Advanced Fatigue Design Criteria

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[A] Discussion of Fatigue FailureProcess through Some Case Studies[B] Approach to Fatigue Failure inAnalysis and Design

[A] Discussion of Fatigue Failure Process through Some Case Studies

Case Study:



A fatigue failure in a bicycle crank spider arm. This was a high quality component that had very high load cycles but was in excellent apparent condition until the final fracture.



The fatigue crack initiated exactly at the location of the maximum tensile bending moment, close to the crank axle on the tensile side of the crank. The crack progressed slowly through the crank arm (dark area) until the remaining fragment was incapable of supporting the bending moment generated by the force on the pedal and the crank arm fractured rapidly. This long term fatigue crack took a considerable time to nucleate from a machining mark between the spider arms on this highly stressed surface. However once initiated propagation was rapid and accelerating as shown in the increased spacing of the 'beach marks' on the surface caused by the advancing fatigue crack.

Fatigue Failure Process

Crack Initiation

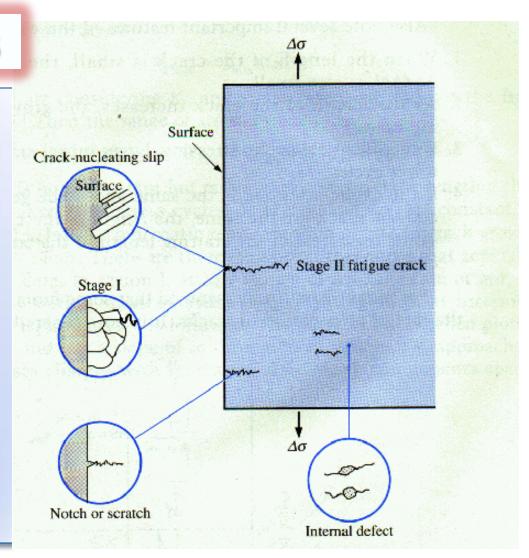
- ☐ Fatigue *always* begins at a crack
- Crack may start at a microscopic inclusion (<.010 in.)
- Crack may start at a "notch", or other stress concentration

Crack Propagation

- Sharp crack creates a stress concentration
- Each tensile stress cycle causes the crack to grow (~10⁻⁸ to 10⁻⁴ in/cycle)

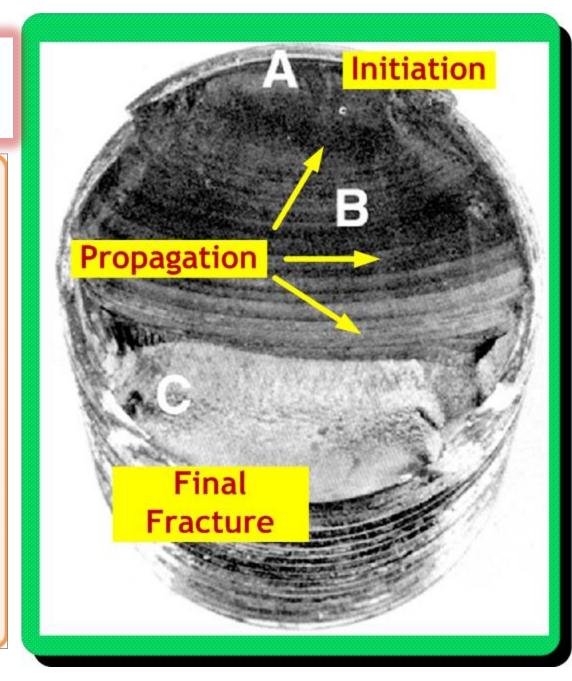
Fracture

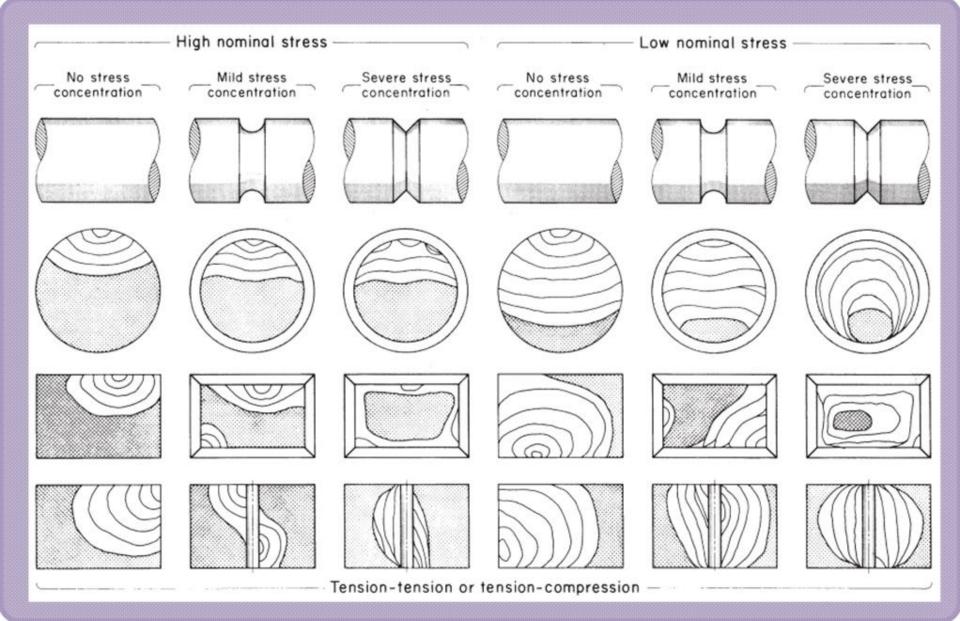
Sudden, catastrophic failure with no warning.

