Name: Key Date: September 10, 2018 Class Time: ______________

Analytic Geometry & Calculus I | Tulsa Community College

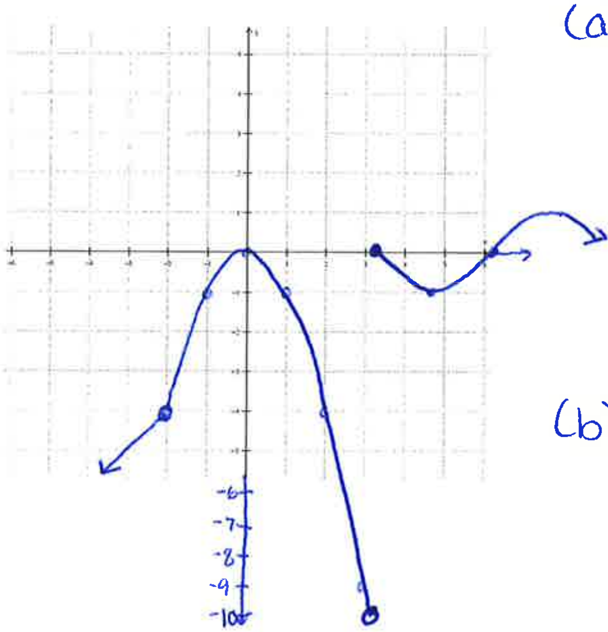
Quiz #2: Continuity and Limits Involving Infinity

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may not use a calculator on this assessment.

1. The piecewise function $f(x) = \begin{cases} x-2, & x < -2 \\ -x^2, & -2 \le x < \pi \\ \sin x, & x \ge \pi \end{cases}$ may or may not have some discontinuities.

You may graph the function for your own benefit, however, I'd like you to (a) explain **why** the only discontinuities that could exist would be at $x = -2$ and $x = \pi$, and nowhere else, AND (b) use the limit definition of continuity to determine whether $f(x)$ is continuous or not at $x = -2$ and $x = \pi$



(a) Linear functions, quadratic functions, + sin x are continuous for all real #s, so the only possible discontinuities occur as we go from the first to the 2nd branch, or 2nd to the 3rd branch, of the piecewise function. We transition from branch to branch at $x=-2$ & $x=\pi$.

(b) $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2} (x-2) = -2-2 = -4.$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2} -x^2 = -(-2)^2 = -4.$

& $f(-2) = -(-2)^2 = -4,$

so $\lim\limits_{x\to -2} f(x) = f(-2) \Leftrightarrow f$ is continuous at $x=-2$.

$\lim\limits_{x\to \pi^-} f(x) = \lim\limits_{x\to \pi^-} -x^2 = -\pi^2 \approx -9.87$

$\lim\limits_{x\to \pi^+} f(x) = \lim\limits_{x\to \pi^+} \sin x = \sin(\pi) = 0$

Since $\lim\limits_{x\to\pi^-} f(x) \ne \lim\limits_{x\to\pi^+} f(x)$, $\lim\limits_{x\to\pi} f(x)$ d.n.e.

$\Rightarrow f$ is not continuous at $x=\pi$.

2. Use your knowledge of continuous functions to determine either that the function $g(x) = \frac{5x - x\cos x}{x^2+9}$ is continuous everywhere, or that it has discontinuities. If it has discontinuities, explain where the discontinuities are located. Please be thorough in your explanation.

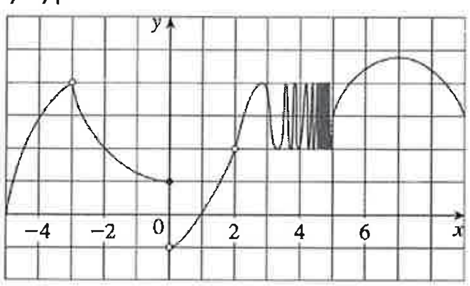
g is continuous everywhere, because (x^2+9) is never equal to 0, and cos x is continuous everywhere, so g is a quotient of continuous functions whose denominator is never 0.

$x^2+9=0$

$\cancel{x^2=-9}$

$x^2 \neq -9$ in the real # system.

