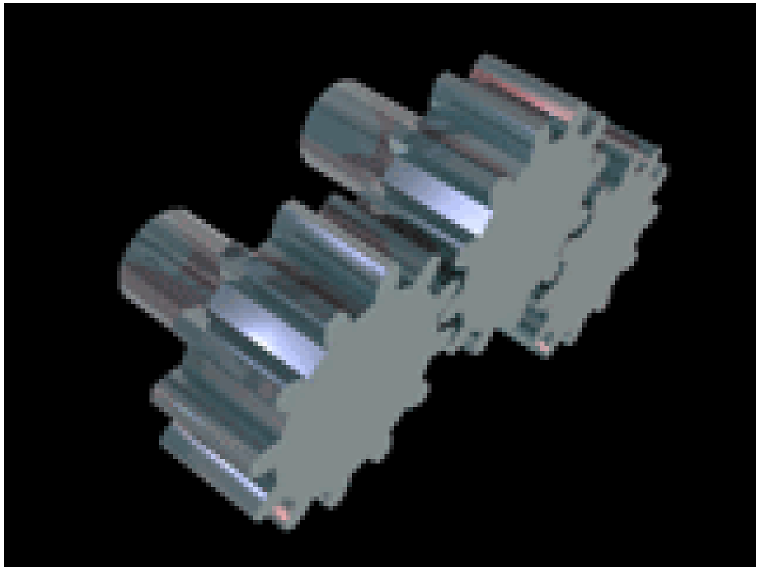
GEAR DESIGN

Reference Books: *(i) Machine Design by R. S. Khurmi*

(ii) Maitra G.M., Prasad L.V., "Hand book of Mechanical Design", II Edition, Tata McGraw-Hill



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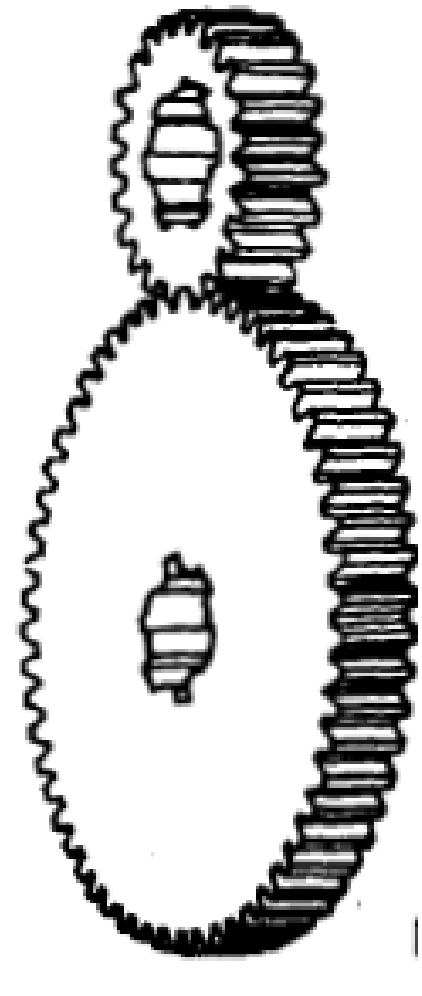
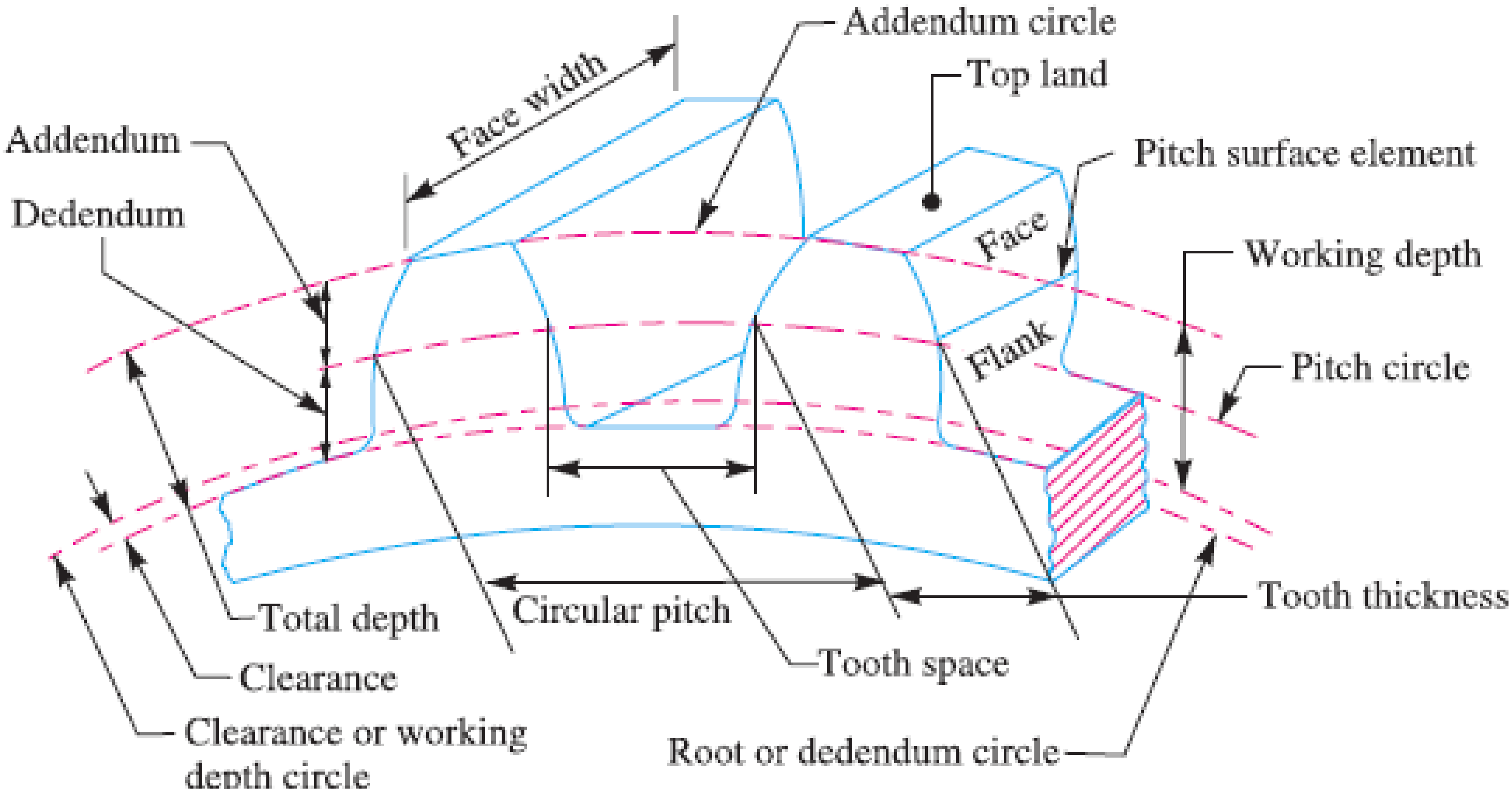
SPUR GEAR DESIGN



Spur Gear Terminology



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Pressure angle or angle of obliquity: *It is the angle between the common normal to two gear* teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by φ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

Circular pitch. *It is the distance measured on the circumference of the pitch circle from* a point of one tooth to the corresponding point on the next tooth. It is usually denoted by *pc.*

Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively; then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Continue…

Diametral pitch. *It is the ratio of number of teeth to the pitch circle diameter in millimetres.*

It denoted by *pd. Mathematically,*

$$\text{Diametral pitch, } p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \cdots\left(\because p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

Module. *It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is* usually denoted by *m.*
Mathematically, Module, $m = D / T$

The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40 and 50.

The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5,5.5, 7, 9, 11, 14, 18, 22, 28, 36 and 45 are of second choice.

