Fundamentals of Forces and Stresses, Strains

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FORCE

- In engineering practice, the machine parts are subjected to various forces which may be due to either one or more of the following:

1. Energy transmitted,

2. Weight of machine,

3. Frictional resistances,

4. Inertia of reciprocating parts,

5. Change of temperature,

6. Un-balance of moving parts.

FORCE

- It is an important factor in the field of Engineering science, which may be defined as **an agent, which produces or tends to produce, destroy or tends to destroy motion.**

Momentum = Mass × Velocity

m = Mass of the body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration, and

t = Time required to change velocity from u to v.

$\therefore$ Change of momentum $= mv - mu$

and rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = m.a \quad \ldots \left(\because \frac{v-u}{t} = a\right)$$

or Force, $F \propto ma$ or $F = k\,m\,a$

where k is a constant of proportionality.

LOAD

- It is defined as **any external force acting upon a machine part**

1. ***Dead or steady load***. A load is said to be a dead or steady load, when it does not change in magnitude or direction.

2. ***Live or variable load.*** A load is said to be a live or variable load, when it changes continually.

3. ***Suddenly applied or shock loads***. A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.

4. ***Impact load.*** A load is said to be an impact load, when it is applied with some initial velocity.

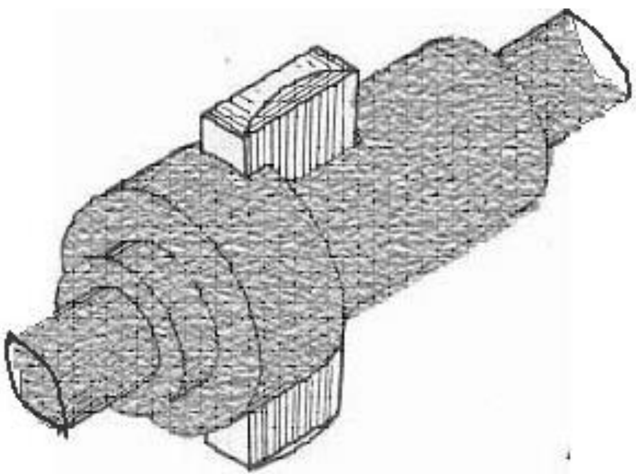
STRESS

- Intensity of **the internal resistant against externally applied load,** measured by force per unit area

Stress, $\sigma = P/A$

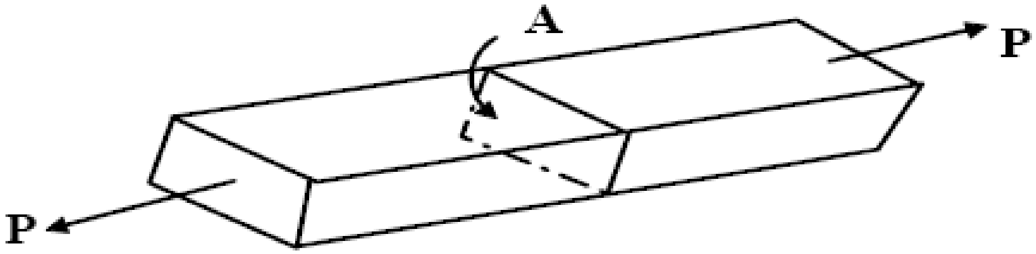
P = Force or load acting on a body, and

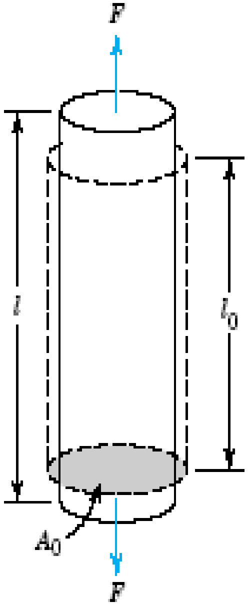
A = Cross-sectional area of the body.



