THEORY OF GEARS

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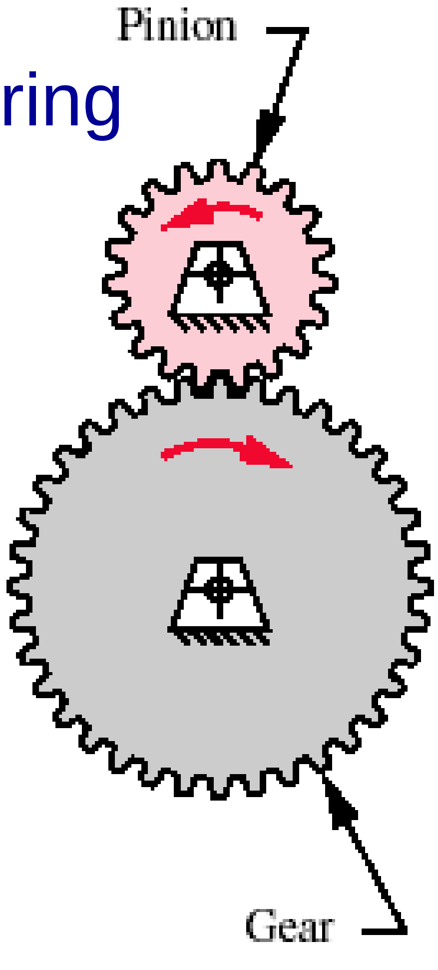
Outline

- Gear Theory
  - Fundamental Law of Gearing
  - Involute profile
- Nomenclature
- Gear Trains
- Loading
- Stresses



Fundamental Law of Gearing

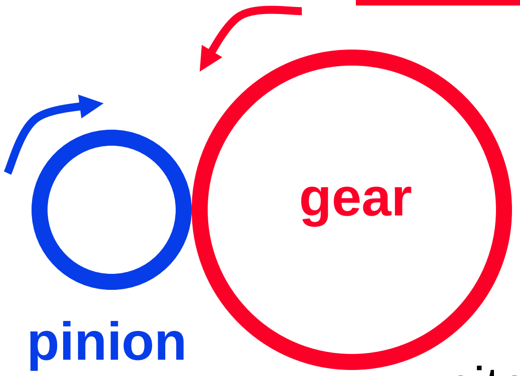
functionally, a gear set is a device to exchange torque for velocity

$$P = T\omega$$

the angular velocity ratio of the gears of a gear set must remain constant throughout the mesh

What gear tooth shape can do this?

click here!

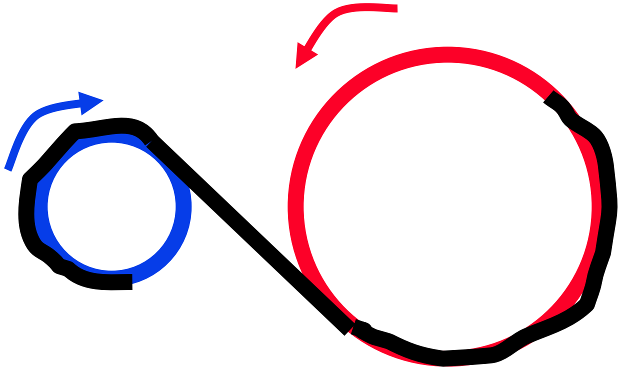


$$m_V = \frac{\omega_{out}}{\omega_{in}} = \frac{r_{out}}{r_{in}}$$

pitch circle, pitch diameter d, pitch point

Towards the Involute Profile

A belt connecting the two cylinders



line of action, pressure angle ϕ

The Involute Profile

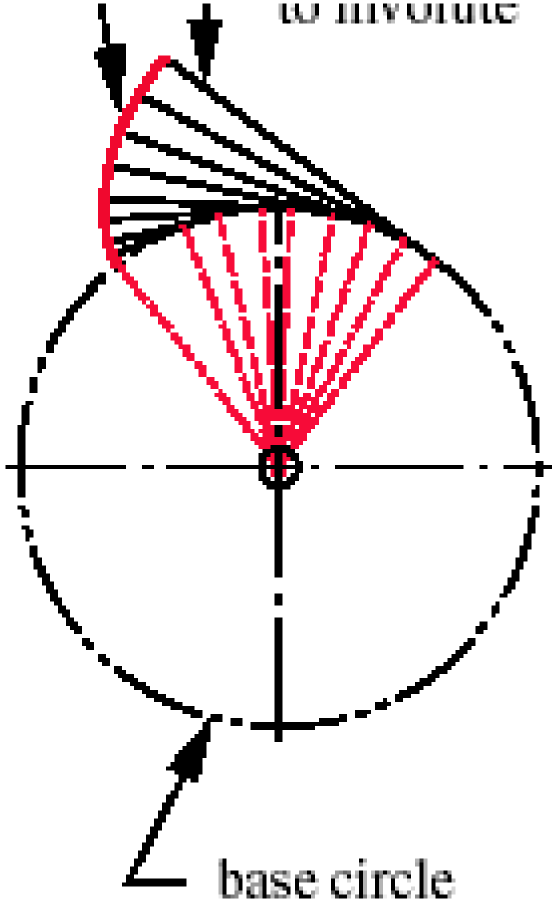
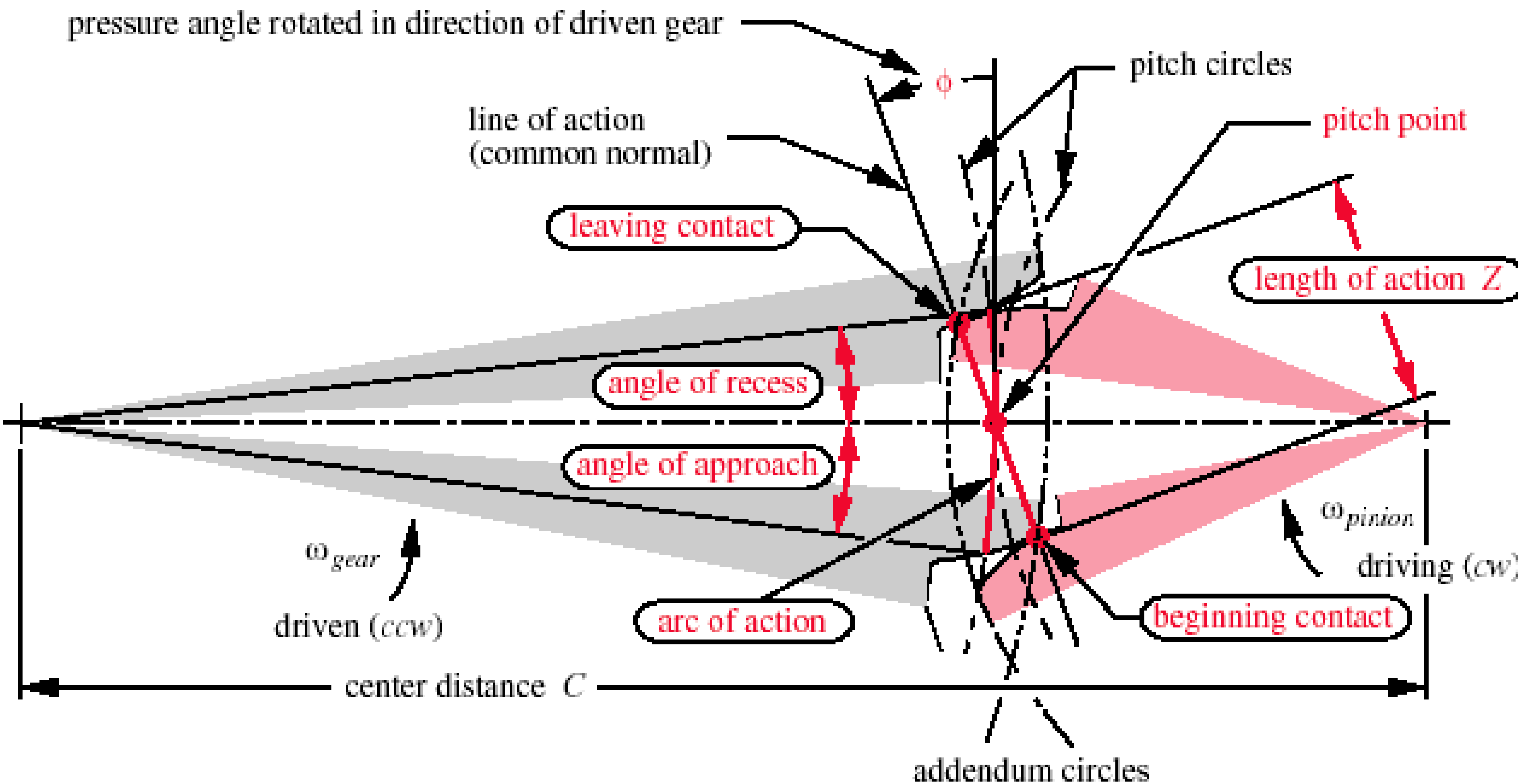


