

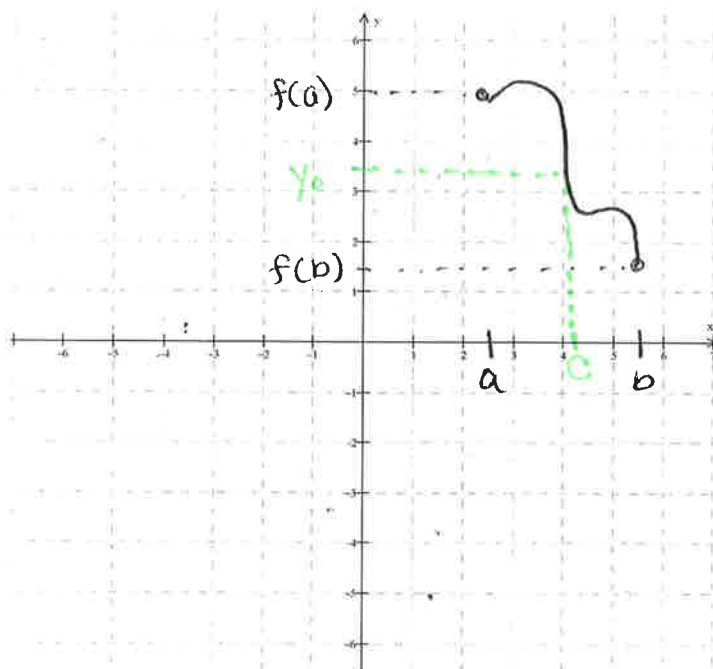
Quiz 3: Intermediate Value Theorem and Introduction to the Derivative Function

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you showing someone else how you got the answer.



You may not use a calculator on this assessment.

1. In your own words, explain the Intermediate Value Theorem.
If you want to, you can graph a function to illustrate your explanation.



If ① f is continuous on $[a, b]$
& ② y_0 is between $f(a)$ & $f(b)$,
then $y_0 = f(c)$ for some c
between $x = a$ & $x = b$.

IVT says that if f is continuous on a closed interval, f attains all of the y -values between $f(a)$ & $f(b)$ for some x -value between $x = a$ & $x = b$.

2. If possible, use the Intermediate Value Theorem to show that $4x^3 - 6x^2 + 3x = -3$ has a solution between $x = 1$ and $x = 2$. Be sure to check the hypothesis of the Intermediate Value Theorem before you use it. If it is NOT possible to use the theorem to prove that $4x^3 - 6x^2 + 3x = -3$ has a solution between $x = 1$ and $x = 2$, explain why it is not possible to use the theorem.

$$\text{Let } f(x) = 4x^3 - 6x^2 + 3x \text{ \& } y_0 = -3.$$

Since f is a polynomial, it is continuous on $(-\infty, \infty)$.

