Name: Key

Date: Tuesday, February 4, 2020 Class Time:____________

Analytic Geometry & Calculus I | Tulsa Community College

Quiz #2: Limits Algebraically, Parts 1 and 2

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use a scientific, nongraphing calculator on this assessment.

Part 1. If possible, evaluate the limit. If the limit does not exist, explain. Then, for each limit, provide a geometric interpretation of the result.

1. $$\lim_{x\to 5} \frac{\sqrt{9-x}-2}{x^2-6x+5}$$ "$\frac{0}{0}$" indet. form

$$= \lim_{x\to 5} \frac{(\sqrt{9-x}-2)(\sqrt{9-x}+2)}{(x-5)(x-1)(\sqrt{9-x}+2)}$$

multiply & divide by the conjugate.

$$= \lim_{x\to 5} \frac{9-x-4}{(x-5)(x-1)(\sqrt{9-x}+2)}$$

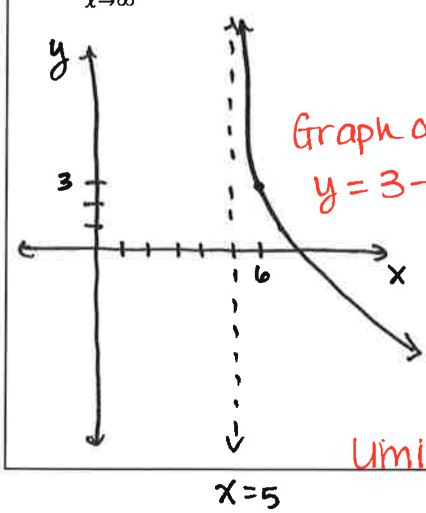
$$= \lim_{x\to 5} \frac{-\cancel{(x-5)}\,1}{\cancel{(x-5)}(x-1)(\sqrt{9-x}+2)}$$

$$= \frac{-1}{(5-1)(\sqrt{9-5}+2)} = -\frac{1}{16}.$$

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ _____.
(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) =$ (5 , $-1/16$).
(c) The function has a vertical asymptote, given by the equation ________________.
(d) The limit of the function does not exist, because ____________________________________.

2. $$\lim_{x\to\infty} [3 - 2\ln(x-5)] = -\infty$$



Graph of $y = 3 - 2\ln(x-5)$

Limit of a Known Function

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ _____
(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) =$ (_____, ______).
(c) The function has a vertical asymptote, given by the equation ________________.
(d) The limit of the function does not exist, because $f(x) = 3-2\ln(x-5) \to -\infty$ as $x \to \infty$.

3. $\lim_{x \to 2} 2x^2 \cos\left(\frac{\pi x}{2}\right)$

Limit of a Continuous Function.

$= 2(2)^2 \cos\left(\frac{\pi \cdot 2}{2}\right)$

$= 8 \underbrace{\cos \pi}_{(-1)} = \boxed{-8}$

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ 2
(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) =$ (______, ______).
(c) The function has a vertical asymptote, given by the equation ________________.
(d) The limit of the function does not exist, because______________________________________.

4. $\lim_{x \to -3} \frac{x^2 + 5x + 6}{2x(x^2 - x - 12)}$ "$\frac{0}{0}$" Indet. form

$= \lim_{x \to -3} \frac{\cancel{(x+3)}(x+2)}{2x(x-4)\cancel{(x+3)}}$ Factor + reduce.

$= \lim_{x \to -3} \frac{x+2}{2x(x-4)}$

$= \frac{-3+2}{2(-3)(-3-4)}$

$= \frac{-1}{-6(-7)} = -\frac{1}{42}$.

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ _____
(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) =$ (-3, $-1/42$).
(c) The function has a vertical asymptote, given by the equation ________________.
(d) The limit of the function does not exist, because______________________________________.

⑤ $\lim_{x\to 4^-} \dfrac{x^2+5x+6}{2x(x^2-x-12)}$

$= \lim_{x\to 4^-} \dfrac{\cancel{(x+3)}(x+2)}{2x(x-4)\cancel{(x+3)}}$

$= \lim_{x\to 4^-} \dfrac{(x+2)}{2x(x-4)}$ "$\frac{6}{0}$" $\to \pm\infty$

Infinite Limit

$\lim_{x\to 4^-} (x+2) = 6$ pos #

$\lim_{x\to 4^-} 2x = 8$ pos #

$\lim_{x\to 4^-} (x-4) = 0$ tiny neg #

$\Rightarrow \lim_{x\to 4^-} \dfrac{x+2}{2x(x-4)} = \boxed{-\infty}$

$\dfrac{(\text{pos\#})}{(\text{pos\#})(\text{tiny neg\#})} \to -\infty.$

Choose one of the following geometric interpretations of the result you just found.

