Name: Key Date: August 29, 2018 Class Time: ____________

Analytic Geometry & Calculus I | Tulsa Community College

Taking It to the Limit!

Quiz #1: Limits numerically, graphically, and algebraically

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use a calculator on this assessment

1. Use your calculator to complete the table. Then make an educated guess for the following limits.

(a) $\lim_{x\to 0}\frac{e^{2x}-1}{x} = \boxed{2}$

x	$f(x)=\frac{e^{2x}-1}{x}$
-0.1	≈ 1.8127
-0.01	≈ 1.9801
-0.001	≈ 1.9980
0	Undefined
0.001	≈ 2.0020
0.01	≈ 2.0201
0.1	≈ 2.2140

Approaching 2 from the left

Approaching 2 from the right

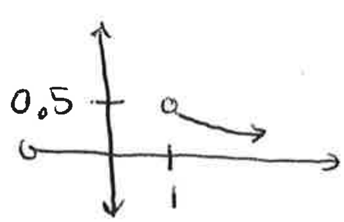
(b) $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) = \boxed{0.5}$

x	$f(x)=\frac{1}{\ln x}-\frac{1}{x-1}$
1	Undefined
1.001	≈ 0.4999
1.01	≈ 0.4992
1.1	≈ 0.4921

Approaching 0.5 from the right

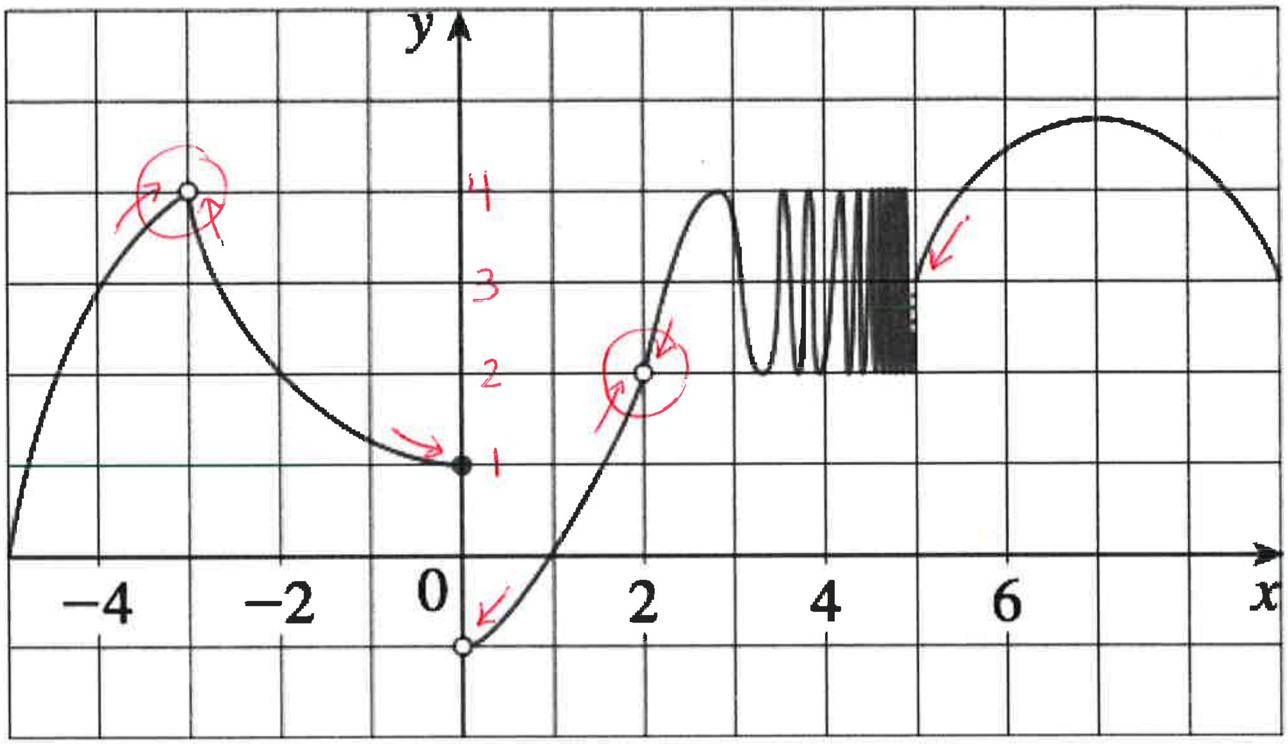
2. Describe what the results of 1(b) tell us about the graph of the function $f(x)=\frac{1}{\ln x}-\frac{1}{x-1}$.