Form of Gear Teeth

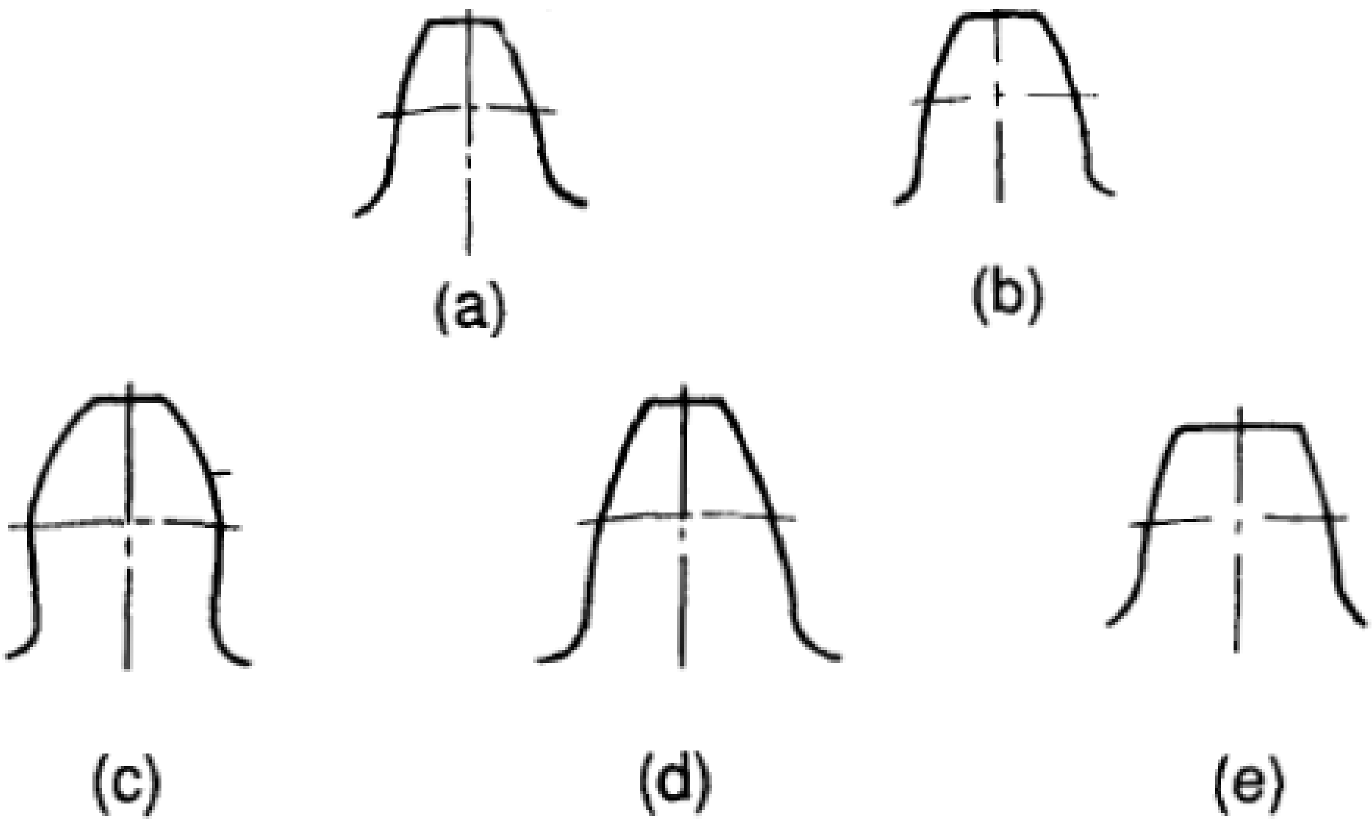

Fig. 2.7 Comparison of gear tooth profiles
(a) Cycloidal, **(b)** Involute, **(c)** $14\frac{1}{2}^{\circ}$ full-depth involute,
(d) **20°** full-depth involute, *(e)* 20' stub involute

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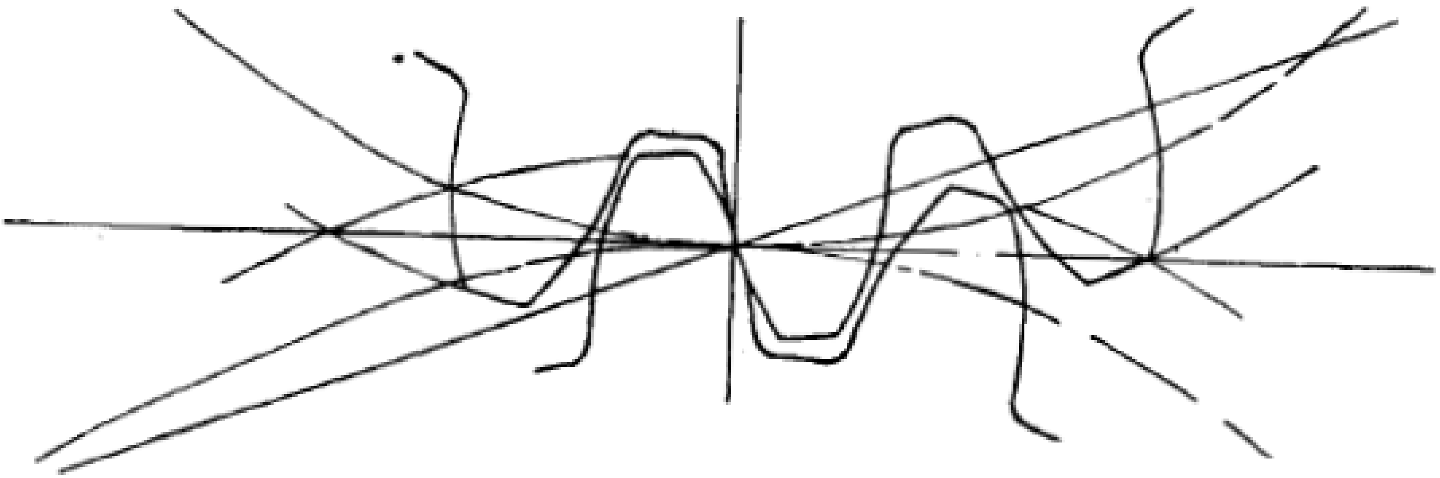
- The $14\frac{1}{2}°$ ***composite system is used for general purpose gears. It is stronger but has no interchangeability.***

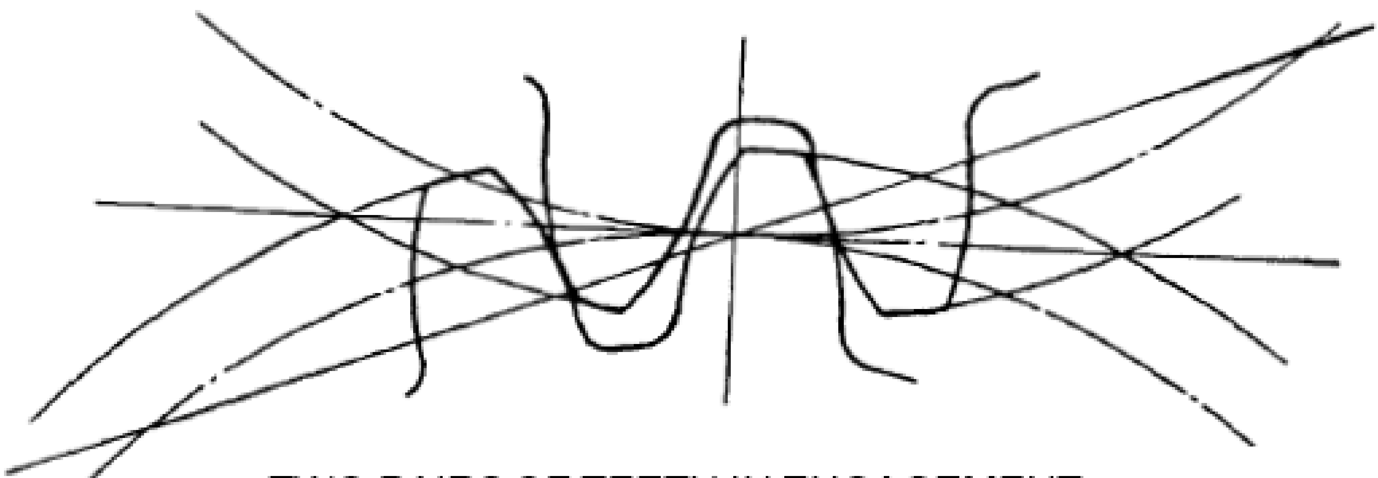
- The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the ° $14\frac{1}{2}°$ ***full depth involute system was developed for use with gear hobs for spur and*** helical gears.

- The tooth profile of the **20°** ***full depth involute system may be cut by hobs. The increase of the*** pressure angle from 114 /2° to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The **20°** ***stub involute system has a strong tooth to take heavy loads.***