FIGURE 11-3

Profile of the Involute Profile



pressure angle, line of action, length of action, addendum

Involute in Action

video from http://www.howstuffworks.com

Nomenclature

$$p_c = \frac{\pi d}{N}$$

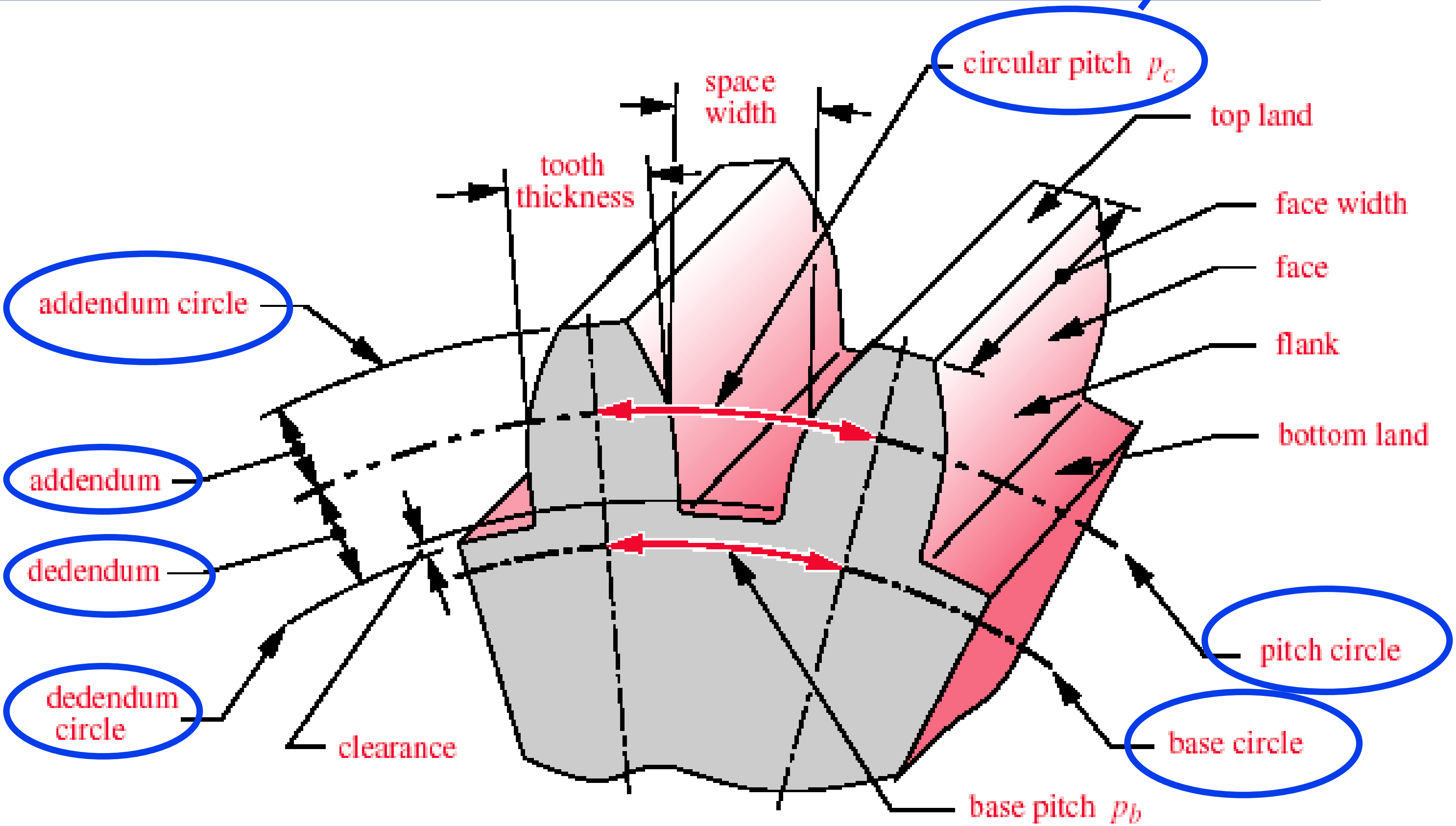


Figure 11-8

Pitches, Etc.

circular pitch (mm, in.) $$p_c = \frac{\pi d}{N}$$

base pitch (mm, in.) $$p_b = p_c \cos\phi$$

diametral pitch (teeth/in.) $$p_d = \frac{N}{d}$$

module (mm/teeth) $$m = \frac{d}{N}$$

Velocity Ratio

pitches must be equal for mating gears, therefore

$$m_V = \pm \frac{r_{out}}{r_{in}} = \pm \frac{N_{out}}{N_{in}}$$

Contact Ratio

average number of teeth in contact at any one time

=

length of action divided by the base pitch, or,

$$m_p = \frac{p_d Z}{\pi \cos\phi}$$

$$Z = \sqrt{\left(r_p + a_p\right)^2 - \left(r_p \cos\phi\right)^2} + \sqrt{\left(r_g + a_g\right)^2 - \left(r_g \cos\phi\right)^2} - C \sin\phi$$