Near $x=1$, $y=\frac{1}{\ln x}-\frac{1}{x-1}$ approaches 0.5 (or $\frac{1}{2}$).



(As x approaches 1 from the right, f(x) approaches 0.5.)

3. For the function f whose graph is given below, state the values of the given quantity, if it exists. If it does not exist, explain why.



(a) $\lim_{x\to -3^-} f(x) = 4$	(e) $\lim_{x\to 0^-} f(x) = 1$	(i) $\lim_{x\to 2} f(x) = 2$
(b) $\lim_{x\to -3^+} f(x) = 4$	(f) $\lim_{x\to 0^+} f(x) = -1$	(j) $f(2)$ undefined (hole at $x=2$)
(c) $\lim_{x\to -3} f(x) = 4$	(g) $\lim_{x\to 0} f(x)$ d.n.e.	(k) $\lim_{x\to 5^+} f(x) = 3$
(d) $f(-3)$ undefined	(h) $f(0) = 1$	(l) $\lim_{x\to 5^-} f(x)$ d.n.e.

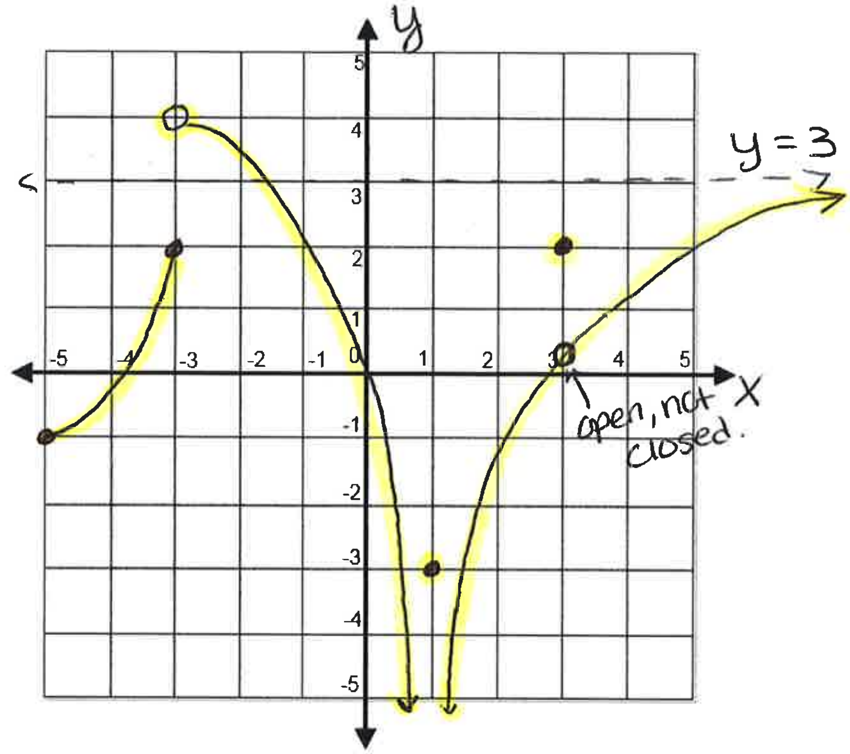
(hole at $x=-3$)

because $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

because $f(x)$ oscillates to the left of $x=5$.

4. Sketch the graph of a function – not a relation – that satisfies the following:

(i) The domain of f is $[-5, \infty)$.
(ii) $f(3) = 2$
(iii) $f(1) = -3$
(iv) $f(-5) = -1$
(v) $f(-3) = 2$
(vi) $\lim_{x\to\infty} f(x) = 3$
(vii) $\lim_{x\to-3^-} f(x) = 2$
(viii) $\lim_{x\to-3^+} f(x) = 4$
(ix) $\lim_{x\to1} f(x) = -\infty$

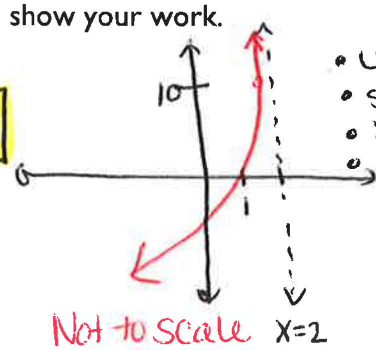


There are many correct answers.

5. Find the limits algebraically, using your knowledge of continuous functions, limit laws, and techniques for evaluating 0/0 indeterminate forms. As always, show your work.

(a) $\lim_{x\to2^-}[10 - 2\ln(6-3x)] = \boxed{\infty}$

(see graph of transformed $y = \ln x$ function)



- y-axis reflection
- shift to the right 2 units
- x-axis reflection
- shift up 10 units

(Limit of a known function.)

