Design of Pressure Vessel

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Design of Pressure Vessel of Following;

1.Thin

(i) Cylindrical

(ii) Spherical

2. Thick;

Brittle & Ductile Material by Lame's Equation

Introduction

- The pressure vessels (*i.e*. cylinders or tanks) are used to store fluids under pressure. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of a pressure vessel means an explosion which may cause loss of life and property.
- The material of pressure vessels may be brittle such as cast iron, or ductile such as mild steel.

Classification

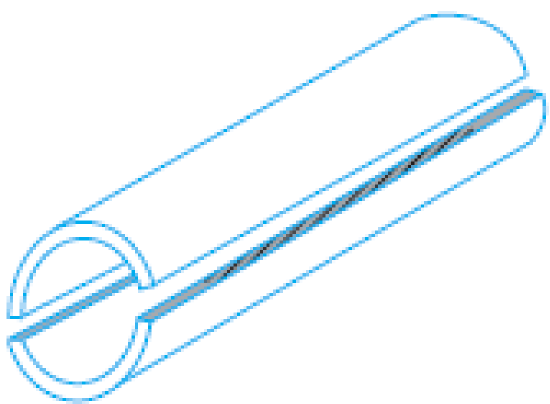
- ***According to the dimensions*.** The pressure vessels, according to their dimensions, may be classified as ***thin shell*** or ***thick shell*.**
- If the wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d), then it is called a ***thin shell*.**
- On the other hand, if the wall thickness of the shell is greater than 1/10 of the diameter of the shell, then it is said to be a ***thick shell*.**
- Thin shells are used in boilers, tanks and pipes, whereas thick shells are used in high pressure cylinders, tanks, gun barrels etc.

Design of Thin Cylindrical Pressure Vessel

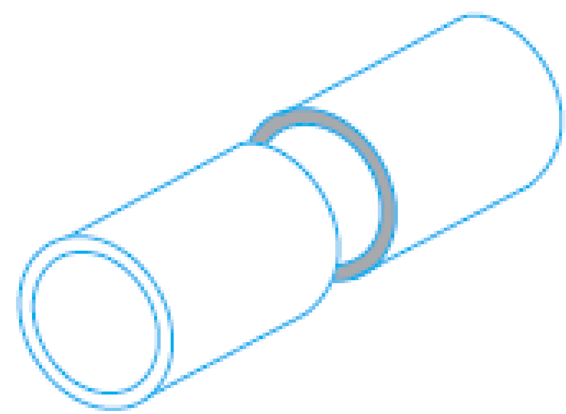
Failure of Thin Pressure Vessel (Cylindrical)

1. It may fail along the longitudinal section (*i.e.* circumferentially) splitting the cylinder into two troughs, as shown in Fig. (*a*).

2. It may fail across the transverse section (*i.e.* longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig. (*b*).



(*a*) Failure of a cylindrical shell along the longitudinal section.



(*b*) Failure of a cylindrical shell along the transverse section.

Notation Identification

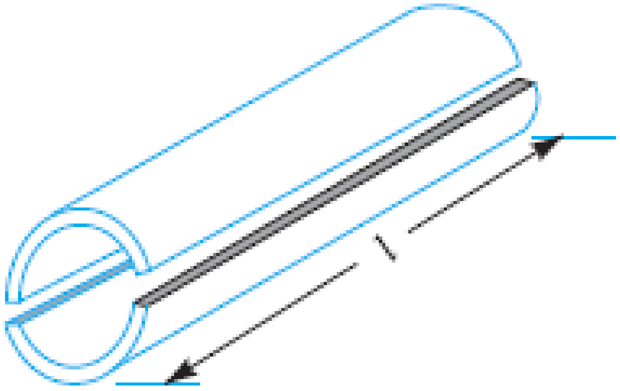
- p = Intensity of internal pressure,
- d = Internal diameter of the cylindrical shell,
- l = Length of the cylindrical shell,
- t = Thickness of the cylindrical shell, and
- $\sigma t1$ = Circumferential or hoop stress for the material of the cylindrical shell.
- $\sigma t2$ = Longitudinal stress for the material of the cylindrical shell.

