Design based on Strength, Rigidity, Fatigue and Creep

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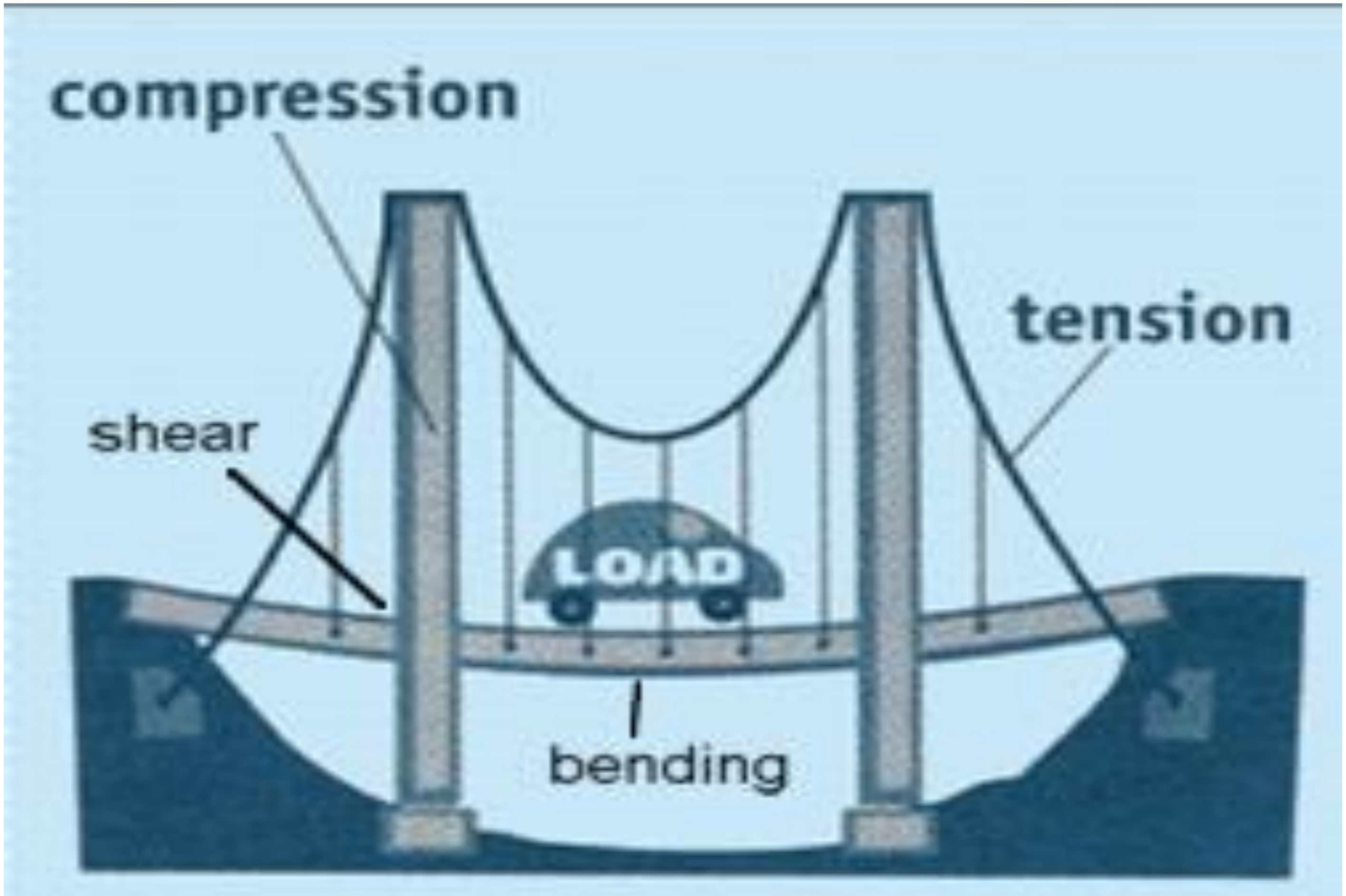
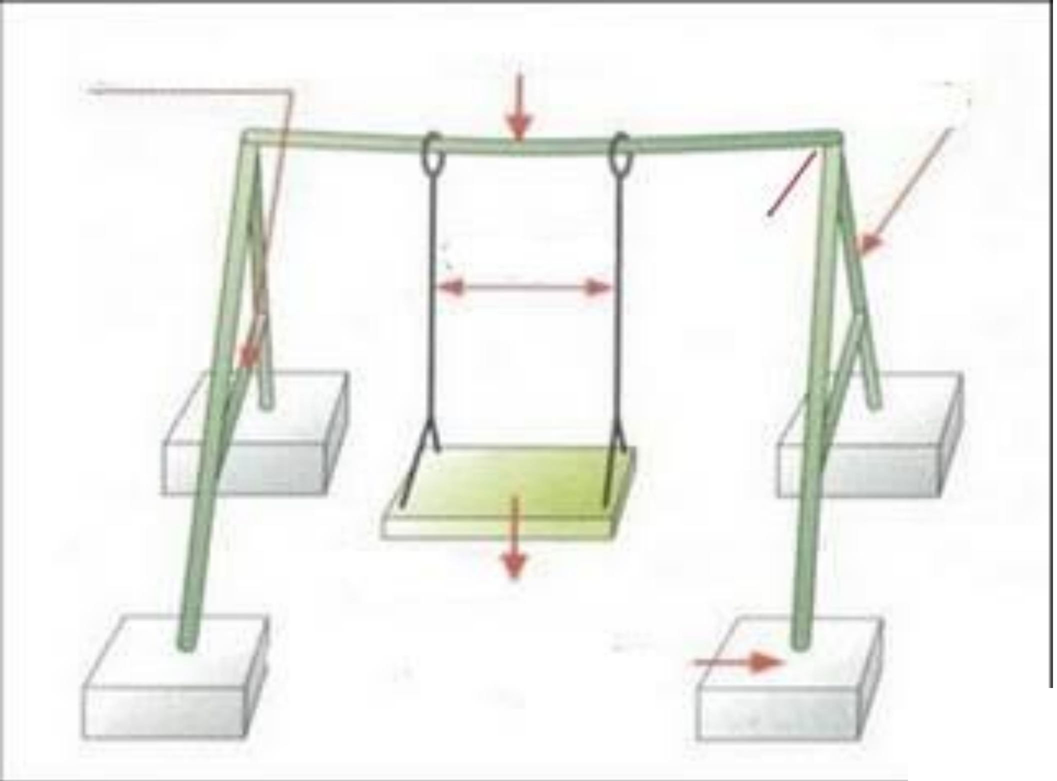
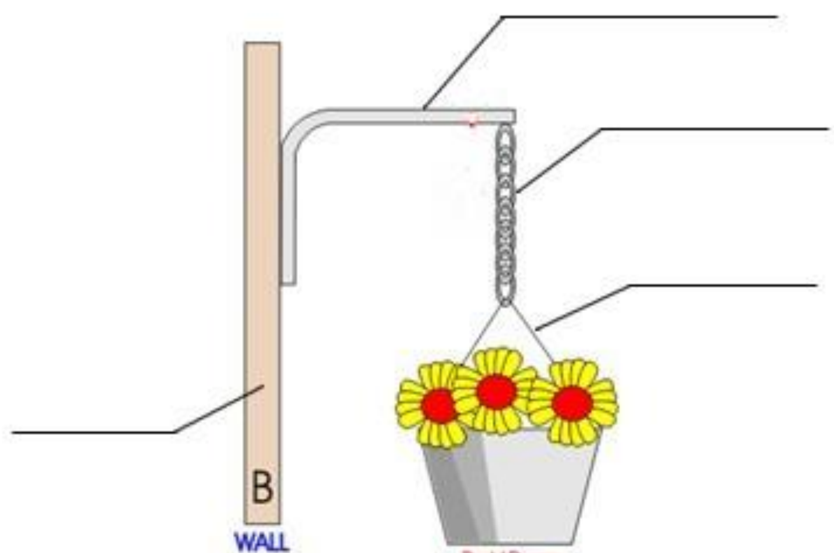
Concept of Forces & Stresses

