideal thermal } \eta &= 1 - \frac{1}{8.12^{(1.4-1)}} = 1 - \frac{1}{8.12^{0.4}} \\ &= 1 - \frac{1}{2.311} \\ &= 1 - 0.433 \\ &= 0.567 \\ &= \mathbf{56.7\%} \end{aligned}$$

$$\begin{aligned} \text{Relative efficiency} &= \frac{\text{Actual thermal efficiency}}{\text{Ideal thermal efficiency}} \\ &= \frac{0.22}{0.567} \\ &= 0.388 \\ &= \mathbf{38.8\%} \end{aligned}$$

Example 5.8 One kilogram of air is taken through a constant volume cycle. At the commencement of the adiabatic compression, the pressure and temperature are 103 kN/m^2 and 100°C respectively. The adiabatic compression has a volume ratio of 6:1. The maximum pressure of the cycle is 3.45 MN/m^2 . Determine for the cycle

- the pressure, volume and temperature at each of the cycle process change points
- the heat transferred to the air
- the heat rejected by the air
- the ideal thermal efficiency
- the work done
- the mean effective pressure

For the air, take $R = 0.287 \text{ kJ/kg K}$, $\gamma = 1.4$.

SOLUTION

First draw a diagram (Fig. 5.15).

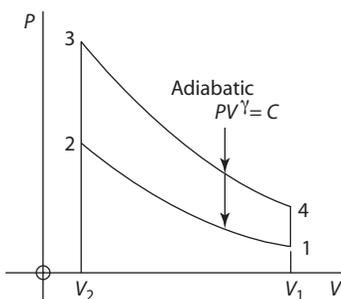


Fig. 5.15 Diagram for Example 5.8

(a)

For point 1

$$\text{Pressure} = P_1 = 103 \text{ kN/m}^2; \text{ temperature} = t_1 = 100 \text{ }^\circ\text{C}$$

$$P_1 V_1 = mRT_1 \text{ and } T_1 = 100 + 273 = 373 \text{ K}$$

$$\therefore V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.287 \times 373}{103} = 1.039 \text{ m}^3$$

For point 2

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\begin{aligned} \therefore P_2 &= P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 103 \times 6^{1.4} \\ &= 103 \times 12.27 \\ &= 1263.8 \text{ kN/m}^2 \end{aligned}$$

$$V_2 = \frac{V_1}{6} = \frac{1.039}{6} = 0.173 \text{ m}^3$$

$$P_1 V_1 / T_1 = P_2 V_2 / T_2$$

$$\begin{aligned} \therefore T_2 &= \frac{P_2 V_2}{P_1 V_1} T_1 = \frac{1263.8}{103} \times \frac{0.173}{1.039} \times 373 \\ &= 762 \text{ K} \end{aligned}$$

$$t_2 = 762 - 273 = 489 \text{ }^\circ\text{C}$$

This result could also have been achieved by using

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{(\gamma-1)/\gamma} = \left(\frac{V_2}{V_1} \right)^{(\gamma-1)}$$

For point 3

$$V_3 = V_2 = 0.173 \text{ m}^3, P_3 = 3450 \text{ kN/m}^2$$

$$P_3 / T_3 = P_2 / T_2$$

$$\therefore T_3 = \frac{P_3}{P_2} T_2 = \frac{3450}{1263.8} \times 762 = 2080 \text{ K}$$

$$t_3 = 2080 - 273 = 1807 \text{ }^\circ\text{C}$$

For point 4

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$\begin{aligned} \therefore P_4 &= P_3 \left(\frac{V_3}{V_4} \right)^\gamma = 3450 \times \left(\frac{1}{6} \right)^{1.4} \\ &= \frac{3450}{12.27} \\ &= 281.2 \text{ kN/m}^2 \end{aligned}$$

$$V_4 = V_1 = 1.039 \text{ m}^3$$

$$P_4/T_4 = P_1/T_1$$

$$\therefore T_4 = \frac{P_4}{P_1} T_1 = \frac{281.2}{103} \times 373 = 1018 \text{ K}$$

$$t_4 = 1018 - 272 = 745 \text{ }^\circ\text{C}$$

This result could also have been achieved by using

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4} \right)^{(\gamma-1)/\gamma}$$

Tabulated results

Change point	Pressure (kN/m ²)	Volume (m ³)	Temperature (°C)
1	103	1.039	100
2	1 263.8	0.173	489
3	3450	0.173	1807
4	281.2	1.039	745

(b)

$$\text{Now } c_v = \frac{R}{(\gamma-1)} = \frac{0.287}{(1.4-1)} = \frac{0.287}{0.4} = 0.717 \text{ kJ/kg K}$$

Heat transferred to the air between state points 2 and 3 is

$$\begin{aligned} mc_v(T_3 - T_2) &= 1 \times 0.717 \times (2080 - 762) \\ &= 1 \times 0.717 \times 1318 \\ &= 945 \text{ kJ} \end{aligned}$$

(c)

Heat rejected from the air between state points 3 and 4 is

$$\begin{aligned} mc_v(T_4 - T_3) &= 1 \times 0.717 \times (1080 - 373) \\ &= 1 \times 0.717 \times 645 \\ &= 462.5 \text{ kJ} \end{aligned}$$

(d)

$$\begin{aligned} \text{Ideal thermal } \eta &= \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}} \\ &= 1 - \frac{\text{Heat rejected}}{\text{Heat received}} \\ &= 1 - \frac{462.5}{945} \\ &= 1 - 0.489 \\ &= 0.511 \\ &= 51.1\% \end{aligned}$$

Alternatively

$$\begin{aligned}
 \text{Ideal thermal } \eta &= 1 - \frac{1}{r_v^{(\gamma-1)}} \\
 &= 1 - \frac{1}{6^{0.4}} \\
 &= 1 - \frac{1}{2.048} \\
 &= 1 - 0.489 \\
 &= 0.511 \\
 &= \mathbf{51.1\%}
 \end{aligned}$$