Circumferential (Hoop Stress) Failure

- The total force acting on a longitudinal section (*i.e.* along the diameter *X-X*) of the shell = Intensity of pressure × Projected area = $p \times d \times l$
- The total resisting force acting on the cylinder walls

  $= \sigma t1 \times 2t \times l$ ...

  (i.e. two sections)



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- Comparing equations;

$$\sigma_{t1} \times 2t \times l = p \times d \times l$$

$$\sigma_{t1} = \frac{p \times d}{2t} \quad \text{or} \quad t = \frac{p \times d}{2\,\sigma_{t1}}$$

Longitudinal Stress Failure

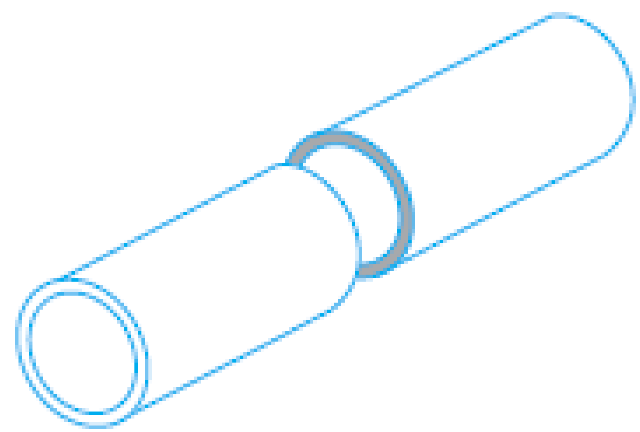
- The total force acting on the transverse section (*i.e*. along *Y-Y*)

= Intensity of pressure × Cross-sectional area

$$= p \times \frac{\pi}{4}(d)^2$$

- Total resisting force;

$$= \sigma_{t2} \times \pi\, d.t$$



Continue...

- Comparing equations;

$$\sigma_{t2} \times \pi\, d.t = p \times \frac{\pi}{4}(d)^2$$

$$\therefore \quad \sigma_{t2} = \frac{p \times d}{4\,t} \quad \text{or} \quad t = \frac{p \times d}{4\,\sigma_{t2}}$$

If η_c is the efficiency of the circumferential joint, then

$$t = \frac{p \times d}{4\sigma_{t2} \times \eta_c}$$

Example: 1

- *A thin cylindrical pressure vessel of 1.2 m diameter generates steam at a pressure of 1.75 N/mm2. Find the minimum wall thickness, if (a) the longitudinal stress does not exceed 28 MPa; and (b) the circumferential stress does not exceed 42 MPa*.

Solution: 1

Solution. Given : $d = 1.2$ m $= 1200$ mm ; $p = 1.75$ N/mm^2 ; $\sigma_{t2} = 28$ MPa $= 28$ N/mm^2 ; $\sigma_{t1} = 42$ MPa $= 42$ N/mm^2

(a) When longitudinal stress (σ_{t2}) does not exceed 28 MPa

We know that minimum wall thickness,

$$t = \frac{p \cdot d}{4\,\sigma_{t2}} = \frac{1.75 \times 1200}{4 \times 28} = 18.75 \text{ say } 20 \text{ mm} \quad \textbf{Ans.}$$

(b) When circumferential stress (σ_{t1}) does not exceed 42 MPa

We know that minimum wall thickness,

$$t = \frac{p \cdot d}{2\,\sigma_{t1}} = \frac{1.75 \times 1200}{2 \times 42} = 25 \text{ mm} \quad \textbf{Ans.}$$

Example: 2

- *A thin cylindrical pressure vessel of 500 mm diameter is subjected to an internal pressure of 2 N/mm2. If the thickness of the vessel is 20 mm, find the hoop stress, longitudinal stress and the maximum shear stress*.

Solution: 2

- Given : d = 500 mm ; p = 2 N/mm2 ;

  t = 20 mm

1. Hoop stress

$$\sigma_{t1} = \frac{p \cdot d}{2t} = \frac{2 \times 500}{2 \times 20} = 25 \text{ N/mm}^2 = 25 \text{ MPa} \quad \textbf{Ans.}$$

2. Longitudinal stress

$$\sigma_{t2} = \frac{p \cdot d}{4t} = \frac{2 \times 500}{4 \times 20} = 12.5 \text{ N/mm}^2 = 12.5 \text{ MPa} \quad \textbf{Ans.}$$

3. Maximum shear stress

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{25 - 12.5}{2} = 6.25 \text{ N/mm}^2 = 6.25 \text{ MPa} \quad \textbf{Ans.}$$

Failure of Thin Pressure Vessel (Spherical)

- Consider a thin spherical shell subjected to an internal pressure as shown in Fig.