Strength & Rigidity



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Brief Review about Fundamentals of Fatigue

Introduction

- Fatigue failure results mainly due to variable loading or more precisely due to cyclic variations in the applied loading or induced stresses.
- Example like;

  (i) human beings get fatigue when a specific task is repeatedly performed, in a similar manner

  (ii) metallic components subjected to variable loading get fatigue, which leads to their premature failure under specific conditions
- Fatigue loading is primarily the type of loading which causes cyclic variations in the applied stress or strain on a component. Thus any variable loading is basically a fatigue loading.

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In reality most mechanical components experience variable loading due to;

- Change in the magnitude of applied load

  Example: punching or shearing operations-

- Change in direction of load application

  Example: connecting rod

- Change in point of load application

  Example: rotating shaft

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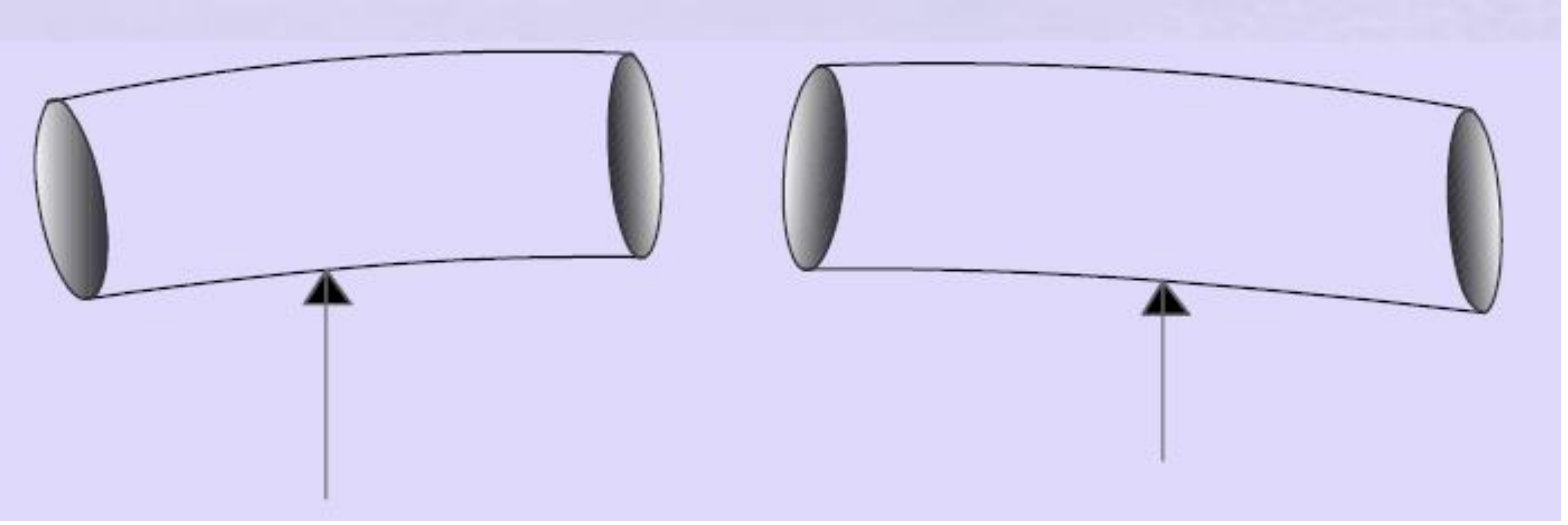
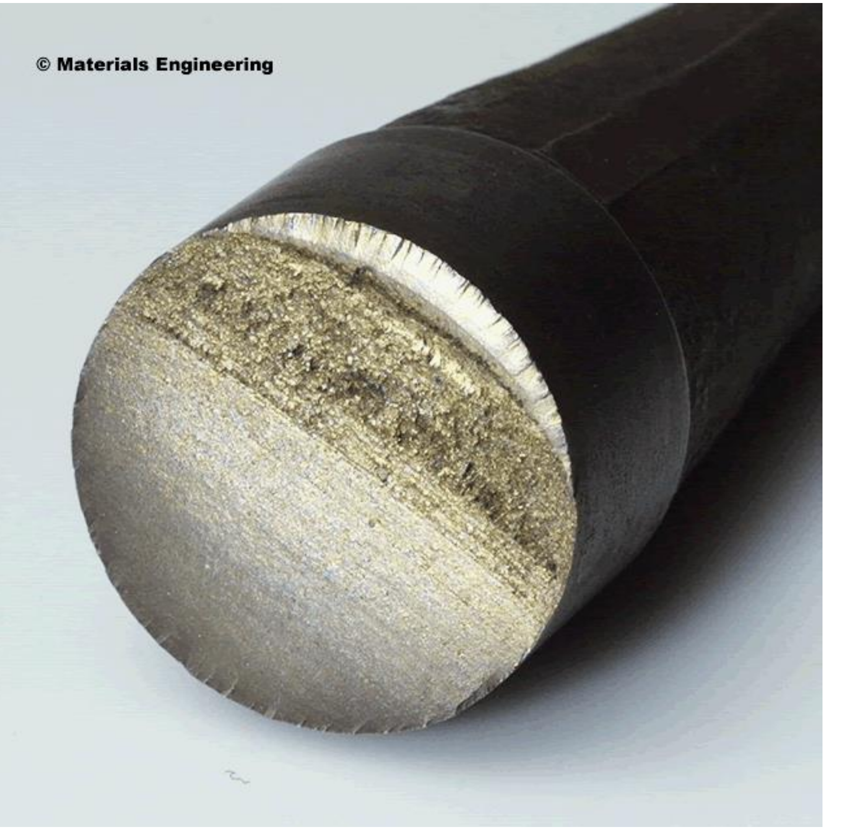