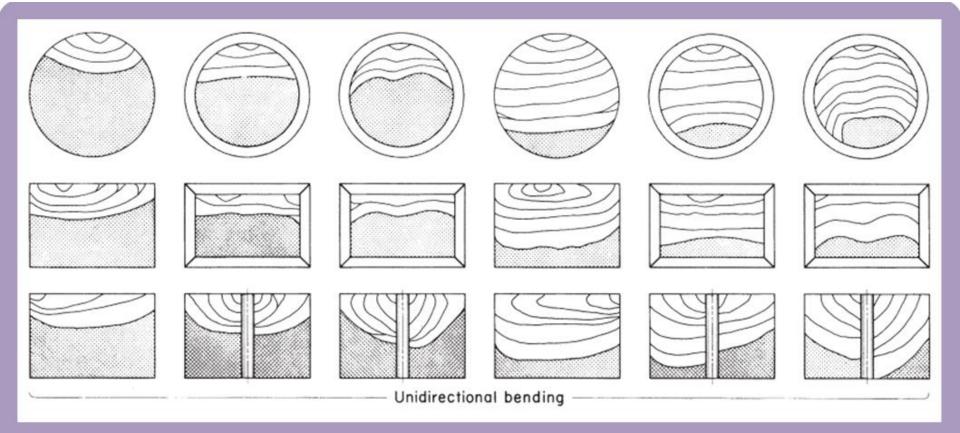


Examples of Fatigue Failures

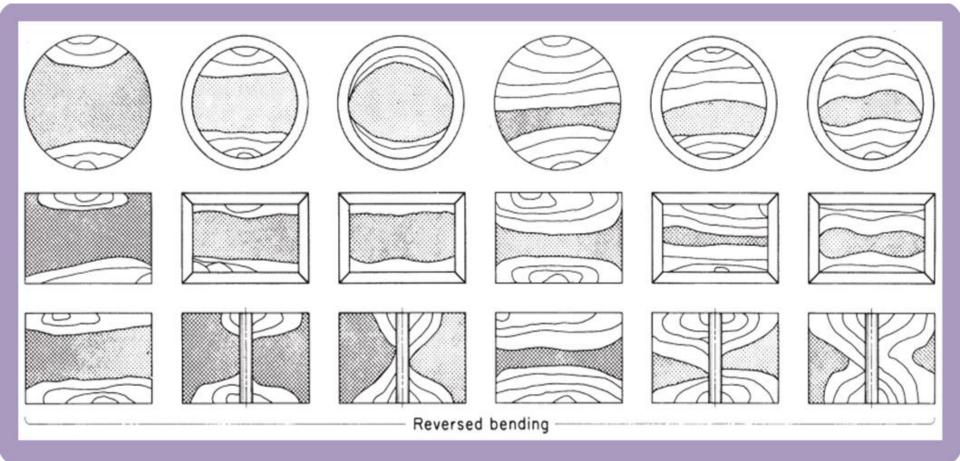
Fatigue failure of a bolt due to repeated unidirectional bending. The failure started at the thread root at A, propagated across most of the cross section shown by the beach marks at B, before final fast fracture at C. (From ASM Handbook, Vol. 12:Fractography, ASM International, Materials Park, OH 44073-0002, Fig. 50, p. 120. Reprinted by permission of ASM International ®, www.asminternational.org.)



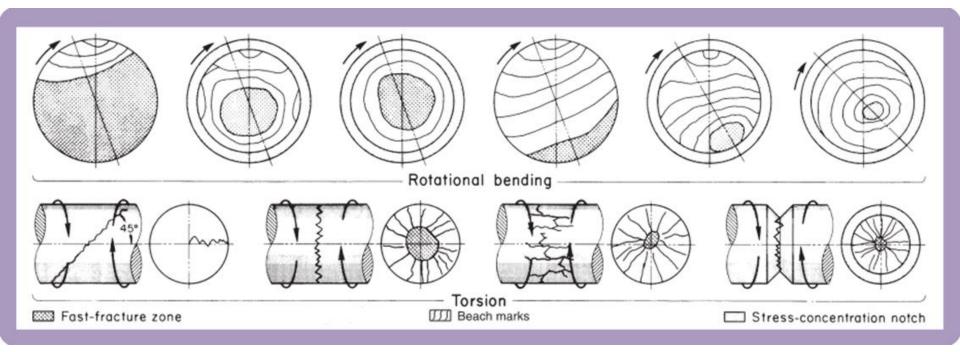




Schematics of fatigue fracture surfaces produced in smooth and notched components with round and rectangular cross sections under various loading conditions and nominal stress levels. (From ASM Handbook, Vol. 11: Failure Analysis and Prevention, ASM International, Materials Park, OH 44073-0002, Fig 18, p. 111.Reprinted by permission of ASM International®, www.asminternational.org.)



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Fatigue fracture of an AISI 4320 drive shaft. The fatigue failure initiated at the end of the keyway at points B and progressed to final rupture at C. The final rupture zone is small, indicating that loads were low. (From ASM Handbook, Vol.11: Failure Analysis and Prevention, ASM International, Materials Park, OH 44073-0002, fig 18, p. 111. Reprinted by permission of ASM International, www.asminternational.org.)



Fatigue fracture surface of an AISI 8640 pin. Sharp corners of the mismatched grease holes provided stress concentrations that initiated two fatigue cracks indicated by the arrows. (From ASM Handbook, Vol.12: Fractography, ASM International, Materials Park, OH 44073-0002, Figg 520, p. 331. Reprinted by permission of ASM International®, www.asminternational.org.)

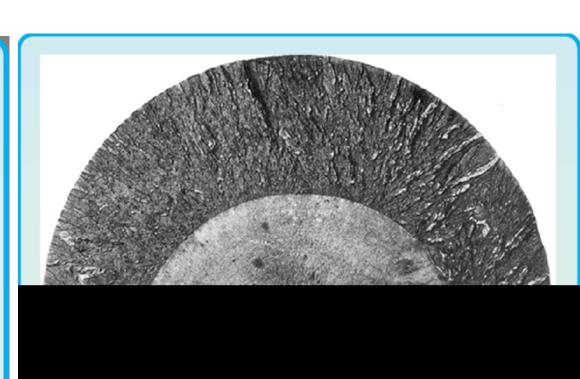


Fatigue fracture surface of a forged connecting rod of AISI 8640 steel. The fatigue crack origin is at the left edge, at the flash line of the forging, but no unusual roughness of the flash trim was indicated. The fatigue crack progressed halfway around the oil hole at the left, indicated by the beach marks, before final fast fracture occurred.

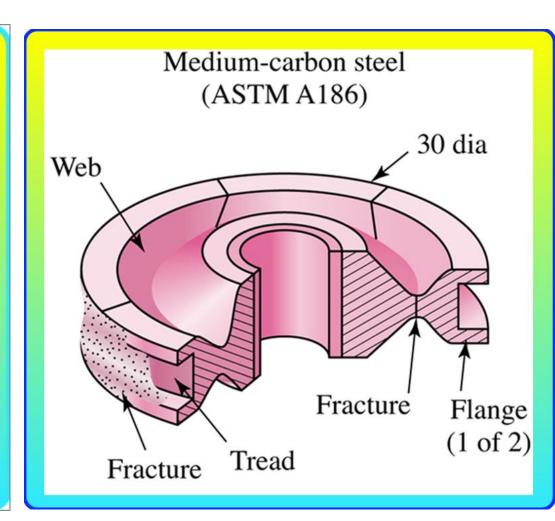


Note the pronounced shear lip in the final fracture at the right edge. (From ASM Handbook, Vol. 12: Fractography, ASM International, Materials Park, OH 44073-0002, fig 523, p. 332. Reprinted by permission of ASM International®, www.asminternational.org.)