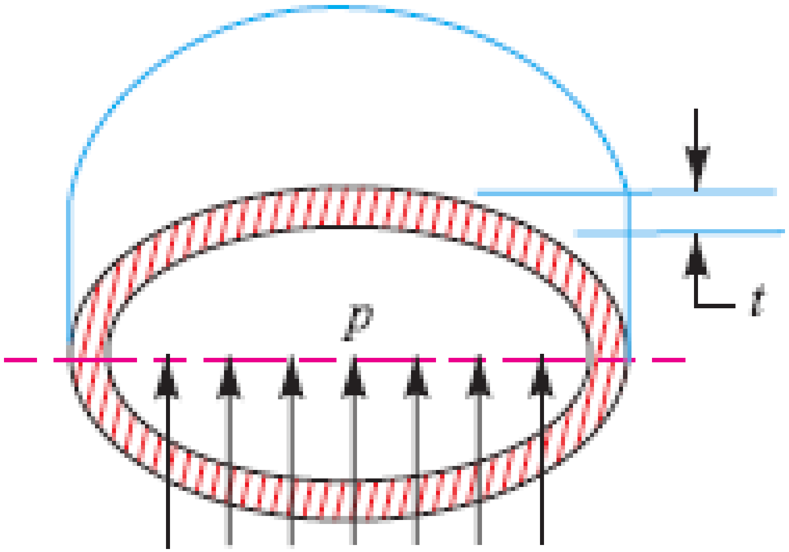
  Let *V* = Storage capacity of the shell,

  p = Intensity of internal pressure,

  d = Diameter of the shell,

  t = Thickness of the shell,

  σ*t* = Permissible tensile stress for the she material.



In designing thin spherical shells, we have to determine;

1. Diameter of the shell, and
2. Thickness of the shell.

Design Procedure

1. Diameter of the shell

From the storage capacity of the shell,

$$V = \frac{4}{3} \times \pi r^3 = \frac{\pi}{6} \times d^3 \quad \text{or} \quad d = \left(\frac{6V}{\pi}\right)^{1/3}$$

2. Thickness of the shell

force tending to rupture the shell along the centre of the sphere or bursting force

$$= \text{Pressure} \times \text{Area} = p \times \frac{\pi}{4} \times d^2$$

and resisting force of the shell

= Stress × Resisting area = $\sigma t \times \pi\, d.t$

Continue…

Combining equations;

$$p \times \frac{\pi}{4} \times d^2 = \sigma_t \times \pi\, d.t$$

or

$$t = \frac{p.d}{4\,\sigma_t}$$

If η is the efficiency of the circumferential joints of the spherical shell, then

$$t = \frac{p.d}{4\,\sigma_t.\eta}$$

Example: 3

- *A spherical vessel 3 metre diameter is subjected to an internal pressure of 1.5 N/mm2. Find the thickness of the vessel required if the maximum stress is not to exceed 90 MPa. Take efficiency of the joint as 75%.*

Solution: 3

- Given: d = 3 m = 3000 mm ;

  p = 1.5 N/mm2 ; σt = 90 MPa = 90 N/mm2 η = 75% = 0.75

Calculating Thickness;