$$f(1) = 4(1)^3 - 6(1)^2 + 3(1) = 1$$

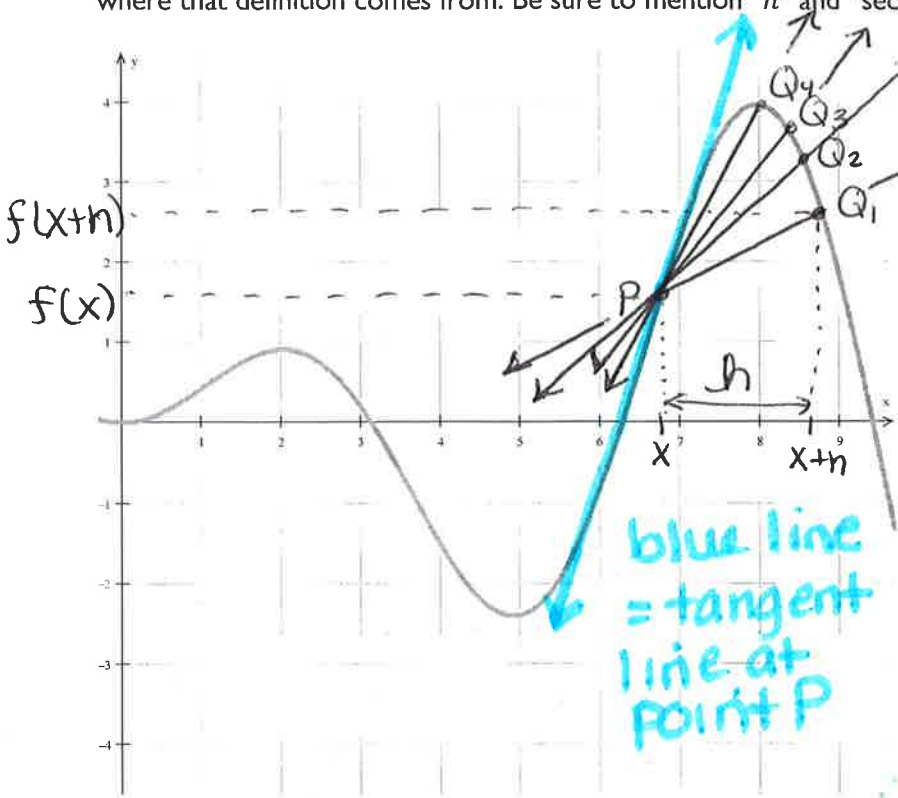
$$f(2) = 4(2)^3 - 6(2)^2 + 3(2) = 14$$

Since y_0 is not between $f(1) = 1$ & $f(2) = 14$, the IVT does not apply; IVT cannot be used to show that $f(x) = -3$ has a solution on the interval between $x = 1$ & $x = 2$.

Skill #: F8

Score:

3. Write down the limit definition of the derivative. Then explain – you may use the graph below to help you illustrate – where that definition comes from. Be sure to mention "h" and "secant" and "slope."



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The difference quotient

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

represents the slope of the secant line through P & Q.