where C=center distance=(N_g+N_p)*1/p_d*1/2

Minimum # of Teeth

minimum # of teeth to avoid undercutting with gear and rack

$$N_{\min} = \frac{2}{\sin^2 \phi}$$

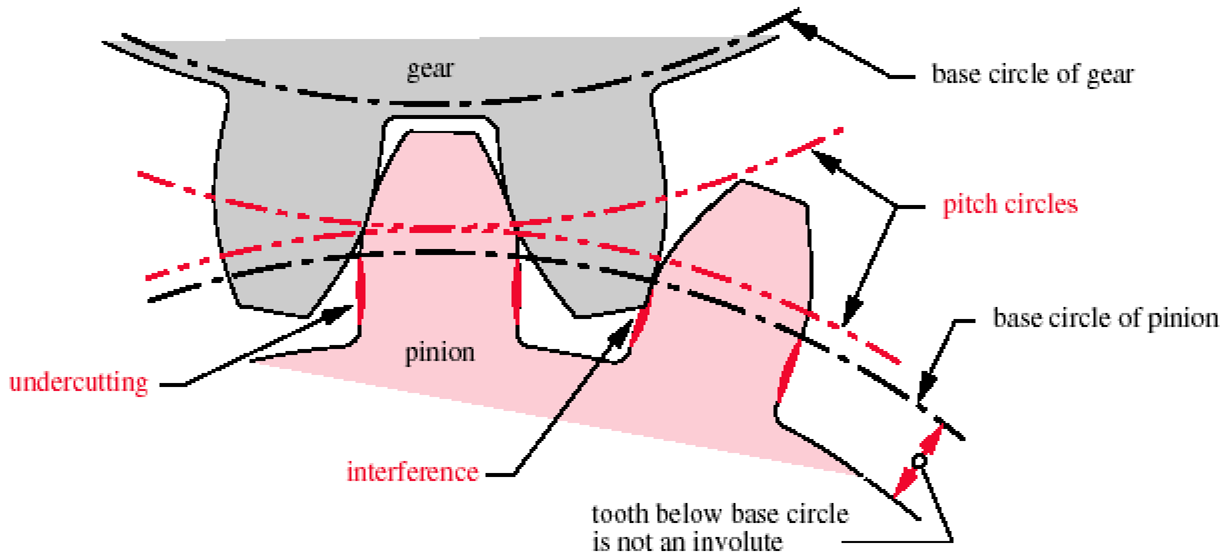


FIGURE 11-11

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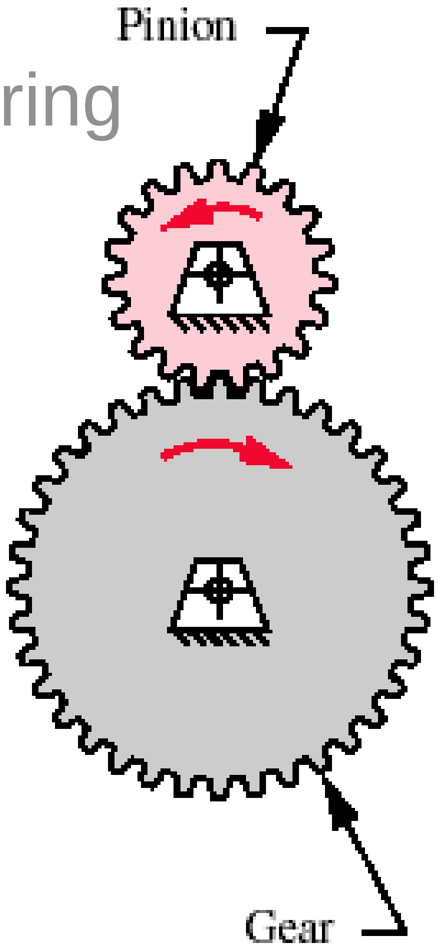
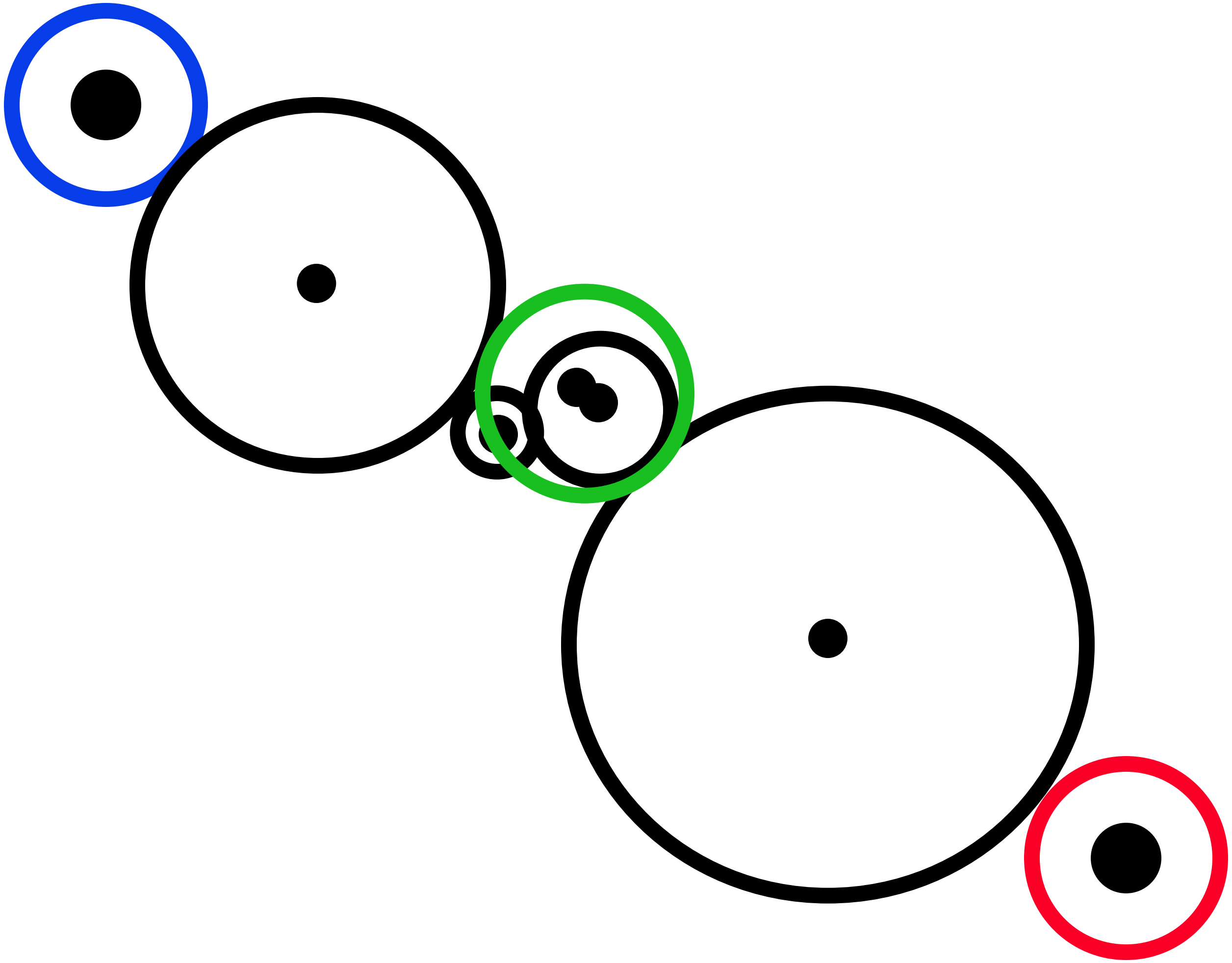
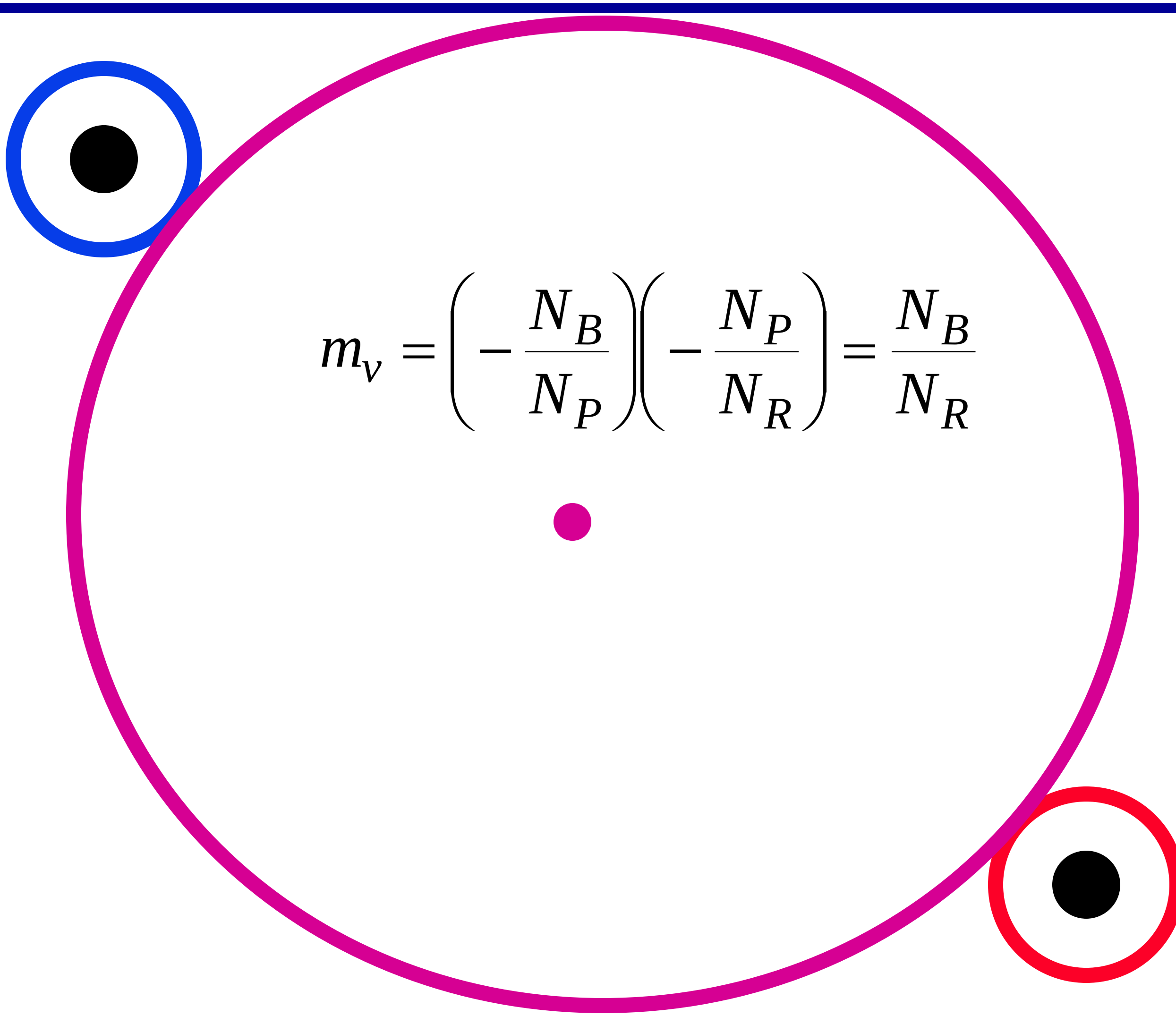