Gear Motion Mechanism



ONE PAIR OF TEETH IN ENGAGEMENT



TWO PAIRS OF TEETH IN ENGAGEMENT

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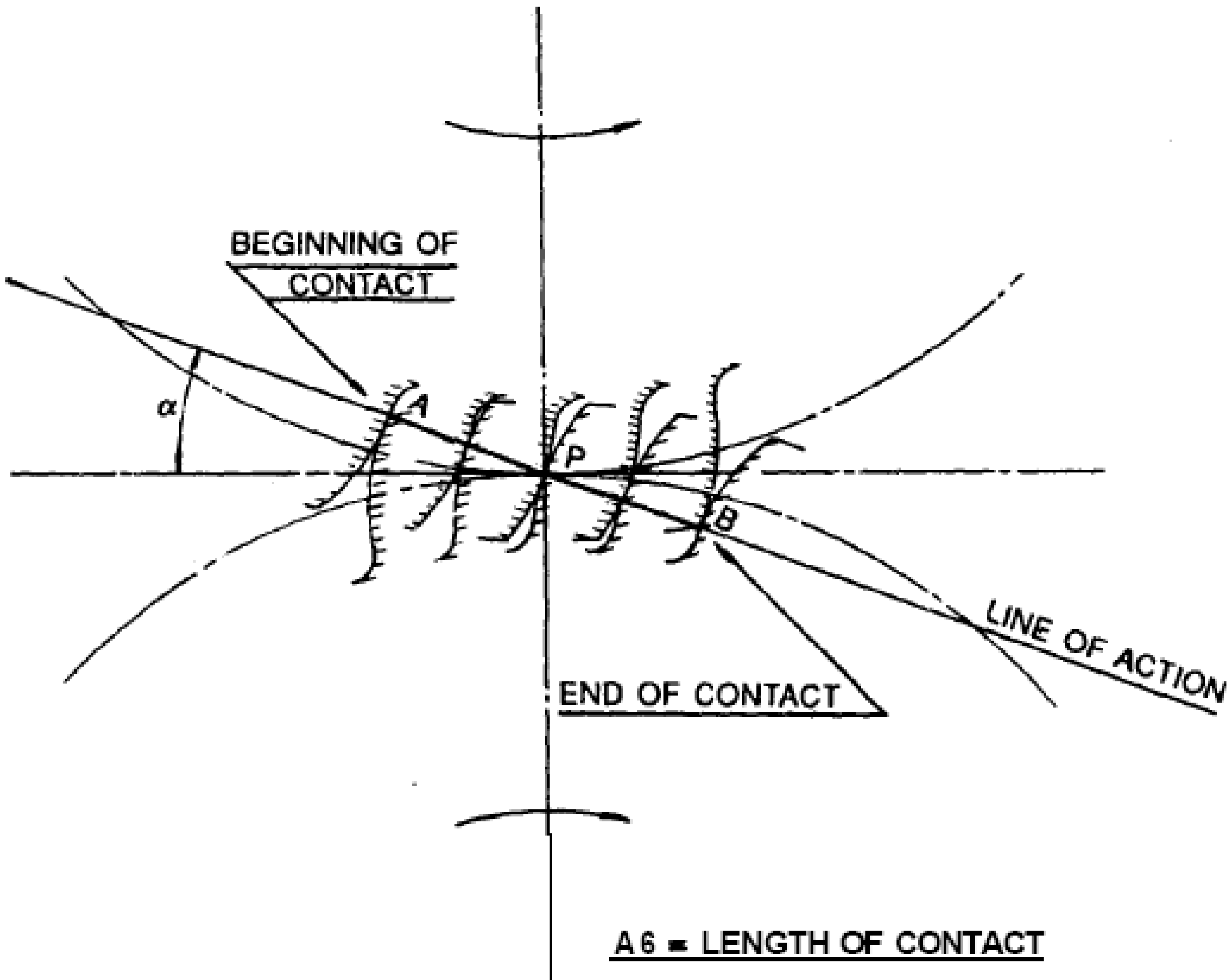

A 6 = LENGTH OF CONTACT

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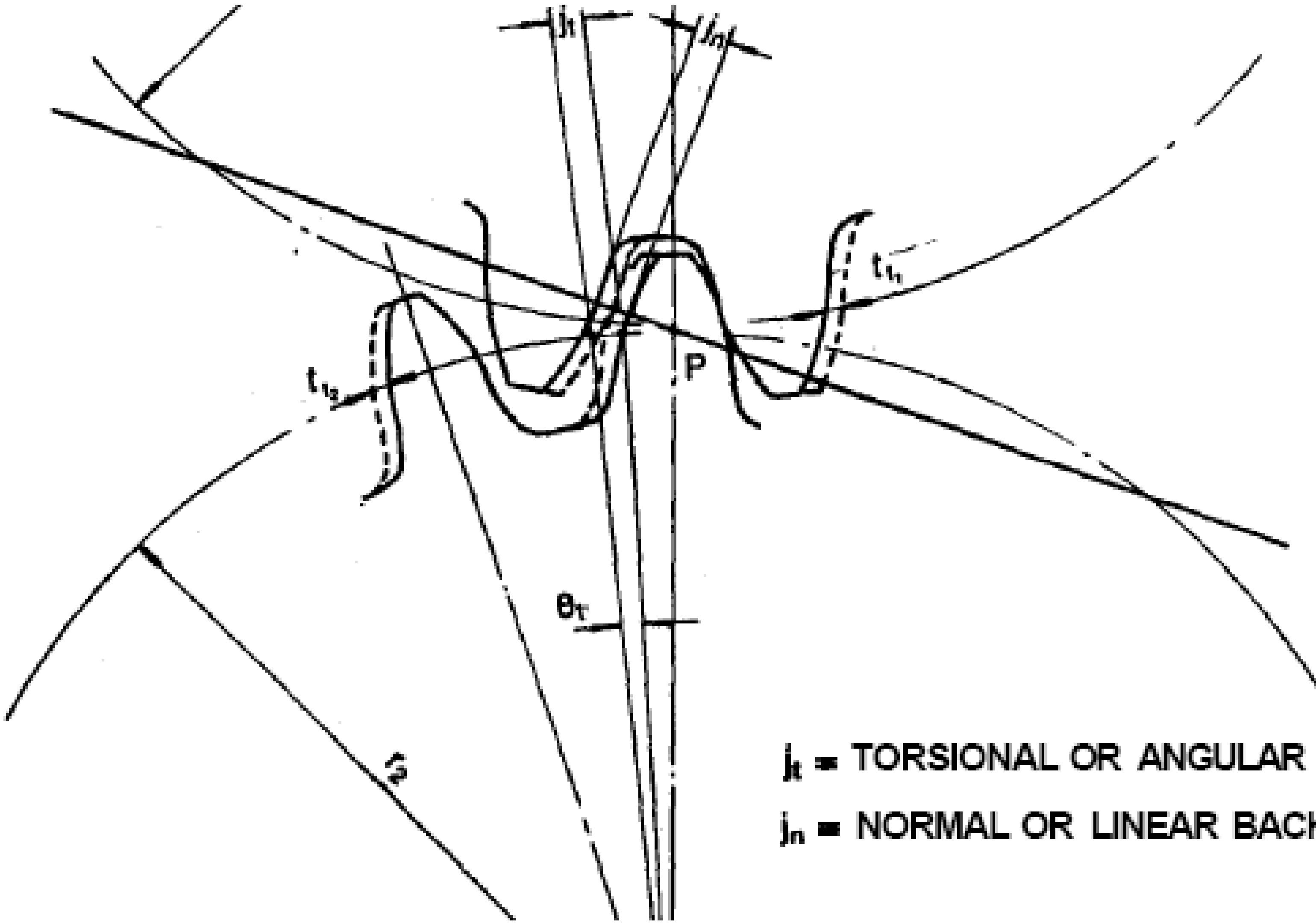
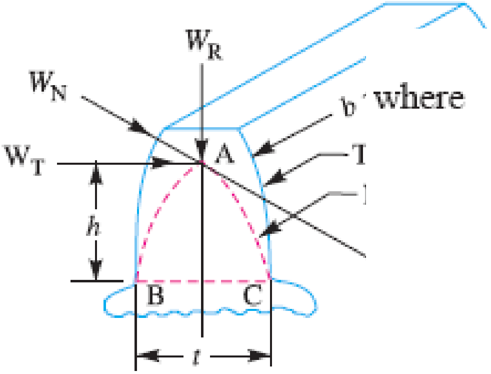


j_t = TORSIONAL OR ANGULAR BACKLASH

j_n = NORMAL OR LINEAR BACKLASH

Beam Strength of Spur Gear Teeth – Lewis Equation



$$\sigma_w = M.y / I \qquad ...(i)$$

where M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth $= b.t^3/12$,

b = Width of gear face.

Substituting the values for M, y and I in equation (*i*), we get

$$\sigma_w = \frac{(W_T \times h)\ t/2}{b.t^3/12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

or

$$W_T = \sigma_w \times b \times t^2 / 6\ h$$

In this expression, t and h are variables depending upon the size of the tooth (*i.e.* the circular pitch) and its profile.

Let $t = x \times p_c$, and $h = k \times p_c$; where x and k are constants.

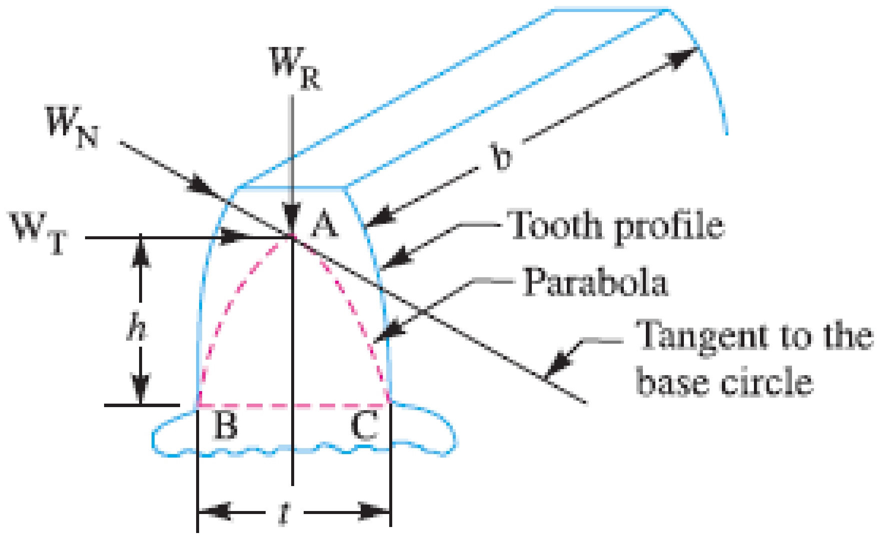
$$\therefore \quad W_T = \sigma_w \times b \times \frac{x^2 . p_c^2}{6k.p_c} = \sigma_w \times b \times p_c \times \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi m . y \qquad ...(\because p_c = \pi m)$$

The quantity y is known as ***Lewis form factor*** or ***tooth form factor*** and W_T (which is the tangential load acting at the tooth) is called the ***beam strength of the tooth.***

Design of Spur Gear



$$W_T = \sigma_w \,.\, b \,.\, p_c \,.\, y = \sigma_w \,.\, b \,.\, \pi m \,.\, y \qquad ...(\because p_c = \pi m)$$

$$y = 0.124 - \frac{0.684}{T}, \text{ for } 14\tfrac{1}{2}^\circ \text{ composite and full depth involute system.}$$

$$= 0.154 - \frac{0.912}{T}, \text{ for } 20^\circ \text{ full depth involute system.}$$

$$= 0.175 - \frac{0.841}{T}, \text{ for } 20^\circ \text{ stub system.}$$

$$\sigma_w = \sigma_o \times C_v$$

σ_o = Allowable static stress, and

C_v = Velocity factor.