Fatigue fracture surface of a 200mm (8-in) diameter piston rod of an alloy steel steam hammer used for forging. This is an example of a fatigue fracture caused by pure tension where surface stress concentrations are absent and a crack may initiate anywhere in the cross section. In this instance, the initial crack formed at a forging flake slightly below center, grew outward symmetrically, and ultimately produced a brittle fracture without warning. (From ASM Handbook, Vol. 12: Fractography, ASM International, Materials Park, OH44073-0002, fig 570, p. 342. Reprinted by permission of ASM International, www.asminternational.org.)



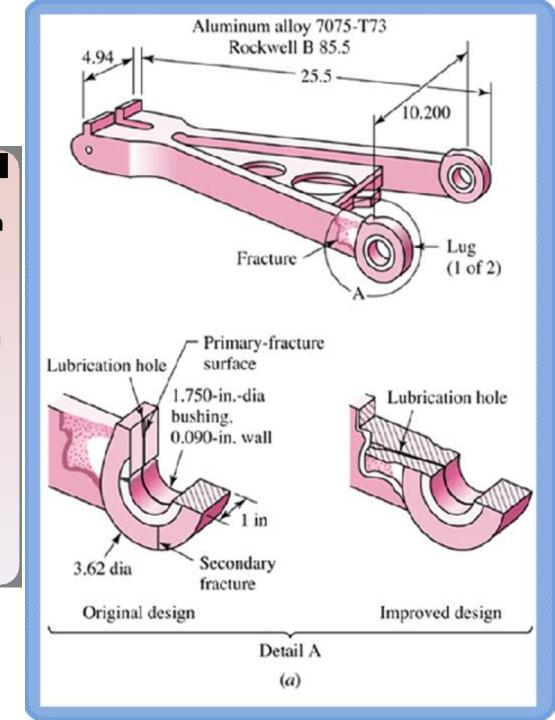
Fatigue failure of an ASTM A186 steel double-flange trailer wheel caused by stamp marks. (a) Coke-oven car wheel showing position of stamp marks and fractures in the rib and web. (From ASM Handbook, Vol.11: Failure Analysis and Prevention, ASM International, Materials Park, OH 44073-0002, fig 51, p. 130. Reprinted by permission of ASM International®, www.asminternational.org.)



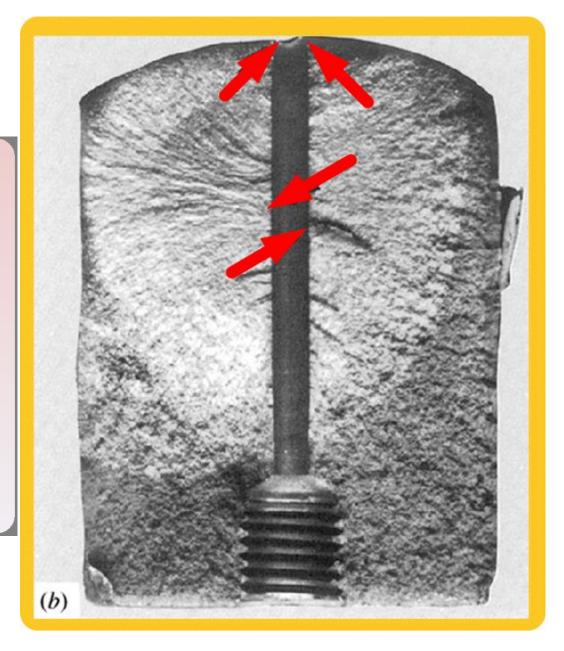


Fatigue failure of an ASTM A186 steel double-flange trailer wheel caused by stamp marks. (b) Stamp mark showing heavy impression and fracture extending along the base of the lower row of numbers. (From ASM Handbook, Vol.11: Failure Analysis and Prevention, ASM International, Materials Park, OH 44073-0002, fig 51, p. 130. Reprinted by permission of ASM International, www.asminternational.org.)

Aluminum alloy 7075-T73 landinggear torque-arm assembly redesign to eliminate fatigue fracture at a lubrication hole. (a) Arm configuration, original and improved design (dimensions given in inches). (From ASM Handbook, Vol. 11: Failure Analysis and Prevention, ASM International, Materials Park, wall OH 44073-0002, fig 23, p. 114. Reprinted By permission of ASM International, www.asminternational.org.)



Aluminum alloy 7075-T73 landing-gear torque-arm assembly redesign to eliminate fatigue fracture at a lubrication hole. (b) Fracture surface where arrows indicate multiple crack origins. (From ASM Handbook, Vol. 11: Failure Analysis and Prevention, ASM International, Materials Park, wall OH 44073-0002, fig 23, p. 114. Reprinted By permission of ASM International, www.asminternational.org.)



Summary

- ☐ Thus far we've studied **STATIC FAILURE** of machine elements.
- The second major class of component failure is due to DYNAMIC LOADING
 - Repeated stresses
 - Alternating stresses
 - Fluctuating stresses
- The ultimate strength of a material (**S**_u) is the **maximum stress** a material can sustain before failure assuming the load is applied only once and held.
- □ Fatigue strength Resistance of a material to failure under cyclic loading.
- A material can also **FAIL** by being loaded repeatedly to a stress level that is **LESS** than (S_u)
 - Fatigue failure

[B] Approaches to Fatigue Failure in Analysis and Design

Approach to fatigue Failure in Analysis and Design

- **☐** Fatigue-Life Methods
- ☐ Fatigue and the Endurance Limit
- **☐** Fatigue Fracture Surface
- ☐ Damage Tolerance Design
- ☐ Varying Fluctuating Cumulative Fatigue damage
- **☐** Fatigue Map

Fatigue-Life Methods

Three major fatigue life methods used in design and analysis for safe life estimation:

- 1. Stress life method (S-N Curves)
- 2. Strain life method (ε-N Curve)
- 3. Linear elastic fracture mechanics method

The above methods predict the life in number of cycles to failure, *N*, for a specific level of loading. :

- 1. Low cycle Fatigue: $1 \le N \le 10^3$ cycles
- 2. High cycle fatigue $N > 10^3$ cycles

1. Stress-Life Method

- **□** based on stress levels only
- ☐ It is the least accurate approach, especially for low-cycle applications.
- Most traditional method:
 - easiest to implement for a wide range of design applications
 - ample supporting data
 - □ represents high-cycle applications adequately

2. Strain-Life Method

- ☐ Involves more detailed analysis of the plastic deformation at localized regions where the stresses and strains are considered for life estimates.
- **□** Good for low-cycle fatigue applications.
- Some uncertainties exist in the results.