Simple Gear Trains

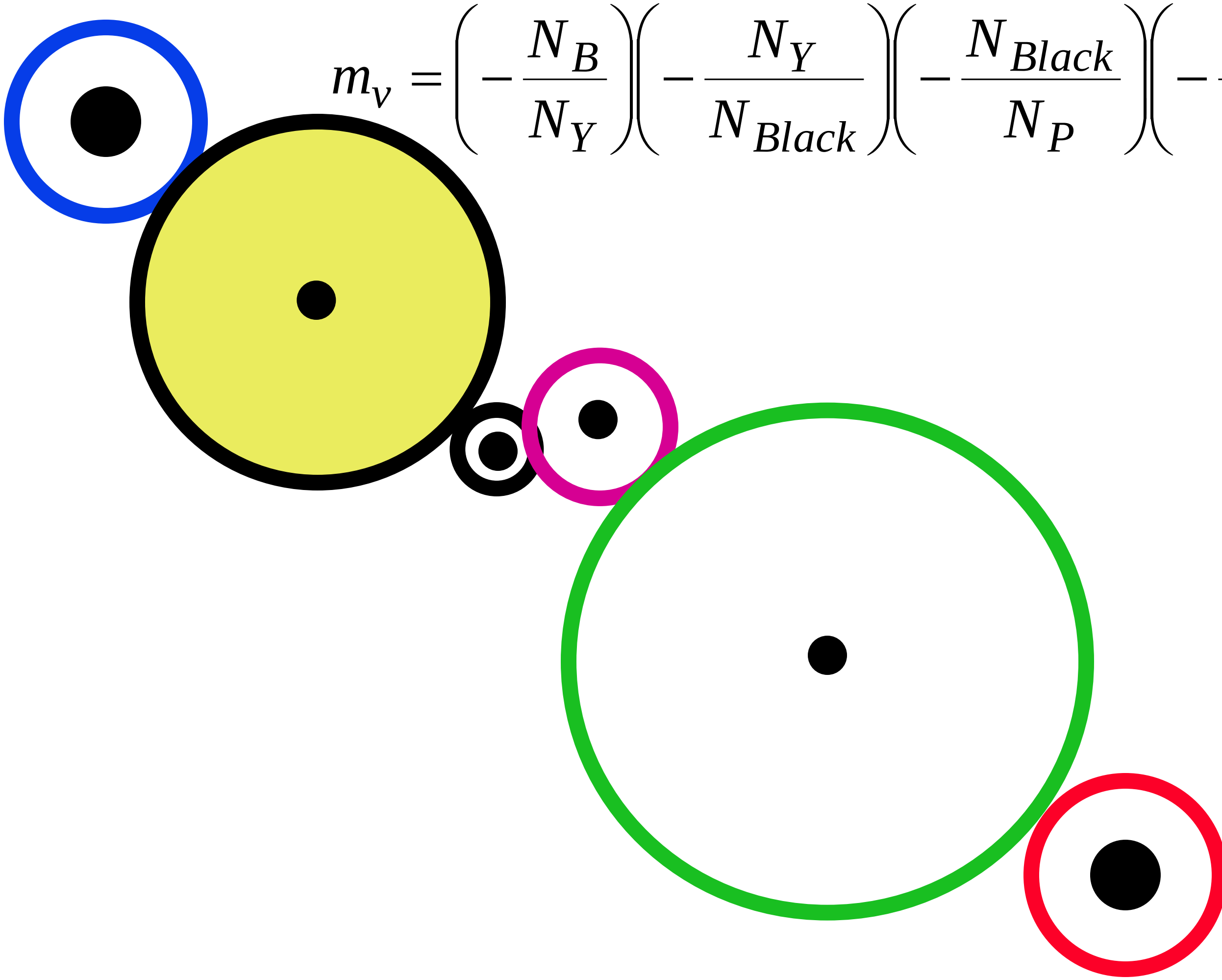


Simple Gear Train



$$m_v = \left(-\frac{N_B}{N_P} \right) \left(-\frac{N_P}{N_R} \right) = \frac{N_B}{N_R}$$

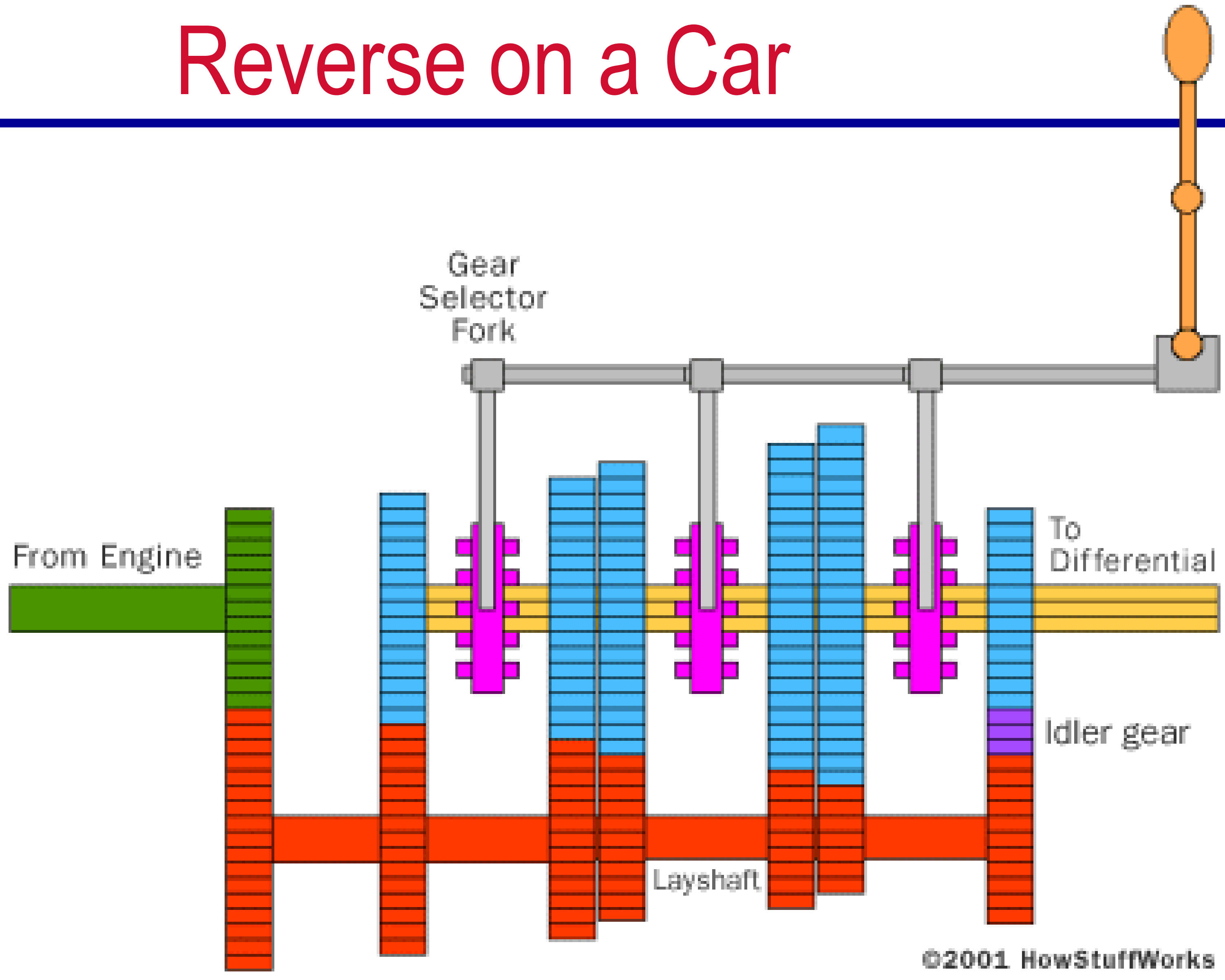
$$m_v = \left(-\frac{N_B}{N_Y}\right)\left(-\frac{N_Y}{N_{Black}}\right)\left(-\frac{N_{Black}}{N_P}\right)\left(-\frac{N_P}{N_G}\right)\left(-\frac{N_G}{N_R}\right) = -\frac{N_B}{N_R}$$