$C_v = \dfrac{3}{3 + v}$, for ordinary cut gears operating at velocities upto 12.5 m / s.

$= \dfrac{4.5}{4.5 + v}$, for carefully cut gears operating at velocities upto 12.5 m/s.

$= \dfrac{6}{6 + v}$, for very accurately cut and ground metallic gears operating at velocities upto 20 m / s.

$= \dfrac{0.75}{0.75 + \sqrt{v}}$, for precision gears cut with high accuracy and operating at velocities upto 20 m / s.

$= \left(\dfrac{0.75}{1 + v}\right) + 0.25$, for non-metallic gears.

Dynamic Tooth Load Checking

$$W_D = W_T + W_I = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

K = A factor depending upon the form of the teeth.

= 0.107, for $14\frac{1}{2}°$ full depth involute system.

= 0.111, for 20° full depth involute system.

= 0.115 for 20° stub system.

Static Tooth Load Checking

Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e . b . p_c . y = \sigma_e . b . \pi\, m . y$$

For safety, against tooth breakage, the static tooth load (WS) should be greater than the dynamic load (WD)

Wear Tooth Load Checking

$W_w = D_p.b.Q.K$

W_w = Maximum or limiting load for wear in newtons,

D_p = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2T_G}{T_G + T_P}, \text{ for external gears}$$

$$= \frac{2 \times V.R.}{V.R. - 1} = \frac{2T_G}{T_G - T_P}, \text{ for internal gears.}$$

$V.R.$ = Velocity ratio = T_G / T_P,

K = Load-stress factor (also known as material combination factor) in N/mm^2.

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

σ_{es} = Surface endurance limit in MPa or N/mm^2,

ϕ = Pressure angle,

E_P = Young's modulus for the material of the pinion in N/mm^2, and

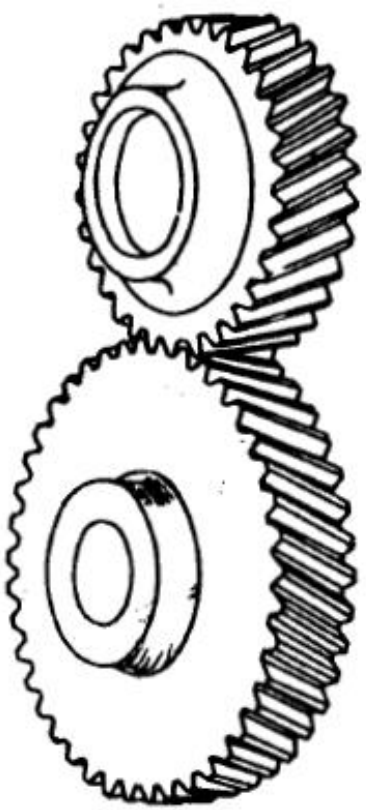
E_G = Young's modulus for the material of the gear in N/mm^2.

The wear load (Ww) should not be less than the dynamic load (WD).

Parallel Axis Helical Gear Design

Introduction

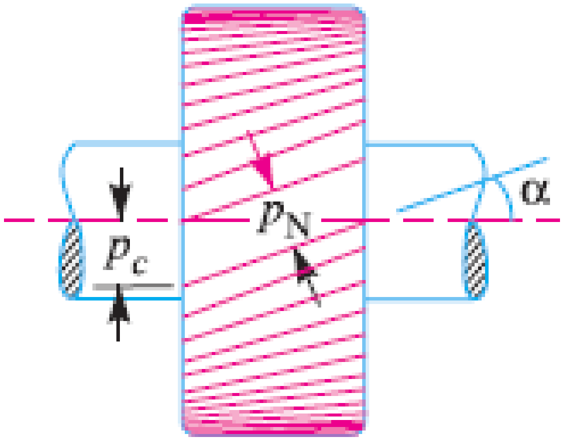
- A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears.
- The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing.
- This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission



Helical Gear Terminology

- **1. *Helix angle.*** *It is a constant angle made by the helices with the* axis of rotation.
- **2. *Axial pitch.*** *It is the distance, parallel to the axis, between similar* faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by *pc. The axial pitch may also be defined as the circular pitch* in the plane of rotation or the diametral plane.
- **3. *Normal pitch.*** *It is the distance between similar faces of adjacent* teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by *pN. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane* perpendicular to the teeth. Mathematically, normal pitch,

$$pN = pc \cos \alpha$$



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- **4. Face Width:**

$$b = \frac{1.15\ p_c}{\tan\ \alpha} = \frac{1.15 \times \pi m}{\tan\ \alpha} \quad ...(\because p_c = \pi m)$$

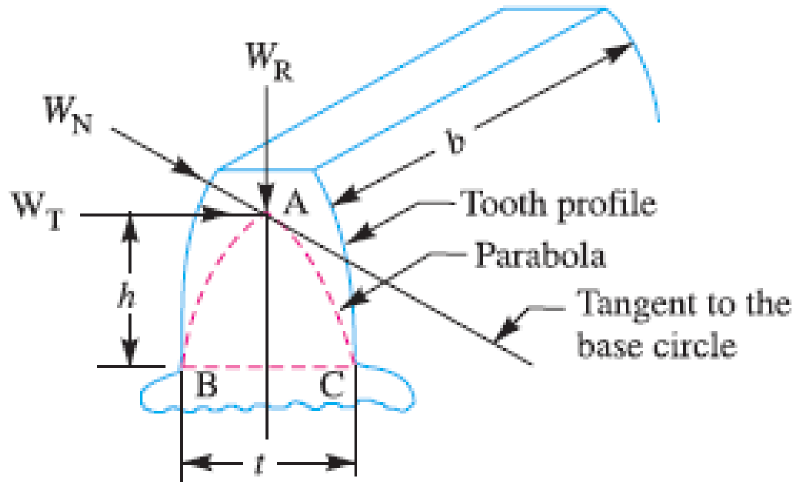
b = Minimum face width, and
m = Module.

5. Formative or Equivalent Number of Teeth for Helical Gears

$$T_E = T / \cos^3 \alpha$$

T = Actual number of teeth on a helical gear, and
α = Helix angle.

Beam Strength of Helical Gear Teeth – Lewis Equation



$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

$$C_v = \frac{6}{6+v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15+v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1+v} + 0.25, \text{ for non-metallic gears.}$$

Dynamic Tooth Load Checking

$$W_D = W_T + \frac{21\,v\,(b.C\cos^2\alpha + W_T)\cos\alpha}{21\,v + \sqrt{b.C\cos^2\alpha + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

K = A factor depending upon the form of the teeth.

= 0.107, for $14\frac{1}{2}°$ full depth involute system.

= 0.111, for 20° full depth involute system.

= 0.115 for 20° stub system.

Static Tooth Load Checking

$$W_S = \sigma_e . b . \pi\, m . y'$$

For safety, against tooth breakage, the static tooth load (WS) should be greater than the dynamic load (WD)

Wear Tooth Load Checking

$$W_w = \frac{D_p . b . Q . K}{\cos^2 \alpha}$$

The wear load (Ww) should not be less than the dynamic load (WD).

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2T_G}{T_G + T_P}, \text{ for external gears}$$

$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4} \left[\frac{1}{E_P} + \frac{1}{E_G} \right]$$

ϕ_N = Normal pressure angle.