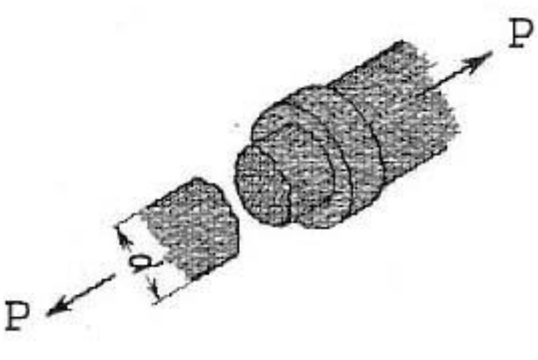
Some basic issues of simple stresses

- **Tensile stress and Strain**



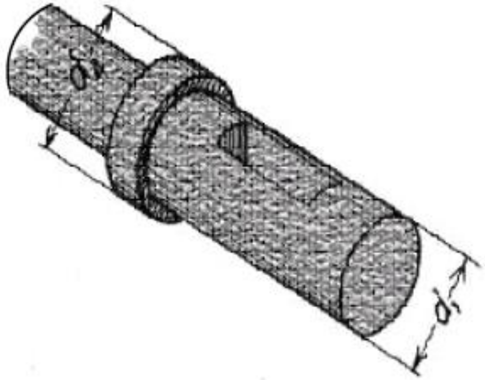


$$\sigma_t = \frac{P}{A}$$

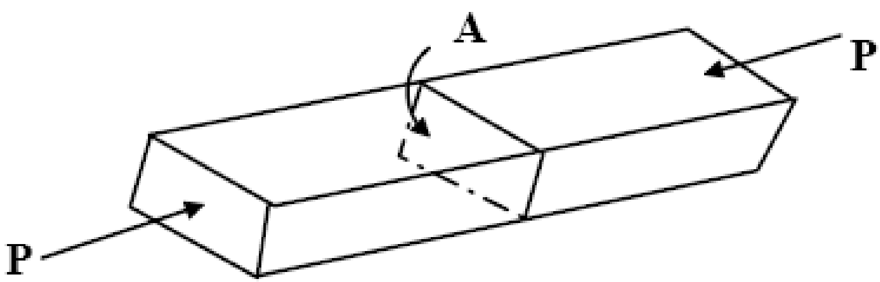


$$\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}$$

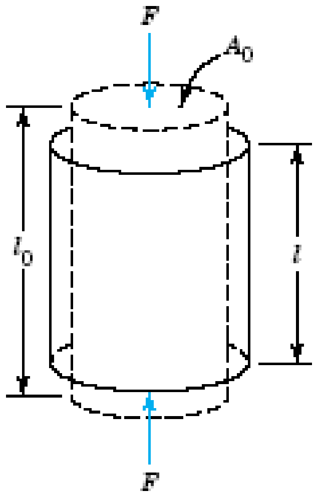
$$\left(\frac{\pi}{4}(d_4^{\ 2}-d_1^{\ 2})\right)\sigma_c = P$$



- **Compressive stress and Strain**



$$\sigma_c = \frac{P}{A}$$



$$\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}$$

Since lo > li , negative strain

Hook’s Law

$$\sigma \propto \varepsilon \qquad \text{or} \qquad \sigma = E.\varepsilon$$

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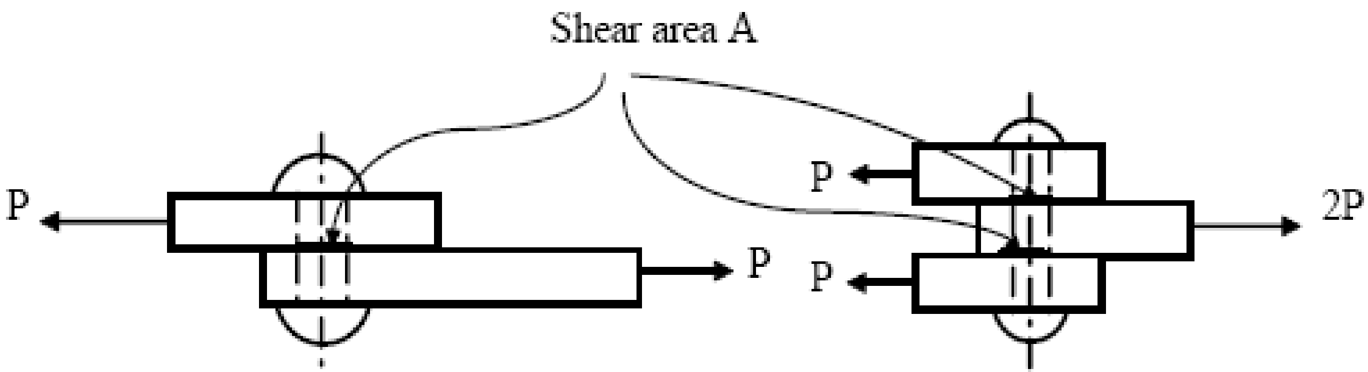
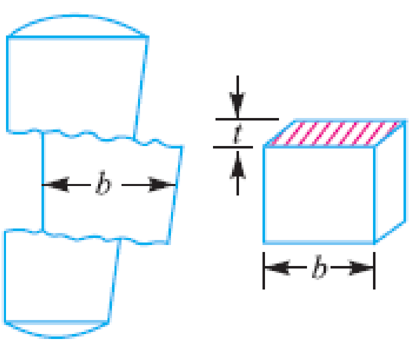
$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