(e)

$$\begin{aligned}
 \oint W &= \text{Heat received} - \text{Heat rejected} \\
 &= 945 - 462.5 \\
 &= \mathbf{482.5 \text{ kJ}}
 \end{aligned}$$

Alternatively

$$\begin{aligned}
 \oint W &= \frac{(P_3V_3 - P_2V_2)}{(\gamma - 1)} - \frac{(P_2V_2 - P_1V_1)}{(\gamma - 1)} \\
 &= \frac{(P_3V_3 - P_2V_2) - (P_2V_2 - P_1V_1)}{(\gamma - 1)} \\
 &= \frac{[(3450 \times 0.173) - (281.2 \times 1.039)] - [(1263.8 \times 0.173) - (103 \times 1.039)]}{(1.4 - 1)} \\
 &= \frac{[596.8 - 292.2] - [218.6 - 107]}{0.4} \\
 &= \frac{[304.6 - 111.6]}{0.4} = \frac{193}{0.4} \\
 &= \mathbf{482.5 \text{ kJ}}
 \end{aligned}$$

(f)

$$\begin{aligned}
 \text{Mean effective pressure} &= \frac{\oint W}{(V_1 - V_2)} \\
 &= \frac{482.5}{(1.039 - 0.173)} \\
 &= \frac{482.5}{0.866} \\
 &= \mathbf{557.16 \text{ kN/m}^2}
 \end{aligned}$$

Example 5.9 The pressure, volume and temperature at the beginning of the compression of a constant volume cycle are 101 kN/m^2 , 0.003 m^3 and 18°C , respectively. The maximum pressure of the cycle is 4.5 MN/m^2 . The volume ratio of the cycle is $19:1$. The cycle is repeated 3000 times/min . Determine for the cycle

- the pressure, volume and temperature at each of the cycle process change points
- the thermal efficiency
- the theoretical output in kilowatts
- the mean effective pressure
- the Carnot efficiency within the cycle temperature limits

Take $c_p = 1.006 \text{ kJ/kg K}$, $c_v = 0.716 \text{ kJ/kg K}$. The diagram is as Fig. 5.16.

(a)

$P_1 = 101 \text{ kN/m}^2$, $V_1 = 0.003 \text{ m}^3$, $t_1 = 18^\circ\text{C}$, all given.

For process 1–2

$$\frac{V_1}{V_2} = r_v = 9$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad \text{and} \quad \gamma = c_p/c_v = 1.006/0.716 = 1.405$$

$$\begin{aligned} \therefore P_2 &= P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 101 \times 9^{1.405} \\ &= 101 \times 21.91 \\ &= 2212.9 \text{ kN/m}^2 \\ &= 2.2129 \text{ MN/m}^2 \end{aligned}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \quad \text{and} \quad T_1 = 18 + 273 = 291 \text{ K}$$

$$\begin{aligned} \therefore T_2 &= T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 291 \times 9^{(1.405-1)} \\ &= 291 \times 9^{0.405} \\ &= 291 \times 2.435 \\ &= 708.6 \text{ K} \end{aligned}$$

$$\begin{aligned} t_2 &= 708.6 - 273 \\ &= 435.6^\circ\text{C} \end{aligned}$$

$$\begin{aligned} V_2 &= V_1/9 = 0.003/9 \\ &= 0.00033 \text{ m}^3 \end{aligned}$$

For process 2–3

$$V_3 = V_2 = 0.00033 \text{ m}^3$$

$$\frac{P_3}{T_3} = \frac{P_2}{T_2} \quad \text{and} \quad P_3 = 4.5 \text{ MN/m}^2$$

$$\begin{aligned}\therefore T_3 &= T_2 \frac{P_3}{P_2} = 708.6 \times \frac{4.3}{2.2129} \\ &= \mathbf{1441 \text{ K}} \\ t_3 &= 1441 - 273 \\ &= \mathbf{1168 \text{ }^\circ\text{C}}\end{aligned}$$

For process 3–4

$$V_4 = V_1 = \mathbf{0.003 \text{ m}^3}$$

$$\begin{aligned}P_4 &= P_3 \left(\frac{V_3}{V_4} \right)^\gamma = \frac{500}{9^{1.405}} \\ &= \frac{4500}{21.91} \\ &= \mathbf{205.4 \text{ kN/m}^2}\end{aligned}$$

$$\begin{aligned}T_4 &= T_3 \left(\frac{V_3}{V_4} \right)^{(\gamma-1)} = \frac{1441}{9^{0.405}} \\ &= \frac{1441}{2.435} \\ &= \mathbf{591.8 \text{ K}}\end{aligned}$$

$$\begin{aligned}t_4 &= 591.8 - 273 \\ &= \mathbf{318.8 \text{ }^\circ\text{C}}\end{aligned}$$

Tabulated results

Change point	Pressure (kN/m ²)	Volume (m ³)	Temperature (°C)
1	101	0.003	18
2	2 212.9	0.000 33	435.6
3	4 500	0.000 33	1168
4	205.4	0.003	318.8

(b)

$$\text{Thermal } \eta = 1 - \frac{1}{r_v^{(\gamma-1)}} \text{ and } r_v = \mathbf{9}$$

$$\text{Now } \gamma = \frac{c_p}{c_v} = \frac{1.006}{0.716} = \mathbf{1.405}$$

$$\begin{aligned}\therefore \text{Thermal } \eta &= 1 - \frac{1}{9^{(1.405-1)}} \\ &= 1 - \frac{1}{9^{0.405}} \\ &= 1 - \frac{1}{2.439} \\ &= 1 - 0.41 \\ &= 0.59 \\ &= \mathbf{59\%}\end{aligned}$$

Alternatively

$$\begin{aligned}\text{Thermal } \eta &= 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \\ &= 1 - \frac{(591.8 - 291)}{(1441 - 708.6)} \\ &= 1 - \frac{300.8}{732.4} \\ &= 1 - 0.41 \\ &= \mathbf{0.59}\end{aligned}$$