A rotating shaft with a bending load applied
it is a good example of fully reversible load (worst case of fatigue)

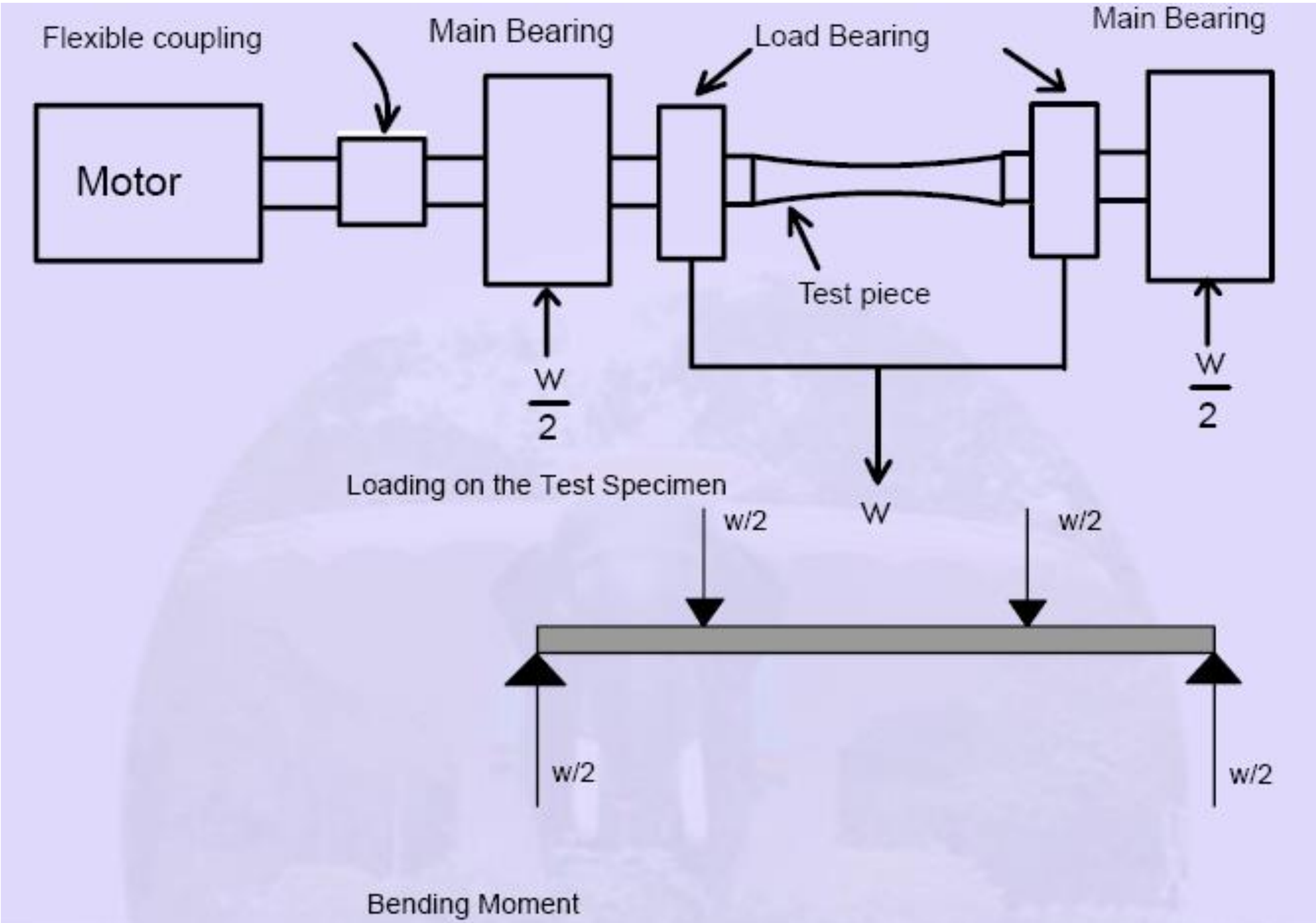
Fatigue Failure Stages

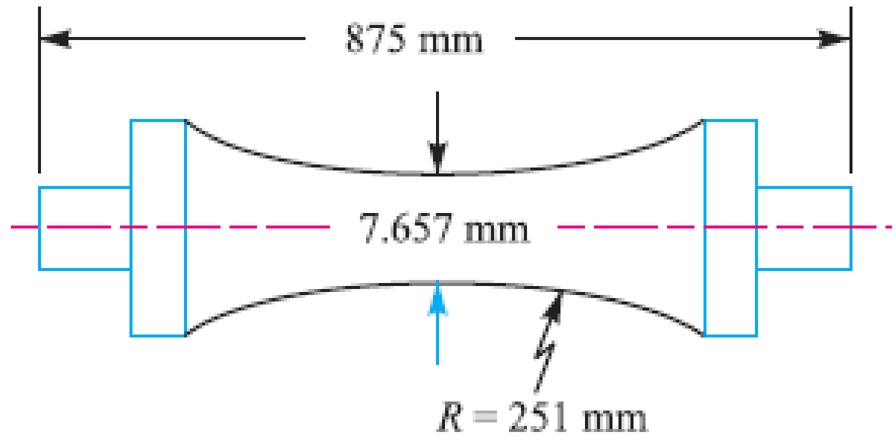
Three stages are involved in fatigue failure namely;