Problem of Helical Gear Design

- *A helical gear speed reducer is to be designed with rated power of 75 KW at pinion speed of 1200 RPM and pitch cirlce diameter of 0.16 m.. The velocity ratio is 3. Take the teeth are 20 full depth involute in normal plane with helix angle 29 and module in standard plane 8 mm.*
- σo = *175 Mpa, C = 250000 N/m*

Solution

Given:

P = 75 KW

Np = 1200 RPM

VR = 3

$\varphi = 20°$

$\alpha = 29°$

m = 8mm

σo = *175 Mpa, C = 250000 N/m*

Step 1 Basic Calculation

Calculation of ;

- Velocity v
- Module in normal plane mn = m.cos α

Step 2 Lewis Equation

Calculation of;

- Wt from power
- Cv
- Formative no. of teeth Tpe
- Lewis form factor y’

Lewis Equation;

$$WT = (\sigma o \times Cv)\ b.\pi\ (mn).y'$$

Checking for Design

(i) Dynamic Load checking

$$W_D = W_T + \frac{21\, v\, (b.C \cos^2 \alpha + W_T) \cos \alpha}{21\, v + \sqrt{b.C \cos^2 \alpha + W_T}}$$

$$W_S = \sigma_e . b . \pi\, m . y'$$

Take σe = 315 Mpa from design data

- (ii) Wear Load Checking

$$W_w = \frac{D_p . b . Q . K}{\cos^2 \alpha}$$

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2T_G}{T_G + T_P}, \text{ for external gears}$$

Take K= 350000 Mpa from design data

Step 4 Standard Proportion & Result Table

	ϕ	$= 15° \text{ to } 25°$
Helix angle,	α	$= 20° \text{ to } 45°$
Addendum		$= 0.8\, m$ (Maximum)
Dedendum		$= 1\, m$ (Minimum)
Minimum total depth		$= 1.8\, m$
Minimum clearance		$= 0.2\, m$
Thickness of tooth		$= 1.5708\, m$

Design of Bevel Gear

Strength of Bevel Gears

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y' \left(\frac{L - b}{L}\right)$$

σ_o = Allowable static stress,

C_v = Velocity factor,

$= \dfrac{3}{3 + v}$, for teeth cut by form cutters,

$= \dfrac{6}{6 + v}$, for teeth generated with precision machines,

v = Peripheral speed in m / s,

b = Face width,

m = Module,

y' = Tooth form factor (or Lewis factor) for the equivalent number of teeth,

L = Slant height of pitch cone (or cone distance),

$$= \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2}$$

D_G = Pitch diameter of the gear, and

D_P = Pitch diameter of the pinion.

b = L/3

$$y'_P = 0.124 - \frac{0.686}{T}$$

$$y'_G = 0.124 - \frac{0.686}{T_{EG}}$$

$$T_{EP} = T_P \cdot \sec \theta_{P1}$$

$$T_{EG} = T_G \cdot \sec \theta_{P2}$$

$$\theta_{P1} = \tan^{-1}\left(\frac{\sin \theta_S}{V.R + \cos \theta_S}\right)$$

$$\theta_{P2} = \tan^{-1}\left(\frac{\sin \theta_S}{\frac{1}{V.R} + \cos \theta_S}\right)$$

θ_{P1} = Pitch angle for the pinion,

θ_{P2} = Pitch angle for the gear,

θ_S = Angle between the two shaft axes,

Standard Proportions

1. Addendum, $a = 1\,m$
2. Dedendum, $d = 1.2\,m$
3. Clearance $= 0.2\,m$
4. Working depth $= 2\,m$
5. Thickness of tooth $= 1.5708\,m$

where m is the module.

Check for Dynamic Load

$$W_D = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

Checking for Wear Load

$$W_w = \frac{D_P \cdot b \cdot Q \cdot K}{\cos \theta_{P1}}$$

$$Q = \frac{2\, T_{EG}}{T_{EG} + T_{EP}}$$

$$W_S = \sigma_e . b . \pi\, m . y' \left(\frac{L - b}{L} \right)$$

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4} \left[\frac{1}{E_P} + \frac{1}{E_G} \right]$$

Example

A pair of cast iron bevel gears connect two shafts at right angles. The pitch diameters of the pinion and gear are 80 mm and 100 mm respectively. The tooth profiles of the gears are of 14 1/2º composite form. The allowable static stress for both the gears is 55 MPa. If the pinion transmits 2.75 kW at 1100 r.p.m., Design the bevel gear and check the design from the standpoint of dynamic and wear load. Take surface endurance limit as 630 MPa and modulus of elasticity for cast iron as 84 kN/mm2.

Solution

- Given : $\theta S = 90^o$; *DP = 80 mm = 0.08 m ; DG = 100 mm = 0.1 m ; φ = 141/2*° ;
- σOP = σOG = 55 MPa = 55 N/mm2 ;
- *P = 2.75 kW = 2750 W ;*
- *NP = 1100 r.p.m. ;*
- *σes = 630 MPa = 630* N/mm2 ;
- *EP = EG = 84 kN/mm2 = 84 × 103 N/mm2*

Step 1

m= module;

b = L/3 (Taken)

$$v = \frac{\pi D_P \cdot N_P}{60} = \frac{\pi \times 0.08 \times 1100}{60} = 4.6 \text{ m/s}$$

$$C_v = \frac{6}{6 + v} = \frac{6}{6 + 4.6} = 0.566$$

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{80}{100}\right) = 38.66°$$

$$\theta_{P2} = 90° - 38.66° = 51.34°$$

$$T_{EP} = T_P \,.\, \sec \theta_{P1} = \frac{80}{m} \times \sec 38.66° = \frac{102.4}{m}$$

$$T_{EG} = T_G \,.\, \sec \theta_{P2} = \frac{100}{m} \times \sec 51.34° = \frac{160}{m}$$

$$y'_P = 0.124 - \frac{0.684}{T_{EP}} = 0.124 - \frac{0.684 \times m}{102.4}$$

$$= 0.124 - 0.006\,68\, m$$

$$*L = \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2} = \sqrt{\left(\frac{100}{2}\right)^2 + \left(\frac{80}{2}\right)^2} = 64 \text{ mm}$$

WT = P/v *Cs

Step 2

$$W_T = (\sigma_{OP} \times C_v)\, b \times \pi\, m \times y'_P \left(\frac{L - b}{L} \right)$$

TP = D P / m = 80 / 5 = 16

TG = DG / m = 100 / 5 = 20

Step 3 (Checking for Design)

$$W_D = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

$$W_S = \sigma_e.b.\pi\,m.y'\left(\frac{L-b}{L}\right)$$

Ws > Wd

(ii) Wear Load Checking

$$W_w = \frac{D_P \,.\, b \,.\, Q \,.\, K}{\cos\theta_{P1}} = \frac{80 \times 22 \times 1.22 \times 1.687}{\cos 38.66°} = 4640 \text{ N}$$

$$\text{ratio factor}, Q = \frac{2\, T_{EG}}{T_{EG} + T_{EP}} = \frac{2 \times 160/m}{160/m + 102.4/m} = 1.22$$

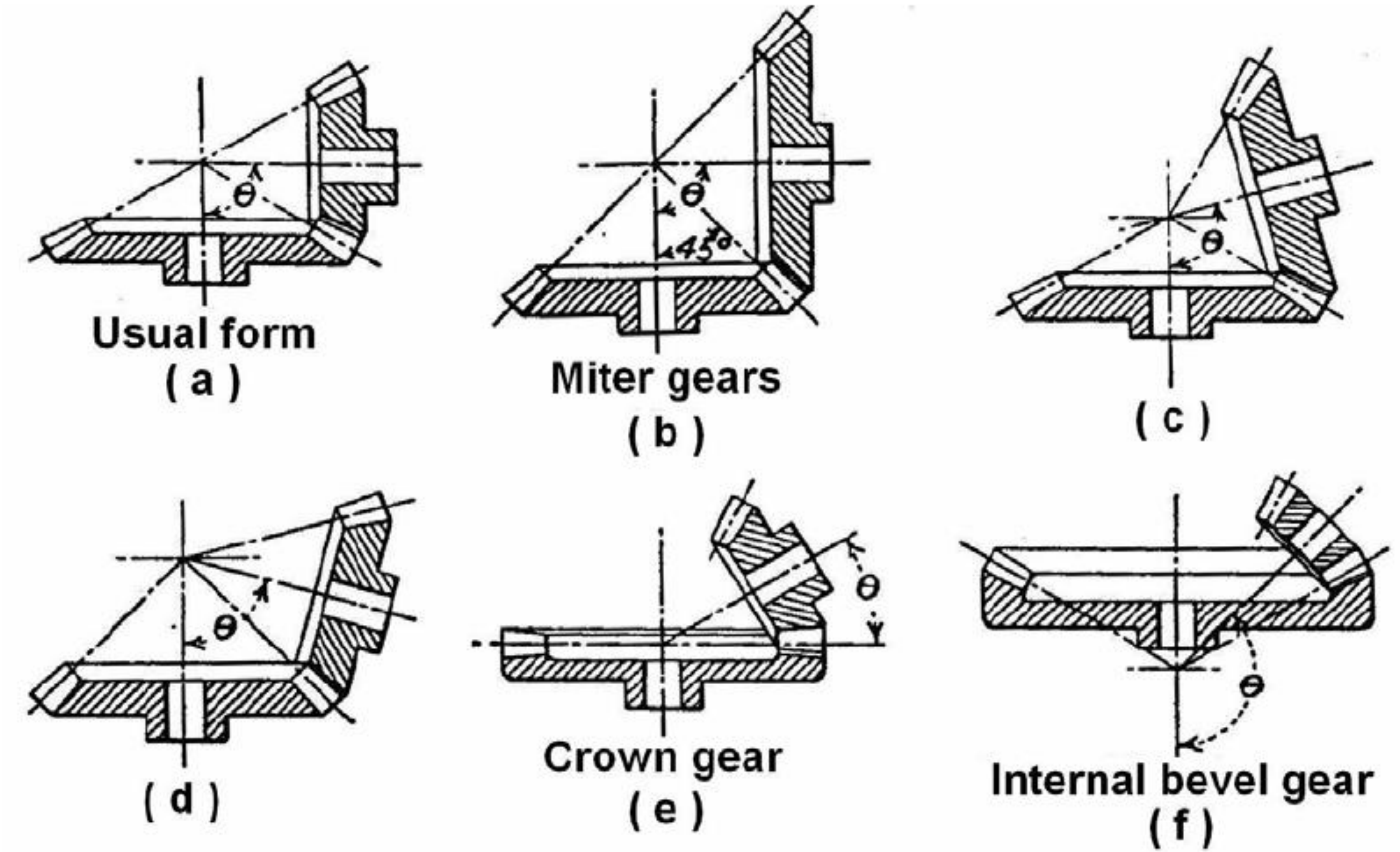
$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

$$= \frac{(630)^2 \sin 14\,{}^1/_2\,°}{1.4}\left[\frac{1}{84 \times 10^3} + \frac{1}{84 \times 10^3}\right] = 1.687$$

Ww > Wt

Step 4 (Result Table)

DIFFERENT TYPES OF BEVEL GEARS



- ***Mitre gears. When equal bevel gears (having equal teeth and equal pitch angles) connect two*** shafts whose axes intersect at right angle, *then they are known as* **mitre gears.**
- ***Angular bevel gears. When the bevel gears connect two shafts whose axes intersect at an*** angle other than a right angle, then they are known as ***angular bevel gears.***

- ***Crown bevel gears. When the bevel gears connect two shafts whose axes intersect at an angle*** greater than a right angle and one of the bevel gears has a pitch angle of 90º, then it is known as a crown gear.
- ***Internal bevel gears. When the teeth on the bevel gear are cut on the inside of the pitch cone,*** then they are known as ***internal bevel gears***

A pair of bevel gears connect two shafts at right angles and transmits 9 kW. Determine the required module and gear diameters for the following specifications :

Particulars	*Pinion*	*Gear*
Number of teeth	*21*	*60*
Material	*Semi-steel*	*Grey cast iron*
Brinell hardness number	*200*	*160*
Allowable static stress	*85 MPa*	*55 MPa*
Speed	*1200 r.p.m.*	*420 r.p.m.*
Tooth profile	$14\frac{1}{2}°$ *composite*	$14\frac{1}{2}°$ *composite*

Worm Gear Design

Introduction

- Worm gears are used for transmitting power between two non-parallel, non-intersecting shafts.
- High gear ratios of 200:1 can be got.

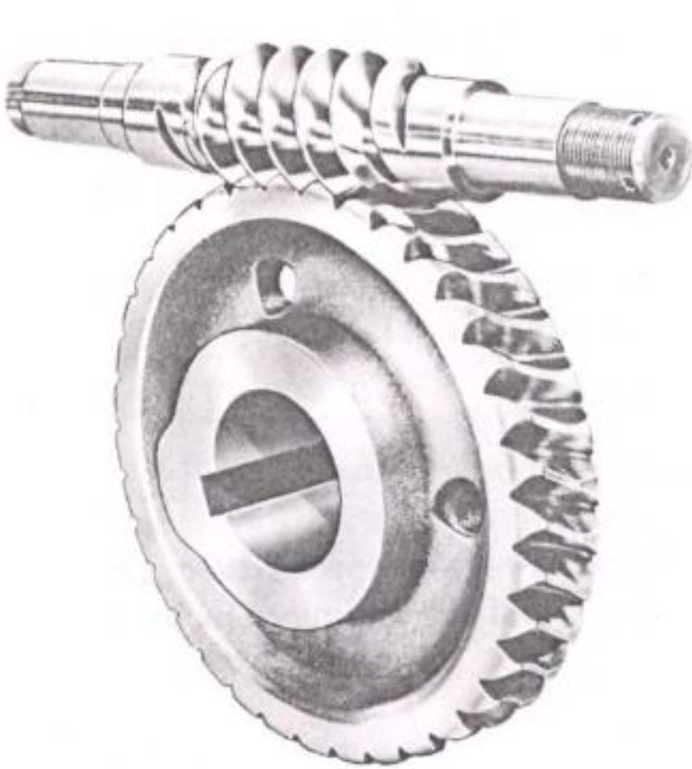


Fig.1. Single enveloping worm gear.

Fig. 2. Double enveloping worm gear.