(c)

$$\begin{aligned}\oint W &= \text{Heat received} - \text{Heat rejected} \\ &= mc_v[(T_3 - T_2) - (T_4 - T_1)]\end{aligned}$$

and

$$\begin{aligned}m &= \frac{P_1 V_1}{RT_1} = \frac{101 \times 0.003}{(1.006 - 0.716) \times 291} \\ &= \frac{101 \times 0.003}{0.29 \times 291} \\ &= \mathbf{0.0036 \text{ kg}}\end{aligned}$$

$$\begin{aligned}\therefore \oint W &= 0.0036 \times 0.716 \times [(1441 - 708.6) - (591.8 - 291)] \\ &= 0.0036 \times 0.716 \times (732.4 - 300.8) \\ &= 0.0036 \times 0.716 \times 431.6 \\ &= \mathbf{1.112 \text{ kJ}}\end{aligned}$$

$$\text{Work done/min} = (1.112 \times 3000) \text{ kJ}$$

$$\begin{aligned}\therefore \text{Theoretical output} &= \frac{1.112 \times 3000}{60} \\ &= \mathbf{55.6 \text{ kW}} \quad (1 \text{ W} = 1 \text{ J/s})\end{aligned}$$

(d)

$$\begin{aligned}\text{Mean effective pressure} &= \frac{\oint W}{(V_1 - V_2)} \\ &= \frac{1.112}{(0.003 - 0.00033)} \\ &= \frac{1.112}{0.00267} \\ &= \mathbf{416.5 \text{ kN/m}^2}\end{aligned}$$

(e)

$$\begin{aligned} \text{Carnot } \eta &= \frac{T_3 - T_1}{T_3} \\ &= \frac{1441 - 291}{1441} \\ &= \frac{1150}{1441} \\ &= 0.798 \\ &= \mathbf{79.8\%} \end{aligned}$$

5.2.5 Diesel Cycle

The Diesel cycle (Fig. 5.16) consists of two adiabatic processes, 1–2 and 3–4, one constant pressure process, 2–3, and one constant volume process, 4–1.

The cycle was conceived of by Rudolf Diesel, a German engineer, born in Paris, who from about 1893 onward pioneered the development of fuel injection oil engines. The diesel engine is named after him. Rudolf Diesel mysteriously disappeared while crossing the English Channel in 1913.

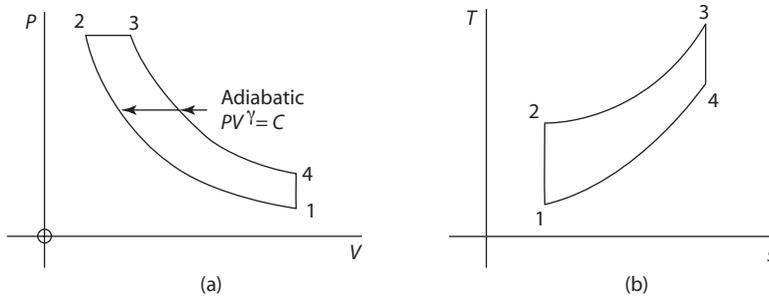


Fig. 5.16 Diesel cycle: (a) P - V diagram; (b) T - s diagram

This cycle is named after Rudolf Diesel (1858–1913). Fig. 5.17 (a) is the P - V diagram of the Diesel cycle; Fig. 5.17(b) is the corresponding T - s diagram. The cycle is arranged as follows:

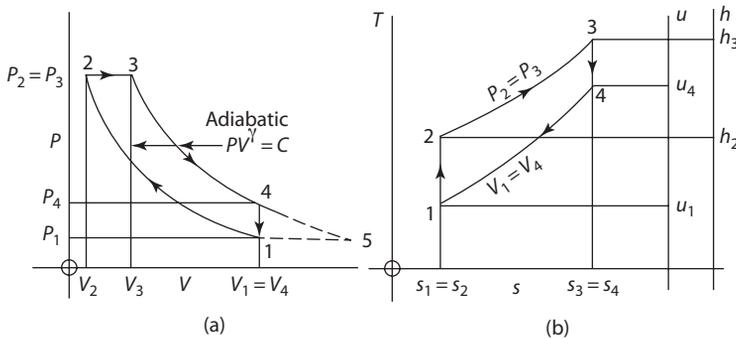


Fig. 5.17 Diesel cycle: (a) P - V diagram; (b) T - s diagram

- **1–2 Adiabatic compression according to the law $P V^\gamma = C$**

Pressure increases from P_1 to P_2 .

Temperature increases from T_1 to T_2 .

Volume decreases from V_1 to V_2 .

Entropy remains constant at $s_1 = s_2$.

- **2–3 Constant pressure heat addition**

Pressure remains constant at $P_2 = P_3$.

Temperature increases from T_2 to T_3 .

Volume increases from V_2 to V_3 .

Entropy increases from s_2 to s_3 .

- **3–4 Adiabatic expansion according to the law $P V^\gamma = C$**

Pressure decreases from P_3 to P_4 .

Temperature decreases from T_3 to T_4 .

Volume increases from V_3 to V_4 .

Entropy remains constant at $s_3 = s_4$.

- **4–1 Constant volume heat rejection**

Volume remains constant at $V_4 = V_1$.

Temperature decreases from T_4 to T_1 .

Pressure decreases from P_4 to P_1 .

Entropy decreases from s_4 to s_1 .

This process completes the cycle and returns the gas to its original state.

This cycle is sometimes called the **modified constant pressure cycle**. Inspection of Fig. 5.17(a) will show the reason. If, instead of cutting off the expansion at 4, the gas were allowed to expand completely to 5, then in order to return the gas to its original state at 1, constant pressure heat rejection would have to take place from 5 to 1.

This is shown dotted. The diagram 1235 is the constant pressure cycle and, by cutting off the part 145, it is modified into the Diesel cycle. In practice, by cutting off part 145 of the cycle, a considerable saving in cylinder volume would be obtained. The area 145 represents a small amount of work which does not really justify the increase of the cylinder volume from V_1 to V_5 .

An analysis of the properties at state points 1, 2, 3 and 4 can be made. Again it is assumed that P_1 , V_1 and T_1 are known.