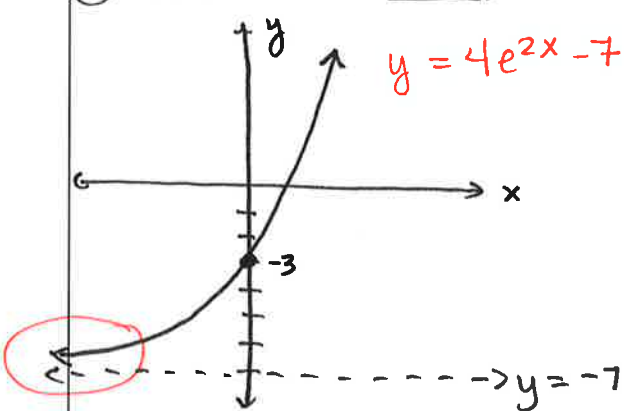
(a) The function is continuous at $x =$ ____

(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) =$ (______, ______).

(c) The function has a vertical asymptote, given by the equation $x = 4$.

(d) The limit of the function does not exist, because ______________________________.

⑥ $\lim_{x\to -\infty} [4e^{2x} - 7] = \boxed{-7}$



Limit of a Known Function

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ ____

(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) =$ (______, ______).

(c) The function has a horizontal asymptote, given by the equation $y = -7$.

(d) The limit of the function does not exist, because ______________________________.

Skill: L4
Score:

Part 2. Use limit laws, the special limits of trigonometric functions we discussed in class, and your knowledge of continuous functions to evaluate the limit below.

7. $\lim_{x \to 0}(\cot(3x) - \csc(3x))$

Hint: Use reciprocal and quotient identities to write the functions in terms of sine and cosine.

$$= \lim_{x \to 0}\left(\frac{\cos(3x)}{\sin(3x)} - \frac{1}{\sin(3x)}\right)$$

Write the functions in terms of sine & cosine.

$$= \lim_{x \to 0} \frac{\cos(3x) - 1}{\sin(3x)}$$

Get a common denominator.

$$= \lim_{x \to 0} \frac{1}{\sin(3x)} \cdot \frac{\cos(3x) - 1}{1} \cdot \frac{3x}{3x}$$

Multiply by $\frac{3x}{3x}$.

$$= \lim_{x \to 0} \left(\frac{3x}{\sin(3x)}\right) \cdot \left(\frac{\cos(3x) - 1}{3x}\right)$$

Regroup.

$$= \left(\lim_{x \to 0} \frac{3x}{\sin(3x)}\right)\left(\lim_{x \to 0} \frac{\cos(3x) - 1}{3x}\right)$$

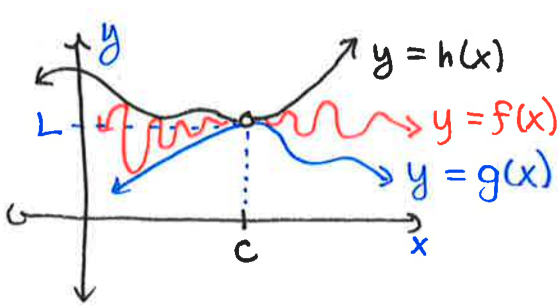
Apply limit law.

$$= 1 \cdot (0) = \boxed{0}.$$

8. State the Squeeze Theorem in your own words.

Suppose $g(x) \le f(x) \le h(x)$ on some interval containing c, except possibly at c.

Then, if $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} f(x) = L$.



9. Assume $(18 - 3x) \le f(x) \le (-2x^2 + 20x - 46)$ for all $x \ge 3$, except possibly at $x = 5$. If possible, use the squeeze theorem to find $\lim_{x\to 5} f(x)$. If it's not possible to find the limit using the squeeze theorem, explain.

Assume $\lim_{x\to 5} f(x)$ exists. Then

$$\lim_{x\to 5} (18-3x) \le \lim_{x\to 5} f(x) \le \lim_{x\to 5} (-2x^2+20x-46)$$

$$18-3(5) \le \lim_{x\to 5} f(x) \le -2(5)^2+20(5)-46$$

$$3 \le \lim_{x\to 5} f(x) \le -50+100-46 = 4$$

Since $\lim_{x\to 5} (18-3x) \ne \lim_{x\to 5} (-2x^2+20x-46)$, the squeeze theorem can't be applied here.

10. Assume $\frac{2(x-1)(x+3)}{(x+3)(x-5)} \le f(x) \le (x^2 + 6x + 10)$ for all $x < 0$, except possibly at $x = -3$. If possible, use the squeeze theorem to find $\lim_{x\to -3} f(x)$. If it's not possible to find the limit using the squeeze theorem, explain.

Assume $\lim_{x\to -3} f(x)$ exists. Then

$$\lim_{x\to -3} \frac{2(x-1)\cancel{(x+3)}}{\cancel{(x+3)}(x-5)} \le \lim_{x\to -3} f(x) \le \lim_{x\to -3} (x^2+6x+10)$$

$$\Leftrightarrow \quad \frac{2(-3-1)}{(-3-5)} \le \lim_{x\to -3} f(x) \le (-3)^2+6(-3)+10$$

$$\Leftrightarrow \quad 1 \le \lim_{x\to -3} f(x) \le 9-18+10 = 1$$

$\Rightarrow \lim_{x\to -3} f(x) = 1$ by the Squeeze Theorem.

Please sign below.

INTEGRITY STATEMENT:
On my personal integrity, I have not given, received, nor witnessed any unauthorized assistance on this quiz.

____
(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill: G1	Skill: L5
Score:	Score: