10 Air Compressors

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OBJECTIVES

After studying this chapter, you will be able:

- To understand the introduction of air compressors and their types like reciprocating and rotary compressors, working etc.
- To understand the operation of reciprocating and rotary air compressors and thermodynamic parameters like P–V diagram, efficiency etc.

10.1 Introduction

There are many processes used in a modern industrial society which have the need for a compressed gas. The majority of cases require the use of air as the compressed gas.

The compressors are of two general types, reciprocating or rotary.

10.2 Use of Compressed Air

The main uses of compressed air are:

1. Automobile vehicles like tires, air brakes etc.
2. Pneumatic appliances like drills, hammers, spray painting equipments, air motors, vacuum cleaners and pumps etc.
3. Conveying agents
4. Water pumping
5. Power plants and sewage plants
6. Foundry industries for sand blasting, blast furnaces etc.
7. Refrigeration and air-conditioning

10.3 Reciprocation Compressor

10.3.1 Introduction, Operation and Efficiency

A single-stage reciprocating air compressor is illustrated in Fig. 10.1. It consists of a piston which reciprocates in a cylinder, driven through a connecting-rod and crank mounted in a crankcase. There are inlet and delivery valves mounted in the head of the cylinder. These valves are usually of the pressure differential type, meaning that they will operate as the result of the difference of pressure across the valve. The operation of this type of compressor is as follows.

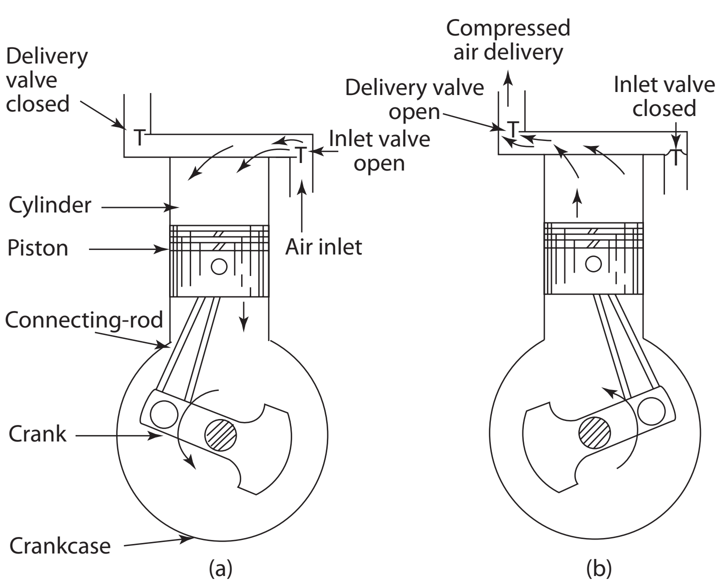

Fig. 10.1 Single-stage air compressor: (a) induction stroke; (b) compression stroke

In Fig. 10.1(a) the piston is moving down the cylinder. Any residual compressed air left in the cylinder after the previous compression will expand, eventually to reach a pressure slightly below intake pressure early on in the stroke. This means that the pressure outside the inlet valve is now higher than on the inside, so the inlet valve will lift off its seat. A stop is provided to limit its lift and to retain it within its valve seating. Thus a fresh charge of air will be aspirated into the cylinder for the remainder of the induction stroke, as it is called. During this stroke the delivery valve will remain closed because the compressed air on the outside of this valve is at a much higher pressure than the induction pressure. In Fig. 10.1 (b) the piston is now moving upwards. At the beginning of this upward stroke, a slight increase in cylinder pressure will have closed the inlet valve. The inlet and delivery valves are now closed, so the pressure of the air will rapidly rise because it is now locked up in the cylinder. Eventually the pressure will become slightly greater than the pressure of the compressed air on the outside of the delivery valve, so the delivery valve will lift. The compressed air is now delivered from the cylinder for the remainder of the stroke. Once again, there is a stop on the delivery valve to limit its lift and to retain it within its seating. At the end of the compression stroke the piston again begins to move down the cylinder, the delivery valve closes, the inlet valve eventually opens and the cycle is repeated.

Air is locked up in the cylinder of a reciprocating compressor, so the pressure during compression can be very high. It is limited by the strength of the various parts of the compressor and the power of the driving motor. Note that with the reciprocating compressor there is intermittent flow of air.

Fig. 10.2(a) shows a theoretical *P–V* diagram for a single-stage reciprocating air compressor, neglecting clearance. The processes are as follows:

- **4–1** Volume of air V_1 aspirated into compressor at pressure P_1 and temperature T_1.
- **1–2** Air compressed according to law $PV^n = C$ from pressure P_1 to pressure P_2· Volume decreases from V_1 to V_2. Temperature increases from T_1 to T_2.
- **2–3** Compressed air of volume V_2 and at pressure P_2 with temperature T_2 delivered from compressor.

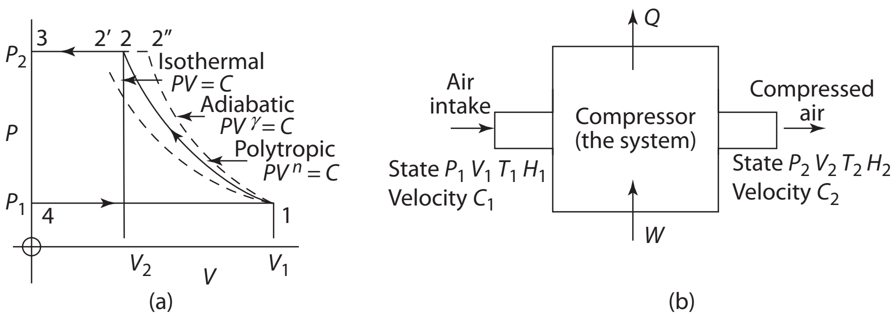

Fig. 10.2 Single-stage air compressor: (a) *P–V* diagram and (b) its theoretical circuit

During compression, due to its excess temperature above the compressor surroundings, the air will transfer some heat to the surroundings. The internal effect of friction is small in the reciprocating compressor, so neglecting friction, the index n is less than γ, the adiabatic index. Work must be input to an air compressor to keep it running, so every effort is made to reduce this input. Inspection of the *P–V* diagram (Fig. 10.2(a)) shows the frictionless adiabatic compression as 1–2″. If compression were along the isothermal 1–2′ instead of the polytropic 1–2, the work done, given by the area of the diagram, would be reduced and, in fact, would be a minimum. Isothermal compression cannot be achieved in practice, but an attempt is made to approach the isothermal case by cooling the compressor either by the addition of cooling fins or a water jacket to the compressor cylinder. For a reciprocating compressor, a comparison between the actual work done during compression and the ideal isothermal work done is made using the **isothermal efficiency**. This is defined as

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work done during compression}}{\text{Actual work done during compression}} \qquad [1]$$

Thus, the higher the isothermal efficiency, the more nearly has the actual compression approached the ideal isothermal compression.

Neglecting the change of potential energy and writing H = enthalpy of the actual mass passing through the compressor, and neglecting any small change of kinetic energy, the energy equation for the reciprocating compressor becomes

$$H_1 + Q = H_2 + W \qquad [2]$$

or

$$W = (H_1 - H_2) + Q \qquad [3]$$

$$= mc_p\,(T_1 - T_2) + \frac{(\gamma - n)}{(\gamma - 1)}\frac{(P_1V_1 - P_2V_2)}{(n-1)} \qquad [4]$$

Now

$$c_p - c_v = R \text{ and } \frac{c_p}{c_v} = \gamma$$

$$\therefore \quad c_v = \frac{c_p}{\gamma}$$

Hence

$$c_p - \frac{c_p}{\gamma} = R$$

$$\therefore c_p\left(1 - \frac{1}{\gamma}\right) = R$$

$$c_p\left(\frac{\gamma - 1}{\gamma}\right) = R$$

or

$$c_p = \frac{R\gamma}{(\gamma - 1)} \qquad [5]$$

Substituting equation [5] in [4]

$$W = \frac{\gamma}{(\gamma-1)}\,mR\,(T_1 - T_2) + \frac{(\gamma - n)}{(\gamma - 1)}\frac{(P_1V_1 - P_2V_2)}{(n-1)}$$

$$= \frac{\gamma}{(\gamma-1)}\,(P_1\,V_1 - P_2\,V_2) + \frac{(\gamma - n)}{(\gamma - 1)}\frac{(P_1V_1 - P_2V_2)}{(n-1)}$$

since $PV = mRT$

$$= \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)}\left[\gamma + \frac{(\gamma - n)}{(n-1)}\right]$$

$$= \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)}\left[\frac{\gamma(n-1) + (\gamma - n)}{(n-1)}\right]$$

$$= \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)}\left[\frac{\gamma n - \gamma + \gamma - n}{(n-1)}\right]$$

$$= \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)}\left[\frac{\gamma n - n}{(n-1)}\right]$$

$$= \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} \frac{n(\gamma - 1)}{(n-1)}$$

$$= \frac{n}{(n-1)} (P_1V_1 - P_2V_2) \quad [6]$$

$$= \frac{n}{(n-1)} mR(T_1 - T_2) \quad [7]$$

since $PV = mRT$.

This result could have been arrived at by summing areas of the P–V diagram in Fig. 10.2(a).

$\oint W$ = Net area of diagram ($\oint W$ means cycle work)

= Area 4123

= Area under 4–1 – Area under 1–2 – Area under 2–3

$$= P_1V_1 - \left[\frac{P_2V_2 - P_1V_1}{(n-1)}\right] - P_2V_2$$

$$= (P_1V_1 - P_2V_2) - \left[\frac{P_2V_2 - P_1V_1}{(n-1)}\right]$$

$$= (P_1V_1 - P_2V_2) + \left[\frac{P_1V_1 - P_2V_2}{(n-1)}\right]$$

$$= \left[1 + \frac{1}{(n-1)}\right] (P_1V_1 - P_2V_2)$$

$$= \left[\frac{n - 1 + 1}{(n-1)}\right] (P_1V_1 - P_2V_2)$$

$$\oint W = \frac{n}{(n-1)} (P_1V_1 - P_2V_2) \quad [8]$$

Equation [8] can be modified to give

$$\oint W = \frac{n}{(n-1)} (P_1V_1 - P_2V_2)$$

$$= \frac{n}{(n-1)} P_1V_1 \left(1 - \frac{P_2V_2}{P_1V_1}\right) \quad [9]$$

Now $P_1 V_1^n = P_2 V_1^n$

$$\therefore \; \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/n}$$

and substituting this into equation [9]

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[1 - \frac{P_2}{P_2}\left(\frac{P_1}{P_2}\right)^{1/n}\right]$$

$$= \frac{n}{(n-1)} P_1 V_1 \left[1 - \frac{P_2}{P_1}\left(\frac{P_2}{P_1}\right)^{-1/n}\right]$$

$$= \frac{n}{(n-1)} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{1-1/n}\right]$$

$$= \frac{n}{(n-1)} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{(n-1)/n}\right] \qquad [10]$$

The solution to this equation will always come out negative, showing that work must be done on the compressor. Only the magnitude of the work done is required from the expression, so it is often written

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \qquad [11]$$

$$= \frac{n}{(n-1)} mRT_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \qquad [12]$$

If the air delivery temperature T_2 is required, this can be obtained by using

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n}$$

or

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(n-1)/n} \qquad [13]$$

10.3.2 Clearance Volume

In practice all reciprocating compressors will have a clearance volume, which is the volume that remains in the cylinder after the piston has reached the end of its inward stroke. Fig. 10.3 shows its effect.

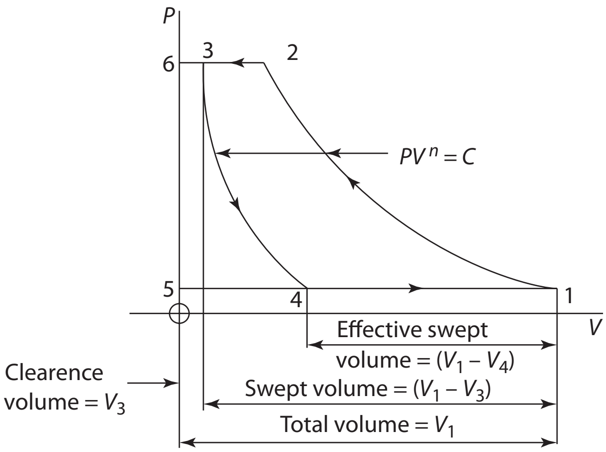

Fig. 10.3 Clearance volume: effect on reciprocating compressor

Commencing at 1 the cylinder is full of intake air, volume V_1, and the piston is about to commence its compression stroke. The air is compressed polytropically according to some law $PV^n = C$ to delivery pressure P_2 and volume V_2. At 2 the delivery valve theoretically opens and for the remainder of the stroke, 2 to 3, the compressed air is delivered from the cylinder. At 3 the piston has reached the end of its inward stroke, so delivery of compressed air ceases at 3. V_3 is the clearance volume and is filled at this stage with compressed air. As the piston begins the intake stroke, this residual compressed air will expand according to some polytropic law $PV^n = C$. It is not until the pressure has reduced to intake pressure at 4 that the inlet valve will begin to open, thus permitting the intake of a fresh charge of air. For the remainder of the intake stroke, a fresh charge is taken into the cylinder. This volume $(V_1 - V_4)$ is called the **effective swept volume**.

The ratio

$$\frac{\text{Effective swept volume}}{\text{Swept volume}} = \frac{(V_1 - V_4)}{(V_1 - V_3)} \qquad [1]$$

is called the **volumetric efficiency**; it is always less than unity because there has to be a clearance volume.

The volumetric efficiency will generally range from about 60 to 85 per cent.

The ratio

$$\frac{\text{Clearance volume}}{\text{Swept volume}} = \frac{V_3}{(V_1 - V_3)} \qquad [2]$$

is the **clearance ratio**. This will generally have a value of between 4 and 10 per cent. The greater the pressure ratio through a reciprocating compressor, the greater the effect of the clearance volume because the clearance air will now expand through a greater volume before intake conditions are reached. But the fixed cylinder size and stroke will mean that the effective swept volume, $(V_1 - V_4)$, will reduce as the pressure ratio increases, so the volumetric efficiency will also reduce.

This can also be shown as follows:

$$\text{Volumetric efficiency} = \frac{(V_1 - V_4)}{(V_1 - V_3)} = \frac{(V_1 - V_3) + (V_3 - V_4)}{(V_1 - V_3)}$$

$$= 1 + \frac{V_3}{(V_1 - V_3)} - \frac{V_4}{(V_1 - V_3)}$$

$$= 1 + \frac{V_3}{(V_1 - V_3)} - \left[\frac{V_4}{(V_1 - V_3)} \times \frac{V_3}{V_3}\right]$$

$$= 1 + \frac{V_3}{(V_1 - V_3)} - \left[\frac{V_3}{(V_1 - V_3)} \times \frac{V_4}{V_3}\right]$$

$$= 1 + \frac{V_3}{(V_1 - V_3)} \left[1 - \frac{V_4}{V_3}\right]$$

$$= 1 - \frac{V_3}{(V_1 - V_3)} \left[\frac{V_4}{V_3} - 1\right]$$

$$= 1 - \frac{V_3}{(V_1 - V_3)} \left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right] \qquad [3]$$

since $V_4/V_3 = (P_2/P_1)^{1/n}$

This again shows that for fixed cylinder conditions, V_1 and V_3, the greater the pressure ratio P_2/P_1, the smaller the volumetric efficiency.

Work done/cycle = Net area 1234

= Area 5126 – Area 5436

Assuming the polytropic index to be the same for both compression and clearance expansion, then

$$\text{Work done/cycle} = \oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$- \frac{n}{(n-1)} P_4 V_4 \left[\left(\frac{P_3}{P_4}\right)^{(n-1)/n} - 1\right] \qquad [4]$$

But $P_4 = P_1$ and $P_3 = P_2$, therefore equation [4] becomes

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] - \frac{n}{(n-1)} P_1 V_4 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$= \frac{n}{(n-1)} P_1 (V_1 - V_4) \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \qquad [5]$$

10.3.3 Actual P–V Diagram

Fig. 10.4 shows an actual compressor diagram, 1234 is the theoretical *P–V* diagram already discussed.

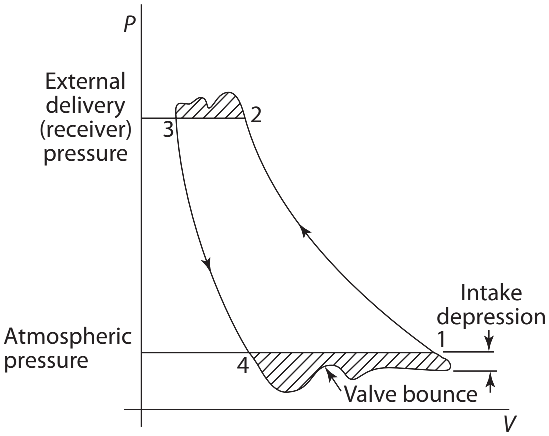

Fig. 10.4 Single-stage air compressor: actual *P–V* diagram

At 4, when the clearance air has reduced to atmospheric pressure, the inlet valve in practice will not open. There are two main reasons for this: there must be a pressure difference across the inlet valve in order to move it and there is the inertia of the inlet valve. Thus, the pressure drops away until the valve is forced off its seat. Some valve bounce will then set in, as shown by the wavy line, and eventually intake will become very nearly steady at some pressure below atmospheric pressure. This negative pressure difference, called the **intake depression**, settles naturally, showing that what is called suction is really the atmospheric air forcing its way into the cylinder against a reduced pressure.

A similar situation occurs at 2, at the beginning of compressed air delivery. There is a pressure rise followed by valve bounce; the pressure then settles at some pressure above external delivery pressure. Compressed air is usually delivered into tank called the **receiver**, so external delivery pressure is sometimes called the **receiver pressure**. Other small effects at inlet and delivery would be gas inertia and turbulence.

The practical effects discussed are responsible for the addition of the two small shaded negative work areas shown in Fig. 10.4. These areas are in addition to the theoretical area 1234.

10.3.4 Free Air Delivery

If the volume of air delivered by an air compressor is reduced to atmospheric temperature and pressure, this volume of air is called the **free air delivery**.

Remember that due to mass flow continuity.