3. (a) Determine the intervals where the function, graphed below, is continuous, and classify the discontinuities by type.



[-5,-3), (-3,0), (0,2), (2,5), (5,9]

x = -3 : Removable discontinuity
x = 0 : Jump discontinuity
x = 2 : Removable discontinuity
x = 5 : Oscillating discontinuity

(b) Can the function be defined or redefined at $x = -3$ to make the resulting function continuous there? If so, how should we define or redefine the function at $x = -3$?

Yes, define f(-3) = 4 to make the resulting function continuous at x = -3.

(c) Can the function be defined or redefined at $x = 5$ to make the resulting function continuous there? If so, how should we define or redefine the function at $x = 5$?

No; the discontinuity at x = 5 cannot be removed because the function oscillates to the left of x = 5.

Skill #: F6
Score:

(e) Use limit(s) to determine whether the function $f(x) = \frac{x^2(x-4)(x+2)}{(x-4)(x+3)}$ has horizontal asymptote(s).

Hint: $f(x) = \frac{x^2(x-4)(x+2)}{(x-4)(x+3)} = \frac{x^4-2x^3-8x^2}{x^2-x-12}$.

$$\lim_{x\to\pm\infty} \frac{(x^4-2x^3-8x^2)/x^2}{(x^2-x-12)/x^2} = \lim_{x\to\pm\infty} \frac{x^2-2x-8}{1-\underbrace{\frac{1}{x}}_{0}-\underbrace{\frac{12}{x^2}}_{0}} = \infty$$

$\Rightarrow$ The function does not have HAs.

(f) Find the x- and y-intercepts of the graph of $f(x) = \frac{x^2(x-4)(x+2)}{(x-4)(x+3)}$.

x-intercept: $0 = \frac{x^2\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)}$

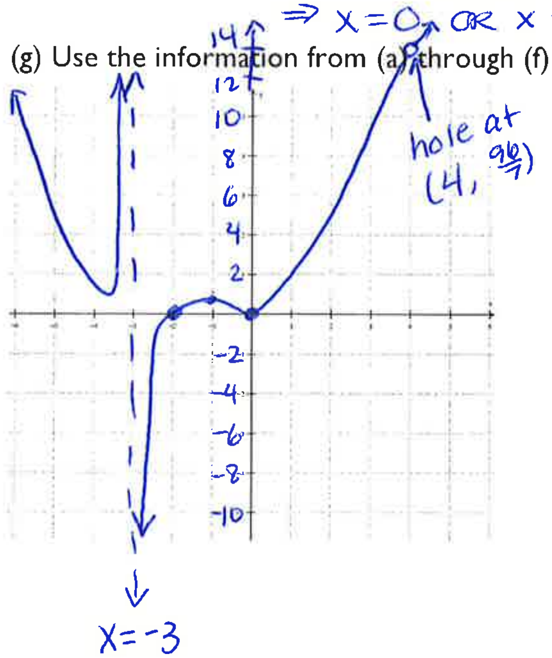
$0 = x^2(x+2)$

$\Rightarrow x = 0$ or $x = -2$. $\Rightarrow (0,0)$ & $(-2,0)$

y-intercept: $f(0) = \frac{0^2(0-4)(0+2)}{(0-4)(0+3)} = 0$

$\Rightarrow (x,y) = (0,0)$

(g) Use the information from (a) through (f) to graph the function $f(x) = \frac{x^2(x-4)(x+2)}{(x-4)(x+3)}$.



hole at $(x,y) = (4, \frac{96}{7})$

$\lim_{x\to -3^+} f(x) = -\infty$

$\lim_{x\to -3^-} f(x) = \infty$

$\lim_{x\to \pm\infty} f(x) = \infty$

intercepts: $(0,0)$, $(-2,0)$

Skill #: F7 Score:	Skill #: G1 Score:

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam."