- -Crack initiation
- -Crack propagation
- -Fracture

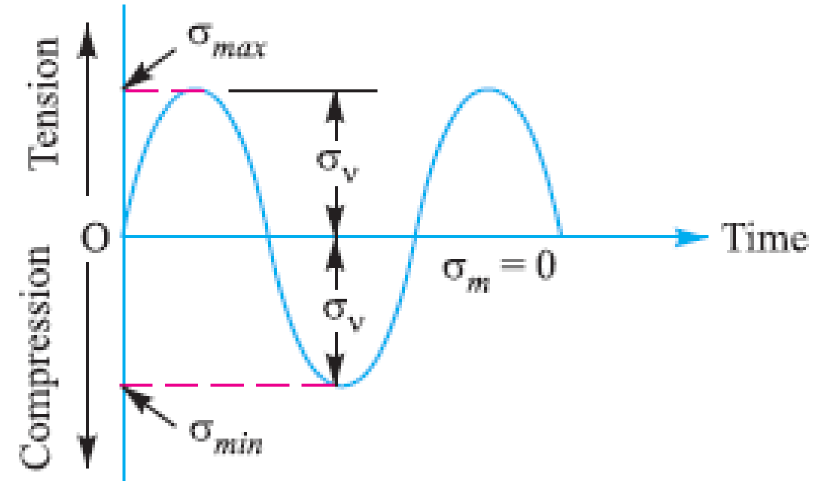


Fatigue Testing

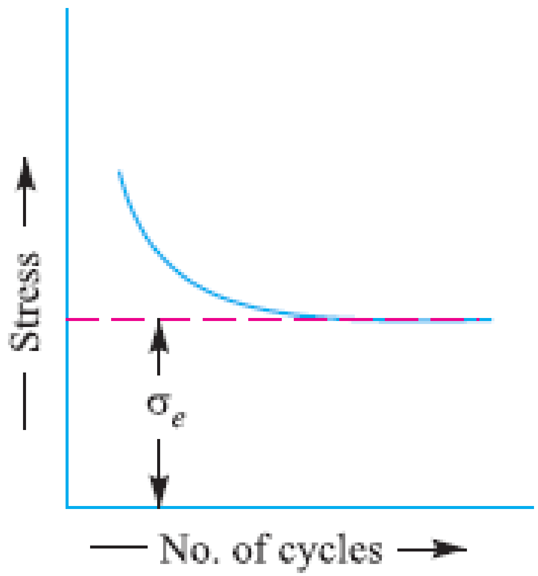




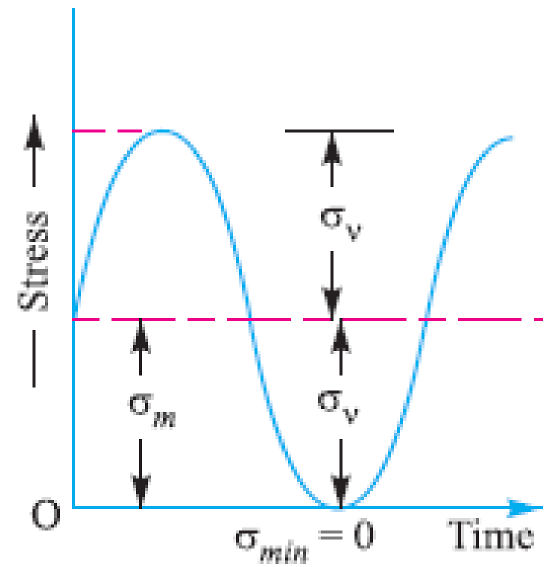
(*a*) Standard specimen.