- P_1, V_1, T_1
- Assume that the volume ratio V_1/V_2 is known.

$$T_1/T_2 = (V_2/V_1)^{\gamma-1}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma-1)} = T_1 r_v^{(\gamma-1)}$$

where $r_v = V_1/V_2 =$ adiabatic compression volume ratio

Also $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1 r_v^\gamma$$

- $P_3 = P_2$ because the pressure remains constant

$$V_3/T_3 = V_2/T_2$$

$$\therefore T_3 = T_2 \frac{V_3}{V_2} = \frac{V_3}{V_2} T_1 r_v^{(\gamma-1)} = \beta T_1 r_v^{(\gamma-1)}$$

where $\beta = V_3/V_2 = \text{cut-off ratio}$

- $T_4/T_3 = (V_3/V_4)^{(\gamma-1)}$

$$\therefore T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{(\gamma-1)}$$

Now $V_3/V_4 = V_3/V_1$, since $V_4 = V_1$, and

$$\frac{V_3}{V_1} = \frac{V_3}{V_2} \frac{V_2}{V_1} = \frac{\beta}{r_v}$$

$$\therefore T_4 = T_3 \left(\frac{\beta}{r} \right)^{(\gamma-1)} = \beta T_1 r_v^{(\gamma-1)} \frac{\beta^{(\gamma-1)}}{r_v^{(\gamma-1)}}$$

or

$$T_4 = \beta^\gamma T_1$$

Also $P_4 V_4^\gamma = P_3 V_3^\gamma$

$$\therefore P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma = P_3 \left(\frac{\beta}{r_v} \right)^\gamma$$

The work done during the cycle may be determined as follows:

- Processes 2–3 and 3–4 are expansions and hence give positive work done.
- Process 1–2 is a compression and hence will give negative work done.
- The net work done during the cycle will be the sum of the work done during these processes.

Hence

$$\oint W = \text{Area under } 2-3 + \text{Area under } 3-4 - \text{Area under } 1-2$$

$$= P_2(V_3 - V_2) + \frac{(P_3 V_3 - P_4 V_4)}{(\gamma-1)} - \frac{(P_2 V_2 - P_1 V_1)}{(\gamma-1)}$$

$$= P_2(V_3 - V_2) + \frac{(P_3V_3 - P_4V_4) - (P_2V_2 - P_1V_1)}{(\gamma - 1)} \quad [1]$$

$$= mR(T_3 - T_2) + \frac{mR}{(\gamma - 1)}[(T_3 - T_4) - (T_2 - T_1)]$$

$$= mR \left[(T_3 - T_2) + \frac{(T_3 - T_4) - (T_2 - T_1)}{(\gamma - 1)} \right] \quad [2]$$

Alternatively, the work done may be obtained from

$$\oint W = \text{Heat received} - \text{Heat rejected}$$

In this cycle, heat is received during constant pressure process 2–3 and rejected during constant volume process 4–1.

No heat is received or rejected during the adiabatic processes.

Hence

$$\oint W = mc_p(T_3 - T_2) - mc_v(T_4 - T_1) \quad [3]$$

Alternatively

$$\oint W = \text{Heat received} \times \text{Thermal } \eta$$

or

$$\oint W = mc_p(T_3 - T_2) \times \text{Thermal } \eta \quad [4]$$

The thermal efficiency may be determined from

$$\begin{aligned} \text{Thermal } \eta &= 1 - \frac{\text{Heat rejected}}{\text{Heat received}} \\ &= 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} \\ &= 1 - \frac{1}{\gamma} \frac{(T_4 - T_1)}{(T_3 - T_2)} \end{aligned} \quad [5]$$

This gives the thermal efficiency in terms of temperatures.

Also, substituting temperatures in terms of T_1 into equation [5]

$$\begin{aligned} \text{Thermal } \eta &= 1 - \frac{1}{\gamma} \frac{(\beta^\gamma T_1 - T_1)}{[\beta T_1 r_v^{(\gamma-1)} - T_1 r_v^{(\gamma-1)}]} \\ &= 1 - \frac{1}{\gamma} \frac{T_1(\beta^\gamma - 1)}{T_1 r_v^{(\gamma-1)}(\beta - 1)} \\ &= 1 - \frac{1}{r_v^{(\gamma-1)}} \frac{(\beta^\gamma - 1)}{\gamma(\beta - 1)} \end{aligned} \quad [6]$$

Now, for a constant pressure process

Heat received or rejected = Change of enthalpy

For a constant volume process

Heat received or rejected = Change of internal energy

So using the T - s chart

Heat received = $h_3 - h_2$

Heat rejected = $u_2 - u_1$

$$\begin{aligned} \text{Thermal } \eta &= 1 - \frac{\text{Heat rejected}}{\text{Heat received}} \\ &= 1 - \frac{(u_2 - u_1)}{(h_3 - h_2)} \end{aligned} \quad [7]$$

Also

$$\begin{aligned} \oint W &= \text{Heat received} - \text{Heat rejected} \\ &= (h_3 - h_2) - (u_2 - u_1) \end{aligned} \quad [8]$$

Once again this gives the work done per unit mass per cycle because the chart will be made out for unit mass of gas.

It should be noted here that the Diesel cycle does not in fact follow the cycle of a modern diesel engine. The actual cycle is more closely approximated by the dual combustion cycle.

Example 5.10 At the beginning of compression of an ideal Diesel cycle the gas has a temperature and pressure of 40°C and 90 kN/m^2 , respectively. The volume ratio of compression is 16:1. The maximum temperature of the cycle is 1400°C . Determine for the cycle

- the pressure and temperature at each of the cycle process change points
- the work done per kilogram of gas
- the thermal efficiency
- the work ratio
- the mean effective pressure
- the Carnot efficiency within the cycle temperature limits

Take $\gamma = 1.4$, $c_p = 1.004 \text{ kJ/kg K}$.