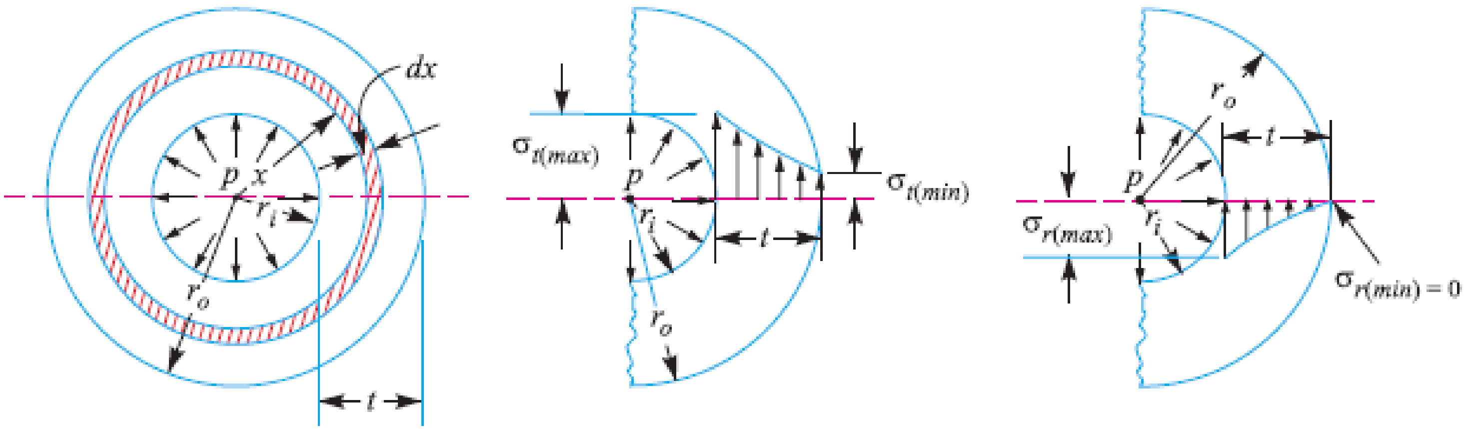
$$t = \frac{p.d}{4\,\sigma_t\,.\,\eta} = \frac{1.5 \times 3000}{4 \times 90 \times 0.75} = 16.7 \text{ say } 18 \text{ mm}$$ **Ans.**

Design of Thick Cylindrical Pressure Vessel

- When a cylindrical shell of a pressure vessel, hydraulic cylinder, gun barrel and a pipe is subjected to a very high internal fluid pressure, then the walls of the cylinder must be made extremely heavy or thick.

- In thin cylindrical shells, we have assumed that the tensile stresses are uniformly distributed over the section of the walls. But in the case of thick wall cylinders the stress over the section of the walls cannot be assumed to be uniformly distributed. They develop both tangential and radial stresses with values which are dependent upon the radius of the element under consideration.

Stress Distribution



(a) Thick cylindrical shell. (b) Tangential stress distribution. (c) Radial stress distribution.

The tangential stress is always a tensile stress whereas the radial stress is a compressive stress.

Lame's Equation

The tangential and radial stress at any radius *x* is;

.

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

.

$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

Continue…

- Considering only internal pressure (*pi* = *p*) only, therefore substituting the value of external pressure, *po* = 0. These Eq. becomes;

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

.

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

Continue…

- The tangential stress is maximum at the inner surface of the shell (*i.e*. when $x = r\,i$) and it is minimum at the outer surface of the shell (*i.e*. when $x = ro$). Substituting the value of $x = ri$ and $x = ro$ in equation

$$\sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} \qquad \sigma_{t(min)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2}$$

Continue…

- The radial stress is maximum at the inner surface of the shell and zero at the outer surface of the shell. Substituting the value of *x* = *ri* and *x* = *ro* in equation

$$\sigma_{r(max)} = -p \text{ (compressive)}$$

$$\sigma_{r(min)} = 0$$

Designing of Thick Cylindrical Shell for Brittle Material

- a thick cylindrical shell of brittle material (*e.g*. cast iron, hard steel and cast Al) with closed or open ends is designed with the maximum normal stress theory failure, the tangential stress induced in the cylinder wall,

$$\sigma_t = \sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

Continue…

- *ro* = *ri* + *t*, so; substituting this value of *ro* in this expression,

$$\sigma_t = \frac{p\,[(r_i + t)^2 + (r_i)^2]}{(r_i + t)^2 - (r_i)^2}$$

$$\sigma_t (r_i + t)^2 - \sigma_t (r_i)^2 = p\,(r_i + t)^2 + p\,(r_i)^2$$

$$(r_i + t)^2 (\sigma_t - p) = (r_i)^2 (\sigma_t + p)$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\sigma_t + p}{\sigma_t - p}$$

Continue…

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}}$$

$$\frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right]$$

Example: 4

- *A cast iron cylinder of internal diameter 200 mm and thickness 50 mm is subjected to a pressure of 5 N/mm2. Calculate the tangential and radial stresses at the inner, middle (radius = 125 mm) and outer surfaces*.