3. Fracture Mechanics Method

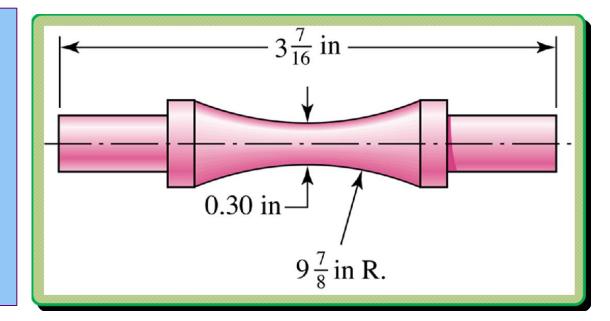
- ☐ Assumes a crack is already present and detected.
- ☐ Predicts crack growth with respect to stress intensity.
- ☐ Most practical when applied to large structures in conjunction with computer codes and a periodic inspection program.

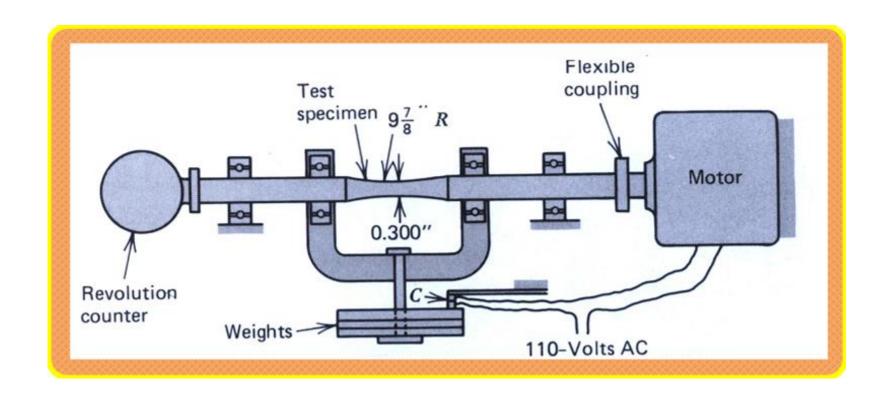
The Stress-Life Method

To determine the strength of materials under the action of fatigue loads, specimens are subjected to repeated or varying forces of specified magnitudes while the cycles or stress reversals are counted to destruction. The most widely used fatigue-testing device is the R. R. Moore high-speed rotating-beam machine. The specimen, shown in Fig.6-9, is very carefully machined and polished, with a final polishing in an axial Direction to avoid circumferential scratches.

Figure 7-9

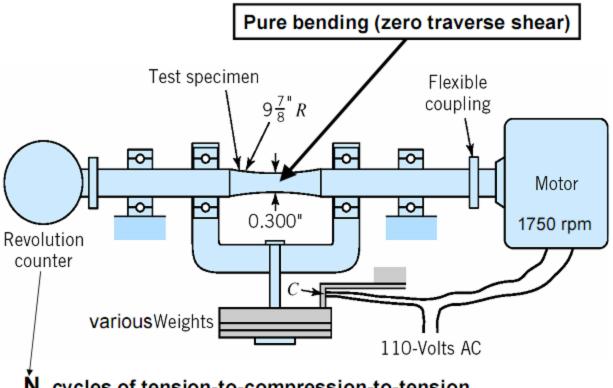
Test-specimen geometry for the R. R. Moore rotatingbeam machine. The bending moment is uniform over the curved at the highest-stressed portion, a valid test of material, whereas a fracture elsewhere (not at the higheststress level) is grounds for suspicion of material flaw.





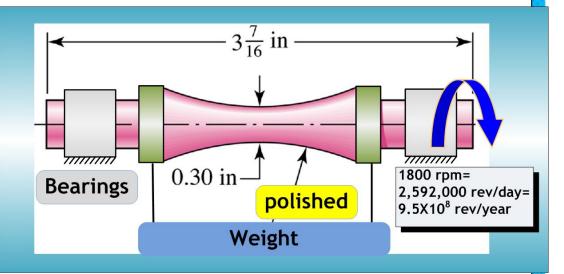
R. R. Moore rotating-beam fatigue testing machine

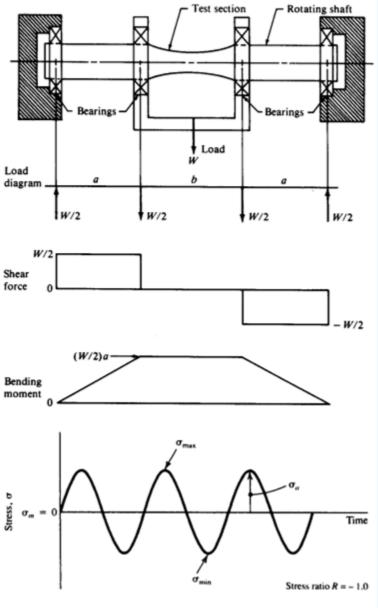
Rotating-beam fatigue-testing machine



N cycles of tension-to-compression-to-tension

R. R. Moore rotating-beam fatigue testing machine





There are essentially two types of fatigue:

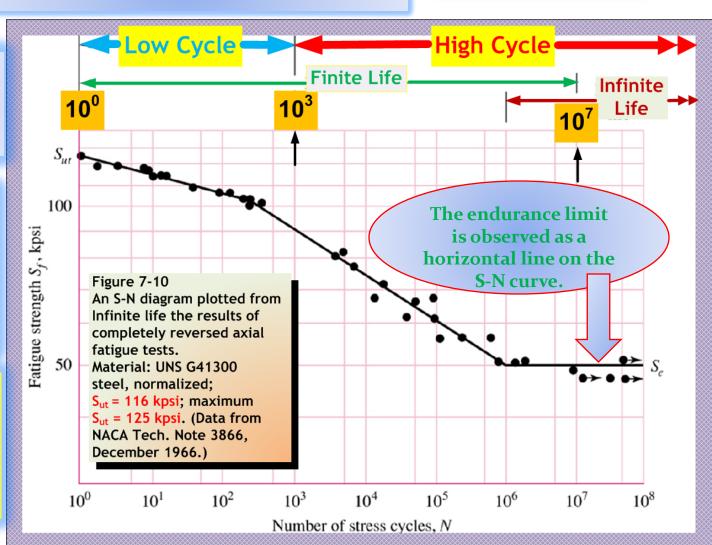
- 1. High Cycle Fatigue (HCF) (Elastic Strain)
- 2. Low Cycle Fatigue (LCF) (Plastic Strain)

Semi-Log Scale

The fatigue strength S_f is the stress level that a material can endure for N cycles.