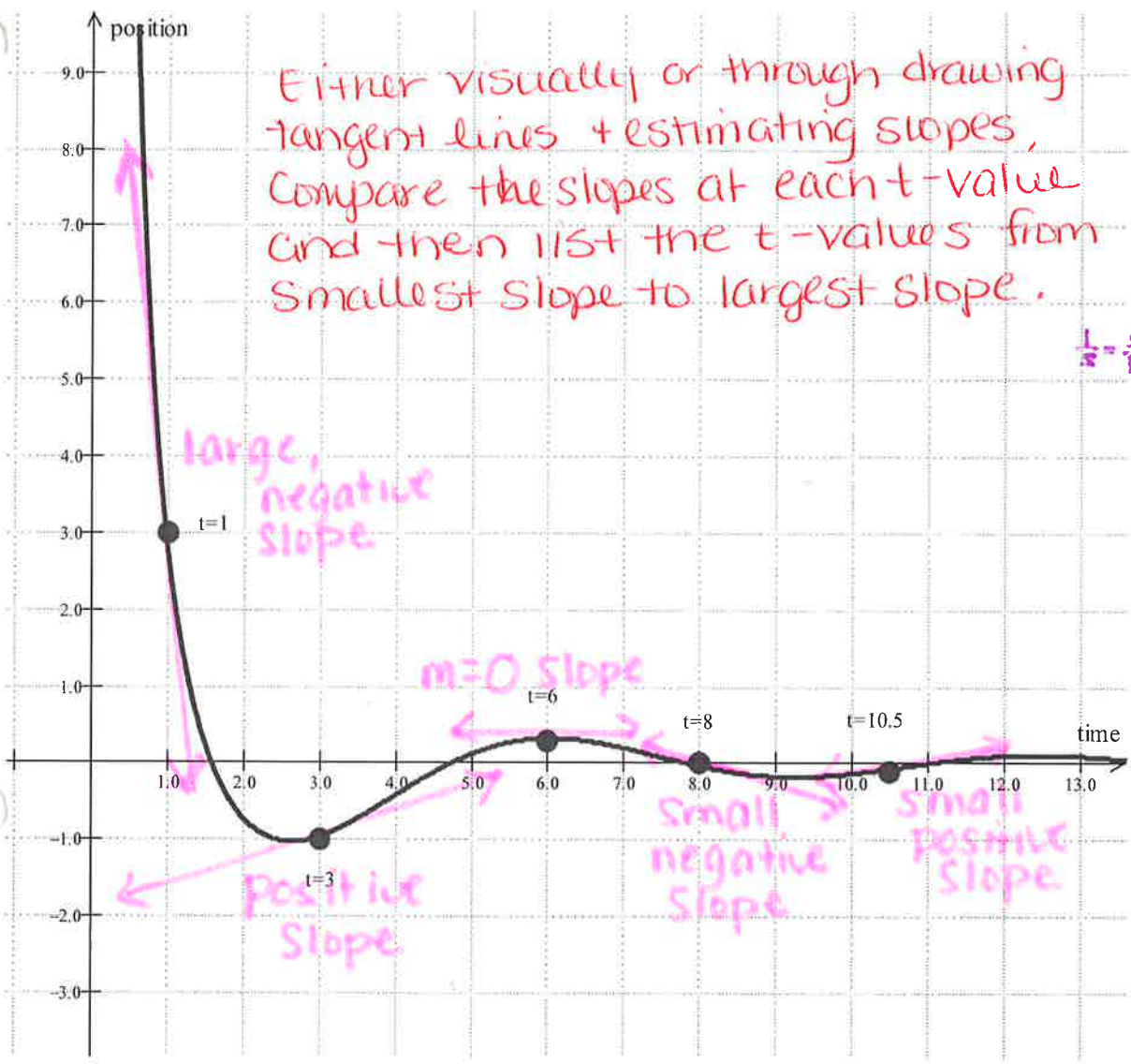
As Q approaches P, h gets closer to zero, and the slope of the secant line approaches the slope of the tangent line at P.

The slope of the tangent line at the point P is given by the derivative.

$$m_{\text{tangent}} = f'(x) = \lim_{h \rightarrow 0} m_{\text{secant}}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

4. In the following position versus time graph, order the following instantaneous velocities (the instantaneous rate of change of position with respect to time) from LEAST (smallest) to MOST (greatest): $t=1$, $t=3$, $t=6$, $t=8$, $t=10.5$.



LEAST (smallest) TO MOST (greatest) velocities: (Be sure to take the sign of the rate of change into consideration as you put these in order!)