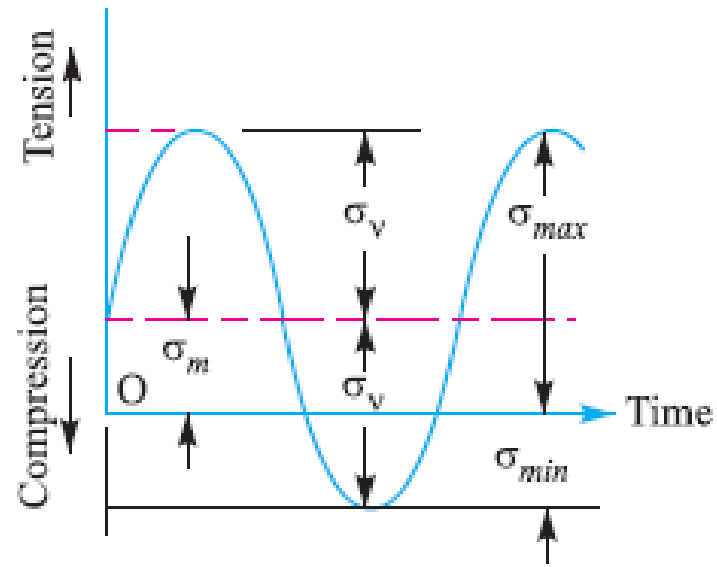
(*b*) Completely reversed stress.



(*c*) Endurance or fatigue limit.



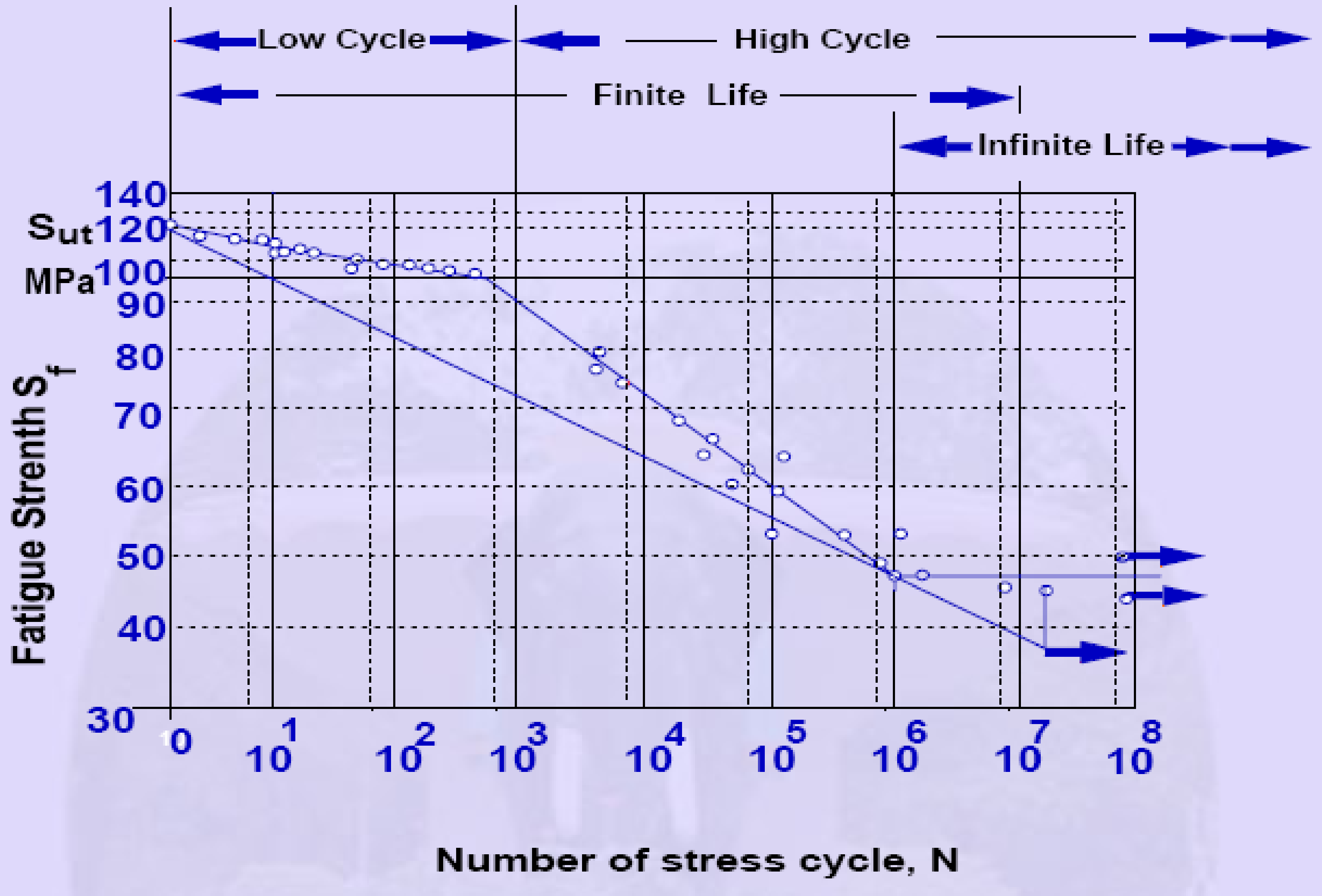
(*d*) Repeated stress.



(*e*) Fluctuating stress.

The S-N Diagram

- Tests on several specimens are conducted under identical conditions with varying levels of stress amplitude.
- The cyclic stress level of the first set of tests is some large percentage of the Ultimate Tensile Stress (UTS), which produces failure in a relatively small number of cycles.
- Subsequent tests are run at lower cyclic stress values until a level is found at which the samples will survive 10 million cycles without failure.
- The results are plotted as an S-N diagram (see the figure) usually on semi-log or on log-log paper, depicting the life in number of cycles tested as a function of the stress amplitude. A.



