Name: Key Date: December 3, 2018 Class Time: ______

Analytic Geometry & Calculus I | Tulsa Community College

Quiz #10 Part 1: Basic Antidifferentiation Rules

Please complete the table of basic antidifferentiation rules below. You may want to differentiate your answers to check your work! When you're finished, come get a completed rule sheet and the second part of your quiz.

$\int e^x\,dx = e^x + C$	$\int \sec x \tan x\,dx = \sec x + C$
$\int \cos x\,dx = \sin x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + C$	$\int \sin x\,dx = -\cos x + C$
$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$	$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\operatorname{arcsec}\left(\frac{\lvert x\rvert}{a}\right) + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int 1\,dx = x + C$
$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\int kf(x)\,dx = k\int f(x)\,dx$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
$\int a^x\,dx = \frac{1}{\ln a}a^x + C$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$ or $\ln\lvert\sec x\rvert + C$
$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$	$\int \csc^2 x\,dx = -\cot x + C$

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Quiz #10 Part 2: Antidifferentiation, Evaluating Definite Integrals, Interpretation of an Integral, Derivatives of Integral Functions, Average Value & the Area Between Two Curves

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use the provided formula sheets for basic antidifferentiation rules, as well a scientific, nongraphing calculator.

Find the antiderivative, using basic rules and algebraic manipulation.

1.

$$\int \left[5 + \frac{2}{3}x^2 + \frac{3}{4}x^3\right] dx$$

$$= 5x + \frac{2}{3} \cdot \frac{1}{3} x^3 + \frac{3}{4} \cdot \frac{1}{4} x^4 + C$$

$$= \boxed{5x + \frac{2}{9}x^3 + \frac{3}{16}x^4 + C}$$

2. $$\int \left[6x^3 - 8x - 7 + \frac{2}{5x^2} - \sqrt{x} + \frac{4}{\sqrt[3]{x^2}}\right] dx$$

$$= \int \left[6x^3 - 8x - 7 + \frac{2}{5}x^{-2} - x^{1/2} + 4x^{-2/3}\right] dx$$

$$= \frac{6}{1} \cdot \frac{1}{4} x^4 - \frac{8}{1} \cdot \frac{1}{2} x^2 - 7x + \frac{2}{5} \cdot \frac{1}{-1} x^{-1} - \frac{2}{3} x^{3/2} + \frac{4}{1}\left(\frac{3}{1}\right) x^{1/3} + C$$

$$= \boxed{\frac{3}{2}x^4 - 4x^2 - 7x - \frac{2}{5x} - \frac{2}{3}\sqrt{x^3} + 12\sqrt[3]{x} + C}$$

3. $\int [3\sin x - 4\cos x + \frac{\sec x \tan x}{6} - \csc^2 x + \frac{2}{5}\csc x \cot x + 5\tan x]\, dx$

$$= \boxed{-3\cos x - 4\sin x + \frac{1}{6}\sec x + \cot x - \frac{2}{5}\csc x + \underbrace{5\ln|\sec x|}_{} + C}$$

This term can also be written as $-5\ln|\cos x|$

4. $\int \left[8^x - \frac{3e^x}{4}\right] dx = \boxed{\frac{8^x}{\ln 8} - \frac{3}{4}e^x + C}$

5. $\int \left[\frac{3}{x\sqrt{x^2-1}} + \frac{6}{\sqrt{49-x^2}} - \frac{4}{x^2+16}\right] dx$

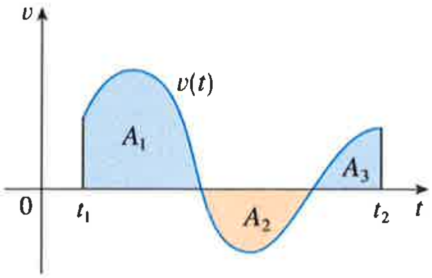
$a^2 = 1 \Rightarrow a = \sqrt{1} = 1$ 　 $a^2 = 49$, $a = \sqrt{49} = 7$ 　 $a^2 = 16$, $a = \sqrt{16} = 4$

$$= 3\left(\frac{1}{1}\right)\text{arcsec}\left(\frac{|x|}{1}\right) + 6\arcsin\left(\frac{x}{7}\right) - 4\left(\frac{1}{4}\right)\arctan\left(\frac{x}{4}\right) + C$$

$$= \boxed{3\,\text{arcsec}|x| + 6\arcsin\left(\frac{x}{7}\right) - \arctan\left(\frac{x}{4}\right) + C}$$

Skill #: I25
Score:

6. The graph of the velocity of an object (measured in meters per second) is shown below..



(a) $A_1, A_2,$ and A_3 are the areas of the regions shown above.
Find the value of $\int_{t_1}^{t_2} v(t)\,dt$ in terms of $A_1, A_2,$ and A_3.

$$\int_{t_1}^{t_2} v(t)\,dt = \left(\begin{array}{c}\text{Area above}\\ \text{the t-axis}\end{array}\right) - \left(\begin{array}{c}\text{Area below}\\ \text{the t-axis}\end{array}\right)$$

$$= \boxed{A_1 - A_2 + A_3}$$

(b) Provide a **real-world interpretation** (not a geometric interpretation!) of $\int_{t_1}^{t_2} v(t)\,dt$.