The stress level at which that material can withstand an infinite number of cycles is called the endurance limit.

Endurance Limit (S_e):
Maximum stress that a
material will endure
without failure for an
infinite number of load
cycles.



Fatigue and The Endurance Limit

S-N Curves for Non-Ferrous Metals (Al-Alloys)

- Note that nonferrous materials often exhibit no endurance limit.
- Eventually these materials will fail due to repeated loading.
- □ To come up with an equivalent endurance limit, designers typically use the value of the fatigue strength S_f at 10⁸ cycles

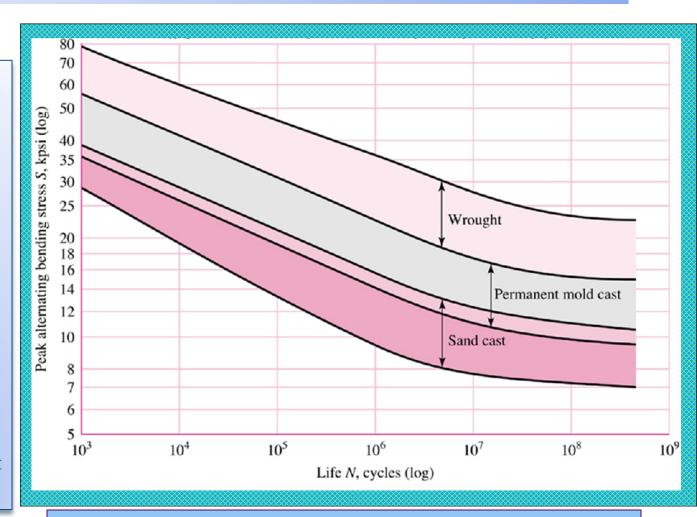
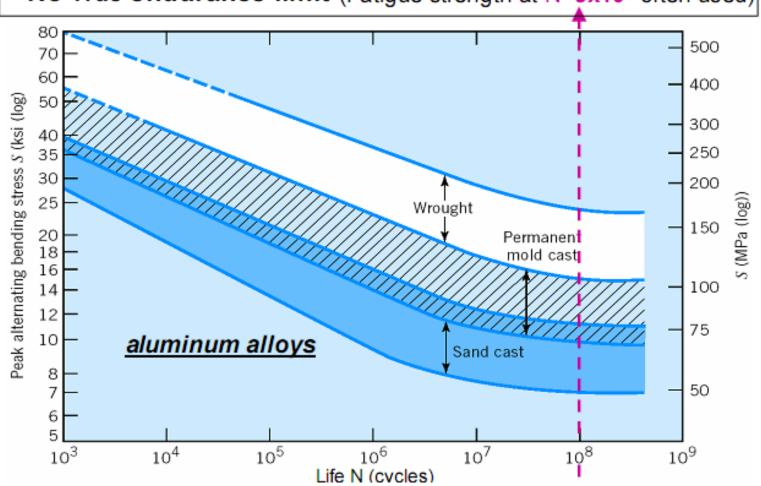


Figure S-N bands for representative aluminum alloys, excluding wrought alloys with S_{ut} < 38 kpsi.

Endurance Limit

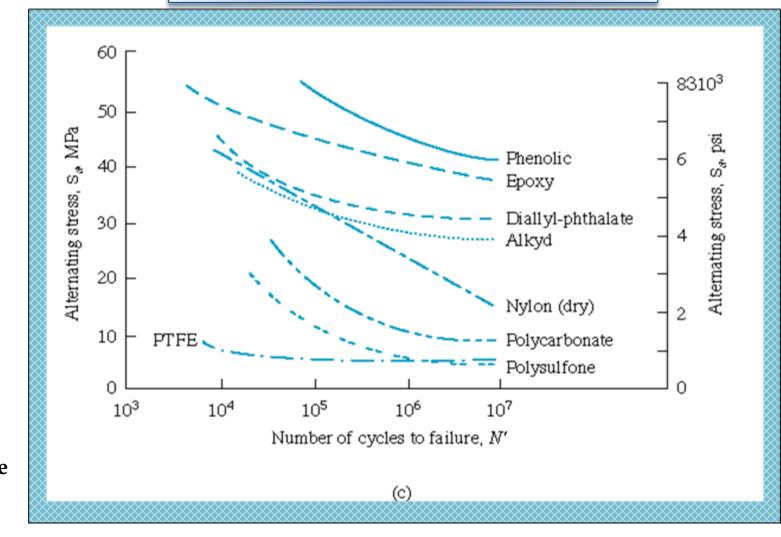
S-N curve for *nonferrous metals*

- No Sharply defined knee and
- •No True endurance limit (Fatigue strength at N=5x108 often used)



S-N Curves for Polymers

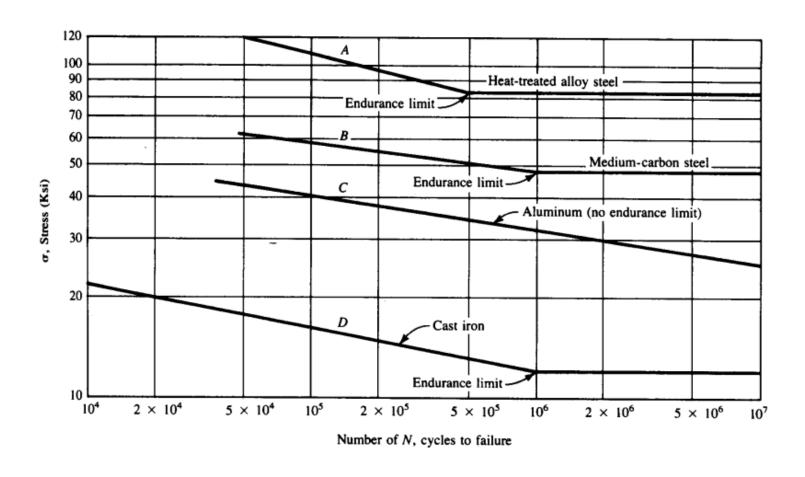
Note that non-ferrous materials often exhibit no endurance limit.



For frequencies < 200Hz, metals are insensitive to frequency; fatigue life in polymers *is* frequency

dependent.

Representative S-N Curves



Note that non-ferrous materials often exhibit no endurance limit.