Low Cycle
High Cycle
Finite Life
Infinite Life
Sut
MPa
140
120
100
90
80
70
60
50
40
30
Fatigue Strenth Sf
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
Number of stress cycle, N

S-N Curve

Continue...

Low Cycle Fatigue

- The body of knowledge available on fatigue failure from N=1 to N=1000 cycles is generally classified as low-cycle fatigue.

High Cycle Fatigue

- High-cycle fatigue, then, is concerned with failure corresponding to stress cycles greater than 10^3 cycles. (Note that a stress cycle (N=1) constitutes a single application and removal of a load and then another application and removal of load in the opposite direction. Thus N= ½ means that the load is applied once and then removed, which is the case with the simple tensile test.

Finite and Infinite Life

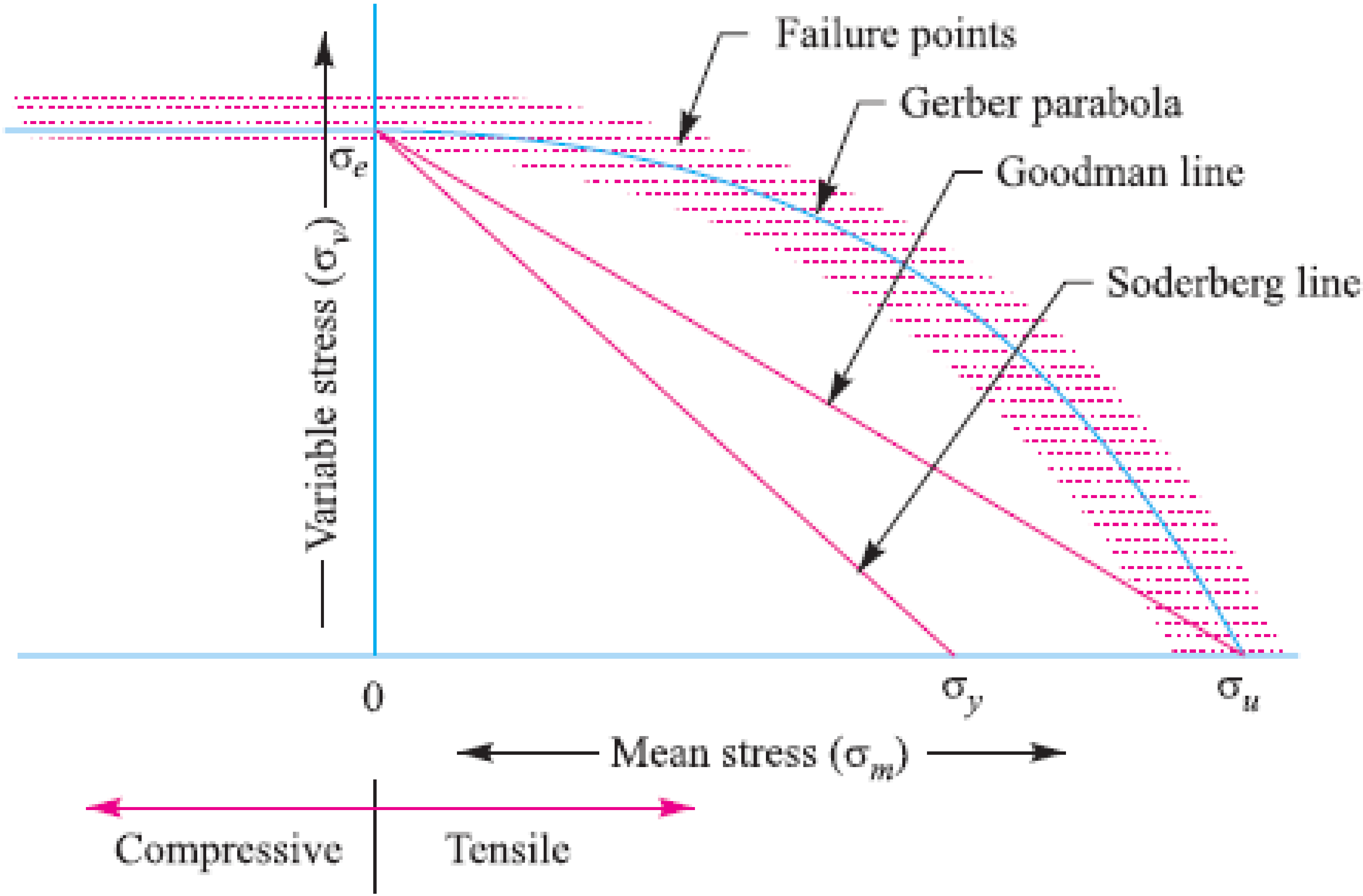
- We also distinguish a finite-life and an infinite-life region. Finite life region covers life in terms of number of stress reversals up to the knee point. (in case of steels) beyond which is the infinite-life region. The boundary between these regions cannot be clearly defined except for specific materials; but it lies somewhere between 10^6 and 10^7 cycles, for materials exhibiting fatigue limit.

Endurance or Fatigue Limit

- In the case of the steels, a knee (flattening or saturation) occurs in the graph, and beyond this knee failure will not occur, no matter how large the numbers of cycles are. The strength (stress amplitude value) corresponding to the knee is called the endurance limit (Se) or the fatigue limit. However the graph never does become horizontal for non-ferrous metals and alloys, hence these materials do not have an endurance limit.
- Endurance or fatigue limit can be defined as the magnitude of stress amplitude value at or below which no fatigue failure will occur, no matter how large the number of stress reversals are, in other words leading to an infinite life to the component or part being stressed. For most ferrous materials Endurance limit (Se) is set as the cyclic stress level that the material can sustain for 10 million cycles.