____

(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

4. Find the following limits. They may or may not be infinite limits.

(a) $\lim_{x\to -3^+} \frac{x^2\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)} = \boxed{-\infty}$ "$\frac{-9}{0}$" → infinite limit

$\lim_{x\to -3^+} x^2 = (-3)^2 = 9$ (pos #)

$\lim_{x\to -3^+} (x+2) = -3+2 = -1$ (neg #)

$\lim_{x\to -3^+} (x+3) = 0$ (tiny pos #)

Try $x = -2.99$: $-2.99 + 3 = 0.01$

$\frac{(\text{pos \#})(\text{neg \#})}{(\text{tiny pos \#})} \to -\infty$

(b) $\lim_{x\to -3^-} \frac{x^2\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)} = \boxed{\infty}$ "$\frac{-9}{0}$" → infinite limit

$\lim_{x\to -3^-} x^2 = 9$ (pos #)

$\lim_{x\to -3^-} (x+2) = -1$ (neg #)

$\lim_{x\to -3^-} (x+3) = 0$ (tiny neg #)

Try $x = -3.001$: $-3.001 + 3 = -0.001$

$\frac{(\text{pos \#})(\text{neg \#})}{(\text{tiny neg \#})} \to \infty$

(c) $\lim_{x\to 4} \frac{x^2\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)} = \frac{16\cdot 6}{7} = \boxed{\frac{96}{7}} \approx 13.7$

(d) Using the function itself and the results of (a), (b), and (c), determine whether the function is continuous at $x = -3$, has a hole in the graph at $x = -3$, or has a vertical asymptote at $x = -3$. Then determine whether the function is continuous at $x = 4$, has a hole in the graph at $x = 4$, or has a vertical asymptote at $x = 4$.

The function has a vertical asymptote at $x = -3$, since $\lim_{x\to -3^\pm} f(x) = \pm\infty$.
The function has a hole in the graph at $x = 4$, since $f(4)$ is undefined while $\lim_{x\to 4} f(x) = \frac{96}{7}$.

If you finish early, here's a challenge problem! **It's worth one extra credit point.**

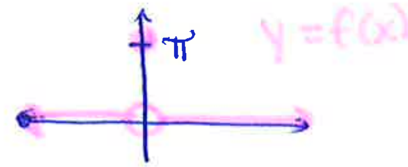
Let $f(x) = \begin{cases} 0 & \text{if } x \neq 0, \\ \pi & \text{if } x = 0. \end{cases}$

a. Find $\lim_{x \to 0} f(x)$.

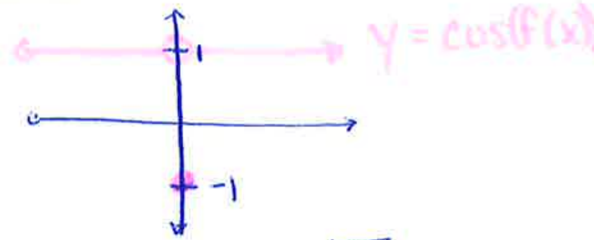
b. Find $\lim_{x \to 0} \cos(f(x))$.

c. Find $\lim_{x \to 0} f(\sin(x))$.

d. Find $\lim_{x \to 0} f(f(x))$.



(a) $\lim_{x \to 0} f(x) = \boxed{0}$



(b) $\lim_{x \to 0} \cos(f(x)) = \boxed{1}$

since $\cos 0 = 1$.

(c) $\lim_{x \to 0} f(\sin(x))$

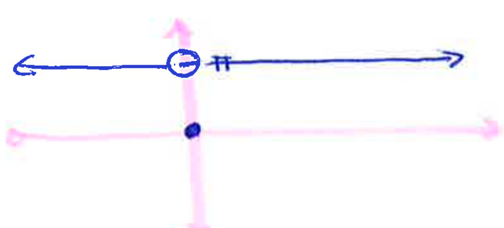
$= f\left(\lim_{x \to 0} \sin x\right)$

$= f(\sin 0)$

$= f(0)$

$= \boxed{\pi}$

$f(f(x)) = \begin{cases} f(0) = \pi & \text{when } x \neq 0 \\ f(\pi) = 0 & \text{when } x = 0 \end{cases}$



(d) $\lim_{x \to 0} f(f(x)) = \boxed{\pi}$