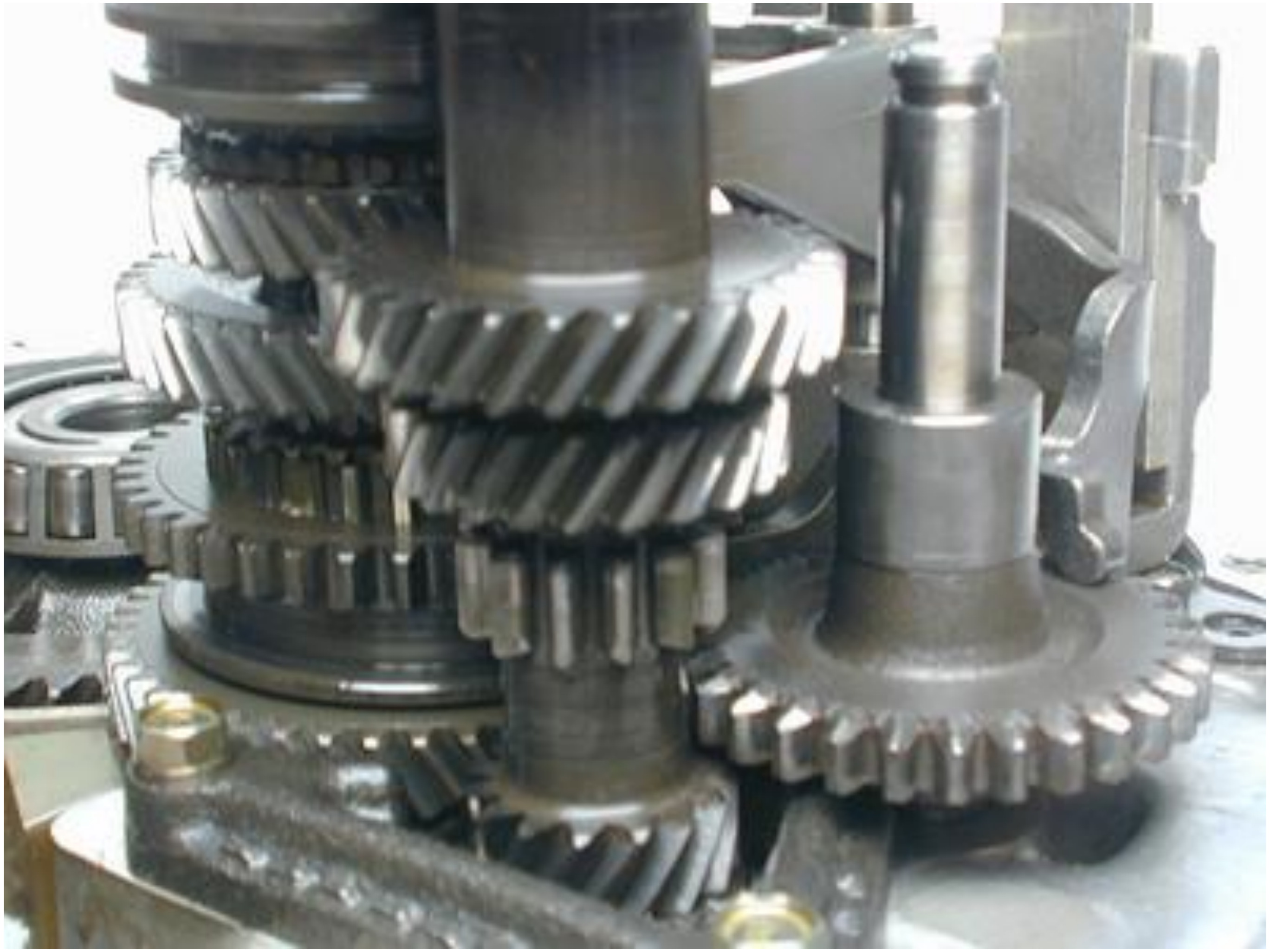
Simple Gear Train

- Fine for transmitting torque between
  - shafts in close proximity
  - when m_v does not need to be too large
- Use third gear ("idler") only for directional reasons (not for gear reduction)

Reverse on a Car



Actual Manual Transmission



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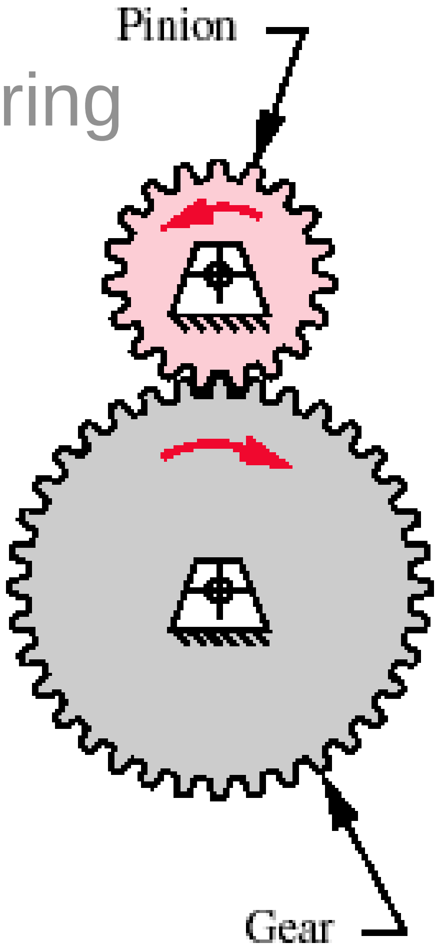


Loading of Gears

W_t= tangential (transmitted) load
Wr= radial load
W= total load

$$W_t = \frac{T_p}{r_p} = \frac{2T_p}{d_p}$$

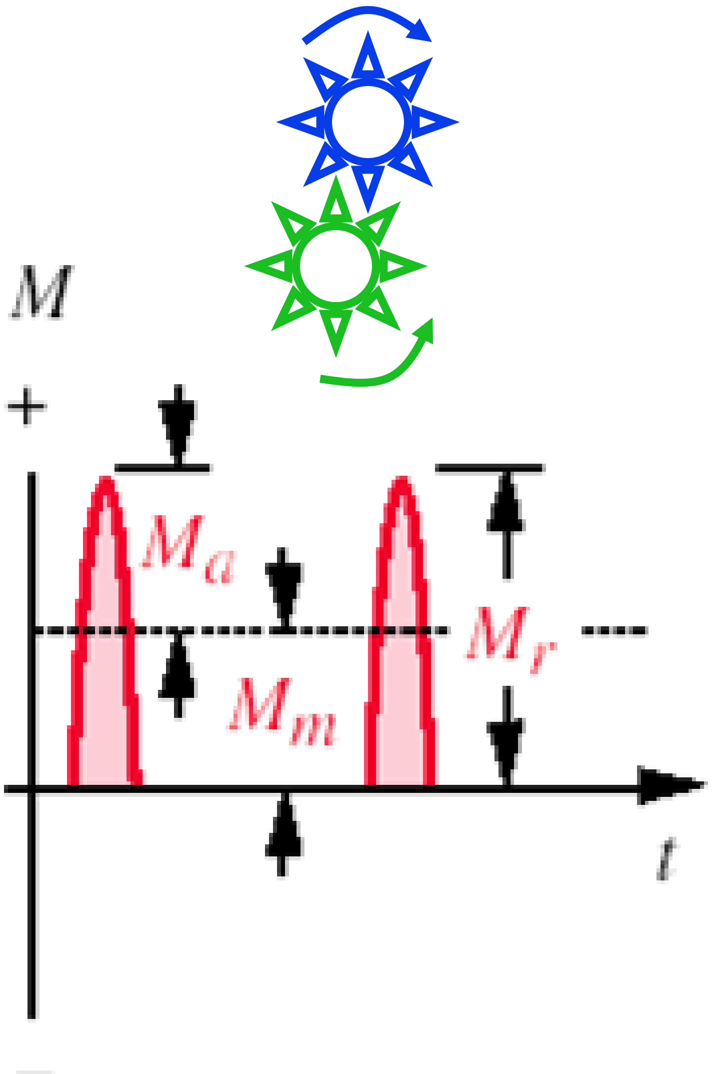
$$W_r = W_t \tan\phi$$

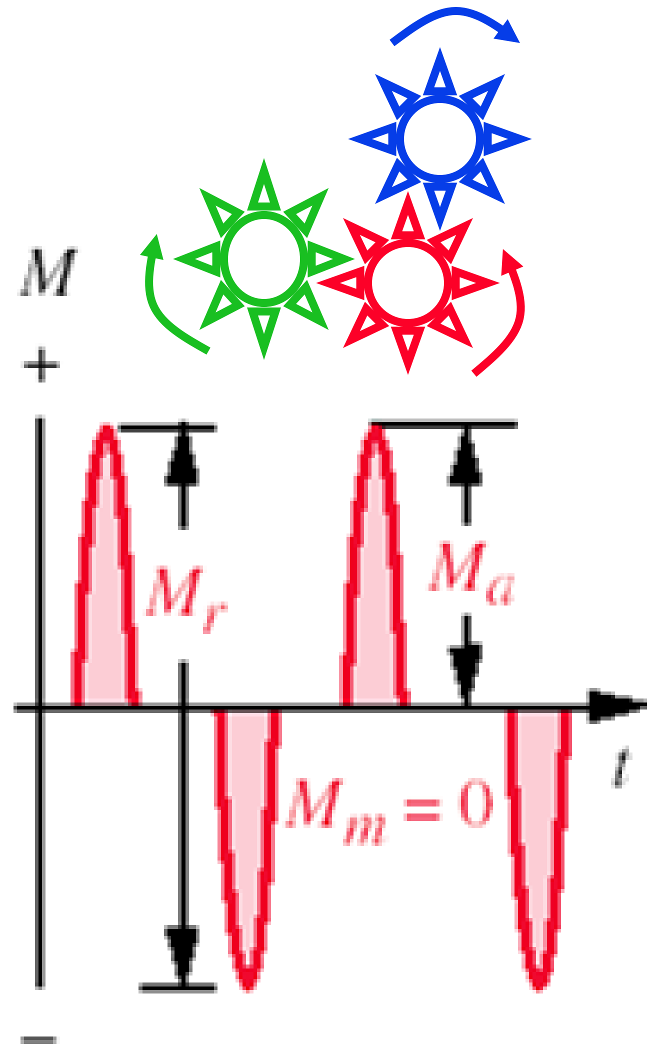
$$W = \frac{W_t}{\cos\phi}$$

Because T_p is constant, should W_t be static for a tooth?