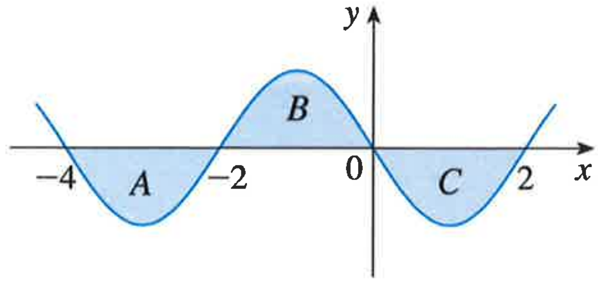
This value represents the displacement of the object, in meters, from time t_1 to time t_2.

7. The current in a wire $I(t)$ is defined at the derivative of the charge $I(t) = Q'(t)$ measured in units of charge per second. What does $\int_0^{10} I(t)\,dt$ represent?

$$\int_0^{10} I(t)\,dt = \int_0^{10} Q'(t)\,dt = Q(10) - Q(0).$$

This value is the net change in Q, the charge in the wire, from time $t=0$ to time $t=10$.

8.



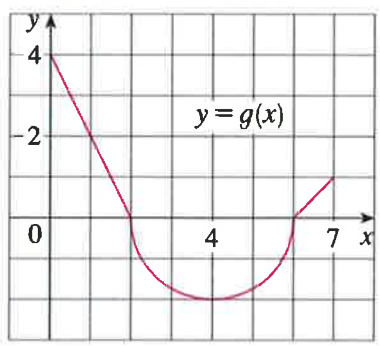
(a) The graph of $y = f(x)$ is shown above. $A, B,$ and C are the areas of the regions shown. Is $\int_{-4}^{2} f(x)\,dx$ positive, negative or zero? Explain your answer.

Negative, b/c $A + C > B$, and $\int_{-4}^{2} f(x)dx = B - A - C$.

(b) Is $\int_{-2}^{2} f(x)\,dx$ positive, negative or zero? Explain your answer.

zero. B appears to equal C.

9. The graph of $y = g(x)$ is shown below. Use the graph to evaluate the integrals.



(a) $\displaystyle\int_0^2 g(x)\,dx = \frac{1}{2}bh = \frac{1}{2}(2)(4) = \boxed{4}$

(b) $\displaystyle\int_0^6 g(x)\,dx = \underbrace{\int_0^2 g(x)\,dx}_{4} + \underbrace{\int_2^4 g(x)\,dx}_{-\underbrace{\frac{1}{2}\pi r^2}_{-\frac{1}{2}\pi(2)^2 = -2\pi.}} = \boxed{4-2\pi}$

(c) $\displaystyle\int_0^7 g(x)\,dx = \int_0^6 g(x)\,dx + \int_6^7 g(x)\,dx$

$= 4 - 2\pi + \underbrace{\frac{1}{2}bh}_{\frac{1}{2}(1)(1)}$

$= \boxed{4 - 2\pi + 1/2}$ OR $\boxed{\frac{9}{2} - 2\pi}$

Evaluate the definite integral using basic rules, algebraic manipulation, and u-substitution.

10.
$$\int_1^3 [5x^3 - 3x^2 + 8]\,dx$$

$$= \left(\frac{5}{4}x^4 - \frac{\cancel{3}}{\cancel{3}}x^3 + 8x\right)\Big|_1^3$$ Antidifferentiate.

$$= \left(\frac{5}{4}x^4 - x^3 + 8x\right)\Big|_1^3$$

$$= \left(\frac{5}{4}(3)^4 - 3^3 + 8(3)\right) - \left(\frac{5}{4}(1)^4 - (1)^3 + 8\cdot 1\right)$$ F(b) − F(a).

$$= \left(\frac{405}{4} - 27 + 24\right) - \left(\frac{5}{4} - 1 + 8\right) = \boxed{90}$$ Simplify.