Delivered mass of air = Intake mass of air [1]

Using the characteristic equation and assuming clearance

$$\frac{P_f V_f}{T_f} = \frac{P_1(V_1 - V_4)}{T_1} = \frac{P_2(V_2 - V_3)}{T_2} \quad [2]$$

If clearance is neglected, equation [1] becomes

$$\frac{P_f V_f}{T_f} = \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \qquad [3]$$

For convenience, P_f and T_f are often taken, as 0.101 325 MN/m^2 (101.325 kN/m^2 = 1.013 25 bar) and 288 K (15°C).

Example 10.1 *A single-stage, single-acting, reciprocating air compressor has a bore of 200 mm and a stroke of 300 mm. It runs at a speed of 500 rev/min. The clearance volume is 5 per cent of the swept volume and the polytropic index is 1.3 throughout. Intake pressure and temperature are 97 kN/m^2 and 20°C, respectively, and the compression pressure is 550 kN/m^2. With the aid of Fig. 10.5, determine*

(a) the free air delivered in m^3/min (free air conditions 101.325 kN/m^2 and 15°C)
(b) the volumetric efficiency referred to the free air conditions
(c) the air delivery temperature
(d) the cycle power
(e) the isothermal efficiency, neglecting clearance

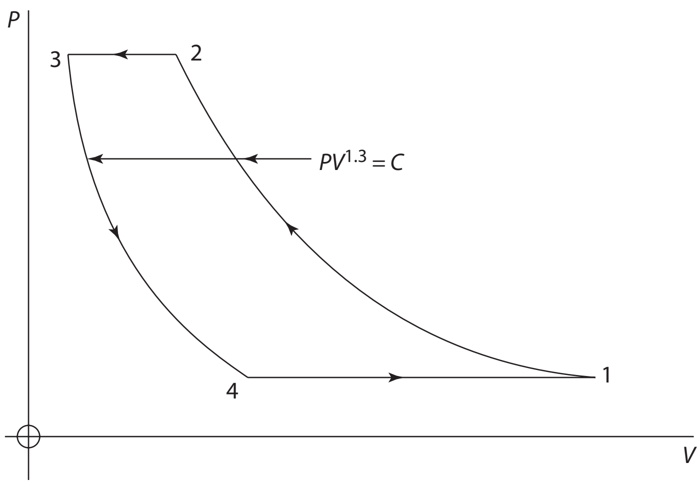

Fig. 10.5 Diagram for Example 10.1

(a)

$$\text{Swept volume, } (V_1 - V_3) = \left[\left(\frac{\pi \times 200^2}{4}\right) \times 300\right] \text{mm}^3$$

$$= \left[\frac{(\pi \times 0.2^2)}{4} \times 0.3\right] \text{m}^3$$

$$= \pi \times \frac{0.4}{4} \times 0.3$$

$$\mathbf{= 0.009\ 425\ m^3}$$

$$V_3 = 0.05(V_1 - V_3) = 0.05 \times 0.009\,425 = 0.000\,471\ \text{m}^3$$

$$V_1 = (V_1 - V_3) + V_3 = 0.009\,425 + 0.000\,471 = 0.009\,896\ \text{m}^3$$

$$V_4 = V_3\left(\frac{P_3}{P_4}\right)^{1/n} = 0.000\,471 \times \left(\frac{550}{97}\right)^{1/1.3} = 0.000\,471 \times 3.8 = 0.001\,79\ \text{m}^3$$

$\therefore$ Effective swept volume $= (V_1 - V_4) = 0.009\,896 - 0.001\,79$

$= \mathbf{0.008\,106\ m^3}$

Effective swept volume/min = 0.008 106 × 500

$= \mathbf{4.053\ m^3}$

Now

$$\frac{P_1(V_1 - V_4)}{T_1} = \frac{P_f V_f}{T_f}$$

$$\therefore\ V_f = \frac{P_1(V_1 - V_4)}{P_f T_1}\ T_f = \frac{97 \times 4.053 \times 288}{101.325 \times 293} = \mathbf{3.814\ m^3/min}$$

(b)

$$\text{Volumetric efficiency} = \frac{V_f}{500(V_1 - V_3)} = \frac{3.814}{500 \times 0.009\,425}$$

$= 0.809$

$= \mathbf{80.9\%}$

(c)

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n-1)/n} = 293 \times \left(\frac{550}{97}\right)^{(1.3-1/1.3}$$

$= 293 \times 5.67^{0.3/1.3}$

$= 293 \times 5.67^{1/4.33}$

$= 293 \times 1.493$

$= \mathbf{437.5\ K}$

$t_2 = 437.5 - 273 = \mathbf{164.5\ °C}$

(d)

$$\text{Cycle power} = \frac{n}{(n-1)} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] \times \frac{500}{60}$$

$$= \frac{1.3}{(1.3-1)} \times 97 \times 0.008\,106 \times \left[\left(\frac{550}{97}\right)^{(1.3-1)/1.3} - 1\right] \times \frac{500}{60}$$

$$= 4.33 \times 97 \times 0.008\,106 \times 0.493 \times \frac{500}{60}$$

$= \mathbf{14\ kW}$

(e)
Neglecting clearance

$$\oint W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

$$\text{Isothermal } \oint W = P_1 V_1 \ln (P_2/P_1)$$

$$\therefore \quad \text{Isothermal efficiency} = \frac{P_1 V_1 \ln(P_2/P_1)}{\frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]}$$

$$= \frac{\ln(P_2/P_1)}{\frac{n}{(n-1)} \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]}$$

$$= \frac{\ln 5.67}{4.33 \times 0.943}$$

$$= \frac{1.735}{4.33 \times 0.493}$$

$$= 0.813$$

$$= \mathbf{81.3\%}$$

10.4 Multi-stage Reciprocating Compressor

10.4.1 Introduction and Operation

If the delivery from a single-stage, reciprocating compressor is restricted, the delivery pressure will increase. But if the delivery pressure is increased too far, certain disadvantages will appear.

Referring to Fig. 10.6, assume that the single-stage compressor is compressing to pressure P_2, the complete cycle is 1234. Clearance air expansion will be 3–4 and the mass flow through the compressor will be controlled by the effective swept volume ($V_1 - V_4$). Assume now that a restriction is placed on delivery. The delivery pressure becomes P_5, say, the cycle becomes 1567 and clearance air expansion becomes 6–7. The mass flow through the compressor is now controlled by effective swept volume ($V_1 - V_7$), which is less than ($V_1 - V_4$). In the limit, assuming the compressor to be strong enough, the compression 1–8 would take place, where V_8 is the clearance volume, in which case there would be no delivery. Consequently, as the delivery pressure for a single-stage, reciprocating compressor is increased, the mass flow through the compressor will increase.

And as the delivery pressure is increased, the delivery temperature will increase. Referring to Fig. 10.6, $T_8 > T_5 > T_2$. If high-temperature air is not a requirement of the compressed air delivered, any increase in temperature represents an energy loss.

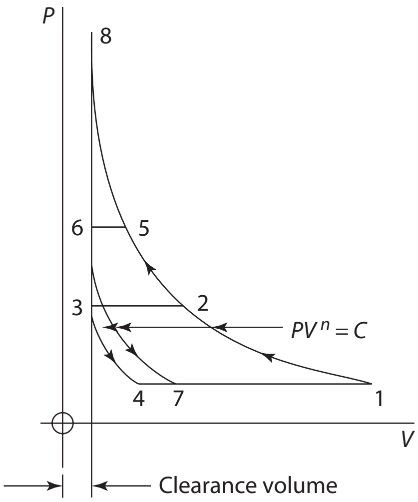

Fig. 10.6 Single-stage compressor: effect of increasing delivery pressure

If a single-stage machine is required to deliver high-pressure air, it will require heavy working parts in order to accommodate the high pressure ratio through the machine. This will increase the balancing problem and the high torque fluctuation will require a heavy flywheel installation.

Such disadvantages as described can largely be overcome by multi-stage compression. This is a series arrangement of cylinders in which the compressed air from the preceding cylinder becomes the intake air for the following cylinder. This is illustrated for a 3-stage compressor in Fig. 10.7.

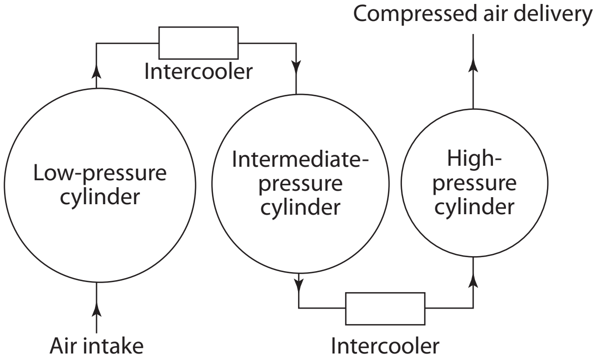

Fig. 10.7 Three-stage compressor

The low pressure ratio in the low-pressure cylinder means that the clearance air expansion is reduced and the effective swept volume of this cylinder is increased. This cylinder controls the mass flow through the machine because it introduces the air into the machine. This means there is greater mass flow through the multi-stage arrangement than through the single-stage machine.

By installing an intercooler between cylinders, in which the compressed air is cooled between cylinders, the final delivery temperature is reduced. This reduction in temperature means a reduction in internal energy of the delivered air (Joule's law). Since this energy must have come from the input energy required to drive the machine, it results in a decrease of input work requirement for a given mass of delivered air.

A multi-stage arrangement of cylinders can also be set up with better balancing and torque characteristics than a single-stage machine. It is common to find machines with either two or three stages of compression. The complexity of the machinery limits the number of stages.

It will be noted that Fig. 10.7 shows how the cylinder diameters decrease as the pressure increases. This is because, as the pressure increases, so the volume of a given mass of gas decreases. There is continuity of mass flow through a compressor, so each following cylinder will require a smaller volume due to its increased pressure range. This reduction in volume is usually accomplished by reducing the cylinder diameter.

Fig. 10.8 illustrates cycle arrangements in the development of the ideal conditions required for multi-stage compression. For simplicity, clearance is neglected. The effect of clearance has been discussed in section 10.3.2. Referring to Fig. 10.8, the overall pressure range is P_1 to P_3. Cycle 8156 is that of the single-stage compressor. Cycles 8147 and 7456 are those of a two-stage compressor without intercooling between cylinders.

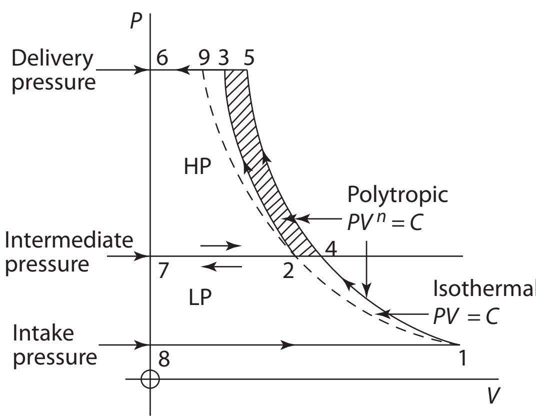

Fig. 10.8 Two-stage compressor: cycle diagram

Cycles 8147 and 7236 are those of a two-stage compressor with perfect intercooling between cylinders. Perfect intercooling means that, after the initial compression in the low-pressure (LP) cylinder, with its consequent temperature rise, the air is cooled in an intercooler back to its original temperature. Referring to Fig. 10.8, this means that $T_2 = T_1$, in which case point 2 lies on the isotherm through 1. This shows that multi-stage compression, with perfect intercooling, approaches more closely the ideal isothermal compression than does single-stage compression.

10.4.2 Ideal Conditions for Multi-Stage Compressor

Consider Fig. 10.8: cycle 8156 is that of a single-stage compressor, neglecting clearance. For this cycle

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_5}{P_1} \right)^{(n-1)/n} - 1 \right] \quad [1]$$

and

$$T_5 = T_1 \left(\frac{P_5}{P_1}\right)^{(n-1)/n} \qquad [2]$$

= delivery temperature

For a two-stage machine, without intercooling between cylinders, 8147 is the low-pressure cycle and 7456 is the high-pressure cycle. For this arrangement

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_4}{P_2}\right)^{(n-1)/n} - 1\right] + \frac{n}{(n-1)} P_4 V_4 \left[\left(\frac{P_5}{P_4}\right)^{(n-1)/n} - 1\right] \qquad [3]$$

This will give the same result as equation [1]. The final delivery temperature will also be as given by equation [2] because there is no intercooling. For the two-stage machine with perfect intercooling, 8147 is the low-pressure cycle and 7236 is the high-pressure cycle.

In this case

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_4}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{n}{(n-1)} P_2 V_2 \left[\left(\frac{P_3}{P_2}\right)^{(n-1)/n} - 1\right] \qquad [4]$$

Delivery temperature is given by

$$T_3 = T_2 \left(\frac{P_3}{P_2}\right)^{(n-1)/n} = T_1 \left(\frac{P_3}{P_2}\right)^{(n-1)/n}, \text{ since } T_2 = T_1 \qquad [5]$$

And since $T_2 = T_1$

$$P_2 V_2 = P_1 V_1 \qquad [6]$$

Also

$$P_4 = P_2 \qquad [7]$$

Substituting equations [6] and [7] in equation [4]

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} + \left(\frac{P_3}{P_2}\right)^{(n-1)/n} - 2\right] \qquad [8]$$

Now Fig. 10.8 shows the shaded area 2453, the work saving which occurs as the result of using an intercooler. As intermediate pressure $P_2 \to P_1$, area 2453 $\to$ 0. And as $P_2 \to P_3$, area 2453 $\to$ 0. This means there exists an intermediate pressure P_2 which makes area 2453 a maximum; this is the condition when $\oint W$ is a minimum.

Inspection of equation [8] shows that for minimum $\oint W$

$$\left(\frac{P_2}{P_1}\right)^{(n-1)/n} + \left(\frac{P_3}{P_2}\right)^{(n-1)/n}$$

must be a minimum because all other parts of the equation are constant in this consideration; P_2 is the variable.

Hence, for minimum $\oint W$

$$\frac{d}{dP_2}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n}+\left(\frac{P_3}{P_2}\right)^{(n-1)/n}\right]=0$$

Differentiating with respect to P_2

$$\frac{1}{P_1^{n-1/n}}\times\left(\frac{n-1}{n}\right)P_2^{(n-1/n)-1}+P_3^{n-1/n}\times-\left(\frac{n-1}{n}\right)P_2^{-[(n-1)/n]-1}=0$$

$$\frac{1}{P_1^{(n-1)/n}}\times\left(\frac{n-1}{n}\right)P_2^{-1/n}=P_3^{(n-1)/n}\times\left(\frac{n-1}{n}\right)P_2^{(-2n+1)/n}$$

$$\frac{P_2^{-1/n}}{P_2^{(-2n+1)/n}}=(P_1P_3)^{(n-1)/n}$$

$$P_2^{-1/n}P_2^{-[(-2n+1)/n]}=(P_1P_3)^{(n-1)/n}$$

$$P_2^{-1/n}P_2^{(2n-1)/n}=(P_1P_3)^{(n-1)/n}$$

$$P_2^{2(n-2)/n}=(P_1P_3)^{(n-1)/n}$$

$$P_2^{(2n-2)/n}=(P_1P_3)^{(n-1)/n}$$

$$\therefore\ P_2^2=P_1P_3 \qquad [9]$$

From which

$$P_2=(P_1P_3)^{1/2}=\sqrt{(P_1P_3)} \qquad [10]$$

and

$$\frac{P_2}{P_1}=\frac{P_3}{P_2} \qquad [11]$$

or, pressure ratio/stage is equal.

P_2 obtained from equation [10] will give the ideal intermediate pressure which, with perfect inter-cooling, will give the minimum $\oint W$.

With these ideal conditions, substituting equations [6], [7] and [11] into equation [4] shows that there is equal work per cylinder.

Hence

$$\oint W=\frac{2n}{(n-1)}P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n}-1\right] \qquad [12]$$

Substituting equation [10] in equation [12]

$$\oint W=\frac{2n}{(n-1)}P_1V_1\left\{\left[\frac{(P_1P_3)^{\frac{1}{2}}}{P_1}\right]^{(n-1)/n}-1\right\}$$

$$= \frac{2n}{(n-1)} P_1 V_1 \left\{ \left[\left(\frac{P_3}{P_1} \right)^{1/2} \right]^{(n-1)/n} - 1 \right\}$$

$$= \frac{2n}{(n-1)} P_1 V_1 \left\{ \left(\frac{P_3}{P_1} \right)^{(n-1)/2n} - 1 \right\}$$

Note that P_3/P_1 is the pressure ratio through the compressor. Now, from the analysis of compressors so far:

for a single-stage machine

$$\oint W = \frac{n}{(n-1)} P_1 V_1 \left\{ \left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right\}$$

for a two-stage machine

$$\oint W = \frac{2n}{(n-1)} P_1 V_1 \left\{ \left(\frac{P_3}{P_2} \right)^{(n-1)/2n} - 1 \right\}$$

So it seems reasonable to assume that for a three-stage machine

$$\oint W = \frac{3n}{(n-1)} P_1 V_1 \left\{ \left(\frac{P_4}{P_1} \right)^{(n-1)/3n} - 1 \right\}$$

and for an x-stage machine

$$\oint W = \frac{xn}{(n-1)} P_1 V_1 \left\{ \left(\frac{P_{x+1}}{P_2} \right)^{(n-1)/xn} - 1 \right\} \quad [13]$$

Note that in each case P_{x+1}/P_1 is the pressure ratio through the compressor. For an ideal compressor, there is equal work per cylinder, so for an x-stage machine

$$\oint W = \frac{xn}{n-1} P_1 V_1 \left\{ \left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right\} \quad [14]$$

Equation [11] is used to determine the intermediate pressures for an x-stage machine running under ideal conditions. It shows that the pressure ratio per stage is equal.

Hence, for an x-stage machine

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \ldots = \frac{P_{x+1}}{P_x} = k, \text{ say} \quad [15]$$

From this

$P_2 = kp_1$

$P_3 = kp_2 = k^2 P_1$

$$P_4 = kP_3 = k^3P_1$$
$$P_{x+1} = kP_x = k^xP_1$$
$$\therefore k^x = \left(\frac{P_{x+1}}{P_1}\right)$$

or

$$k = \sqrt[x]{\left(\frac{P_{x+1}}{P_1}\right)} = \sqrt[x]{\text{(Pressure ratio through compressor)}} \qquad [16]$$

Substituting the value of k in equation [16] will determine the intermediate pressures.

Example 10.2 *A two-stage, single-acting, reciprocating compressor takes in air at the rate of 0.2 m³/s. Intake pressure and temperature are 0.1 MN/m² and 16°C, respectively. The air is compressed to a final pressure of 0.7 MN/m². The intermediate pressure is ideal and intercooling is perfect. The compression index is 1.25 and the compressor runs at 10 rev/s.*

Neglecting clearance, determine

(a) the intermediate pressure
(b) the total volume of each cylinder
(c) the cycle power

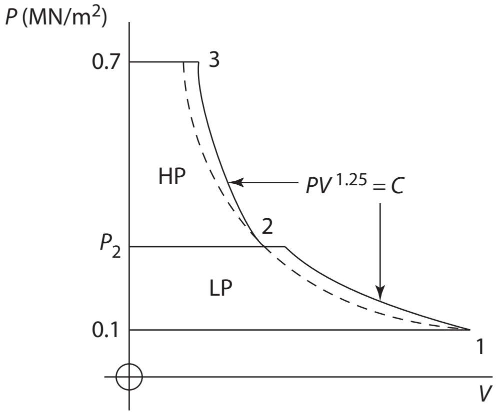

Fig. 10.9 Diagram for Example 10.2

Solution

First draw a diagram (Fig. 10.9).

(a)

$$P_2 = \sqrt{P_1P_2} = \sqrt{0.1 \times 0.7}$$
$$= \sqrt{0.07}$$
$$= \mathbf{0.265\ MN/m^2}$$

(b)

Total volume of LP cylinder = $\frac{0.2}{10}$ = 0.02 m^3 = **20 litres**

Intercooling is perfect, so 2 lies on the isothermal through 1.

$$\therefore P_1V_1 = P_2V_2$$

or

$$V_2 = \frac{P_1V_1}{P_2} = \frac{0.1 \times 0.02}{0.265} = 0.007\,5 \text{ m}^3 = \mathbf{7.5 \text{ litres}}$$

The total volume of the **HP** cylinder is 7.5 litres.

(c)

$$\text{Cycle power} = \frac{2n}{(n-1)} \quad P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] V_1 = \text{vol/s}$$

$$= 2 \times \frac{1.25}{(1.25-1)} \times 0.1 \times 0.2 \times \left[\left(\frac{0.265}{0.1}\right)^{(1.25-1)/1.25} - 1\right]$$

$$= 2 \times \frac{1.25}{0.25} \times 0.02 \times (2.65^{1/5} - 1)$$

$$= 2 \times 5 \times 0.02 \times (1.215 - 1)$$

$$= 10 \times 0.02 \times 0.215$$

$$= 0.043 \text{ MW}$$

$$= \mathbf{42 \text{ kW}}$$

Example 10.3 *A three-stage, single-acting, reciprocating air compressor has a low-pressure cylinder of 450 mm bore and 300 mm stroke. The clearance volume of the low-pressure cylinder is 5 per cent of the swept volume. Intake pressure and temperature are 1 bar and 18°C, respectively; the final delivery pressure is 15 bar. Intermediate pressures are ideal and intercooling is perfect.*

The compression and expansion index can be taken as 1.3 throughout. Determine

(a) the intermediate pressure
(b) the effective swept volume of the low-pressure cylinder
(c) the temperature and the volume of air delivered per stroke at 15 bar
(d) the work done per kilogram of air

Take R *= 0.29 kJ/kg K.*

SOLUTION

First draw a diagram (Fig. 10.10).

(a)

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = k = \sqrt[3]{P_4/P_1}$$

$$k = \sqrt[3]{15/1} = 2.466$$

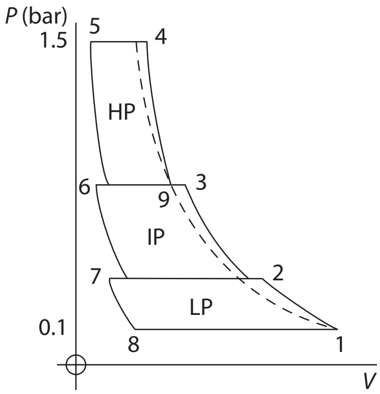

Fig. 10.10 Diagram for Example 10.3

$$\therefore\ P_2 = kP_1 = 2.466 \times 1 = \mathbf{2.466\ bar}$$

$$P_3 = kP_2 = 2.466 \times 2.466 = \mathbf{6.081\ bar}$$

(b)

Swept volume of LP cylinder $= V_1 - V_7$

$$= \pi \times \frac{0.45^2}{4} \times 0.3$$

$$= \pi \times 0.050\,6 \times 0.3$$

$$= \mathbf{0.047\,7\ m^3}$$

$$V_7 = 0.05 \times 0.047\,7 = 0.002\,39\ m^3$$

$$\therefore\ V_1 = (V_1 - V_7) + V_7 = 0.0477 + 0.002\,39 = 0.050\,09\ m^3$$

$$P_7 V_7^{1.3} = P_8 V_8^{1.3}$$

$$\therefore\ V_8 = V_7 \left(\frac{P_7}{P_8}\right)^{1/1.3} = 0.002\,39 \times 2.466^{1/1.3} = 0.004\,78\ m^3$$

The effective swept volume of the low-pressure cylinder is

$$V_1 - V_8 = 0.050\,09 - 0.004\,78$$

$$= 0.045\,31\ m^3$$

$$= \mathbf{45.31\ litres}$$

(c)

$$\frac{T_4}{T_9} = \left(\frac{P_4}{P_9}\right)^{(n-1)/n}$$

$$\therefore\ T_4 = T_9 \left(\frac{P_4}{T_9}\right)^{(n-1)/n}$$

Intercooling is perfect, so $T_9 = T_1$.

$$\therefore\ T_4 = 291 \times 2.466^{0.3/1.3}$$
$$= 291 \times 1.232$$
$$= 358.5 \text{ K}$$
$$t_4 = 358.5 - 273 = \mathbf{85.5°C}$$

The delivery temperature is 85.5 °C.

$$\frac{P_4(V_4 - V_5)}{T_4} = \frac{P_1(V_1 - V_8)}{T_1}$$

$$\therefore\ V_4 - V_5 = \frac{P_1 T_4}{P_4 T_1}(V_1 - V_8)$$
$$= \frac{1}{15} \times \frac{358.5}{291} \times 0.045\ 31$$
$$= 0.003\ 72 \text{ m}^3$$
$$= \mathbf{3.72\ litres}$$

The delivery volume per stroke is 3.72 litres.

(d)

$$\text{Work/kg of air} = \frac{3n}{(n-1)} RT_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$
$$= \frac{3 \times 1.3}{(1.3 - 1)} \times 0.29 \times 291 \times (1.232 - 1)$$
$$= 3 \times 4.33 \times 0.29 \times 291 \times 0.232$$
$$= \mathbf{254.3\ kJ}$$

10.5 Rotary Compressors

10.5.1 Introduction, Types and Operation

There are three basic types of rotary air compressor: the radial or centrifugal compressor, the axial-flow compressor and the positive-displacement compressor or blower.

A general arrangement of a radial compressor is shown is Fig 10.11. It consists of an impeller rotating within a casing, usually at high speed (something like 20 000–30 000 rev/min in some cases). The impeller consists of a disc onto which radial blades are attached. The blades break up the air into cells. The impeller is shrouded by the casing. If the impeller is rotated, the cells of air will also be rotated with the impeller. Centrifugal force means that the air in the cells will move out from the outside edge of the impeller and more air will move into the centre of the impeller to take its place. The centre is called the eye of the impeller. As it moves away from the outside edge of the impeller, the air passes into a diffuser ring which helps to direct it into the volute. The air is also decelerated in the diffuser ring, producing a pressure rise in the air because, theoretically, there is no energy loss to the airstream.

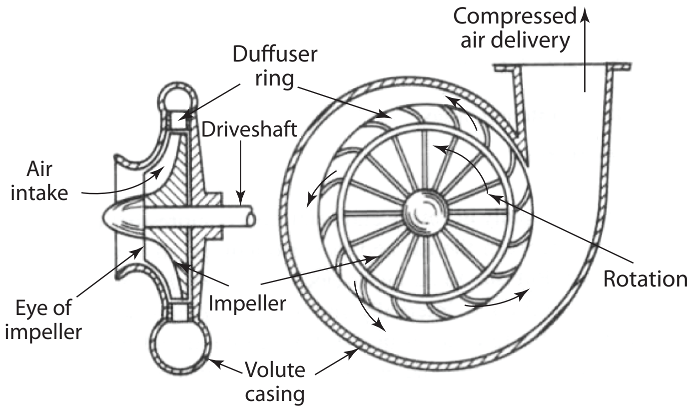

Fig. 10.11 Radial compressor

The volute is the collecting device for this compressor. It section increases round the compressor so that, as the air is collected round the volute, a greater section will be required to pass the increasing quantity of air. A duct leads away from the volute to take the compressed air out of the compressor. This type of compressor, is a continuous-flow device and will deal with large quantities of air through a moderate pressure range. Pressure compression ratios of some 4 or 6:1 are common.

The general arrangement of the axial-flow compressor is shown in Fig. 10.12. In this type of compressor there are alternate rows of fixed and moving blades. The fixed blades are fixed in an outer casing, whereas the moving blades are fixed to a central drum which can be rotated by a driveshaft. The moving blades can be looked at in a simple way as a set of fans in series. These blades progress the air through the compressor; the preceding fan boots the following fan, as it were. The fixed blades act as guide vanes and diffusers. The angles of all blade rows are set such that there is a smooth progression of air from blade row to blade row. The air passes axially along the compressor, hence its name. Air is removed by suitable ducting at the end of the compressor. Once again, this type of compressor runs at high speed (10 000–30 000 rev/min) and generally deals with large quantities of air. Pressure compression ratios of 10:1 or more can be obtained. This compressor design is commonly used in aircraft gas turbines.

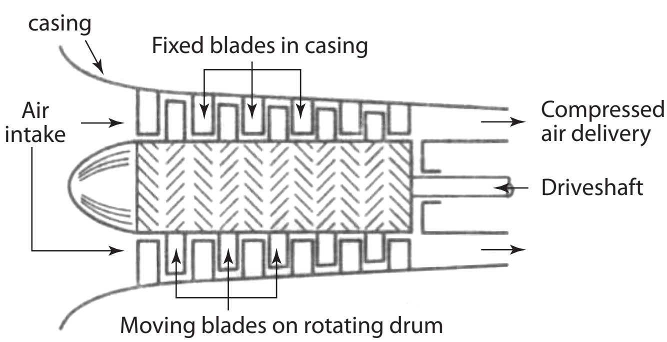

Fig. 10.12 Axial-flow compressor

The compression occurs so rapidly in rotary and axial-flow compressors that there is little time for heat exchange between the gas and its surroundings, so the compression will be very nearly adiabatic. But due to its high velocity through the compressor, the air will encounter considerable friction, internally and with the compressor walls; there will be turbulence and shock due to changes in direction. Friction and turbulence will generate internal energy within the air and produce a temperature higher than the theoretical adiabatic temperature. The index n for a rotary compressor will therefore be greater than the adiabatic index γ. It should be noted that although n is greater than ? in the rotary compressor the compression is nevertheless adiabatic. The adiabatic compression is defined as a compression carried out such that no heat is received or rejected from or to the surroundings during the progress of the compression. Neglecting any small loss of heat to the surroundings, during compression in the rotary compressor, the compression is adiabatic. The temperature increase over the theoretical adiabatic compression temperature is an internal effect, so it does not modify the adiabatic nature of the compression.

Fig. 10.13(a) is a P–V diagram for a rotary compressor; Fig. 10.13(b) is the corresponding T–s diagram. The pressure range is from lower pressure P_1 to higher pressure P_2.

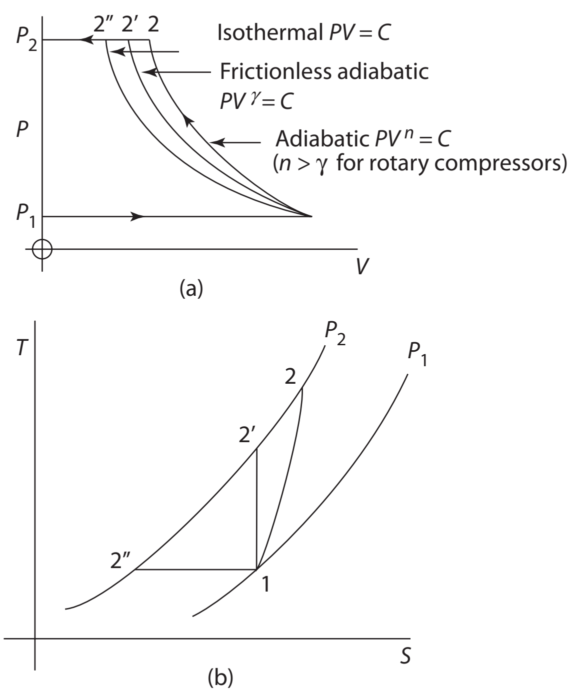

Fig. 10.13 Rotary compressor: (a) P–V diagram; (b) T–s diagram

The actual adiabatic compression is shown as 1–2. The frictionless adiabatic compression (isentropic compression) is shown as 1–2′. The isothermal compression is shown as 1 – 2″. Note that $T_2 > T_{2'} > T_{2''}$ and that $W_{1-2} > W_{1-2'} > W_{1-2''}$.

Consider the steady-flow energy equation applied to the rotary compressor

$$u_1 + P_1 v_1 + \frac{C_1^2}{2} + Q = u_2 + P_2 v_2 + \frac{C_2^2}{2} + W \quad [1]$$

or

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \text{ (since } u + Pv = h) \quad [2]$$

The compression is adiabatic, so $Q = 0$ by definition. And across the compressor rotor there is little or no change of velocity, so the kinetic energy terms can be neglected. Thus, the energy equation becomes

$$h_1 = h_2 + W \quad [3]$$

from which

$$\text{Specific work} = W = (h_1 - h_2)$$
$$= c_p (T_1 - T_2) \quad [4]$$

For a mass flow $\dot{m}$

$$W = \dot{m} c_p (T_1 - T_2) \quad [5]$$

This equation will be negative, showing that work must be done on the compressor.

For a rotary compressor, the ratio

$$\frac{\text{Frictionless adiabatic work}}{\text{Actual adiabatic work}} = \frac{W_{1-2'}}{W_{1-2}} \quad [6]$$

is called the **isentropic efficiency**.

Hence, from equation [6]

$$\text{Isentropic } \eta = \frac{\dot{m} c_p (T_1 - T_{2'})}{\dot{m} c_p (T_1 - T_2)}$$
$$= \frac{(T_1 - T_{2'})}{(T_1 - T_2)} \quad [7]$$

There are several designs of positive-displacement compressor or blower. A well-known design is the Roots blower, illustrated in Fig. 10.14. There are two rotors which mesh together in the same way as gearwheels. Two rotors each with two lobes are shown. The two rotors are driven together through external gearing. Rotors with more than two lobes are sometimes used when an increase in pressure ratio is required. The rotors rotate in a casing.

The operation is as follows. Gas is taken in at the intake and, as the rotors are rotated, a volume of gas V becomes trapped between the rotor and the casing. This volume V is transported to the delivery side of the machine. As the gas is delivered, the compressed gas already on the delivery side compresses it to delivery pressure.

The volume of gas transported is $4V$ per revolution. The pressure ratio through the machine is usually low, say 2:1.

An approximate P–V diagram for the Roots blower is given in Fig. 10.15. From the diagram

$$\text{Work done to compress volume } V = V (P_2 - P_1) \quad [8]$$

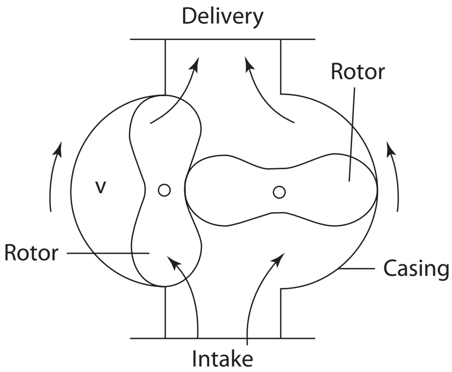

Fig. 10.14 Rotor blower: positive-displacement compressor

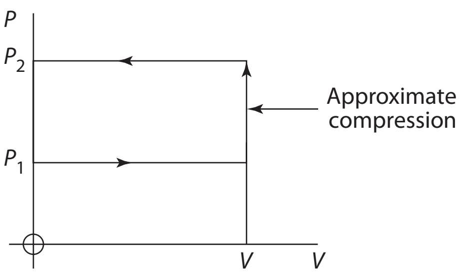

Fig. 10.15 Roots blower: approximate *P–V* diagram

Work done/revolution = $4V\,(P_2 - P_1)$ [9]

(for two-lobe rotors)

Fig. 10.16 is a diagram of a vane pump. It consists of a circular casing in which a drum rotates about a centre eccentric to the centre of the casing. Slots are cut in the drum into which vanes are fitted. During rotation the vanes remain in contact with the casing.

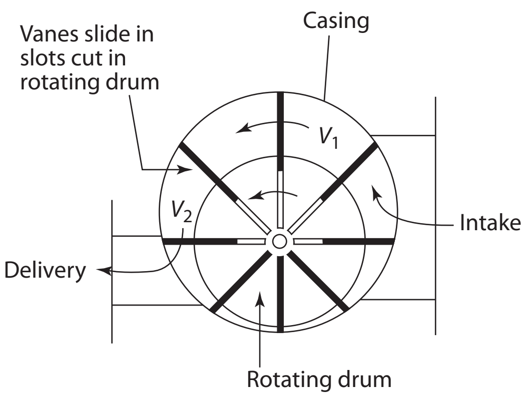

Fig. 10.16 Vane pump

In operation, as the drum rotates, a volume of gas V_1 is trapped between the vanes, the drum and the casing. The space between the drum and casing reduces as delivery is reached and the gas has a reduced volume V_2. The gas has therefore been partially compressed. The remainder of compression is obtained by back pressure from the already compressed gas, as in the Roots blower. An approximate *P–V* diagram for the vane pump is shown in Fig. 10.17.

Compression 1–2, in which the volume is reduced from V_1 to V_2 and pressure is increased from P_1 to P_2, is assumed to be according to the law $PV^\gamma = C$.

Compression 2–3, due to back pressure from the already compressed gas, is assumed to be at constant volume.

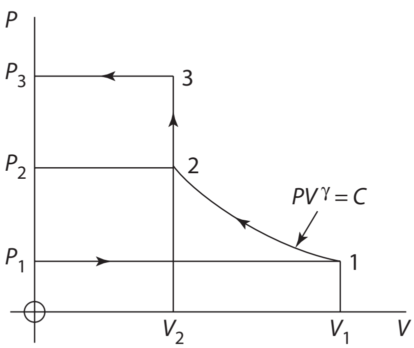

Fig. 10.17 Vane pump: approximate *P–V* diagram

For compression 1–2

$$P_2 V_1^\gamma = P_1 V_1^\gamma \therefore P_2 = P_1 (V_1/V_2)^\gamma \qquad [10]$$

$$\text{Work}_{1-02} = \frac{\gamma}{(\gamma-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \qquad [11]$$

For the compression 2–3

$$\text{Work}_{2-3} = V_2 (P_3 - P_2) \qquad [12]$$

$$\text{Total work}_{1-3} = \frac{\gamma}{(\gamma-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] + V_2 (P_3 - P_2) \qquad [13]$$

If there are *N* vanes, then

$$\text{Work/revolution} = N\left\{l \frac{\gamma}{(\gamma-1)} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] + V_2 (P_3 - P_2)\right\} \qquad [14]$$

10.5.2 Stagnation Temperature and Pressure

If a gas stream is brought to rest, its total energy remains constant, so the loss of kinetic energy of the stream will appear as an increase in its temperature and pressure. This happens during the process of diffusion and in the vicinity of a temperature-sensing bulb suspended in a gas stream. The stream is brought to rest at the bulb, so the temperature recorded is higher than it would be if the device were moving with the stream. The temperature recoded with a fixed bulb is called the **total or stagnation** temperature. If the bulb were moving with the stream it would record the **static** temperature.

Stagnation temperatures can be used in rotary compressors if it is required to include inlet and exit velocities.

Equation [2] of section 10.5.2 shows the energy balance for the rotary compressor

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W \quad [1]$$

For adiabatic compression $Q = 0$.

Also

$$(h_1 - h_2) = c_p(T_1 - T_2)$$

Thus, equation [1] can be rewritten

$$c_pT_1 + \frac{C_1^2}{2} = c_pT_2 + \frac{C_2^2}{2} + W \quad [2]$$

Let

$$c_pT + \frac{C^2}{2} = c_pT_t \quad [3]$$

where T^t = total temperature

From this

$$T_t = T + \frac{C^2}{2c_p} \quad [4]$$

Substituting equation [3] in equation [2]

$$c_pT_{t1} = c_pT_{t2} + W$$

from which

$$W = c_p(T_{t1} - Tt_2) \quad [5]$$

There is assumed to be no energy loss, so it can be assumed that the stream is brought to rest adiabatically. The total or stagnation pressure, P_t, can therefore be determined from

$$\frac{P_1}{P} = \left(\frac{T_t}{T}\right)^{\gamma/(\gamma-1)} \quad [6]$$

from which

$$P_t = P\left(\frac{T_t}{T}\right)^{\gamma/(\gamma-1)} \quad [7]$$

where P = static pressure

Example 10.4 *A rotary compressor has a pressure compression ratio of 5:1. It compresses air at the rate of 10 kg/s. The initial pressure and temperature are 100 kN/m² and 20° C, respectively. The isentropic efficiency of the compressor is 0.85.*

Determine

(a) the final pressure and temperature
(b) the energy, in kilowatts, required to drive the compressor

Take $\gamma = 1.4$ *and* $c_p = 1.005$ *kJ/kg K.*

(a)

Let

$T_{2'}$ = frictionless absolute temperature after compression

Then

$$T_{2'} = T_1 \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 293 \times 5^{0.4/1.4} = 293 \times 5^{1/3.5}$$

$$= 293 \times 1.584$$

$$= \mathbf{464\ K}$$

$$\text{Isentropic efficiency} = \frac{\text{Frictionless adiabatic work}}{\text{Actual adiabatic work}}$$

$$= \frac{\dot{m}c_p(T_{2'} - T_1)}{\dot{m}c_p(T_2 - T_1)}$$

$$= \frac{(T_{2'} - T_1)}{(T_2 - T_1)}$$

$$\therefore \qquad 0.85 = \frac{464 - 293}{T_2 - 293} = \frac{171}{T_2 - 293}$$

$$\therefore \qquad T_2 = 293 + \frac{171}{0.85} = 293 + 201 = \mathbf{494\ K}$$

$$t_2 = 494 - 273 = 221°\text{C}$$

The final temperature is 221°C.

Final pressure = 100 × 5 = **500 kN/m²**

(b)

$$\text{Energy to drive} = \dot{m}c_p(T_1 - T_2)$$

$$= 10 \times 1.005 \times (293 - 494)$$

$$= -10 \times 1.005 \times 201$$

$$= \mathbf{-2020\ kW}$$

The negative sign indicates energy input.

Example 10.5 *A supercharger on a petrol engine deals with an air-fuel mixture of ratio 14:1. It compresses the mixture from a pressure of 93 kN/m² to a pressure of 200 kN/m²; the initial temperature is 15°C. The density of the mixture at the initial conditions is 1.3 kg/m³. The engine uses 0.68 kg fuel/min. The isentropic efficiency of the compressor is 82 per cent. Determine the power absorbed in driving the compressor. Take* γ *for the mixture = 1.38.*

SOLUTION

For the mixture, $R = \frac{P_1V_1}{mT_1}$

and at the initial conditions 1 m^3 has a mass of 1.3 kg.

$$\therefore \quad R = \frac{93\times 1}{1.3\times 288} = \mathbf{0.248\ kJ/kg\ K}$$

For the mixture

$$c_p = \frac{\gamma R}{(\gamma-1)} = \frac{1.38\times 0.248}{(1.38-1)}$$

$$= \frac{1.38\times 0.248}{0.38}$$

$$= \mathbf{0.901\ kJ/kg\ K}$$

$$T_{2'} = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 288\times\left(\frac{200}{93}\right)^{0.38/1.38} = 288\times 2.15^{1/3.63}$$

$$= 288 \times 1.235$$

$$= \mathbf{355.7\ K}$$

$$0.82 = \frac{T_{2'}-T_1}{T_2-T_1}$$

$$\therefore \quad T_2 - T_1 = \frac{T_{2'}-T_1}{0.82} = \frac{355.7-288}{0.82} = \frac{67.7}{0.82} = \mathbf{82.6\ k}$$

For every 1 kg of fuel there will be 15 kg of mixture because the air-fuel mixture has a ratio of 14:1. Hence

Mass flow, $\dot{m} = \frac{0.68}{60}\times 15 = \mathbf{0.17\ kg/s}$

$$\therefore \quad \text{Power absorbed by compressor} = \dot{m}c_p(T_2 - T_1)$$

$$= 0.17\times 0.902\times 82.6$$

$$= \mathbf{12.67\ kW}$$

Example 10.6 *A roots blower has an air capacity of 1 kg/s. The pressure ratio through the blower is 2:1 with an intake pressure and temperature of 1 bar and 70°C, respectively. Determine the power required to drive the blower in kilowatts. Take* R = *0.29 kJ/kg K.*

SOLUTION

$$P_1\dot{V}_1 = \dot{m}RT_1$$

$$\therefore \dot{V}_1 = \frac{\dot{m}RT_1}{P_1} = \frac{1\times 0.29\times 10^3\times 343}{1\times 10^5} = \mathbf{0.995\ m^3/s}$$

$$\therefore \text{Power required} = \dot{V}_1(P_2 - P_1)$$

$$= 0.995\times 10^5$$

$$= 99\,500\ \text{W}$$

$$= \mathbf{99.5\ kW}$$

Example 10.7 *A vane pump has the same air capacity, pressure ratio and intake conditions as the Roots blower of Example 10.6. In the vane pump the volume is reduced to 0.7 of the intake volume and the air is then delivered. Determine the power required to drive the vane pump in kilowatts. Take γ = 1.4.*

SOLUTION

$$P_1V_1^\gamma = P_2V_2^\gamma \qquad \therefore \qquad P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 1 \times \left(\frac{1}{0.7}\right)^{1.4}$$

$$= 1 \times 1.43^{1.4}$$

$$= 1 \times 1.65$$

$$= \mathbf{1.65\ bar}$$

$$\dot{V}_2 = 0.7\,\dot{V}_1 = 0.7 \times 0.955 = \mathbf{0.696\ m^3/s}$$

$$\text{Power} = \frac{\gamma}{(\gamma-1)} P_1\dot{V}_1\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] + \dot{V}_2(P_3 - P_2)$$

$$= \frac{1.4}{(1.4-1)} \times 1 \times 10^5 \times 0.955 \times \left[\left(\frac{1.65}{1}\right)^{0.4/1.4} - 1\right] + 0.696(2 - 1.65) \times 10^5$$

$$= [(3.5 \times 0.995 \times 0.154) + (0.696 \times 0.35)] \times 10^5$$

$$= (0.536 + 0.244) \times 10^5$$

$$= 0.780 \times 10^5$$

$$= 78\ 000\ \text{W}$$

$$= \mathbf{78\ kW}$$

Summary

Air Compressors

Air compressors are devices used to raise the pressure of air and to utilize the compressed air for different industrial applications.

The compressors are of two general types, reciprocating or rotary.

