ANALYTIC GEOMETRY & CALCULUS I

Standards Based Grading Course Standards & Grading Rubric

WHAT DO I NEED TO KNOW BEFORE I BEGIN?

- **G1. Arithmetic, algebra, and trigonometry** – Use precalculus skills including techniques of algebraic manipulation from high school algebra; logarithmic and exponential properties; factoring; techniques for solving algebraic equations; techniques for graphing functions from college algebra and their transformations; knowledge of x- and y-intercepts; right triangle trigonometry and the unit circle; graphs of sine, cosine, and tangent functions, and their inverses; and some trigonometric identities (Right triangle identities, reciprocal identities, Pythagorean identities, double angle formulas for sine and cosine functions, and power reducing formulas).

LIMITS, FUNCTIONS, AND THE LIMIT DEFINITION OF THE DERIVATIVE

LIMITS

- **L2. Limits Graphically. Demonstrate conceptual understanding of a limit**, by estimating limits given a function's graph, or describing the geometric interpretation of a limit calculated algebraically or estimated numerically. Students may also be asked to draw a graph of a function that has given limits and given function values.
- **L3. Limits Numerically. Estimate the limit of a function numerically.** Students will not be permitted to use their graphing calculators. Students are expected to estimate the limit numerically and sketch the graph of the function near the limiting value of x.
- **L4. Limits Algebraically, Part 1. Find the limit of a function given algebraically**, using
    - Knowledge of continuous functions,
    - Knowledge of known functions from college algebra and trigonometry, including logarithmic and exponential functions and their end behavior,
    - Techniques for eliminating a 0/0 indeterminate form, including
        - Factoring and reducing common factors,
        - Multiplying and dividing by the Pythagorean conjugate, and
        - Clearing complex fractions,
    - Techniques for recognizing and evaluating infinite limits.

- **L5. Limits Algebraically, Part 2. Find the limit of a function given algebraically**, using
    - The squeeze theorem (AKA the sandwich theorem), and
    - Known limits involving $\frac{\sin\theta}{\theta}$ and $\frac{1-\cos\theta}{\theta}$, trigonometric identities and limit laws.

FUNCTIONS AND GRAPHING

- **F6. Demonstrate understanding of the relationship between limits and continuity.** Determine the intervals where a function is continuous, given the equation of a function or its graph. Determine the x-values where a given function is discontinuous, given a function or its graph. Classify the different types of discontinuities and provide examples of each type. Graph piecewise functions, and determine the x-values where the function is continuous. Determine if it is possible to define a continuous extension of a discontinuous function, and how to define or redefine to a function to make such functions continuous.
- **F7. Demonstrate understanding of the relationship between limits, vertical asymptotes, horizontal asymptotes, and removable discontinuities.** Evaluate infinite limits algebraically. Demonstrate understanding of the relationship between certain limits and a function's vertical and horizontal asymptotes. Find the vertical asymptotes of a rational function, given the function itself, or a graph of the function. Identify the locations of removable discontinuities, given the formula for the function. Find the horizontal asymptotes of a function, given the function itself, or a graph of the function. Graph rational functions.
- **F8. The Intermediate Value Theorem**. State the theorem, and describe it in your own words. Use the theorem to solve a problem.

LIMIT DEFINITIONS OF THE DERIVATIVE AND INTERPRETATIONS

- **D9. Limit Definitions of the Derivative.** Use one of the limit definitions to compute the derivative of a function algebraically.
- **D10. Conceptual understanding of the derivative.** To demonstrate understanding of the derivative and its interpretations as slope and instantaneous rate of change,
    - State the limit definition of the derivative and its relationship to the slope of the secant line and slope of the tangent line.
    - Explain the relationship between the average rate of change (as a difference quotient) and instantaneous rate of change of a function, geometrically and in applications,
    - Interpret the derivative of a function in words, in an applied problem,
    - Graph $f, f', \text{and/or } f''$, given the graph of one of them, and
    - Given a graph, determine the x-values where a function fails to be differentiable.

DIFFERENTIATION

- **D11. Compute derivatives** $f', f'', f''', f^{(4)}, \ldots, f^{(n)}$ using the following rules:
    - The derivative of a constant function,
    - The power rule,
    - The derivative of a sum,
    - Constant multiple rule,
    - Derivatives of exponential functions,
    - Derivatives of trigonometric functions, and
    - Derivatives of logarithmic functions.
    - (You will be expected to memorize your basic differentiation rules and complete a formula sheet on assessments. You'll want to know these as well as you know your multiplication facts – Doing so will be tremendously helpful as you go on to calculus II, calculus III, and differential equations, as well as upper division coursework.)
- **D12. Compute derivatives** $f', f'', f''', f^{(4)}, \ldots, f^{(n)}$ using **the product rule** and **the quotient rule.**
- **D13. Apply differentiation rules and interpret the derivative** $f'(x_0)$
    - As the slope of the curve or of the tangent line to curve at x_0, and in applications related to slope:
        - Use the derivative to find the equation of the tangent line (AKA standard linear approximation, AKA linearization of a function), and
        - Use the derivative to find the x-values where a function has a given slope,
    - As an instantaneous rate of change (given a position function, a cost function, a function describing simple harmonic motion, etc).