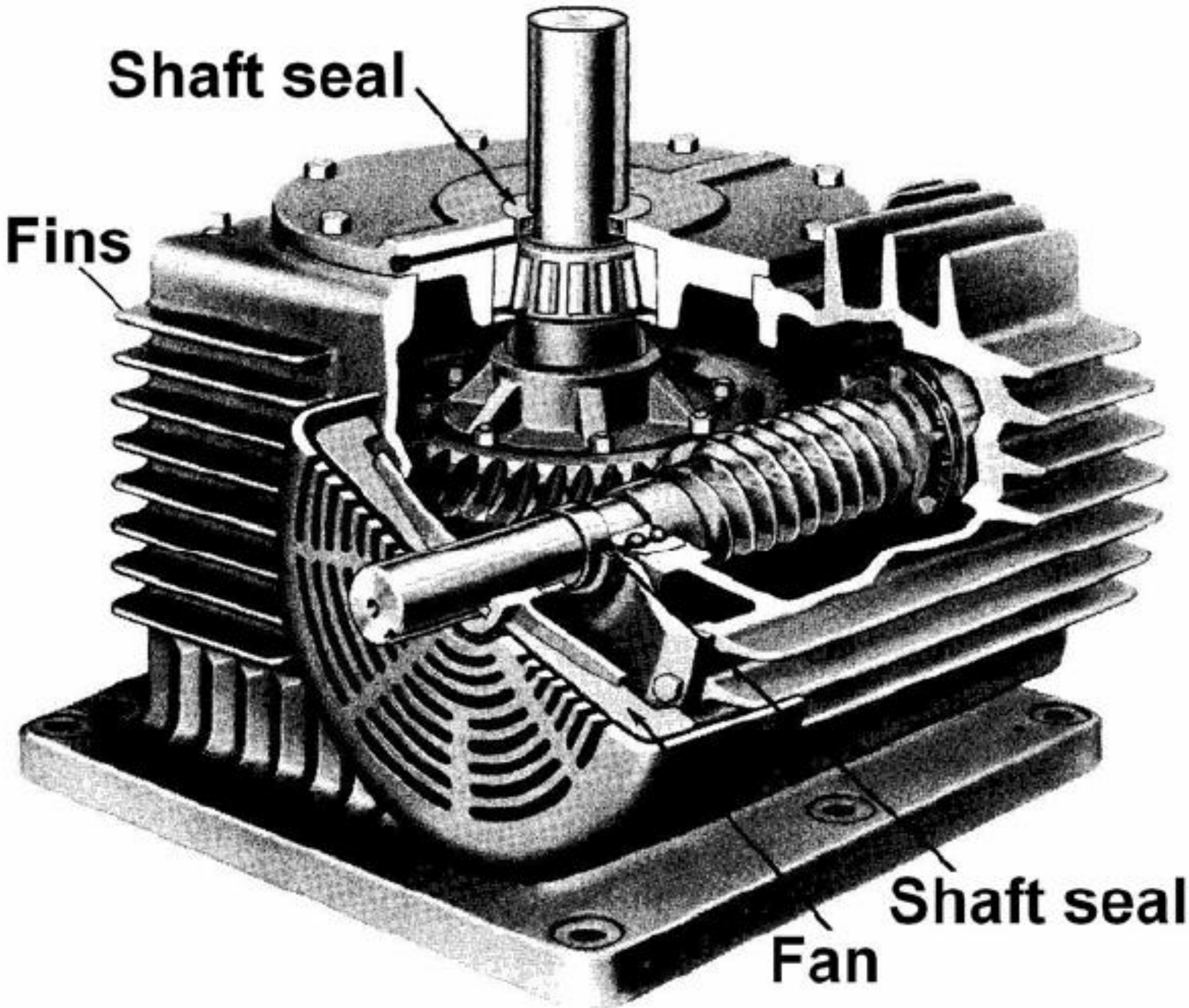
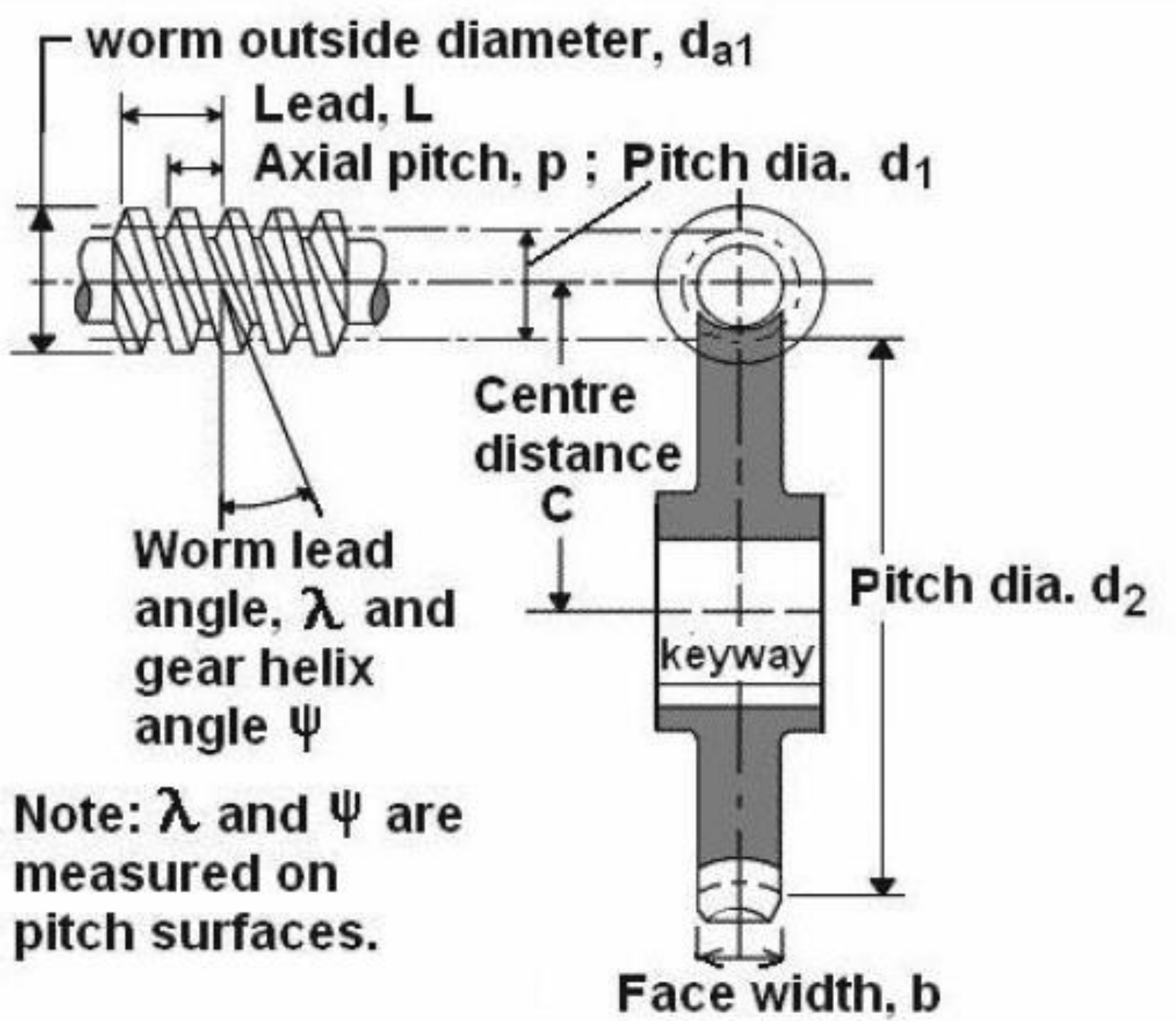
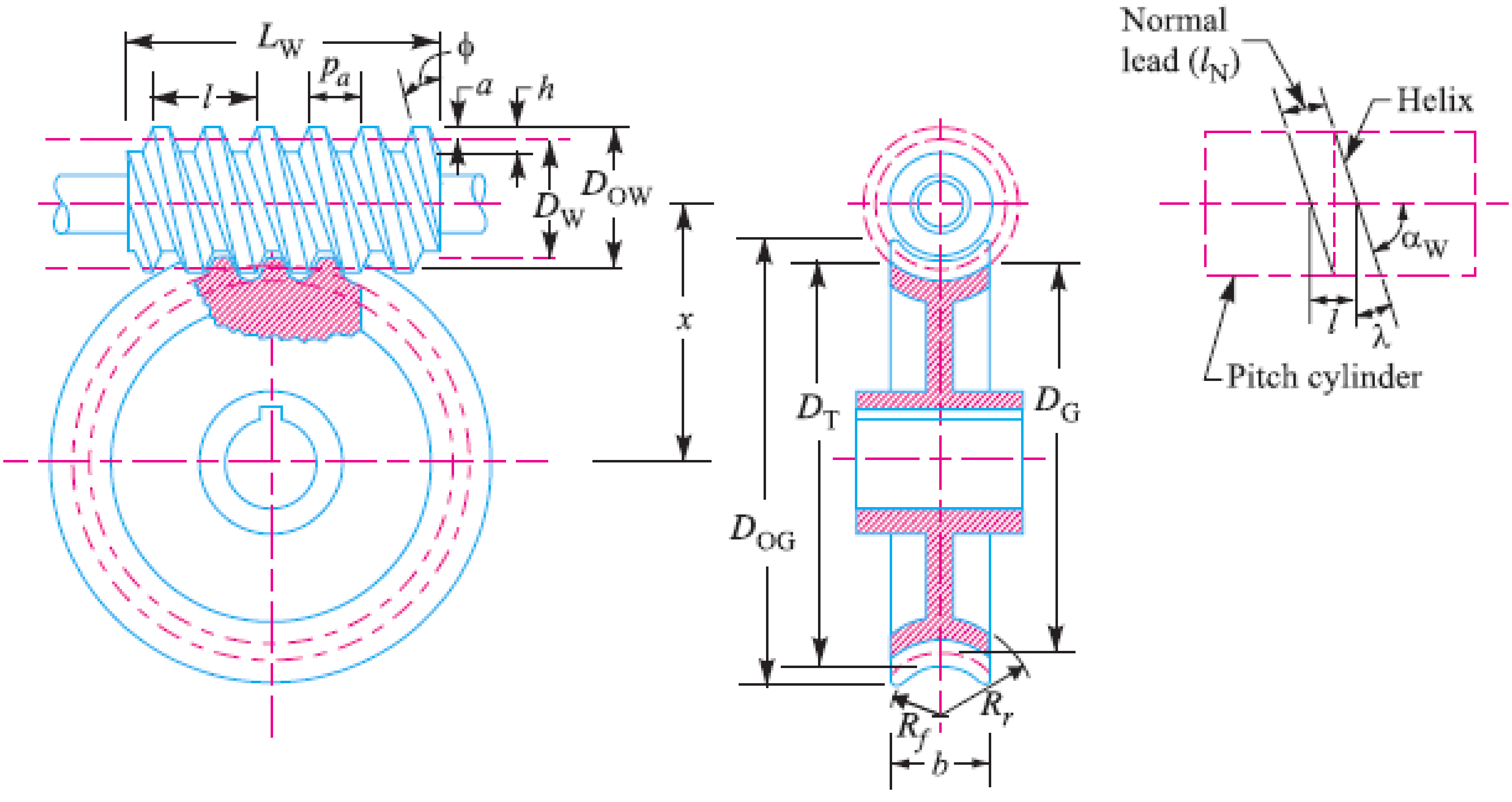


Fig.3 Worm gearbox.

WORM GEARS – GEOMETRY AND NOMENCLATURE





L_W
l
p_a
ϕ
a
h
D_W
D_{OW}
x
D_{OG}
D_T
D_G
R_f
R_r
b
Normal lead (l_N)
Helix
α_W
l
λ
Pitch cylinder

Design a worm gear set to deliver 12 kW from a shaft rotating at 1500 rpm to another rotating at 75 rpm.

Solution:

20° normal pressure angle worm gear is assumed for which the lead angle should not exceed 25° (Table 1) and Z_2 minimum is 21 (Table 2). Allowing 6° lead per thread of the worm, the worm could have 4 or less teeth. $Z_1 = 4$ or quadruple threaded worm is assumed

WORM GEARS – GEOMETRY AND NOMENCLATURE

Table 1. Maximum Worm Lead Angle and Worm Gear Lewis Form Factor for Various Pressure angles.

Pressure Angle Φ_n (Degrees)	Maximum Lead Angle λ (degrees)	Lewis form factor y	Modified Lewis form factor Y
14.5	15	0.100	0.314
20	25	0.125	0.393
25	35	0.150	0.473
30	45	0.175	0.550

WORM GEARS–DESIGN GUIDELINES

Table 2. Minimum number of teeth in the worm gear

Pressure angle φ_n	14.5°	17.5°	20°	22.5°	25°	27.5°	30°
Z_2 minimum	40	27	21	17	14	12	10

$i = n_1 / n_2 = 1500 / 75 = 20 = Z_2 / Z_1$

$\omega_1 = 2\pi n_1 / 60 = 2 x 3.14 x 1500 / 60 = 157$ rad/s

$Z_2 = i \times Z_1 = 20 \times 4 = 80$

A centre distance of 250 mm (as per R10 series) is assumed.

$$\frac{C^{0.875}}{3.0} \leq d_1 \leq \frac{C^{0.875}}{1.7}$$

$d_1 \geq C^{0.875} / 3 = 250^{0.875} / 3 \geq 42$ mm and

$d_1 \leq C^{0.875} / 1.7 = 75$ mm.

d_1 = 72 mm is taken.
Since $d_1 \approx 4p_2$ or circular pitch,
$p_2 = d_1/4 = 72 / 4 = 18$ mm
$m = p / \pi = 18/3.14 = 5.73$ mm take standard module of 6mm.
Hence, $d_2 = m Z_2 = 6 \times 80 = 480$ mm.
Actual centre distance: $C = 0.5 (d_1 + d_2)$
$= 0.5(72+480) = 276$ mm.
Check for $d_1 \leq C^{0.875}/1.7 \leq 80.4$ mm, $d_1 = 80$ mm is taken.