SOLUTION

First draw a diagram (Fig. 5.18)

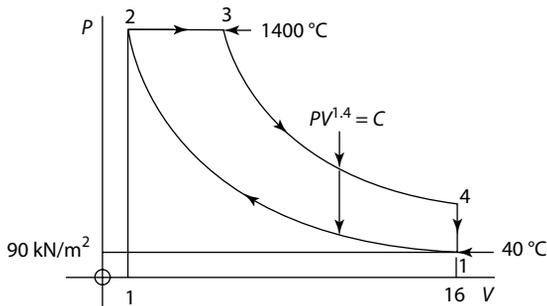


Fig. 5.18 Diagram for Example 5.10

(a)

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 90 \times 16^{1.4} = 90 \times 48.5 = \mathbf{4365 \text{ kN/m}^2}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{(\gamma-1)} \quad \text{and} \quad T_1 = 40 + 273 = \mathbf{313 \text{ K}}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma-1)} = 313 \times 6^{0.4} = 313 \times 3.031 = \mathbf{948.7 \text{ K}}$$

$$t_2 = 675.7 - 273 = \mathbf{402.7 \text{ }^\circ\text{C}}$$

$$P_3 = P_2 = \mathbf{4365 \text{ kN/m}^2}$$

$$t_3 = \mathbf{1400 \text{ }^\circ\text{C}}$$

For the constant pressure process

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad \text{and} \quad T_3 = 1400 + 273 = \mathbf{1673 \text{ K}}$$

$$\therefore V_3 = V_2 \frac{T_3}{T_2} = 1 \times \frac{1673}{948.7} = \mathbf{1.763 \text{ volumes}}$$

For the adiabatic expansion

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$\begin{aligned} \therefore P_4 &= P_3 \left(\frac{V_3}{V_4} \right)^\gamma = 4365 \times \left(\frac{1.763}{16} \right)^{1.4} \\ &= \frac{4365}{9.075^{1.4}} \\ &= \frac{4365}{21.93} \\ &= \mathbf{199.04 \text{ kN/m}^2} \end{aligned}$$

For the constant volume process

$$\frac{P_1}{T_1} = \frac{P_4}{T_4}$$

$$\therefore T_4 = T_1 \frac{P_4}{P_1} = \frac{199.04}{90} = \mathbf{692.2 \text{ K}}$$

$$t_4 = 692.2 - 273 = \mathbf{419.2 \text{ }^\circ\text{C}}$$

Hence

$$P_1 = 90 \text{ kN/m}^2$$

$$t_1 = 40^\circ\text{C}$$

$$P_2 = 4365 \text{ kN/m}^2$$

$$t_2 = 402.7^\circ\text{C}$$

$$P_3 = 4365 \text{ kN/m}^2$$

$$t_3 = 1400^\circ\text{C}$$

$$P_4 = 199.04 \text{ kN/m}^2$$

$$t_4 = 419.2^\circ\text{C}$$

(b)

$$\oint W/kg = c_p(T_3 - T_2) - c_v(T_4 - T_1)$$

$$\text{Now } \frac{c_p}{c_v} = \gamma$$

$$\therefore c_v = \frac{c_p}{\gamma} = \frac{1.004}{1.4} = \mathbf{0.717 \text{ kJ/kg K}}$$

$$\begin{aligned} \therefore \oint W/kg &= 1.004 \times (1673 - 948.7) - 0.717 \times (692.2 - 313) \\ &= (1.004 \times 724.3) - (0.717 \times 379.2) \\ &= 727.2 - 271.9 \\ &= \mathbf{455.3 \text{ kJ}} \end{aligned}$$

(c)

$$\begin{aligned} \text{Thermal } \eta &= 1 - \frac{1}{\gamma} \frac{(T_4 - T_1)}{(T_3 - T_2)} \\ &= 1 - \frac{1}{1.4} \frac{(692.2 - 313)}{(1673 - 948.7)} \\ &= 1 - \frac{379.2}{1.4 \times 727.2} \\ &= 1 - 0.372 \\ &= 0.628 \\ &= \mathbf{62.8\%} \end{aligned}$$

(d)

$$\text{Work ratio} = \frac{\oint W}{\text{Positive work done}}$$

$$\text{Positive work done} = c_p(T_3 - T_2) + \frac{R(T_3 - T_4)}{(\gamma - 1)}$$

$$\text{Also } (c_p - c_v) = R = (1.004 - 0.717) = \mathbf{0.287 \text{ kJ/kg K}}$$

$$\begin{aligned} \therefore \text{Positive work done} &= 1.004 \times (1673 - 948.7) + \frac{0.287 \times (1673 - 692.2)}{(1.4 - 1)} \\ &= (1.004 \times 724.3) + \frac{(0.287 \times 980.8)}{0.4} \\ &= 727.2 + 703.7 \\ &= \mathbf{1430.9 \text{ kJ/kg}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Work ratio} &= \frac{455.3}{1430.9} \\ &= \mathbf{0.318} \end{aligned}$$

(e)

$$\text{Mean effective pressure} = \frac{\oint W}{(V_1 - V_2)}$$

Now, for 1 kg of gas

$$P_1 V_1 = RT_1$$

$$\therefore V_1 = \frac{RT_1}{P_1} = 0.287 \times \frac{313}{90} = \mathbf{0.998 \text{ m}^3}$$

$$\text{Hence } V_2 = \frac{V_1}{16} = \mathbf{0.062 \text{ m}^3}$$

$$\begin{aligned} \therefore \text{Mean effective pressure} &= \frac{455.3}{(0.998 - 0.062)} \\ &= \frac{455.3}{0.936} \\ &= \mathbf{486.4 \text{ kN/m}^2} \end{aligned}$$

(f)

$$\begin{aligned} \text{Carnot } \eta &= \frac{(T_3 - T_1)}{T_3} \\ &= \frac{(1673 - 313)}{1673} \\ &= \frac{1360}{1673} \\ &= 0.813 \\ &= \mathbf{81.3\%} \end{aligned}$$

Example 5.11 In an ideal Diesel cycle the volume ratios of the adiabatic expansion and compression are 7.5:1 and 15:1, respectively. The pressure and temperature at the beginning of compression are 98 kN/m² and 44°C, respectively. The pressure at the end of the adiabatic expansion is 258 kN/m². Determine

(a) the maximum temperature attained during the cycle

(b) the thermal efficiency of the cycle

Take $\gamma = 1.4$.