- Some materials exhibit *endurance limits*, i.e. a stress below which the life is infinite: [fig. 12.8]
 - Steels typically show an endurance limit, = 40% of yield; this is typically associated with the presence of a solute (carbon, nitrogen) that pines dislocations and prevents dislocation motion at small displacements or strains (which is apparent in an upper yield point).
 - Aluminum alloys do not show endurance limits; this is related to the absence of dislocation-pinning solutes.
- At large N_f, the lifetime is dominated by nucleation.
 - Therefore strengthening the surface (shot peening) is beneficial to delay crack nucleation and extend life.

Fatigue Fracture Surface

- Crack Nucleation → stress intensification at crack tip.
- Stress intensity → crack propagation (growth);
 - stage I growth on shear planes (45°), strong influence of microstructure
 - stage II growth normal to tensile load (90°) weak influence of microstructure.
- Crack propagation → catastrophic, or ductile failure at crack length dependent on boundary conditions, fracture toughness.
- Flaws, cracks, voids can all act as crack nucleation sites, especially at the surface.

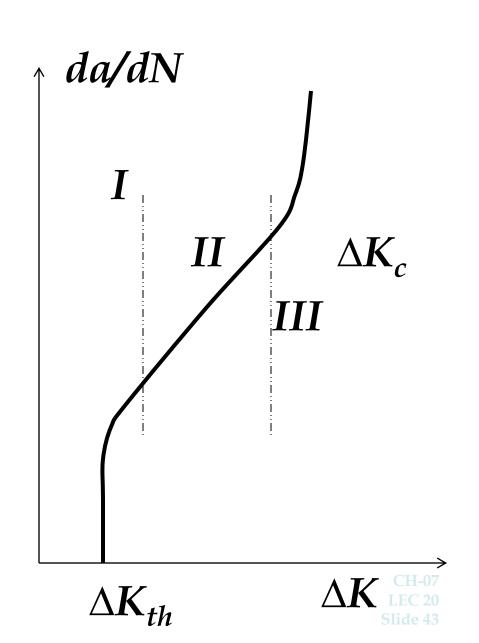
- Therefore, smooth surfaces increase the time to nucleation; notches, stress risers decrease fatigue life.
- Dislocation activity (slip) can also nucleate fatigue cracks.
- Dislocation slip -> tendency to localize slip in bands.
- Persistent Slip Bands (PSB's) characteristic of cyclic strains.
- Slip Bands -> extrusion at free surface
- Extrusions -> intrusions and crack nucleation.

Fatigue Crack Propagation

- Crack Length := a. Number of cycles := NCrack Growth Rate := da/dNAmplitude of Stress Intensity := $\Delta K = \Delta \sigma \sqrt{c}$.
- Define three stages of crack growth, I, II and III, in a plot of da/dN versus ΔK .
- Stage II crack growth: application of linear elastic fracture mechanics.
- Can consider the crack growth rate to be related to the applied stress intensity.
- Crack growth rate somewhat insensitive to R (if R<o) in Stage II
- Environmental effects can be dramatic, e.g. H in Fe, in increasing crack growth rates.

Fatigue Crack Propagation

- Three stages of crack growth, I, II and III.
- Stage I: transition to a finite crack growth rate from no propagation below a threshold value of ΔK.
- Stage II: "power law" dependence of crack growth rate on ΔK.
- Stage III: acceleration of growth rate with ΔK, approaching catastrophic fracture.



*Paris Law

• Paris Law:
$$\frac{dc}{dN} = A(\Delta K)^m$$

- *m* ~ 3 (steel); *m* ~ 4 (aluminum).
- Crack nucleation ignored!
- Threshold ~ Stage I
- The threshold represents an endurance limit.
- For ceramics, threshold is close to K_{IC} .
- Crack growth rate increases with *R* (for *R*>o).

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*Striations-mechanism

- Striations occur by development of slip bands in each cycle, followed by tip blunting, followed by closure.
- Can integrate the growth rate to obtain cycles as related to cyclic stress-strain behavior

$$N_{II} = \int_{c_0}^{c_f} \frac{dc}{dc/dN} \qquad N_{II} = \int_{c_0}^{c_f} \frac{dc}{A\alpha^m (\Delta \sigma \sqrt{c})^m}$$

*Striations, contd.

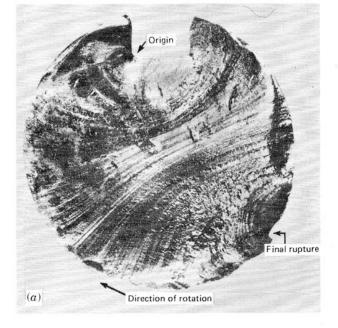
• Provided that m>2 and α is constant, can integrate.

$$N_{II} = \frac{A^{-1}(\overline{\alpha}\Delta\sigma)^{-m}}{(m/2)-1} \left[c_0^{1-(m/2)} - c_f^{1-(m/2)} \right]$$

• If the initial crack length is much less than the final length, $c_o < c_f$, then approximate thus:

$$N_{II} = \frac{A^{-1}(\overline{\alpha}\Delta\sigma)^{-m}}{(m/2)-1}c_0^{1-(m/2)}$$

 Can use this to predict fatigue life based on known crack



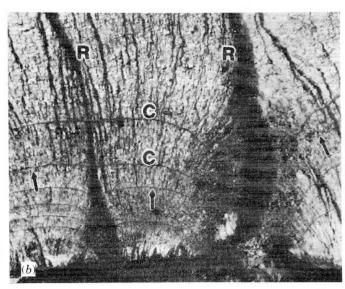


FIGURE 12.3 Fatigue fracture markings. (a) Rotating steel shaft. Center of curvature of earlier "beach markings" locate crack origin at corner of keyway. (By permission from D. J. Wulpi, How Components Fail, copyright American Society for Metals, 1966.) (b) Clam shell markings (C) and ratchet lines (R) in aluminum. Arrows indicate crack propagation direction (Photo courtesy of R. Jaccard). (c) Fatigue bands in high-impact polystyrene toilet seat. (d) Beach markings in South Carolina.

Damage Tolerance Design

- S-N (stress-cycles) curves = basic characterization.
- Old Design Philosophy = <u>Infinite Life design</u>: accept empirical information about fatigue life (S-N curves); apply a (large!) safety factor; retire components or assemblies at the pre-set life limit, e.g. $N_f=10^7$.
- *Crack Growth Rate characterization ->
- *Modern Design Philosophy (Air Force, not Navy carriers!) = <u>Damage Tolerant design</u>: accept presence of cracks in components. Determine life based on prediction of crack growth rate.