Reciprocation Compressor

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work done during compression}}{\text{Actual work done during compression}}$$

Work done without clearance

$$\oint W = \frac{n}{(n-1)} P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$= \frac{n}{(n-1)} mRT_1\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Clearance Ratio

$$\frac{\text{Clearance volume}}{\text{Swept volume}} = \frac{V_3}{(V_1 - V_3)}$$

Volumetric efficiency =

$$1 - \frac{V_3}{(V_1 - V_3)}\left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right]$$

Work done with clearance

$$\oint W = \frac{n}{(n-1)} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] - \frac{n}{(n-1)} P_1V_4 \left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$= \frac{n}{(n-1)} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Free Air Delivery

If the volume of air delivered by an air compressor is reduced to atmospheric temperature and pressure, this volume of air is called the free air delivery.

Multi-stage Reciprocating Compressor

$$\oint W = \frac{xn}{n-1} P_1V_1 \left\{\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right\}$$

Rotary Compressors

$$\text{Isentropic } \eta = \frac{\dot{m}c_p(T_1 - T_{2'})}{\dot{m}c_p(T_1 - T_2)}$$

$$= \frac{(T_1 - T_{2'})}{(T_1 - T_2)}$$

If there are N vanes, then

$$\text{Work/revolution} = N\left\{l\,\frac{\gamma}{(\gamma-1)} P_1V_1 \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] + V_2(P_3 - P_2)\right\}$$

$$P_t = P\left(\frac{T_t}{T}\right)^{\gamma(\gamma-1)}$$

where P = static pressure

Questions

1. A single-stage, single-acting air compressor compresses 7 litres of air per second from a pressure of 0.101 3 MN/m^2 to a pressure of 1.4 MN/m^2. Compression follows the law $PV^{1.3} = C$. The mechanical efficiency of the compressor is 82 per cent. The effect of clearance can be neglected.

   Determine the power required to drive the compressor.

   [3.12 kW]

2. A single-stage, single-acting, reciprocating air compressors has a bore and stroke of 150 mm. The clearance volume is 6 per cent of the swept volume and the speed is 8 rev/s. Intake pressure is 100 kN/m^2 and the delivery pressure is 550 kN/m^2. The polytropic index is 1.32 throughout. Determine
   (a) the theoretical volumetric efficiency referred to intake conditions
   (b) the volume of air delivered per second at 550 kN/mn^2
   (c) the air power of the compressor.

   [(a) 84.2%; (b) 4.9 litres; (c) 3.75 kW]

3. A three-stage, single-acting, reciprocating air compressor takes in 0.5 m^3 of air per second at a pressure of 100 kN/m^2 and at a temperature of 20°C. The air is compressed to a delivery pressure of 2 MN/m^2. The intermediate pressures are ideal and intercooling between stages is perfect. The compression index can be taken as 1.25 in all stages. The compressor runs at 8.5 rev/s. Neglecting clearance, determine
   (a) the intermediate pressures
   (b) the total volume of each cylinder
   (c) the air power of the compressor

   [(a) 271.4 kN/m^2, 736.6 kN/m^2; (b) 0.058 8 m^3, 0.021 7 m^3, 0.008 m^3; (c) 165.8 kW]

4. A two-stage, single-acting, reciprocating air compressor has a law-pressure cylinder 250 mm diameter with a 250 with a 250 mm stoke. The clearance volume of the low-pressure cylinder is 5 per cent of the stroke volume of the cylinder. The intake pressure and temperature are 100 kN/m^2 and 18° C, respectively. Delivery pressure is 700 kN/m^2 and the compressor runs at 5 rev/s. The polytropic index is 1.3 throughout. The intermediate pressure is ideal and intercooling is complete. The overall efficiency of the plant, including the electric driving motor is 70 per cent. Take $R = 0.29$ kJ/kg K.

   Determine
   (a) the air mass flow rate through the compressor
   (b) the energy input to drive the motor

   [(a) 0.069 kg/s; (b) 18.1 kW]

5. A two-stage, single-acting, reciprocating air compressor delivers 0.07 m^3 of free air per second (free air condition 101.325 kN/m^2 and 15° C). Intake conditions are 95 kN/m^2 and 22° C. Delivery pressure from the compressor is 1300 kN/m^2. The intermediate pressure is ideal and there is perfect intercooling. The compression index is 1.25 in both cylinders. The overall mechanical and electrical is 75 per cent. Neglecting clearance, determine
   (a) the energy input to the driving motor
   (b) the heat transfer per second in the intercooler
   (c) the percentage saving in work by using a two-stage intercooled compressor instead of a single-stage compressor.

   Take $c_p = 1.006$ kJ/kg K, $R = 0.287$ kJ/kg K.

   [(a) 29.1 kW; (b) 6.99 kJ/s; (c) 13%]

6. A rotary compressor is used as a supercharger to an aero-engine. When flying at a particular altitude the engine uses 4.6 kg of fuel per minute. The air-fuel mixture is compressed from a pressure of 55 kN/m^2 to a pressure of 100 kN/m^2 with an isentropic efficiency of 87 per cent. The volume of the mixture produced by 1 kg of fuel occupies 11.7 m^3 at 0° C and 101.3 kN/m^2. The air-fuel ratio is 145:1 and the initial temperature of the mixture is –2° C. If γ for the mixture is 1.37, determine the power required to drive the supercharger.

   [67.7 kW]

7. A rotary air compressor has an inlet static pressure and temperature of 100 kN/m^2 and 20° C, respectively. The compressor has an air mass flow rate of 2 kg/s through a pressure ratio of 5:1. The isentropic efficiency of compression is 85 per cent. Exit velocity from the compressor is 150 m/s. Neglecting change of velocity through the compressor, determine the power required to drive the compressor. Estimate the total temperature and pressure at exit from the compressor.

   Take $\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.

   [404 kW, 232.2° C, 541.4 kN/m^2]

8. A vane pump aspirates 1.25 m^3 of air and compresses it through an overall pressure ratio of 3:1. The pressure ratio due to volume reduction in the pump is 2:1. Intake pressure and temperature are 95 kN/m^2 and 18° C, respectively. Determine the power required to drive the pump and compare this power with that required to drive a Roots blower which has the same mass flow rate and overall pressure ratio.

   Take $\gamma = 1.4$, $R = 0.287$ kJ/kg K.

   [163.4 kW, 237.5 kW]

Previous Years' GTU Examination Questions

1. Define volumetric efficiency with *P–V* diagram and usual notations. Prove that volumetric efficiency of reciprocating compressor is 1-C [(p2/p1)1/n –1].

   [Dec '08]

2. A single stage, single cylinder reciprocating air compressor with negligible clearance takes 1 m^3 of air per minute at 1.013 bar and 150°C. The delivery pressure is 7 bar. Assuming law of compression $PV^{1.35} = C$, R = 0.287 kJ/kg K, calculate:

   (i) Mass of air delivered per minute
   (ii) Delivery temperature
   (iii) Indicated Power
   (iv) Isothermal efficiency

   [Dec '08]

3. What are the uses of compressed air?

4. A single stage reciprocating compressor takes 1m^3 of air permissible at 1.013 bar and 150°C and delivers it at 7 bar. Assuming that the law of compression is pv 1.35 = constant and the clearance is neglible, calculate the indicated power.

   [Mar '09]

5. Prove that the work done per kg of air in Reciprocating Air Compressor neglecting clearance volume is given by W = RT1 n/(n – 1) [(Rp) (n–1)/n – 1] where Rp = Pressure Ratio.

   [Jun '09]

6. Air is to be compressed in a single stage reciprocating compressor from 1.013 bar and 15°C to 7 bar. Calculate the indicated power required for a free air delivery of 0.3 m^3/min when the compression process is
   (i) Isentropic
   (ii) Reversible Isothermal
   (iii) Polytropic with n = 1.25. What will be the delivery temperature in each case? Neglect clearance.

   [Jun '09]

7. In air compressor air enters at 1.013 bar and 27°C having volume 5.0 m^3/kg and it is compressed to 12 bar isothermally. Determine
   (i) Work done
   (ii) Heat transfer and
   (iii) Change in internal energy.

   [Sep '09]

8. Derive an expression for compressor without clearance W = P * V * loge(P2/P1) for isothermal compression.

   [Sep '09]

9. Single stage air compressor is required to compress 94 m^3 air/min from 1 bar and 25º C to 9 bar. Find the temperature at the end of compression, work done, power required and heat rejected during each of the following processes (i) isothermal (ii) adiabatic (iii) polytropic, following the law pV1.25 = constant. Assume no clearance.

   [Jan '10]

10. Determine the work done in compressing 1 kg of air from a volume of 0.15m^3 at a pressure of 1.0 bar to a volume of 0.05 m^3, when the compression is (i) isothermal and (ii) adiabatic, Take γ =1.4 Also, comment on your answer.

    [Jun '10]

11. State the advantages of multi-stage compressor and explain with *P–V* diagram the working of two-stage compressor.

    [Jun '10]

12. A single cylinder, single acting air compressor has a cylinder diameter of 150 mm and stroke of 300 mm. it draws air into its cylinder at a pressure of 1 bar and temperature 27ºC.This air is then compressed adiabatically to a pressure of 8 bar if the compressor runs at a speed of 120 rpm. Find,
    (i) Mass of the air compressed per cycle
    (ii) Work required per cycle
    (iii) Power required to drive the compressor

    Neglect the clearance volume and take R= 0.287 KJ/kgK.

    [Jun '10]