SOLUTION

First draw a diagram (Fig. 5.19).

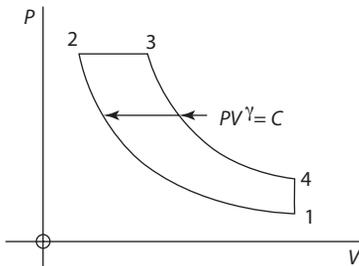


Fig. 5.19 Diagram for Example 5.11

(a)

$$\frac{P_4}{T_4} = \frac{P_1}{T_1} \quad \text{and} \quad T_1 = 44 + 273 = 317 \text{ K}$$

$$\therefore T_4 = T_1 \frac{P_4}{P_1} = 317 \times \frac{258}{98} = 834.6 \text{ K}$$

For the expansion

$$\therefore \frac{T_3}{T_4} = \left(\frac{V_4}{V_3} \right)^{(\gamma-1)}$$

$$\begin{aligned} \therefore T_3 &= T_4 \left(\frac{V_4}{V_3} \right)^{(\gamma-1)} = 834.6 \times 7.5^{0.4} \\ &= 834.6 \times 2.239 \\ &= 1868.7 \text{ K} \end{aligned}$$

$$t_3 = 1868.7 - 273 = 1595.7 \text{ }^\circ\text{C}$$

This is the maximum temperature attained during the cycle.

(b)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{(\gamma-1)}$$

$$\therefore T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma-1)} = 317 \times 15^{0.4} = 317 \times 2.95 = 935 \text{ K}$$

$$\begin{aligned} \text{Thermal } \eta &= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \\ &= 1 - \frac{(834.6 - 317)}{1.4(1868.7 - 935)} \\ &= 1 - \frac{517.6}{1.4 \times 933.7} \\ &= 1 - 0.396 \\ &= 0.604 \\ &= 60.4\% \end{aligned}$$

Example 5.12 An oil engine works on the ideal Diesel cycle. The overall volume ratio of compression is 11:1 and constant pressure energy addition ceases at 10 per cent of the stroke. The pressure and temperature at the commencement of compression are 96 kN/m² and 18°C, respectively. The engine uses 0.05 m³ of air per second. Determine

(a) the thermal efficiency of the cycle

(b) the indicated power of the cycle

Take $\gamma = 1.4$.

SOLUTION

First draw a diagram (Fig. 5.20).

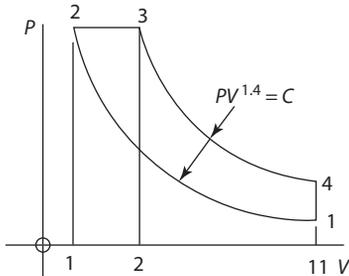


Fig. 5.20 Diagram for Example 5.12

(a)

Swept volume = $11 - 1$

$$= \mathbf{10 \text{ volumes}}$$

10% of swept volume = $10 \times 0.1 = \mathbf{1 \text{ volume}}$

∴ Constant pressure energy addition ceases at $1 + 1 = \mathbf{2 \text{ volumes}}$

$$\text{Thermal } \eta = 1 - \frac{1}{r_v^{(\gamma-1)}} \frac{1}{\gamma} \frac{(\beta^\gamma - 1)}{(\beta - 1)}$$

and $r_v = V_1/V_2 = 11$, $\beta = V_3/V_2 = 2$.

$$\begin{aligned} \therefore \text{Thermal } \eta &= 1 - \frac{1}{11^{(1.4-1)}} \times \frac{1}{1.4} \times \frac{(2^{1.4} - 1)}{(2 - 1)} \\ &= 1 - \frac{1}{11^{0.4}} \times \frac{(2.64 - 1)}{1.4} \\ &= 1 - \frac{1.64}{2.61 \times 11.4} \\ &= 1 - 0.449 \\ &= 0.551 \\ &= \mathbf{55.1\%} \end{aligned}$$

(b)

Let $V_1 - V_2 = 0.05 \text{ m}^3$

then $V_2 = 0.05 \times 0.1 = \mathbf{0.005 \text{ m}^3}$

From this

$$V_1 = 11 \times 0.005 = \mathbf{0.055 \text{ m}^3}$$

$$V_3 = 2V_2 = 2 \times 0.005 = \mathbf{0.01 \text{ m}^3}$$

$$V_4 = V_1 = \mathbf{0.055 \text{ m}^3}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 96 \times 11^{1.4} = 96 \times 28.7 = 2755 \text{ kN/m}^2$$

$$P_3 V_3^\gamma = P_4 V_4^\gamma \text{ and } P_3 = P_2$$

$$\therefore P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma = 2760 \times \left(\frac{2}{11} \right)^{1.4} = \frac{2760}{5.5^{1.4}} = \frac{2760}{10.88} = 253.7 \text{ kN/m}^2$$

$\oint W/s =$ indicated power

$$\begin{aligned} &= P_2(V_3 - V_2) + \frac{(P_3V_3 - P_4V_4) - (P_2V_2 - P_1V_1)}{(\gamma - 1)} \\ &= 2755 \times (0.01 - 0.005) \\ &\quad + \frac{[(2755 \times 0.01) - (253.7 \times 0.005)] - [(2755 \times 0.005) - (96 \times 0.005)]}{(1.4 - 1)} \\ &= (2755 \times 0.005) + \frac{(27.55 - 13.95) - (13.78 - 5.28)}{0.4} \\ &= 13.78 + \frac{(13.6 - 8.5)}{0.4} = 13.78 + \frac{5.1}{0.4} \\ &= 13.78 + 12.75 \\ &= 26.53 \text{ kW} \end{aligned}$$

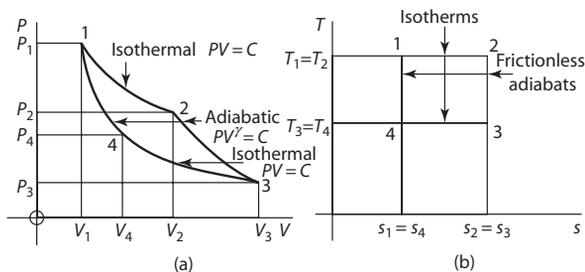
Summary

Heat Engine

The thermodynamic heat engine is a device in which energy is supplied in the form of heat and some of this energy is transformed into work. Working substance is basically a gas or mixture of gases used for heat transfer and capable of absorbing and storing heat and released when need arises. Heat engine is also called converting machine. Heat engines are of I.C. engines and E.C. engines.