Solution: 4

Given : *di* = 200 mm or *ri* = 100 mm ;

- *t* = 50 mm ;
- *p* = 5 N/mm2
- *ro* = *ri* + *t* = 100 + 50 = 150 mm

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right] \quad \sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

Continue…

- ***Tangential stresses at the inner, middle and outer surfaces***

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Tangential stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{t(inner)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} = \frac{5\,[(150)^2 + (100)^2]}{(150)^2 - (100)^2} = 13\ \text{N/mm}^2 = 13\ \text{MPa}\ \textbf{Ans.}$$

Tangential stress at the middle surface (*i.e.* when $x = 125$ mm),

$$\sigma_{t(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 + \frac{(150)^2}{(125)^2}\right] = 9.76\ \text{N/mm}^2 = 9.76\ \text{MPa}\ \textbf{Ans.}$$

and tangential stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{t(outer)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2} = \frac{2 \times 5\,(100)^2}{(150)^2 - (100)^2} = 8\ \text{N/mm}^2 = 8\ \text{MPa}\ \textbf{Ans.}$$

Continue…

- ***Radial stresses at the inner, middle and outer surfaces***

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

∴ Radial stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{r(inner)} = -p = -5\ \text{N/mm}^2 = 5\ \text{MPa (compressive)}$$ **Ans.**

Radial stress at the middle surface (*i.e.* when $x = 125$ mm)

$$\sigma_{r(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 - \frac{(150)^2}{(125)^2}\right] = -1.76\ \text{N/mm}^2 = -1.76\ \text{MPa}$$

$$= 1.76\ \text{MPa (compressive)}$$ **Ans.**

radial stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{r(outer)} = 0$$ **Ans.**

Designing of Thick Cylindrical Shell for Ductile Material

- In case of cylinders made of ductile material, Lame's equation is modified according to maximum shear stress theory.

Maximum principal stress at the inner surface,

$$\sigma_{t\,(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum principal stress at the outer surface,

$$\sigma_{t(min)} = -p$$

$\therefore$ Maximum shear stress,

Continue…

$$\tau = \tau_{max} = \frac{\sigma_{t(max)} - \sigma_{t(min)}}{2} = \frac{\dfrac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} - (-p)}{2}$$

$$= \frac{p\,[(r_o)^2 + (r_i)^2] + p\,[(r_o)^2 - (r_i)^2]}{2[(r_o)^2 - (r_i)^2]} = \frac{2\,p\,(r_o)^2}{2[(r_o)^2 - (r_i)^2]}$$

$$= \frac{p\,(r_i + t)^2}{(r_i + t)^2 - (r_i)^2} \qquad \ldots (\because r_o = r_i + t)$$

Continue…

$$\tau(r_i + t)^2 - \tau(r_i)^2 = p(r_i + t)^2$$

$$(r_i + t)^2 (\tau - p) = \tau(r_i)^2$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\tau}{\tau - p}$$

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}}$$

$$\frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1\right]$$

The value of shear stress (τ) is usually taken as one-half the tensile stress (σ*t*). Therefore the above expression may be written as;

$$t = r_i \left[\sqrt{\frac{\sigma_t}{\sigma_t - 2p}} - 1\right]$$

Example: 5

- A ductile thick cylindrical shell of internal diameter 150 mm has to withstand an internal fluid pressure of 50 N/mm2. Determine its thickness so that the maximum shear stress in the section does not exceed 150 MPa.

Solution: 5

Solution Hints:

Given Data: di = 150 mm, p = 50 N/mm2, τ = 150 MPa

Use Following equation as shear stress for ductile material is given

$$t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1 \right]$$

Answer = ?

Example: 6

- Find the thickness of the flat end ductile cover plates for a 1 N/mm2 boiler that has a diameter of 600 mm. The limiting tensile stress in the boiler shell is 40 MPa

Solution: 6

Solution Hints:

Given Data: di = 600 mm, p = 1 N/mm2, = σ_t MPa

Use Following equation as shear stress for ductile material is given

$$t = r_i \left[\sqrt{\frac{\sigma_t}{\sigma_t - 2p}} - 1 \right]$$

Answer = ?