- $t = 1$

(large in magnitude, negative velocity)
- $t = 8$

(small, negative velocity)
- $t = 6$

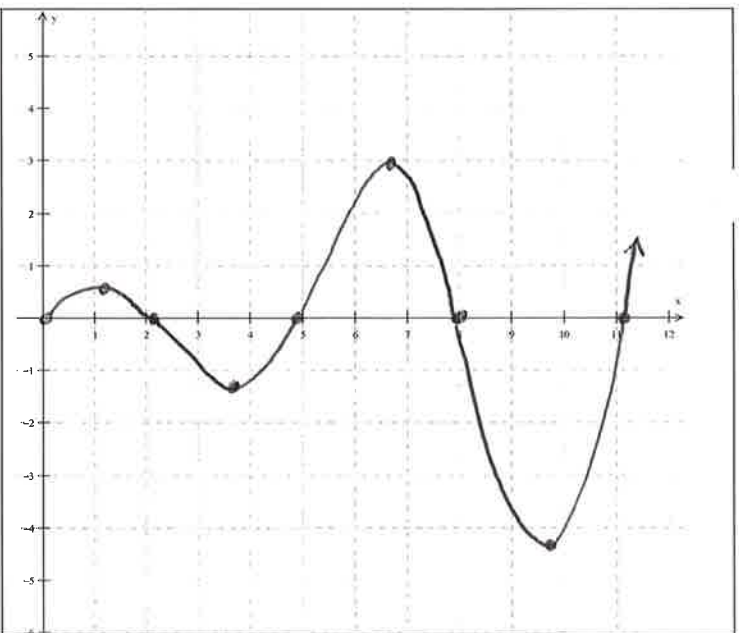
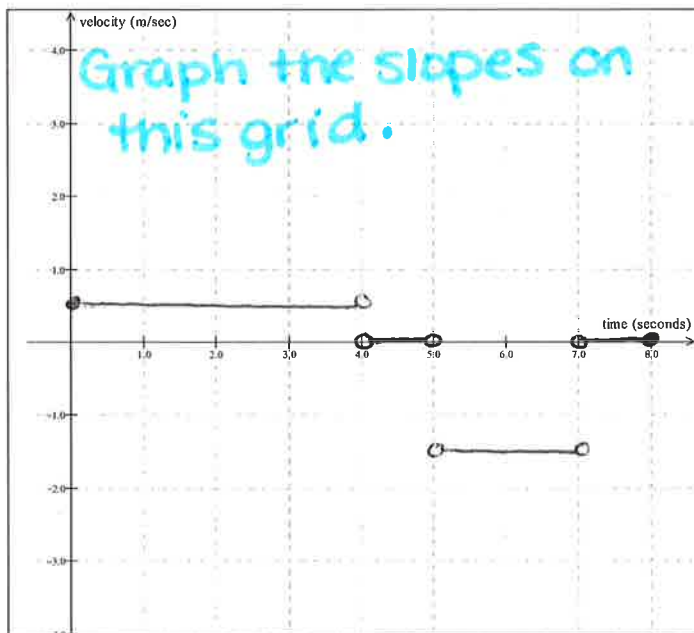
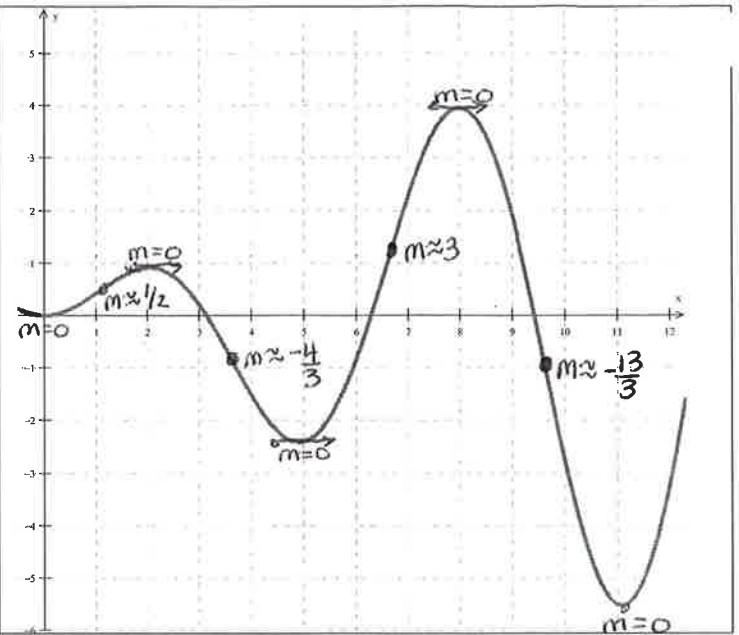
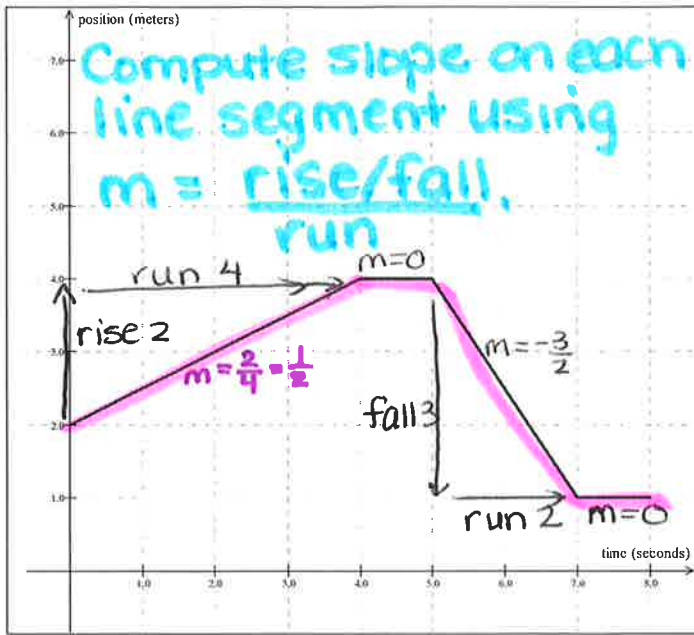
(velocity = 0)
- $t = 10.5$