Carnot Cycle

Carnot cycle: (a) P - V diagram; (b) T - s diagram



$$\oint W = mR \ln \frac{V_2}{V_1} (T_1 - T_3)$$

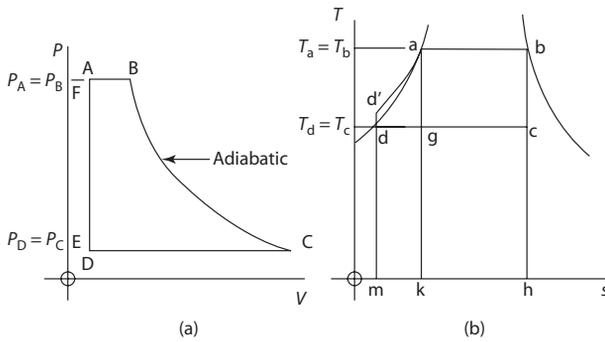
$$\text{Carnot thermal } \eta = \frac{\text{Max. abs. temp.} - \text{Min. abs. temp.}}{\text{Max. abs. temp.}} = 1 - \frac{1}{r_v^{\gamma-1}}$$

$$\text{Work ratio} = \frac{\text{Net work done}}{\text{Positive work done}}$$

$$= \frac{mR \ln \frac{V_2}{V_1} (T_1 - T_3)}{mRT_1 \ln \frac{V_2}{V_1} + \frac{mR(T_1 - T_3)}{\gamma - 1}}$$

Rankine Cycle

Rankine cycle: (a) P-V diagram; (b) T-s diagram

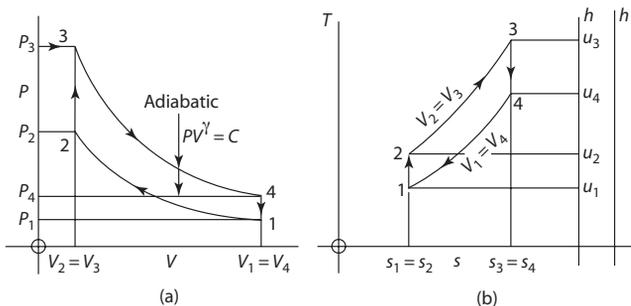


$$\text{Work done} = \frac{n}{n-1} (P_1 V_1 - P_2 V_2)$$

$$\text{Rankine } \eta = \frac{h_c - h_d}{h_c - h_e}$$

Otto Cycle

Constant volume cycle: (a) P-V diagram; (b) T-s diagram

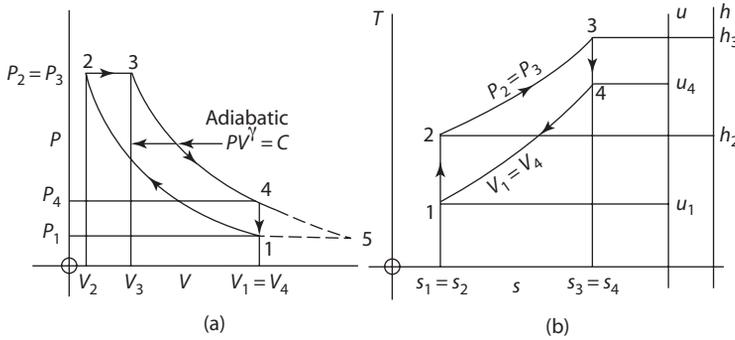


$$\oint W = \frac{mR}{(\gamma - 1)} [(T_3 - T_4) - (T_2 - T_1)]$$

$$\text{Thermal efficiency} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{(u_4 - u_1)}{(u_3 - u_2)}$$

Diesel Cycle

Diesel cycle: (a) P - V diagram: (b) T - s diagram



$$\oint W = mR \left[(T_3 - T_2) + \frac{(T_3 - T_4) - (T_2 - T_1)}{\gamma - 1} \right]$$

$$\text{Thermal } \eta = 1 - \frac{\text{Heat rejected}}{\text{Heat received}} = 1 - \frac{1}{\gamma} \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Questions

- Steam at a pressure of 30 bar and temperature of 250°C is fed to a steam turbine from a boiler. In the turbine the steam is expanded isentropically to a pressure of 1 bar. The steam is then exhausted into a condenser where it is condensed but not undercooled. The condensate is then pumped back into the boiler. Determine
 - the dryness fraction of the steam after expansion
 - the Rankine efficiency
- Steam at 1.7 MN/m² and with a temperature of 250°C is fed into a steam engine in which it is expanded adiabatically to a release pressure of 0.35 MN/m². From this pressure it is released at constant volume to a condenser pressure of 50 kN/m². The steam is exhausted from the engine into the condenser where it is condensed but not undercooled. The condensate is pumped back into the boiler. The steam flow rate is 1500 kg/h. Determine
 - the power output of the engine
 - the Rankine efficiency
 - the Carnot efficiency for the same temperature limits of the cycle

[(a) 0.823, (b) 23.88%]

[(a) 187 kW; (b) 17.4%; (c) 32.25%]

3. The temperatures of the two isothermal processes of a Carnot cycle are 350°C and 15°C, respectively. Determine the thermal efficiency of the cycle.

[53.7%]

4. 0.23 kg of gas is taken through a Carnot cycle whose temperature limits are 300°C and 50°C. If the volume ratio of the isothermal processes is 2.5:1, determine for the cycle

- (a) the thermal efficiency
- (b) the net work done
- (c) the work ratio

Take $R = 0.28$ kJ/kg K, $\gamma = 1.4$.

[(a) 43.6%; (b) 14.75 kJ; (c) 0.2]

5. The high temperature of a Carnot cycle is 400°C and the cycle has a thermal efficiency of 55 per cent. The volume ratio of the isothermal processes is 2.8:1. Determine for the cycle

- (a) the low temperature
- (b) the volume ratio of the adiabatic processes
- (c) the overall volume ratio

Take $\gamma = 1.4$.

[(a) 30°C; (b) 7.34:1; (c) 20.6:1]

6. The pressures at the beginning and end of the adiabatic compression of a constant volume cycle are 103.5 kN/m² and 1.85 MN/m², respectively. Determine for the cycle

- (a) the overall volume ratio
- (b) the thermal efficiency

Take $\gamma = 1.4$.

[(a) 7.87:1; (b) 56.1%]

7. An internal combustion engine working on the constant volume cycle has a cylinder bore of 100 mm and a stroke of 95 mm. The clearance volume is 0.001 m³. Determine for the cycle

- (a) the ideal thermal efficiency
- (b) the actual thermal efficiency if the relative efficiency is 45%

Take $\gamma = 1.4$.

[(a) 57.4%; (b) 25.8%]

8. The pressure, volume and temperature at the beginning of the compression of a constant volume (Otto) cycle are 105 kN/m², 0.002 m³ and 25°C, respectively. The maximum temperature of the cycle is 1250°C. The volume ratio of the cycle is 8:1. The cycle is repeated 4000 times/min. Determine for the cycle

- (a) the theoretical output in kilowatts
- (b) the thermal efficiency
- (c) the mean effective pressure
- (d) the Carnot efficiency within the same temperature limits

Take $c_p = 1.007$ kJ/kg K, $c_v = 0.717$ kJ/kg K.[(a) 54.3 kW; (b) 57%; (c) 465.1 kN/m²; (d) 80.4%]

9. In an ideal Diesel cycle the pressure and temperature at the beginning of the adiabatic compression are 98.5 kN/m² and 60°C, respectively. The maximum pressure attained during the cycle is 4.5 MN/m² and the heat received during the cycle is 580 kJ/kg gas. Determine for the cycle

- (a) the volume ratio of compression
- (b) the temperature at the end of compression

- (c) the temperature at the end of heat reception
- (d) the temperature at the end of the adiabatic expansion
- (e) the net work done per kilogram of gas
- (f) the thermal efficiency
- (g) the work ratio
- (h) the mean effective pressure
- (i) the Carnot efficiency within the cycle temperature limits

[(a) 15.3:1; (b) 719°C; (c) 1297°C; (d) 360°C; (e) 365 kJ/kg; (f) 63%; (g) 0.435; (h) 402 kN/m²; (i) 79%]

Previous Years' GTU Examination Questions

1. Derive an expression for efficiency of Carnot cycle.

[Dec '08]

2. In an Otto cycle the compression ratio is 8. The temperatures at the beginning of compression and at the end of heat supply are 310 K and 1600 K respectively. Assume $\gamma = 1.4$ and $C_v = 0.717$ kJ/kg K .

Find: (i) Heat supplied (ii) Efficiency of the cycle.

[Dec '08]

3. The compression ratio of an oil engine working on Diesel Cycle is 15. Cut off takes place at 12% of the working stroke. The air is drawn in to cylinder at 100 kPa and 270°C. Assume $C_p = 1.006$ kJ/kg K and $C_v = 0.717$ kJ/kg K. Calculate:

- (i) Temperature at the end of compression
- (ii) Pressure at the end of compression
- (iii) Air std. efficiency of the cycle

[Dec '08]

4. Derive Efficiency Equation for Diesel Cycle.

[Mar '09]

5. Explain the essential elements of a Heat Engine.

[Jun '09]

6. Show that the efficiency of Otto cycle is a function of Compression Ratio only.

[Jun '09]

7. In air standard Otto Cycle the maximum and minimum temperatures are 1673 K and 288 K. The heat supplied per kg of air is 800 KJ. Calculate.

- (i) The Compression Ratio
- (ii) Efficiency
- (iii) Max & min pressures

Take $C_v = 0.718$ KJ/kg K & $\gamma = 1.4$ for air.

[Jun '09]

8. Derive an expression for efficiency of Otto Cycle.

[Sep '09]

9. A hot air engine works on Carnot cycle with thermal efficiency of 70%. If final temperature of air is 20°C, determine initial temperature.

[Sep '09]

10. In an ideal Diesel Cycle the temperature at the beginning and at the end of Compression are 57°C and 603°C respectively. The temperatures at beginning and end of expansion are 1950°C and 870°C respectively. Determine the ideal efficiency of the cycle if pressure at beginning is 1.0 bar. Calculate: maximum pressure in the cycle. [Sep '09]
11. Derive thermal efficiency formulae for Rankine Cycle. [Jan '10]
12. Define heat engine. What are the essential requirements of heat engine? [Jan '10]
13. Determine the compression ratio, the cycle efficiency, and the ratio of maximum to minimum pressure in an air standard Otto Cycle from following data: Minimum temperature = 25°C ; Maximum temperature = 1500°C ; Heat supplied per kg of air = 900 kJ. Take $C_v = 0.718 \text{ kJ/kg K}$ & $\gamma = 1.4$. [Jan '10]
14. An engine operating on the ideal Diesel Cycle has a maximum pressure of 44 bar and a maximum temperature of 1600°C . The pressure and temperature of air at the beginning of the compression stroke are 1 bar and 27°C respectively. Find the air standard efficiency of the cycle. For air take $\gamma = 1.4$. [Jun '10]
15. Draw P–V diagram for an ideal Diesel Cycle and derive an expression for its air standard efficiency in terms of temperatures only. [Jun '10]
16. For the same compression ratio the air standard efficiency of Otto Cycle is greater than that of Diesel cycle. Justify the statement. [Jun '10]