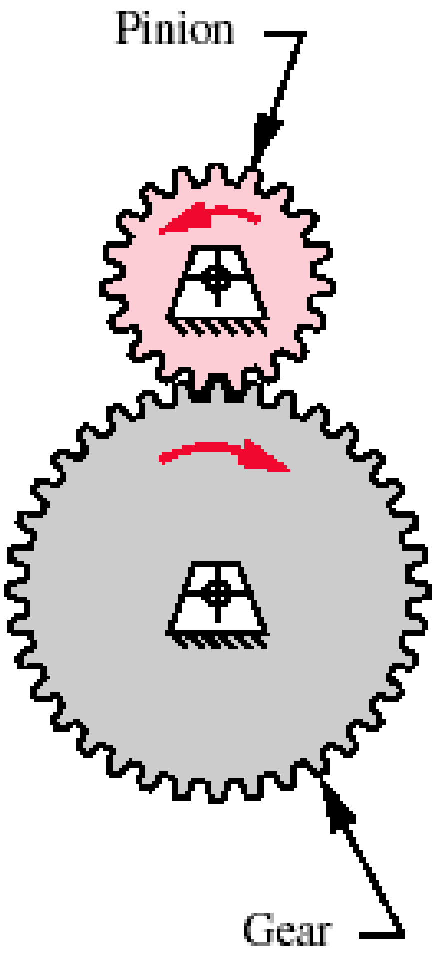
(see board for figure or page 711 in book)

Gear vs. Idler





Loading Example



Given:

- pinion is driving gear with 50hp at 1500 rpm
- N_p=20
- m_v=2 (a.k.a., m_g=2)
- p_d=4 /in.
- ϕ=20 degrees

Find:

Loading and effects of inputs on loading

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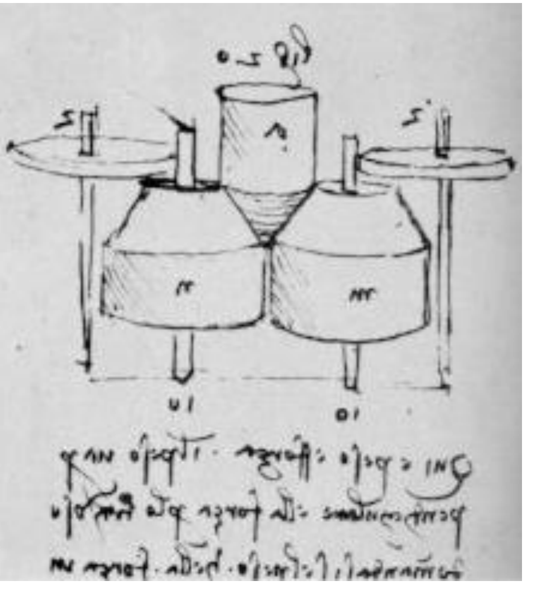


Gear Failure

Fatigue Loading

from bending of teeth

- infinite life possible
- failure can be sudden

Surface Failure

from contact b/n of teeth

- infinite life not possible
- failure is gradual

Lewis, 1892, first formulation of gear tooth fatigue failure

$$\sigma_b = \frac{W_t \cdot p_d}{F \cdot Y}$$

AGMA Gear Stress Formula

many assumptions: see page 714…

…including that the contact ratio $1<m_p<2$

$$\sigma_b = \frac{W_t \cdot p_d}{F \cdot J} \cdot \frac{K_a K_m}{K_v} K_s K_B K_I$$

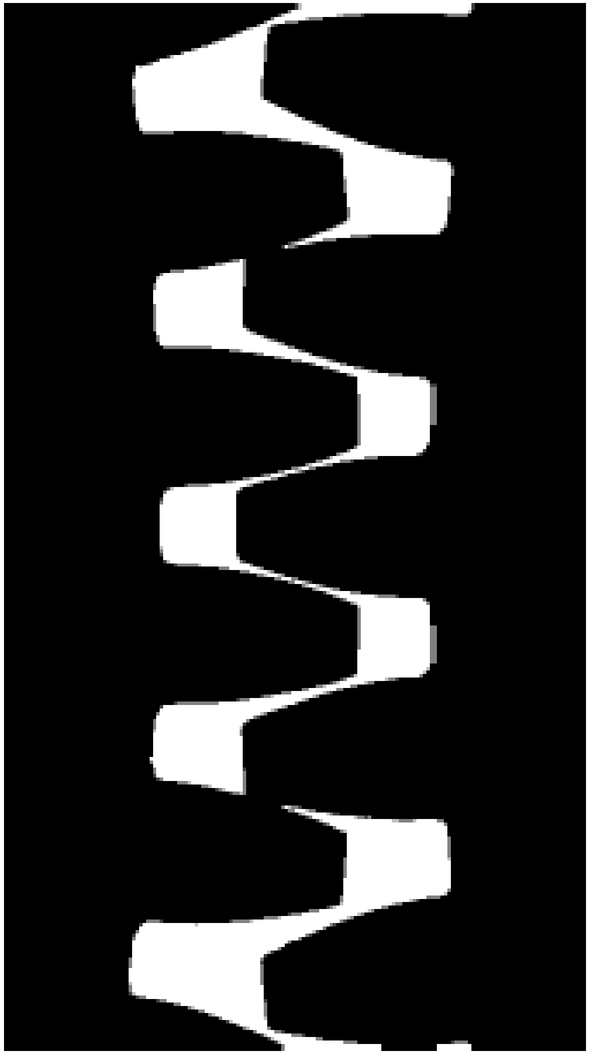
J K_v K_m K_a K_s K_B K_I

Bending Strength Geometry Factor

- from tables on pgs. 716-718
- Inputs
  - pinion or gear
  - number of teeth
  - pressure angle
  - long-addendum or full-depth
  - tip loading or HPSTC

J K_v K_m K_a K_s K_B K_I

Tip vs. HPSCT Loading



Finding J for the Example

- pinion is driving gear with 50hp at 1500 rpm
- $N_p=20$, $N_g=40$
- $p_d=4$ /in.
- $\phi=20$ degrees
- $W_t=420$ lb.

$J_p=0.34$, $J_g=0.38$

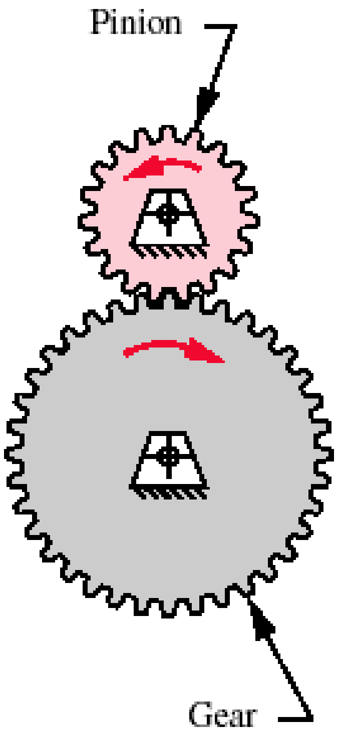


Table 11-9 AGMA Bending Geometry Factor J for 20°, Full-Depth Teetl

Gear teeth	Pinion teeth 12		14		17		21		26		
	P	G	P	G	P	G	P	G	P	G	P
12	U	U									
14	U	U	U	U							
17	U	U	U	U	U	U					
21	U	U	U	U	U	U	0.33	0.33			
26	U	U	U	U	U	U	0.33	0.35	0.35	0.35	
35	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.3
55	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.4
135	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.4

Dynamic (Velocity) Factor

to account for tooth-tooth impacts and resulting vibration loads

from Figure 11-22 or from Equations 11.16-11.18 (pages 718-719)

Inputs needed:

- Quality Index (Table 11-7) →
- Pitch-line Velocity

  V_t = (radius)(angular speed in radians)