Shear stress and Strain



$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi\, d^2}$$

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$



Shear strain is defined as the tangent of the strain angle q.

$$\tau \propto \phi \quad \text{or} \quad \tau = C \cdot \phi \quad \text{or} \quad \tau / \phi = C$$

τ = Shear stress,

ϕ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

Concept of Poisson’s Ratio

Longitudinal Strain:

$$\varepsilon_x = \frac{\Delta x}{x} = \frac{x1 - x0}{x0}$$

Lateral Strain:

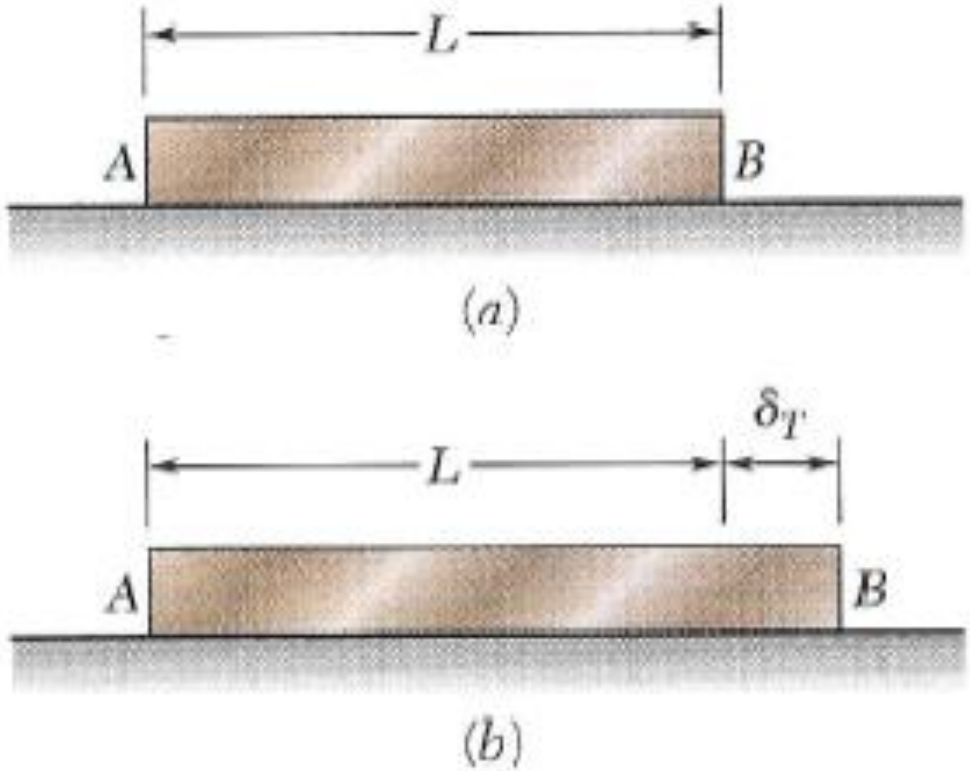
$$\varepsilon_y = \frac{\Delta y}{y} = \frac{y1 - y0}{y0} \qquad \varepsilon_z = \frac{\Delta z}{z} = \frac{z1 - z0}{z0}$$

Poission’s Ratio:

$$\mu_{yx} = \frac{-\varepsilon_y}{\varepsilon_x} \qquad \mu_{zx} = \frac{-\varepsilon_z}{\varepsilon_x}$$