Combined Steady and Variable Stress

- The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. as functions of variable stress (σ_v) and mean stress (σ_m).
- The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.



Failure points
Gerber parabola
Goodman line
Soderberg line
σ_e
Variable stress (σ_v)
0
σ_y
σ_u
Mean stress (σ_m)
Compressive
Tensile

Example

- *A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m2 and – 150 MN/m2. Determine the value of minimum ultimate strength according to 1.Gerber relation; 2. Goodman relation; and*

  3. Soderberg relation.

  Take yield strength = 0.55 Ultimate strength;

  Endurance strength = 0.5 Ultimate strength; and

  factor of safety = 2.

Solution

Given : σ1 = 300 MN/m2 ;

σ2 = – 150 MN/m2 ;

σ*y* = 0.55 σ*u* ; σ*e* = 0.5 σ*u* ; *F.S*. = 2

σ*u* = Minimum ultimate strength in MN/m2.

Calculating; the mean or average stress, and variable stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

.

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

According to Gerber relation

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\,\sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\,\sigma_u$$

or $$(\sigma_u)^2 - 900\,\sigma_u - 22\,500 = 0$$

$$\therefore \quad \sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$= 924.35$ MN/m^2 **Ans.** ...(Taking +ve sign)

According to Goodman relation

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\ \sigma_u} = \frac{525}{\sigma_u}$$

$$\sigma_u = 2 \times 525 = 1050\ \text{MN/m}^2$$ **Ans.**

According to Soderberg relation

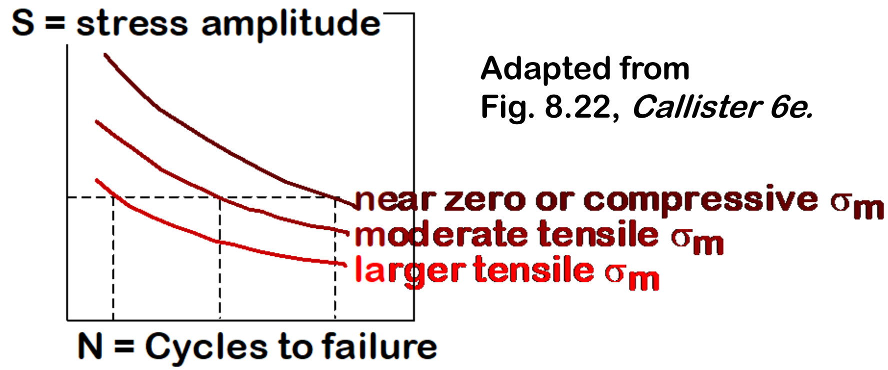
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{75}{0.55\,\sigma_u} + \frac{255}{0.5\,\sigma_u} = \frac{586.36}{\sigma_u}$$