Table 11-7
Recommended Gear Quality Numbers for Pitch Line Velocity

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

J K_v K_m K_a K_s K_B K_I

Dynamic Factor

$$B = \frac{(12 - Q_v)^{2/3}}{4} \text{ for } 6 \le Q_v \le 11$$

$$A = 50 + 56(1 - B)$$

$$K_v = \left(\frac{A}{A + \sqrt{V_t}} \right)^B \quad (US) \qquad K_v = \left(\frac{A}{A + \sqrt{200 V_t}} \right)^B \quad (SI)$$

$V_{t,max} = (A + Q_v - 3)^2$

J K_v K_m K_a K_s K_B K_I

Determining K_v for the Example

V_t=(1500rev/min)(2.5 in)(1ft/12in)(2π rad/1rev)=1964 ft/min

(well below $V_{t,max}$)

therefore, Q_v=10

$$B = \frac{(12-10)^{2/3}}{4} = 0.397$$

$$A = 50 + 56(1-0.397) = 83.8$$

$$K_v = \left(\frac{83.8}{83.8 + \sqrt{1964}} \right)^{0.397} = 0.845$$

Table 11-7

Recommended Gear Quality Numbers for Pitch Line Velocity

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

Load Distribution Factor K_m

to account for distribution of load across face

Table 11-16

Load Distribution Factors K_m

Face Width in	(mm)	K_m
<2	(50)	1.6
6	(150)	1.7
9	(250)	1.8
≥20	(500)	2.0

$8/p_d < F < 16/p_d$

can use $12/p_d$ as a starting point

$J \quad K_v \quad K_m \quad K_a \quad K_s \quad K_B \quad K_I$

K_m for the Example

$F=12/p_d=3$ in.

Table 11-16

Load Distribution Factors K_m

Face Width in	(mm)	K_m
<2	(50)	1.6
6	(150)	1.7
9	(250)	1.8
≥20	(500)	2.0

$K_m=1.63$

Application Factor, K_a

to account for non-uniform transmitted loads

Table 11-17 Application Factors K_a

Driving Machine	Driven Machine: Uniform	Driven Machine: Moderate Shock	Driven Machine: Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light Shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium Shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

for example, assume that all is uniform

J K_v K_m K_a K_s K_B K_I

Other Factors

K_s **to account for size**

K_s=1 unless teeth are very large

K_B **to account for gear with a rim and spokes**

K_B=1 for solid gears

K_I **to account for extra loading on idler**

K_I=1 for non-idlers, K_I=1.42 for idler gears

J K_v K_m K_a K_s K_B K_I

Back to the Example

$$\sigma_b = \frac{W_t \cdot p_d}{F \cdot J} \cdot \frac{K_a K_m}{K_v} K_s K_B K_I$$

$$\sigma_{b_{gear}} = \frac{(420)(4)}{(3)(0.38)} \cdot \frac{(1)(1.63)}{(0.845)} (1)(1)(1) = 2842\ psi$$

$$\sigma_{b_{pinion}} = \frac{(420)(4)}{(3)(0.34)} \cdot \frac{(1)(1.63)}{(0.845)} (1)(1)(1) = 3177\ psi$$

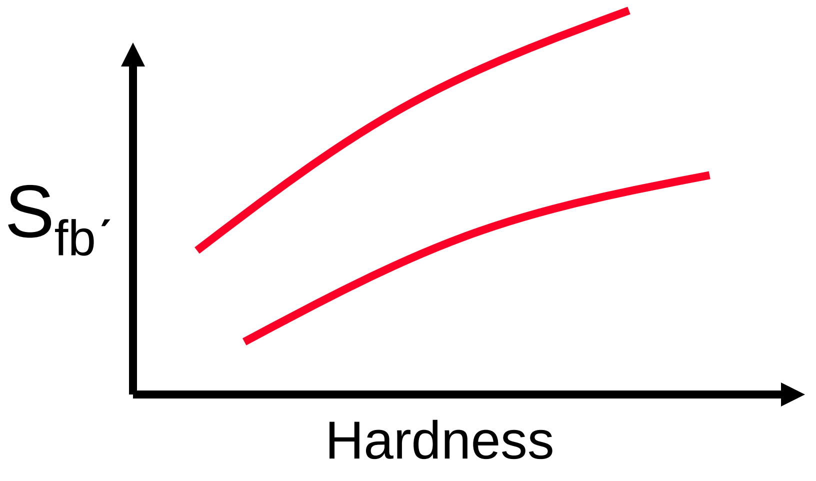
Compared to What??

σ_b is great, but what do I compare it to?

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$$

similar concept to S_e, but particularized to gears

$S_{fb'}$ ➔ Table 11-20
Figure 11-25 for steels



Life Factor K_L

to adjust test data from 1E7 to any number of cycles

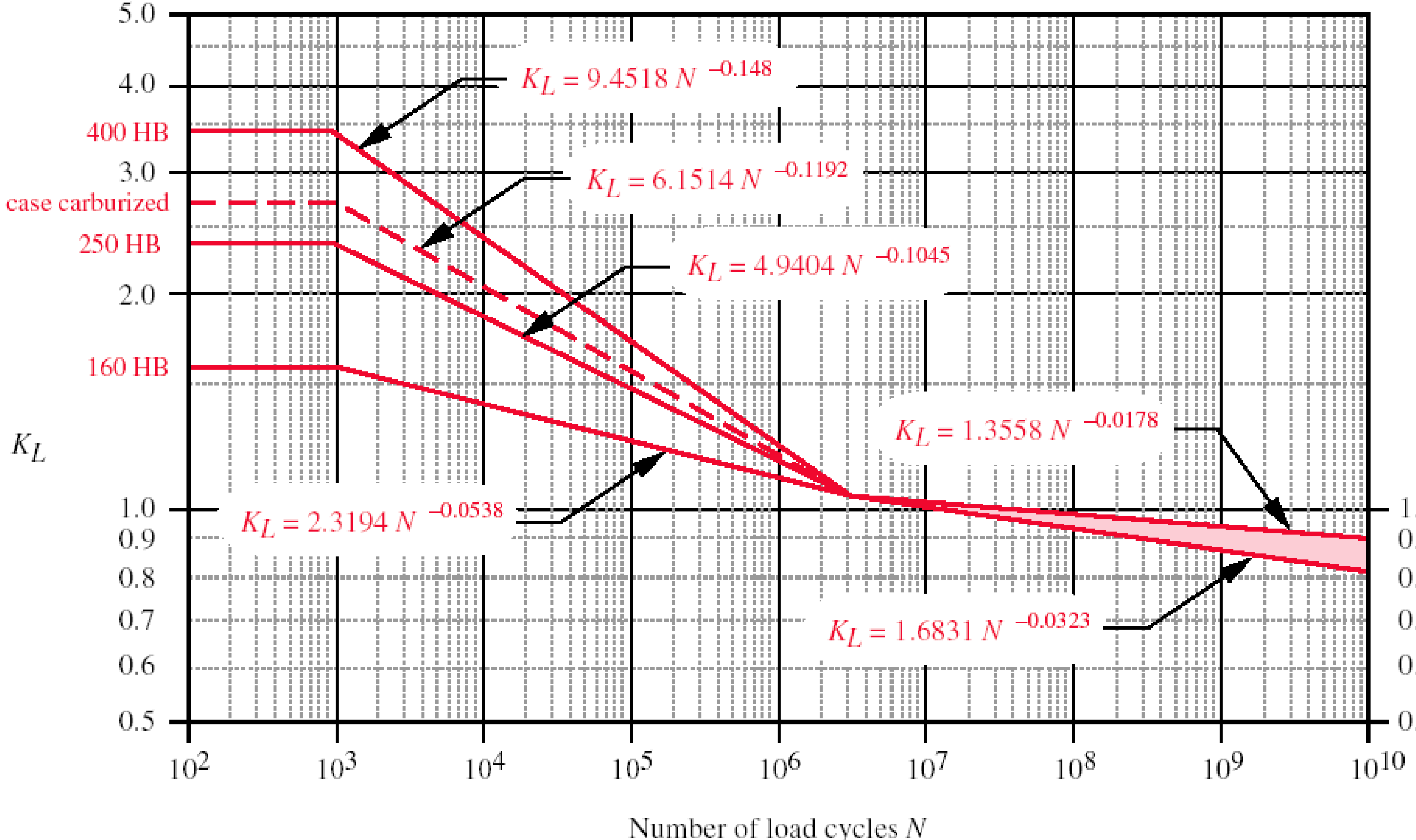


FIGURE 11-24*

Temperature & Reliability Factors

$K_T = 1$ if $T < 250°F$

see Equation 11.24*a* if $T > 250°F$

Table 11-19

AGMA Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Fatigue Bending Strength for Example