Cyclic Stres-Strain Relation

- Cyclic strain control complements cyclic stress characterization: applicable to thermal fatigue, or fixed displacement conditions.
- Cyclic stress-strain testing defined by a controlled strain range, $\Delta \varepsilon_{pl}$.
- Soft, annealed metals tend to harden; strengthened metals tend to soften.
- Thus, many materials tend towards a fixed cycle, i.e. constant stress, strain amplitudes.

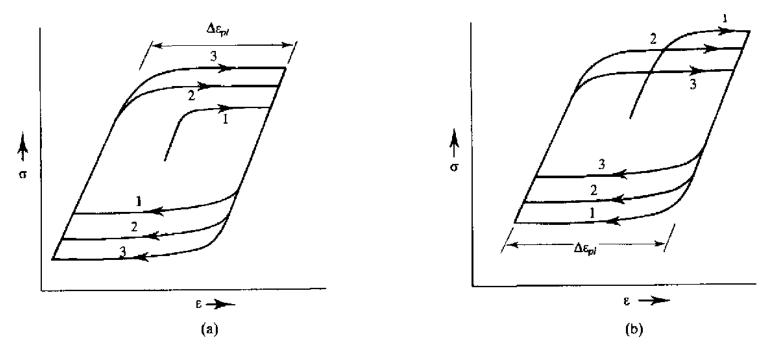


Figure 12.24 Cyclical stress-strain behavior for (a) a material that cyclically hardens and (b) one that softens. The tests are conducted for a fixed plastic strain range ($\Delta \epsilon_{pl}$). For a material that cyclically hardens, the stress amplitude necessary to maintain a stipulated strain amplitude increases. (The numbers on the curves indicate successive cycles.) Conversely, this stress amplitude decreases for a material that softens cyclically. The phenomena are accompanied by respectively increasing and decreasing areas of the stress-strain loops generated during cyclical straining.

- Large number of cycles typically needed to reach asymptotic hysteresis loop (~100).
- Softening or hardening possible

Cyclic stress-strain

- Wavy-slip materials generally reach asymptote in cyclic stress-strain: planar slip materials (e.g. brass) exhibit history dependence.
- Cyclic stress-strain curve defined by the extrema, i.e. the "tips" of the hysteresis loops. [Courtney fig. 12.27]
- Cyclic stress-strain curves tend to lie below those for monotonic tensile tests.
- Polymers tend to soften in cyclic straining.

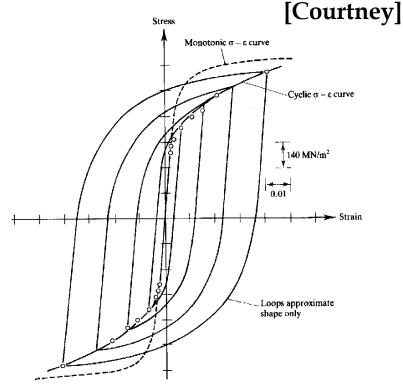


Figure 12.27
The monotonic and cyclic stress-strain curves for a 4340 steel. The hysteresis loops shown are the steady-state ones, obtained for cyclically straining at different plastic strain ranges. A curve drawn through the loop tips (i.e., through the steady-state stress amplitudes) defines the cyclical stress-strain curve. This steel cyclically softens; that is, the cyclical stress-strain curve lies below the monotonic one. (From R. W. Landgraf, ASTM STP467, Philadelphia, Pa., 1970, p. 3.)

- Strain is a more logical independent variable for characterization of fatigue. [fig. 12.11]
- Define an *elastic strain range* as $\Delta \varepsilon_{el} = \Delta \sigma / E$.
- Define a plastic strain range, $\Delta \varepsilon_{pl}$.
- Typically observe a change in slope between the elastic and plastic regimes. [fig. 12.12]
- Low cycle fatigue (small N_f) dominated by plastic strain: high cycle fatigue (large N_f) dominated by elastic strain.

Strain control of fatigue

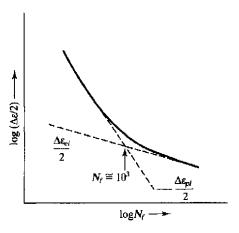


Figure 12.11 In a strain-controlled fatigue test the cyclic strain amplitude ($\Delta \varepsilon/2$) can be related to the number of cycles to failure, N_f . During high-cycle fatigue ($N_f \gtrsim 10^3$) most of the macroscopic strain is elastic, and the slope of log ($\Delta \varepsilon/2$) vs. log N_f is less negative than it is during low-cycle fatigue ($N_f \lesssim 10^3$). For the latter, most of the applied strain is permanent. Note that $\Delta \varepsilon/2 = (\Delta \varepsilon_{el} + \Delta \varepsilon_{pl})/2$; $\Delta \varepsilon_{el} > \Delta \varepsilon_{pl}$

during high-cycle fatigue and $\Delta \varepsilon_{nl} > \Delta \varepsilon_{el}$

for low-cycle fatigue.

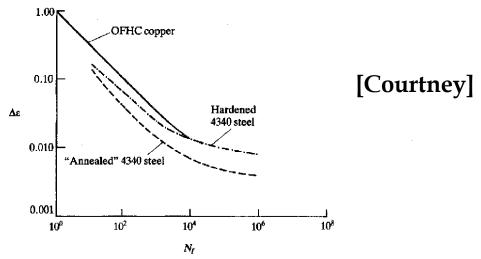


Figure 12.12 Relationship between strain range ($\Delta \epsilon$) and N_f for several engineering metals. The low-cycle behavior of the differently heat-treated alloy steels does not differ all that much. However, the hardened 4340 steel manifests longer fatigue lifetimes during high-cycle fatigue. The ductile copper alloy, which work hardens rapidly, is a superior fatigue-resistant material for low-cycle applications. However, it would perform poorly compared to either of the steels in high-cycle applications. (Data from R. C. Boettner, C. Laird, and A. J. McEvily, Jr., Trans TMS-AIME, 233, 379, 1965, and S. S. Manson, Exp. Mech., 5, 193, 1965.)

Cyclic Strain control: low cycle

- Constitutive relation for cyclic stress-strain: $\Delta \sigma = K'(\Delta \varepsilon)^{n'}$
- $n' \approx 0.1-0.2$
- Fatigue life: Coffin Manson relation:

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon_f' \left(2N_f \right)^c$$

- ε_f ~ true fracture strain; close to tensile ductility
- $c \approx -0.5 \text{ to } -0.7$
- c = -1/(1+5n'); large $n' \rightarrow$ longer life.