For Isotropic Materials: $\mu_y = \mu_z = \mu$

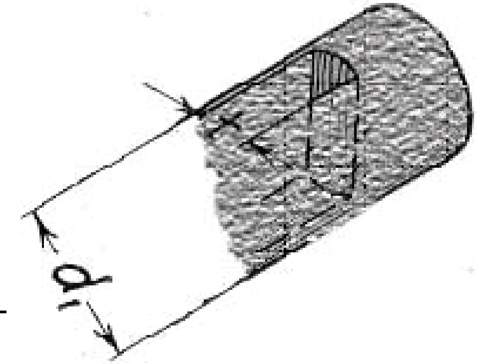
Thermal stress and Strain



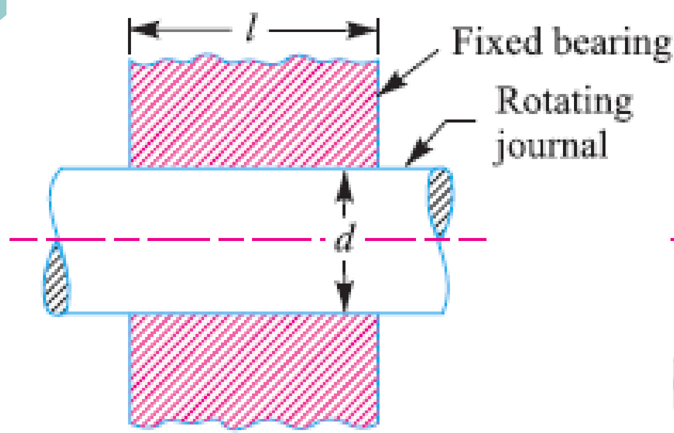
Strain caused by temperature changes. α is a material characteristic called the ***coefficient of thermal expansion***.

$$\varepsilon_T = \alpha \Delta T$$

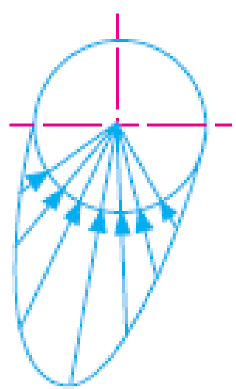
○ Bearing stress



A localized compressive stress at the surface of contact between two members of a machine part, that are relatively at rest is known as ***bearing stress*** or ***crushing stress***



(a) Journal supported in a bearing.



(b) Distribution of bearing pressure.

$$p_b = \frac{P}{l.d}$$

p_b = Average bearing pressure,

P = Radial load on the journal,

l = Length of the journal in contact, and

d = Diameter of the journal.

Working stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the ***working stress*** or ***design stress.*** It is also known as ***safe*** or ***allowable stress.***

Factor of Safety (Safety Factor)

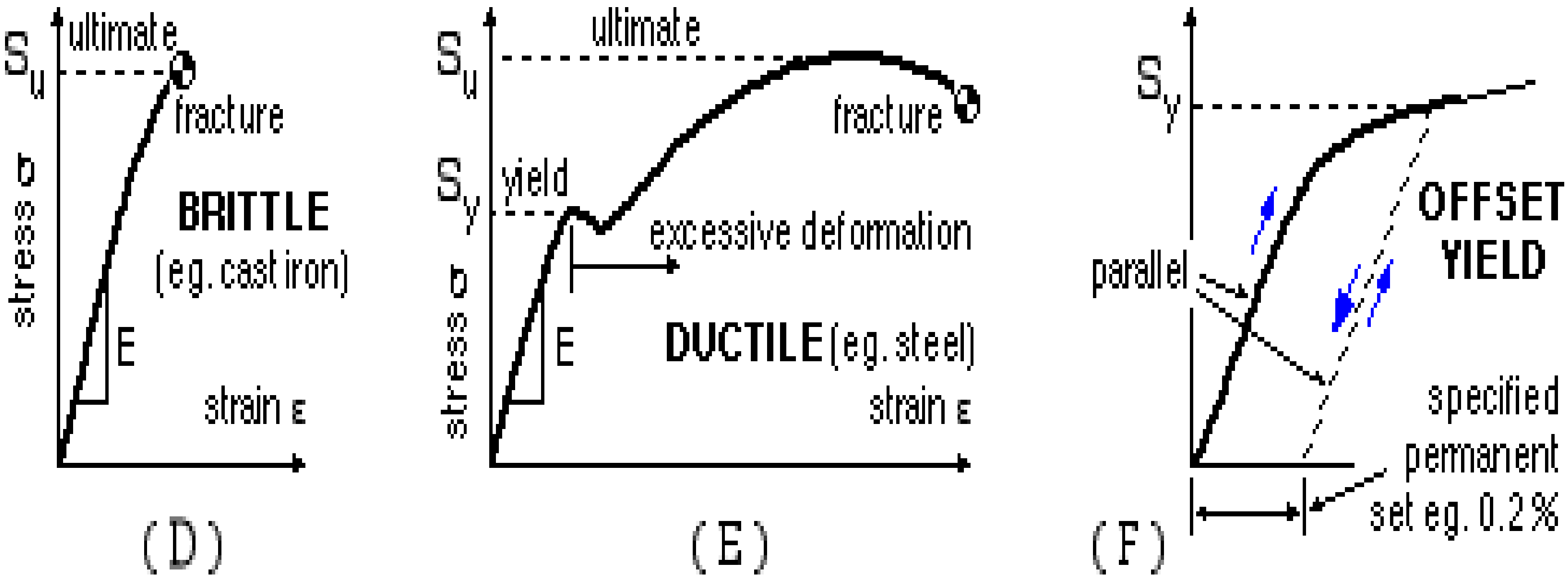
Eg: If a component needs to withstand a load of 100 N and a FoS of 3 is selected then it is designed with strength to support 300 N.