Class 40 Cast Iron
450 rpm, 8 hours/day, 10 years
T = 80°F
Reliability=99%

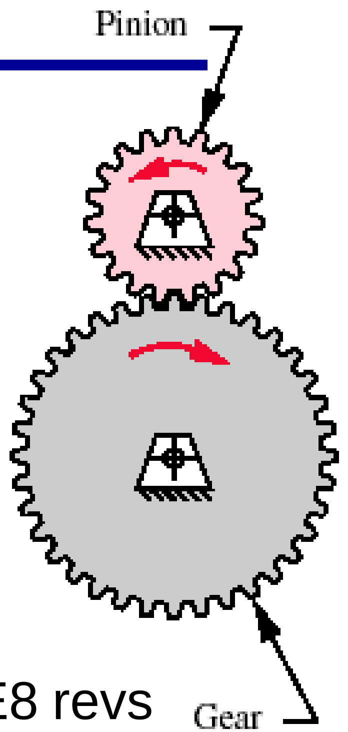


Table 11-20 ⟶ $S_{fb}' = 13\text{ksi}$

N=(450 rev/min)(60 min/hr)(8 hr/dy)(250 dy/yr)(10 yrs)=5.4E8 revs

$K_L = 1.3558N^{-0.0178} = 0.948$

S_{fb} = (0.948)13ksi = 12.3 ksi

Safety Factor

$$N_b = \frac{S_{fb}}{\sigma_b}$$

$N_{b\,pinion}$ =(12300 psi)/(3177psi)= 3.88

$N_{b\,gear}$ =(12300 psi)/(2842psi)= 4.33

Gear Failure

Fatigue Loading

from bending of teeth

- infinite life possible
- failure can be sudden

Surface Failure

from contact b/n of teeth

- infinite life not possible
- failure is gradual

Buckingham pioneered this work

$$\sigma_c = C_p \sqrt{\frac{W_t}{F \cdot I \cdot d} \frac{C_a C_m}{C_v} C_s C_f}$$

equal to 1

same as for bending

d= pitch diameter of smaller gear

Geometry Factor I

considers radius of curvature of teeth and pressure angle

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}\right) d_p}$$

$$\rho_p = \left[\left(r_p + \frac{1 + x_p}{p_d}\right)^2 - \left(r_p \cos\phi\right)^2\right]^{1/2} - \frac{\pi}{p_d}\cos\phi \qquad \rho_g = C\sin\phi \mp \rho_p$$

top sign is for external gears, bottom for internal

For example: ρ_p=0.691, ρ_g=1.87, I=0.095

Elastic Coefficient

considers differences in materials

Table 11-18 AGMA Elastic Coefficient C_p in Units of $[psi]^{0.5}$ ($[MPa]^{0.5}$)*†

Pinion Material	E_p psi (MPa)	Gear Material: Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30E6 (2E5)	2 300 (191)	2 180 (181)	2 160 (179)	2 100 (174)	1 950 (162)	1 900 (158)
Malleable Iron	25E6 (1.7E5)	2 180 (181)	2 090 (174)	2 070 (172)	2 020 (168)	1 900 (158)	1 850 (154)
Nodular Iron	24E6 (1.7E5)	2 160 (179)	2 070 (172)	2 050 (170)	2 000 (166)	1 880 (156)	1 830 (152)
Cast Iron	22E6 (1.5E5)	2 100 (174)	2 020 (168)	2 000 (166)	1 960 (163)	1 850 (154)	1 800 (149)
Aluminum Bronze	17.5E6 (1.2E5)	1 950 (162)	1 900 (158)	1 880 (156)	1 850 (154)	1 750 (145)	1 700 (141)
Tin Bronze	16E6 (1.1E5)	1 900 (158)	1 850 (154)	1 830 (152)	1 800 (149)	1 700 (141)	1 650 (137)

For example, C_p=1960 psi

Surface-Fatigue Strengths

σ_c is great, but what do I compare it to?

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc'}$$

from Table 11-21

same as for bending

Life Factor C_L

to adjust test data fro 1E7 to any number of cycles

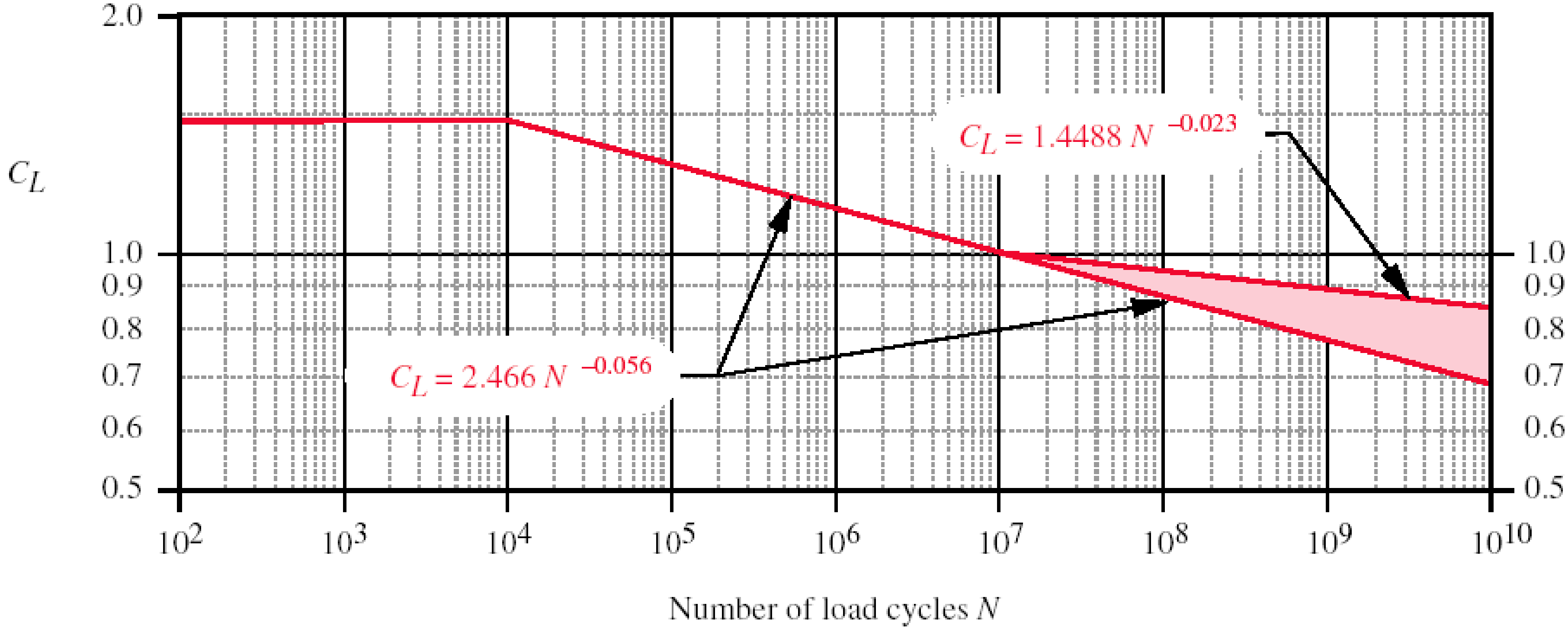


FIGURE 11-26*

AGMA Surface-Fatigue Strength Life Factor C_L

Hardness Ratio C_H

to account for pitting resistance

when pinion is harder than gear, then gear is cold-worked

only apply C_H to the cold-worked gear

when $HB_p = HB_G$, then $C_H = 1$

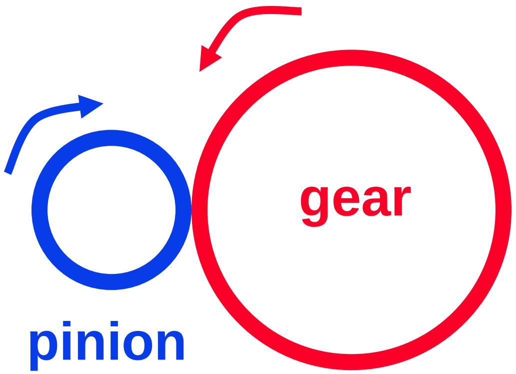
Safety Factor for Loading

stress is based on the square root of loading, therefore:

$$N_C = \left(\frac{S_{fc}}{\sigma_C} \right)^2$$

Gear Design

Gear Design



Same for Pinion and Gear

- p_d (p_c), ϕ, F
- Power
- W, W_t, W_r
- V_t
- N_c

Different for Pinion and Gear

- d, N
- T, ω ,
- N_b

Gear Design Strategy

Given or Set:

gear ratio

Power, ω, T

Properties of Gears to Determine:

d_p, d_g, N_p, N_g

p_d, F -OR- Safety Factors

(the others are given)

W_t

Bending & Surface Stresses

Safety Factors or p_d and F