Alt. method

OR $\lim_{x\to2^-}[10-2\underbrace{\ln(6-3x)}_{(-\infty)}] = \infty$ b/c $\lim_{x\to2^-}\ln(6-3x) = -\infty$

So $10 - 2(\text{large neg \#}) = \text{large pos \#}$.

(b) $\lim_{x\to\frac{1}{2}}(x^2 - 4)\sin(2x)$

(Limit of a continuous function.)

$= \left(\left(\frac{1}{2}\right)^2 - 4\right)\sin\left(2\cdot\frac{1}{2}\right)$ Substitute $x = 1/2$.

$= \left(\frac{1}{4} - \frac{4}{1}\cdot\frac{4}{4}\right)\sin(1)$

$= \boxed{\frac{-15}{4}\sin(1)} \approx -3.156$ Simplify.

(c) $\lim_{x\to4}\frac{3-\sqrt{5+x}}{x-4} = \lim_{x\to4}\frac{(3-\sqrt{5+x})(3+\sqrt{5+x})}{(x-4)(3+\sqrt{5+x})}$

"$\frac{0}{0}$" Indet. form.

(Multiply by the conjugate)

$= \lim_{x\to4}\frac{9-(5+x)}{(x-4)(3+\sqrt{5+x})}$

$= \lim_{x\to4}\frac{9-5-x}{(x-4)(3+\sqrt{5+x})}$

Factor + reduce.

$= \lim_{x\to4}\frac{-\cancel{(x-4)}\,1}{\cancel{(x-4)}(3+\sqrt{5+x})}$

Evaluate the limit.

$= \frac{-1}{3+\sqrt{5+4}} = \boxed{-\frac{1}{6}}$

(d) $\lim_{x\to2}\frac{x^2+2x-8}{x^2-x-2} = \lim_{x\to2}\frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$

"$\frac{0}{0}$" Indet. form.

(Factor + Reduce)

Evaluate the limit.

$= \frac{2+4}{2+1}$

$= \frac{6}{3} = \boxed{2}$

** If you reassess over L4, be sure to study the "clearing complex fractions" technique as well. All 5 types of techniques appear on the reassessments.

6. It is known that $-4x^2+4x-1 \le f(x) \le 2x^2-9$ for values of x near 2. Is it possible to find $\lim_{x\to 2} f(x)$ using the Squeeze Theorem? If it is possible, find that limit using the Squeeze Theorem. If is not possible to find that limit with the information given, explain your reasoning.

$$\lim_{x\to 2}(-4x^2+4x-1) \le \lim_{x\to 2} f(x) \le \lim_{x\to 2}(2x^2-9)$$ provided the limits exist.

$$\Leftrightarrow \quad -4(2)^2+4(2)-1 \le \lim_{x\to 2} f(x) \le 2(2)^2-9$$

$$\Leftrightarrow \quad -9 \le \lim_{x\to 2} f(x) \le -1$$

It is not possible to determine $\lim_{x\to 2} f(x)$ because $\lim_{x\to 2}(-4x^2+4x-1) \ne \lim_{x\to 2}(2x^2-9)$.

7. Determine the limit using the special limits involving trigonometric functions, if the limit exists. If it does not exist, explain.

$$\lim_{x\to 0} \frac{\tan(2x)[1-\cos(3x)]}{x^2}$$

Write $\tan(2x) = \frac{\sin(2x)}{\cos(2x)}$.

$$= \lim_{x\to 0} \frac{\sin(2x)}{\cos(2x)} \cdot \frac{1-\cos(3x)}{x^2} \cdot \frac{2}{2} \cdot \frac{3}{3}$$

Multiply by $\frac{2}{2}$ & $\frac{3}{3}$.

$$= \lim_{x\to 0} \frac{\sin(2x)}{2x} \cdot \frac{1-\cos(3x)}{3x} \cdot \frac{6}{\cos(2x)}$$

Factor.

$$= \left[\lim_{x\to 0} \frac{\sin(2x)}{2x}\right]\left[\lim_{x\to 0} \frac{1-\cos(3x)}{3x}\right]\left[\lim_{x\to 0} \frac{6}{\cos(2x)}\right]$$

Limit of the product is the product of the limits.

$$= \underbrace{1}_{\text{(special limit)}} \cdot \underbrace{0}_{\text{(special limit)}} \cdot \frac{6}{\cos 0}$$

Evaluate limits, using special limits.

$$= 1 \cdot 0 \cdot 6 = \boxed{0}$$

Simplify.

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam."

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(signature)

If you can't sign this in good conscience, please don't. Come speak to me.