Is used to provide a **design margin** over the **theoretical design capacity** to allow for uncertainty in the design process.

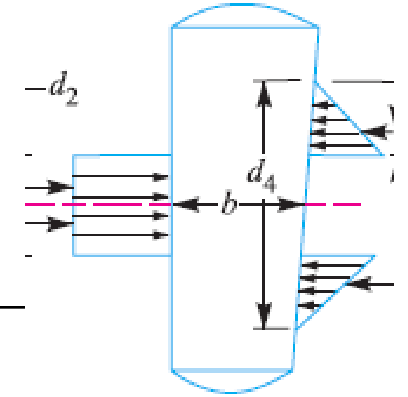
In the calculations,
Material strengths,
Manufacturing process

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

Factor of Safety (Safety Factor)

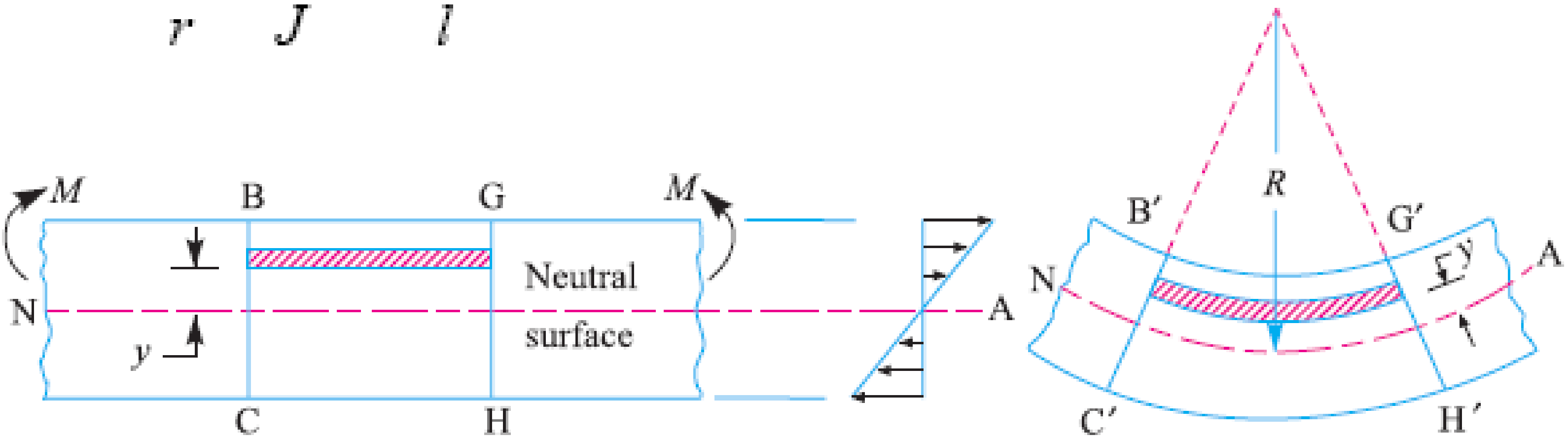


$$\text{FoS} = \frac{\text{Strength of the component } (S_u, S_y)}{\text{Stress in the component due to the actual load}}$$



Bending stress

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l}$$



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

$$\sigma = y \times \frac{E}{R}$$

$$\sigma = \frac{M}{I} \times y = \frac{M}{I / y} = \frac{M}{Z}$$

Torsional Shear stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion*****.** The stress set up by torsion is known as ***torsional shear stress*****.** It is zero at the centroidal axis and maximum at the outer surface.

.

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l}$$

τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

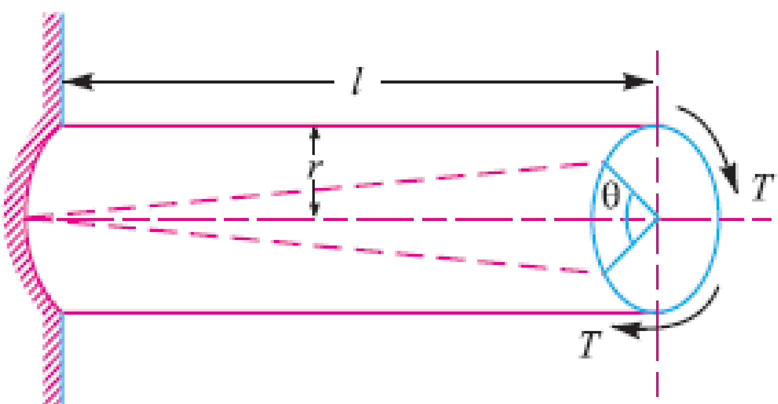
T = Torque or twisting moment,

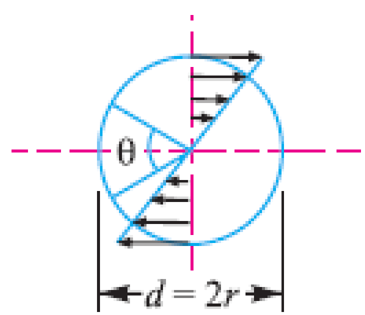
J = Second moment of area of the section about its polar axis or polar moment of inertia,

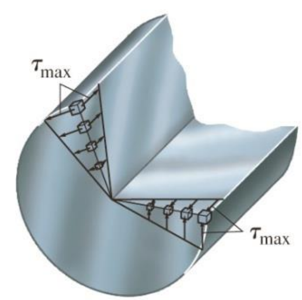
C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.

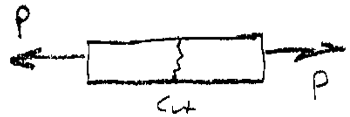
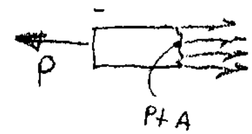
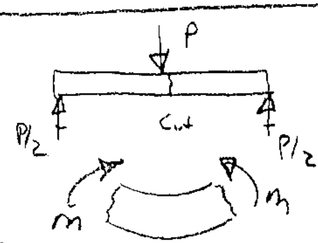
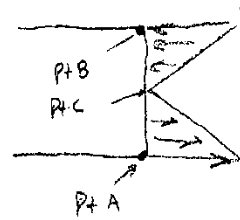
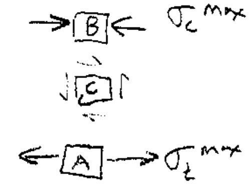
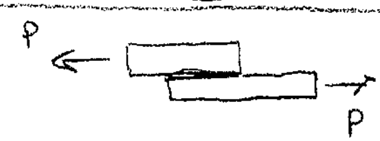
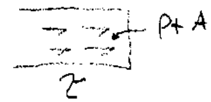
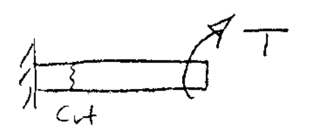
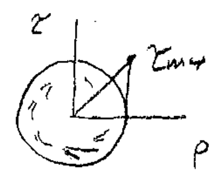
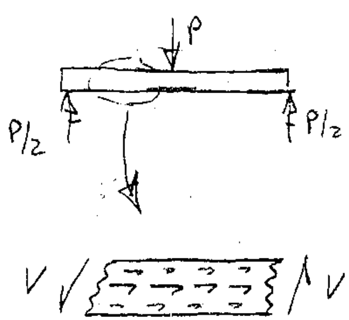
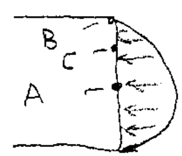






Shear stress varies linearly along each radial line of the cross section.
(b)

Quick Review: Basic Types of Stress

Type of Stress	Example Loading	Stress Profile	Stress Element
1) Direct Normal (tension)		 $\sigma_t = P/A$	
2) Bending Stress		 $\sigma_b = \frac{My}{I}$ $\sigma_{max} = \frac{Mc}{I}$	
3) Direct Shear		 $\tau = \frac{P}{A_s}$	
4) Torsion Shear	 T = torque	 $\tau = \frac{T\rho}{J}$ $\tau_{max} = \frac{Tc}{J}$ @ outer surface	 $\tau = \frac{Tc}{J}$ if outer surface
5) Beam Shear		 $\tau = \frac{VQ}{It}$	 τ_{max} @ A

RJ
4/14/00

Variable stress in Machine Parts

In reality most mechanical components experience variable loading due to;

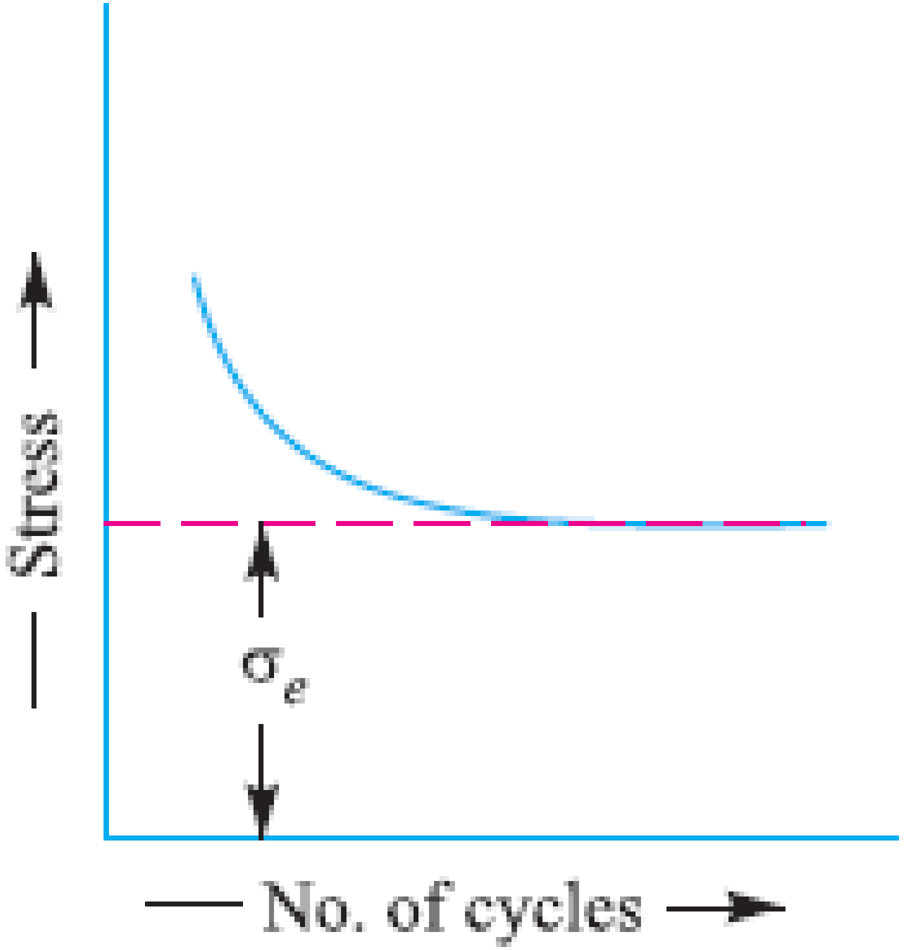
- Change in the magnitude of applied load

  Example: punching or shearing operations-
- Change in direction of load application

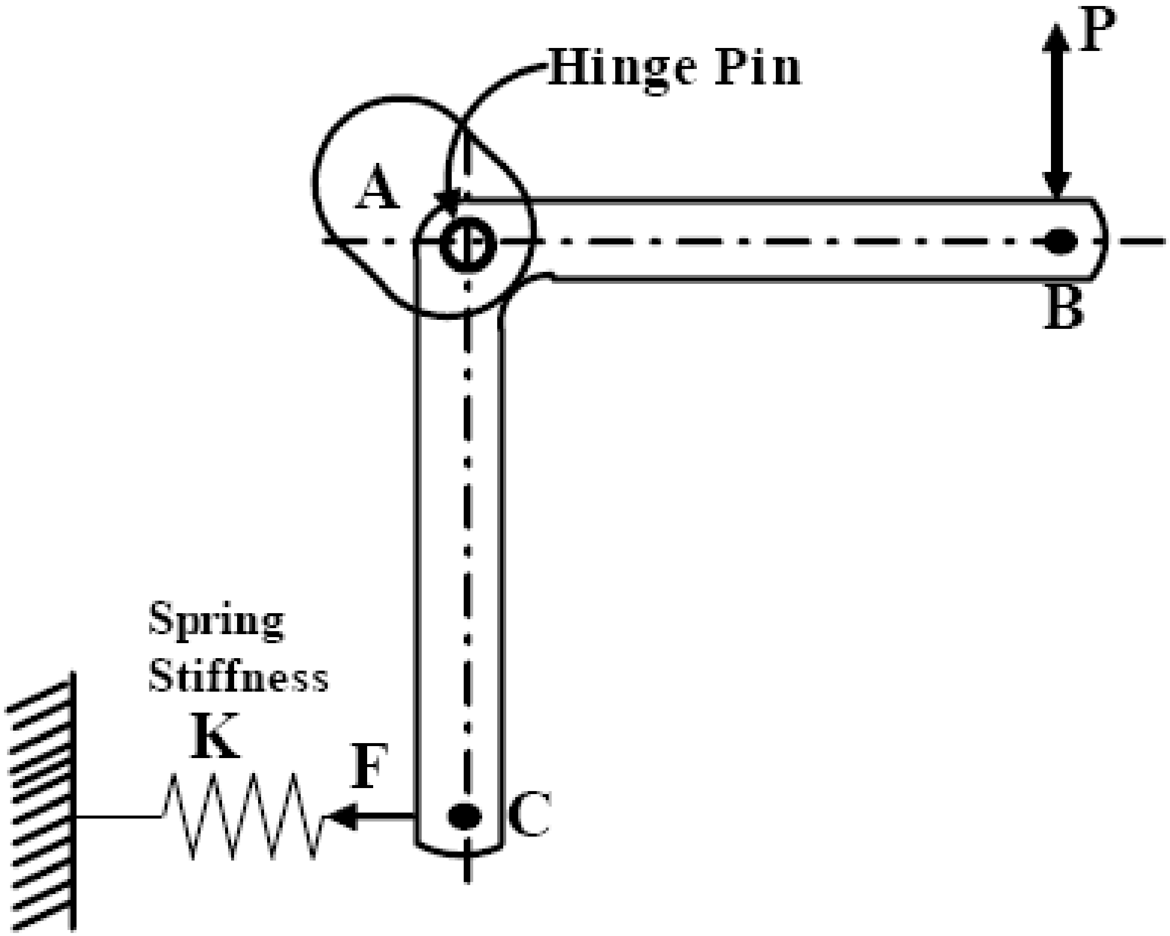
  Example: connecting rod
- Change in point of load application

  Example: rotating shaft

Concept of Fatigue Strength



Review Exercise: Identify the stresses in Each Parts



Lever arms AB and AC	-	Bending stresses
Hinge pin	-	Shear and bearing stresses.
Spring	-	Shear stress.

Tutorial: Stress Fundamentals

Find the maximum length of steel wire that can hang without breaking. Breaking stress=7.9X1012dyne/cm2. Density of steel =7.9g/cc

Solution

$$\text{Breaking stress} = \frac{\text{weight of wire}}{\text{Area}} = \frac{(Al)\rho g}{\text{A}} = l\rho g$$

$$\text{or } l = \frac{\text{Breaking stress}}{\rho g} = \frac{7.9 \times 10^{12}}{7.9 \times 980}$$

$$l = 1.02 \times 10^{9} cm$$

Innovative Practical Questions:

Q Why solids are more elastic than gases?

Q Which is more elastic: water or air, why? Steel or Rubber why?

Q Why does spring balance shows wrong reading after long use ?

Q Why a spring is made of steel and not of copper ?