$C = 0.5(d_1 + d_2) = 0.5(80+480) = 280$ mm
Lead = $N_{tw} \times p_a = 4 \times 18.84 = 75.36$ mm
$\tan \lambda = L / \pi d_1 = 75.36 / 3.14 \times 72 = 0.3333$
$\lambda = 18.43° = \psi$
$\omega_2 = (2\pi n_2 /60) = (2 \times 3.14 \times 75/60) = 7.85$ rad/s
$V_2 = \omega_2 r_2 = 7.85 \times (0.5 \times 480) \times 10^{-3} = 1.884$ m/s
$F_t = 1000W/ V = 1000 \times 12/ 1.884 = 6370$ N
$b \leq 0.5\, d_{a1}$, $b \leq 0.5(d_1 + 2m) \leq 0.5 \times (80+2 \times 6) \leq 46$
b = 45 mm is assumed.
Y = 0.393 from Table 1 for $\varphi_n = 20°$

Table 1. Maximum Worm Lead Angle and Worm Gear Lewis Form Factor for Various Pressure angles.

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14.5	**15**	**0.100**	**0.314**
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25	**35**	**0.150**	**0.473**
30	**45**	**0.175**	**0.550**

$$F_d = F_{2t}\left(\frac{6.1+V_2}{6.1}\right) = 6370x\left(\frac{6.1+1.884}{6.1}\right) = 8133N$$

Choosing phosphor bronze for the gear and heat treated C45 steel for the ground worm,

$[\sigma_b] = 80$ MPa from Table 3
Beam strength of the worm gear

$$F_b = [\sigma_b]\, bmY = 80x45x6x0.393$$
$$= 8489 \text{ N}$$

WORM GEARS – BENDING AND SURFACE FATIGUE STRENGTHS

Table 3. Permissible stress in bending fatigue

Material of the gear	$[\sigma_b]$ MPa
Centrifugally cast Cu-Sn bronze	23.5
Phosphor bronze	80
Aluminium alloys Al-Si	11.3
Zn Alloy	7.5
Cast iron	11.8

F_b (8489) > F_d (8133) Hence the design is safe from bending fatigue consideration. Check for the wear strength.

$$F_w = d_2 \, b \, K_w = 480 \times 45 \times 0.518 = 11189 \text{ N}$$

K_w = 0.518 for steel worm vs bronze worm gear with $\lambda < 25^\circ$ from Table 4.
F_w (11189) > F_d (8133), the design is safe from wear strength consideration.

Table 4. Worm Gear Wear Factors K_w

Material		K_w (MPa)		
Worm	Gear	$\lambda<10°$	$\lambda<25°$	$\lambda>25°$
Steel, 250 BHN	Bronze	0.414	0.518	0.621
Hardened steel (Surface 500 BHN)	Bronze	0.552	0.690	0.828
	Chill-cast Bronze	0.828	1.036	1.243
Cast iron	Bronze	1.036	1.277	1.553

AGMA recommendation for the axial length of the Worm is, L_w

$$L_w = p_a\left(4.5+\frac{Z_2}{50}\right) = 18.84x\left(4.5+\frac{80}{50}\right) = 115\,mm$$

Worm Velocity $V_1 = \omega_1 r_1 = 157 \times 0.04 = 6.28$ m/s

$$V_s = \frac{V_1}{\cos\lambda} = \frac{6.28}{\cos 18.43°} = 6.62\ m/s$$

From the graph 1, for $V_s = 6.62$ m/s $f = 0.025$

WORM GEARS –FRICTION

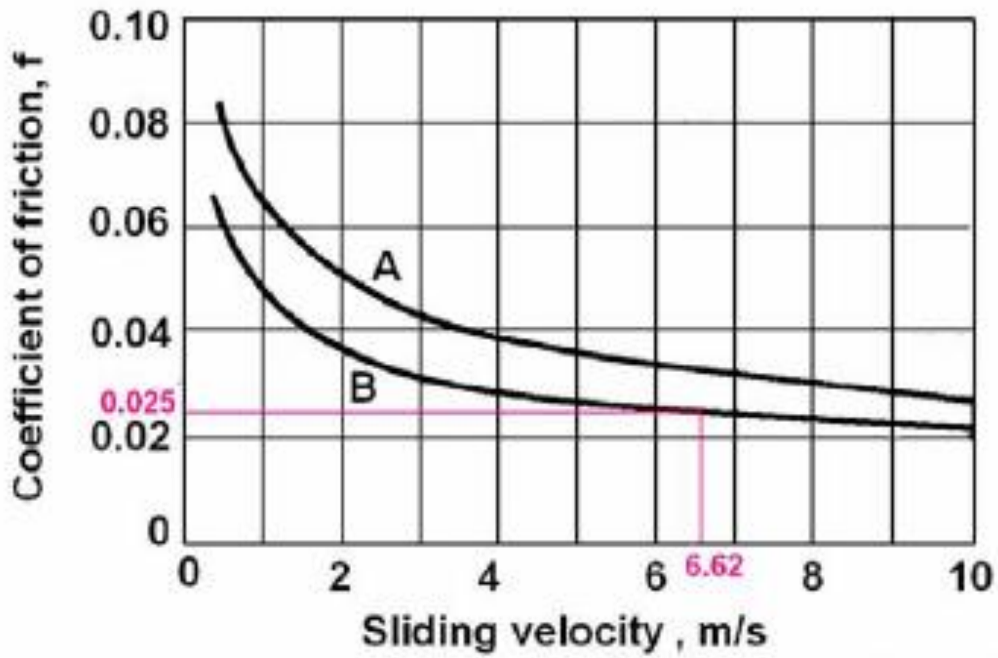

Fig.1. Firction of well lubricated worm gears, A for cast iron worm and worm gear and B for case hardened steel worm and phospor bronze worm gear.

$$F_n = \frac{F_{t2}}{\cos\varphi_n \cos\lambda} = \frac{6370}{\cos 20^\circ \cos 18.43^\circ} = 7145\,\text{N}$$

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$$

$$= \frac{\cos 20^\circ - 0.025\tan 18.43^\circ}{\cos 20^\circ + 0.025\cot 18.43^\circ}$$

$$= 0.918$$

Heat generated during operation: $H_g = (1-\eta)W$
$= (1-0.918)12000 = 984$ Nm

Surface area A for conventional housing designs may be roughly estimated from the equation:

$$A = 14.75\ C^{1.7}$$

Where A is in m^2 and C (the distance between the shafts) is in m.

$$A = 14.75 \times 0.28^{1.7} = 1.694\,m^2$$

Heat generated during operation: H_g

From Fig. 2 the $C_H = 32$ Nm/s/m^2/°C for n_1=1500rpm

WORM GEARS –THERMAL CAPACITY

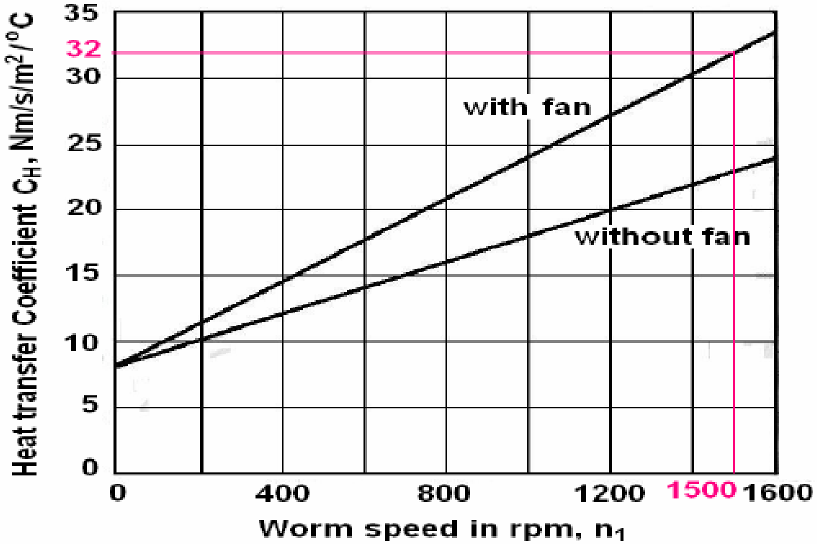

Fig. 2 . Influence of worm speed on heat transfer.

$H_d = C_H\ A\ (T_o - T_a)$ assuming $T_a = 35°C$
$= 32 \times 1.694 \times (T_o - 35°) = 54.21\ (T_o - 35°)$
$= H_g = 984$ Nm

$T_0 = 53.2°\ C < 93°\ C$ permissible for oil.

Hence the design is OK