(small, positive velocity)
- $t = 3$

(largest positive velocity of points shown)

Remember, when we're looking at a position graph, slope & velocity are the same thing!

5. Given the following graphs of position functions $f(t)$ and $g(t)$, sketch the graphs of the velocity functions $f'(t)$ and $g'(t)$ below.



6. If $V(r)$ represents the volume of a spherical balloon in cubic inches as a function of its radius, also in inches, what does $V'(1) = 4\pi$ mean in terms of the volume and radius of the balloon? (Don't mention slope! This question is about the real-world interpretation of the derivative.)

$V'(1)$ represents the instantaneous rate of change in volume when radius is 1 in

$V'(1) = 4\pi$ ~~cm~~ in^3/in means that volume is increasing by $4\pi \text{ in}^3$ for every one inch increase in the radius, when the radius is 1 inch.

Skill #: D10
Score:

7. (a) Use the limit definition of the derivative to **prove** that the derivative of $f(x) = 2x^3 + x$ is $f'(x) = 6x^2 + 1$.
 Hint: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (If you need more space, use the back of this page.)

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x+h)^3 + (x+h) - (2x^3 + x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) + x + h - 2x^3 - x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 + \cancel{x+h} - \cancel{2x^3} - \cancel{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{6x^2 + 6xh + 2h^2 + 1}{1} \\
 &= \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2 + 1) \\
 &= 6x^2 + 1.
 \end{aligned}$$

Substitute $f(x+h)$ & $f(x)$.

Expand $(x+h)^3$ & simplify.

Distribute 2 & simplify.

Factor out h , & reduce.

Evaluate the limit as $h \rightarrow 0$.

* To expand $(x+h)^3$, use the hint I gave you.

- (b) Find $f'(-4)$. What does the value $f'(-4)$ represent geometrically?

$$\begin{aligned}
 f'(-4) &= 6(-4)^2 + 1 \\
 &= 6(16) + 1 \\
 &= 96 + 1 \\
 &= 97.
 \end{aligned}$$

$f'(-4)$ is the slope of the tangent line to $y = f(x)$ at $x = -4$.

(Substitute $x = -4$ & simplify.)

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam.

(signature)

Skill #: G1
Score:

Skill #: D9
Score:

If you can't sign this in good conscience, please don't. Come speak to me.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. This is essential for ensuring the integrity of the financial data and for providing a clear audit trail.

2. The second part of the document focuses on the role of the accounting department in providing timely and accurate financial information to management. This information is crucial for making informed decisions and for identifying areas for improvement.

3. The third part of the document addresses the challenges of managing financial data in a complex and rapidly changing environment. It highlights the need for robust systems and processes to ensure data accuracy and security.

4. The fourth part of the document discusses the importance of regular communication and collaboration between the accounting department and other departments. This helps to ensure that all parties are aware of the latest financial information and can work together to address any issues.

5. The fifth part of the document concludes by emphasizing the need for continuous improvement and innovation in financial management. This involves staying up-to-date with the latest technologies and best practices to ensure the organization remains competitive.



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