    D10 and D13 are often confused. Here's how they're different. D13 is about the derivative at a point, while D10 is about conceptual understanding of the derivative in general and the derivative as a function. D10 looks at the derivative function holistically while D13 focuses on the derivative in specific cases. For example, standard D10 questions may require you to demonstrate that understanding by articulating what the limit definition means, and by considering the relationships between graphs of functions, while D13 may ask you to find the time or the position when the velocity of some object is zero. When preparing for these standards and reassessments, remember that D10 is *global* or *general*, while D13 is *specific.* Both require some computation and interpretation.
- **D14.** Compute derivatives using **the chain rule***. ***The most advanced "basic differentiation rule."
- **D15.** Compute derivatives using **implicit differentiation** and **logarithmic differentiation**.

- **D16.** State the derivative of f^{-1} in terms of the derivative of f. Use this fact to find the slope of f^{-1} at a given point, when given the equation of the function f.
- **D17.** Use derivative rules and implicit differentiation to **set up and solve related rates problems.**
- **D18**. Compute **derivatives, function values, and limits involving inverse trigonometric functions**. This will require the use of basic differentiation rules in addition to the six new rules.
- **D19.** Compute differentials. Use differentials to solve application problems. Compute the increment of y and compare it to the differential of y at a given point. Use linearization as an approximation to the original function. Use differentials to approximate function values, such as $\sqrt{25.02}$.

APPLICATIONS OF DIFFERENTIATION

- **A20. Evaluate limits involving indeterminate forms**, using L'Hopital's Rule and algebraic manipulation. The following indeterminate quotients, product, powers, and difference may be included: $\frac{0}{0}, \pm\frac{\infty}{\infty}, \infty \cdot 0, 1^{\infty}, 0^0, \infty^0, \infty - \infty$. Knowledge of limits and the behavior of transformations of functions from college algebra and trigonometry may be necessary.
- **A21. Mean Value Theorem**. State the theorem,and determine whether the theorem applies. Use the Mean Value Theorem to solve a problem.
- **A22.** Use the Extreme Value Theorem, the first derivative test, or the second derivative test to **find absolute and relative extrema of a function.**
- **A23.** Set up and **solve applied optimization problems**. State the constraint equation (if applicable) and the objective function.
- **A24. Graph a function** using intercepts, limits at $\pm\infty$, critical points, points of inflection, the first derivative and second derivative. Find the critical points of a function and **determine the open intervals where a function is increasing or decreasing** using a sign chart. Find the points of inflection of a function and **determine the open intervals where a function's graph is concave upward or concave downward** using a sign chart. Use the graph of $f, \text{or } f'$ to determine extrema, points of inflection, open intervals where f is increasing or decreasing, and open intervals where the graph of f is concave upward or concave downward.

ANTIDIFFERENTIATION, INTEGRATION, AND THE FUNDAMENTAL THEOREM

- **I25. Find antiderivatives**, using basic rules and u-substitution. (You will be expected to memorize your basic antidifferentiation rules and complete a formula sheet on assessments. You'll want to know these as well as you know your multiplication facts – Doing so will be tremendously helpful as you go on to calculus 2, calculus 3, and differential equations, as well as upper division coursework.)
- **I26. Differential equations.** Use antidifferentiation to find general solutions of differential equations. Find solutions to initial value problems. Understand the geometric interpretation of this.

- **I27. Demonstrate an understanding of sigma notation and the Riemann sum.** Approximate the area under a curve using a Riemann sum. Find the exact area under the curve using the limit of a Riemann sum.
- **I28. Evaluate definite integrals**, using basic rules, u-substitution, the Fundamental Theorem of Calculus, and using geometric formulas for area. Find the average value of a function on an interval using a definite integral.
- **I29. Interpret the integral**
    - Geometrically – as area between the function and the x-axis (area above the x-axis minus the area below the x-axis), and
    - As a net change in the antiderivative function (using correct units).
- **I30. Find derivatives** of functions, using properties of integrals, the chain rule, and the Fundamental Theorem of Calculus, when given a function defined by an integral.
- **I31. Hyperbolic Functions.** State the derivative and antiderivative rules for the hyperbolic functions. Compute derivatives and antiderivatives involving hyperbolic functions. Apply your knowledge of hyperbolic functions to problems about slope, instantaneous rates of change, relative extrema, and area under curves.

GRADING RUBRIC

4: You have demonstrated mastery of this skill. You have exhibited mastery. You have demonstrated full understanding of the concepts involved. You clearly show all steps of your reasoning. Your notation is flawless. You wrote clear prose, and have made no algebraic errors. A 4 is awarded for flawless work.

3.5: You have demonstrated mastery of this skill. You have demonstrated full understanding of the concepts involved, but you may have made a *very small* algebraic error (a sign error, for example) that does not change the nature of the problem.

3: You have a firm grasp of this skill. You have demonstrated full or nearly full understanding of the concepts involved, but you may have failed to show your work or adequately explain your reasoning, you may have used notation incorrectly, and/or made a slight algebraic error. A 3 is awarded for "very good" work.

2: You have demonstrated some conceptual understanding of this skill. You may be confused about some aspect of the concept or skill, did not completely answer the question, did not use consistent notation, used terminology incorrectly, failed to adequately explain your work, and/or made more than one slight algebraic error. A 2 is awarded for work that demonstrates adequate understanding for proceeding to the next course.

1: You have demonstrated weak or no conceptual understanding. You may be confused about the skill and/or concept, used incorrect reasoning, and/or made one or more algebraic errors that completely change the nature of the problem.

0: You left the problem blank, did not attend class, or wrote something down that is completely unrelated to the problem.