11.
$$\int_1^{\pi^2/4} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

u-substitution

u

du

$$u = \sqrt{x} = x^{1/2}$$

$$du = \frac{1}{2}x^{-1/2}\,dx$$

$$du = \frac{1}{2\sqrt{x}}\,dx$$

$$2\,du = \frac{1}{\sqrt{x}}\,dx$$

$$u\left(\frac{\pi^2}{4}\right) = \sqrt{\frac{\pi^2}{4}} = \frac{\pi}{2}$$ New upper bound

$$u(1) = \sqrt{1} = 1$$ New lower bound

$$= \int_1^{\pi/2} \cos u\,du$$ Write the integral in terms of u.

$$= [\sin u]\Big|_1^{\pi/2} = \sin\frac{\pi}{2} - \sin 1$$ Antidifferentiate. F(b) − F(a).

$$= \boxed{1 - \sin 1}$$ Simplified.

12. $\int_0^1 \frac{5x^4}{(2x^5+7)^4}\,dx$

u-substitution

$u = 2x^5 + 7$ u

$du = 10x^4\,dx$ du

$\frac{1}{2}du = 5x^4\,dx$

$u(1) = 2(1)^5 + 7 = 9$ New upper bound

$u(0) = 2(0)^5 + 7 = 7$ New lower bound

$= \int_7^9 \frac{1}{u^4} \cdot \frac{1}{2}\,du$ Integral in terms of u

Antidifferentiate.

F(b) - F(a)

$= \frac{1}{2}\int_7^9 u^{-4}\,du = \frac{1}{2}\left(-\frac{1}{3}\right)u^{-3}\Big|_7^9 = -\frac{1}{6(9)^3} - \left(-\frac{1}{6(7)^3}\right)$

$= \boxed{-\frac{1}{4{,}374} + \frac{1}{2058}}$

13. $\int_1^2 \frac{3x-5}{x^2}\,dx$

$= \int_1^2 \left(\frac{3\cancel{x}}{x^{\cancel{2}}} - \frac{5}{x^2}\right)dx$ Separate the numerator.

$= \int_1^2 \left[3\cdot\frac{1}{x} - 5x^{-2}\right]dx$ Simplify & prepare to antidifferentiate.

$= \left[3\ln|x| - \frac{5x^{-1}}{-1}\right]\Big|_1^2$ Antidifferentiate.

$= \left(3\ln 2 + \frac{5}{2}\right) - \left(3\underbrace{\ln 1}_{0} + 5\right)$ F(b) - F(a).

$= \boxed{3\ln 2 - \frac{5}{2}}$ Simplified.

14. Given

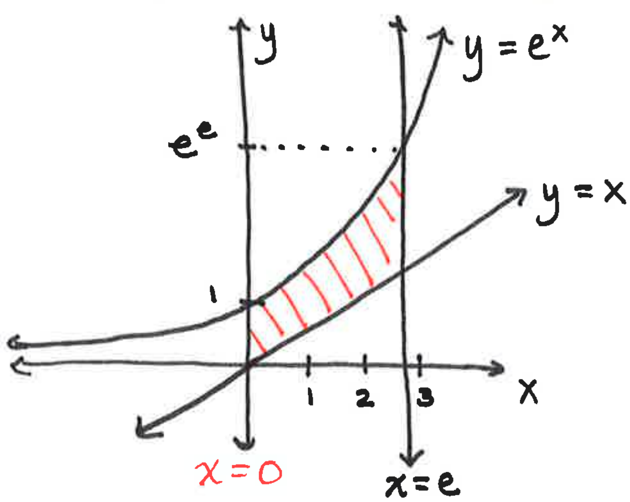
$$y(x) = \int_4^{\tan x} \sqrt{t+5}\,dt,$$

find $y'(x)$.

$$y'(x) = \sqrt{\tan x + 5}\;\underbrace{\frac{d}{dx}[\tan x]}_{\sec^2 x}$$

$$= \boxed{\sec^2 x\sqrt{\tan x + 5}}$$

15. Set up **BUT DO NOT EVALUATE** the integral representing the area between $f(x) = e^x$ and $g(x) = x$, and the vertical lines $x = 0$ and $x = e$. (You may wish to graph the functions in order to identify the top and bottom functions.)



(Not to scale)

$$\text{Area} = \int_a^b [\underbrace{f(x)}_{\text{top function}} - \underbrace{g(x)}_{\text{bottom function}}]\,dx$$

$$= \boxed{\int_0^e [e^x - x]\,dx}$$

16. Set up **BUT DO NOT EVALUATE** the integral representing the average value of

$$f(x) = 5x^2 - \cos x \text{ on } [-3, 3].$$

$$f_{avg} = \frac{\int_a^b f(x)\,dx}{b-a}$$

$$= \frac{\int_{-3}^{3} [5x^2 - \cos x]\,dx}{3-(-3)}$$

$$= \boxed{\frac{1}{6}\int_{-3}^{3} [5x^2 - \cos x]\,dx}$$

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam.

____

(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill #: G1	**Skill #: I27**
Score:	**Score:**