Cyclic Strain control: high cycle

- For elastic-dominated strains at high cycles, adapt Basquin's equation: $\sigma_a = E \frac{\Delta \varepsilon_e}{2} = \sigma_f'(2N)^b$
- Intercept on strain axis of extrapolated elastic line = σ_f/E .
- High cycle = elastic strain control: slope (in elastic regime) = b = -n'/(1+5n') [Courtney, fig. 12.13]
- The high cycle fatigue strength, σ_f , scales with the yield stress \Rightarrow high strength good in high-cycle

Strain amplitude - cycles

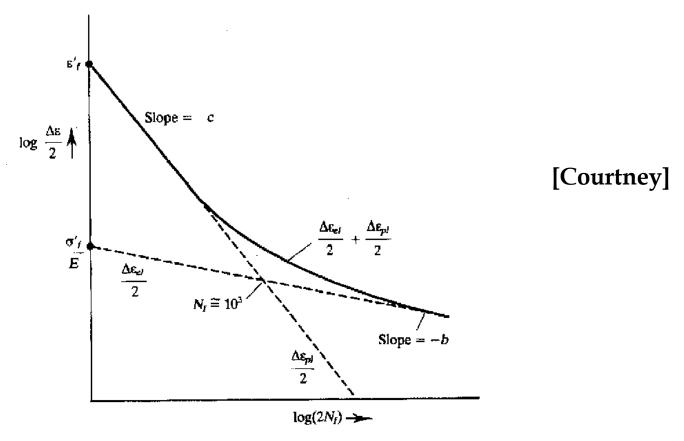


Figure 12.13 A schematic of the strain amplitude–number of stress (strain) reversals as given by Eq. (12.4). At low N_f values, $\Delta \varepsilon \cong \Delta \varepsilon_{pl}$, and the slope of this line on logarithmic coordinates is -c with an intercept (at $\log{(2N_f)}=0$) of ε_f' . At high cycles the logarithmic slope is -b (b < c), and the extrapolation of this portion of the line (where $\Delta \varepsilon \cong \Delta \varepsilon_{el}$) produces the intercept σ_f'/E .

Total strain (plastic+elastic) life

- Low cycle = plastic control: slope = c
- Add the elastic and plastic strains.

$$\frac{\Delta}{2} = \frac{\Delta \varepsilon_{el}}{2} + \frac{\Delta \varepsilon_{pl}}{2} = \frac{\sigma_f'}{E} \left(2N_f \right)^{-b} + \varepsilon_f' \left(2N_f \right)^{-c}$$

- Cross-over between elastic and plastic control is typically at N_f = 10³ cycles.
- Ductility useful for low-cycle; strength for high cycle
- Examples of Maraging steel for high cycle endurance, annealed 4340 for low cycle fatigue strength.

Varying-Fluctuating-Cumulative fatigue Damage

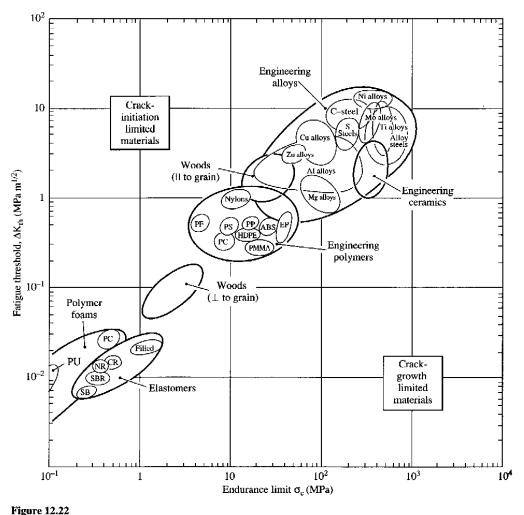
*Variable Stress/Strain Histories

- When the stress/strain history is stochastically varying, a rule for combining portions of fatigue life is needed.
- Palmgren-Miner Rule is useful: n_i is the number of cycles at each stress level, and N_{fi} is the failure point for that stress.

[Ex. Problem 12.2]

$$\sum_{i} \frac{n_i}{N_{f_i}} = 1$$

Fatigue Property Map



A material property chart displaying the fatigue threshold stress intensity (ΔK_{th} , obtained at R=0) vs. endurance limit (σ_e , appropriate for R=-1). Although these two properties correlate for the several material classes, there are some subtleties. Ceramics, for example, have relatively high values of the ratio $\sigma_e/\Delta K_{th}$. Thus, they are more prone to crack-growth-limited fatigue fracture (extrinsic fatigue, cf. Fig.

12.21). Conversely, materials having high values of ΔK_{th} vis-à-vis σ_e (e.g., some of the tough metals) are more prone to intrinsic fatigue, which involves nucleation of the fatigue cracks that result in fracture (also see Fig. 12.21). (Adapted from N. A. Fleck, K. J. Kang, and M. F. Ashby, "The Cylic Properties of Engineering Materials," Acta Metall, et Mater., 42, 365, Copyright 1994, with permission from Elsevier Science.)

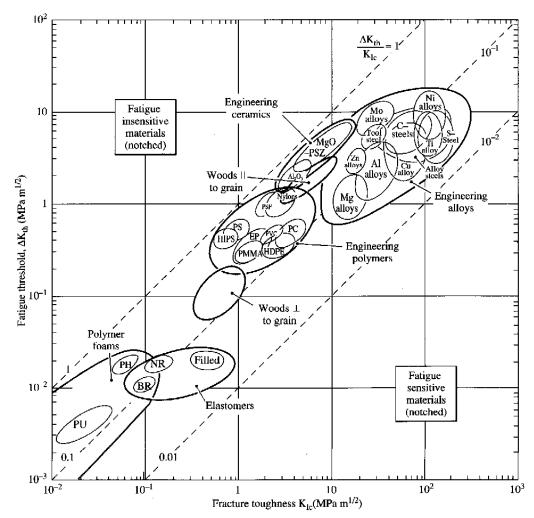


Figure 12.23

A material property chart illustrating the relationship between the fatigue threshold stress intensity (ΔK_{th} , values shown are for R=0) and fracture toughness (K_{Ic}) for several material classes. Materials having ΔK_{th} values close to K_{Ic} are fatigue "insensitive," meaning that preexisting cracks they contain propagate readily. Design with such materials (e.g., ceramics), therefore, is based on a fracture toughness criterion. Most of the metals have high values of K_{Ic} relative to ΔK_{th} . Thus, metals containing preexisting cracks are prone to have these cracks propagate under cyclic loading; most metals, therefore, are considered fatigue "sensitive." (Adapted from N. A. Fleck, K. J. Kang, and M. F. Ashby, "The Cylic Properties of Engineering Materials," Acta Metall. et Mater., 42, 365, Copyright 1994, with permission from Elsevier Science.)