$$\sigma_u = 2 \times 586.36 = 1172.72 \text{ MN/m}^2$$ **Ans.**

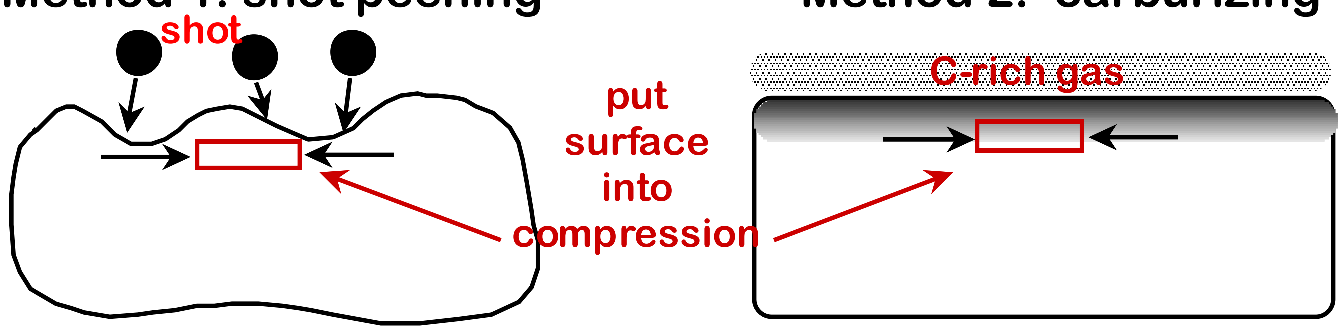
IMPROVING FATIGUE LIFE

1. Impose a compressive surface stress (to suppress surface cracks from growing)

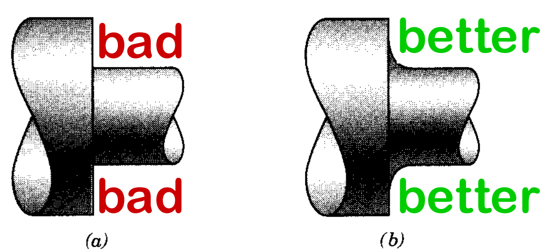


Adapted from Fig. 8.22, *Callister 6e.*

--Method 1: shot peening **--Method 2: carburizing**



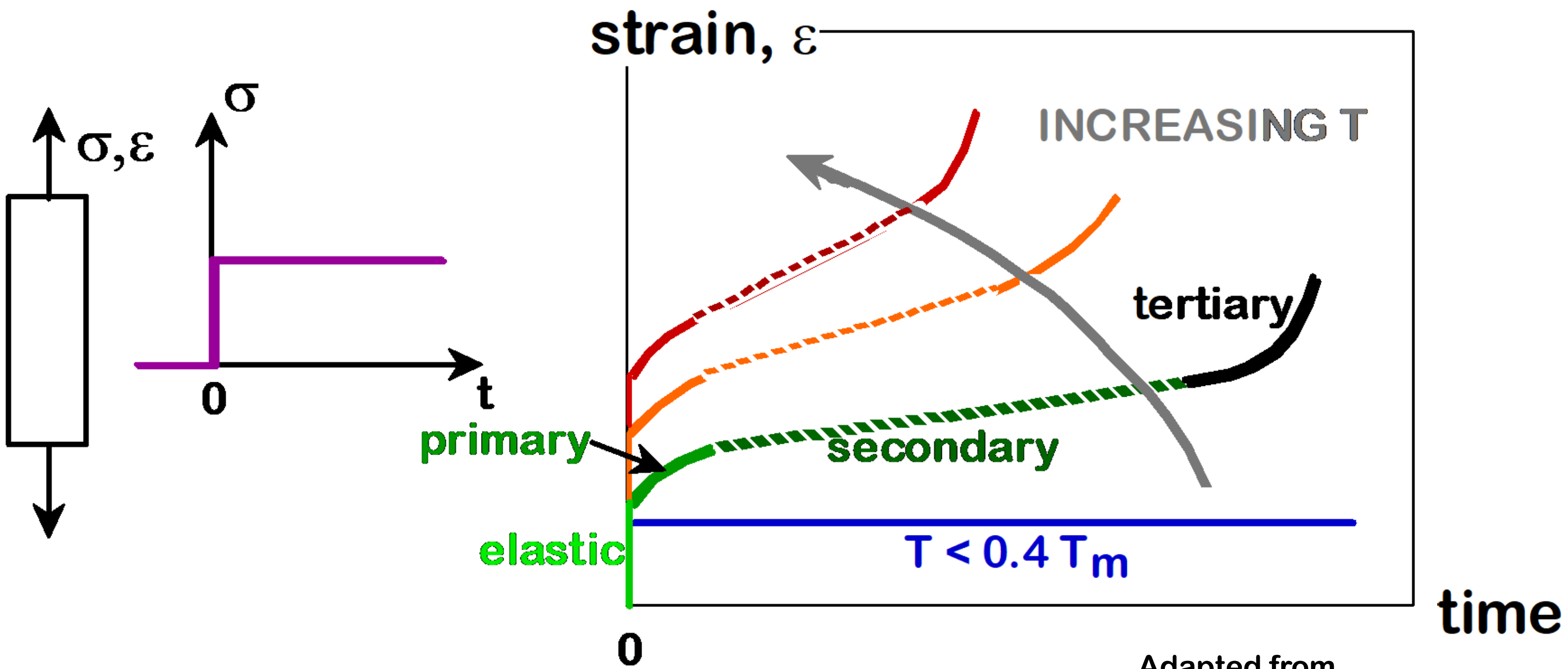
2. Remove stress concentrators.



Adapted from Fig. 8.23, *Callister 6e.*

CREEP

- Occurs at elevated temperature, $T > 0.4\ T_{melt}$
- Deformation changes with time.



Adapted from
Figs. 8.26 and 8.27,
Callister 6e.

- Primary: creep strain increase slows down (strain hardening)
- Secondary: linear, steady-state creep (strain hardening balanced by recovery)
- Tertiary: grain boundary separation, formation of voids, etc.

SECONDARY CREEP

- Most of component life spent here.
- Strain rate is constant at a given T, σ

  --strain hardening is balanced by recovery

$$\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

strain rate ($\dot{\varepsilon}_s$)

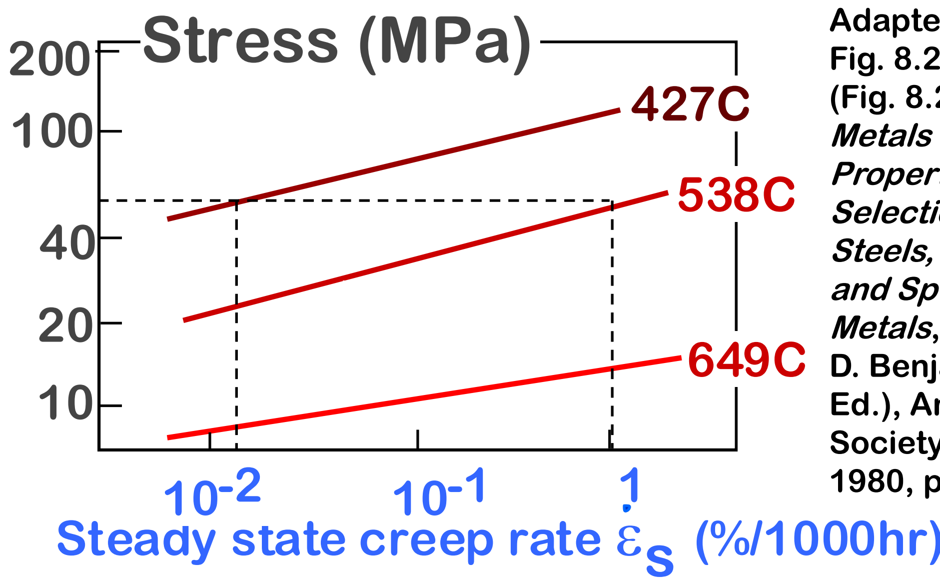
material const. (K_2)

applied stress (σ)

stress exponent (material parameter) (n)

activation energy for creep (material parameter) (Q_c)

- Strain rate increases for larger T, σ



Adapted from Fig. 8.29, *Callister 6e.* (Fig. 8.29 is from *Metals Handbook: Properties and Selection: Stainless Steels, Tool Materials, and Special Purpose Metals*, Vol. 3, 9th ed., D. Benjamin (Senior Ed.), American Society for Metals, 1980, p. 131.)

A typical creep behavior is presented in the diagram: