Name: Key   Date: February 13, 2020   Class Time:____________

Analytic Geometry & Calculus I | Tulsa Community College

Quiz #3: Continuity, Limits Involving Infinity, The Intermediate Value Theorem, and the Limit Definition of the Derivative

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

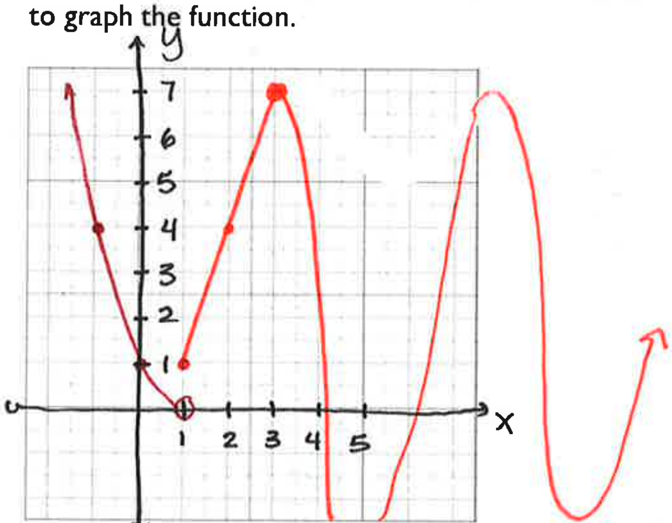
You may use a calculator on this assessment

1. State the limit definition of continuity at the point $x = a$.

The function f is continuous at $x=a$ if $\lim_{x \to a} f(x)$ exists, $f(a)$ exists, and they are equal to each other.

$$\lim_{x \to a} f(x) = f(a)$$

2. The piecewise function $f(x) = \begin{cases} x^2 - 2x + 1, & x < 1 \\ 3x - 2, & 1 \le x < 3 \\ 6\cos(x-3) + 1, & x \ge 3 \end{cases}$ may or may not have some discontinuities. While you're not required to graph the function, you may choose to use the grid below to graph the function.



(a) Explain **why** the only possible discontinuities of the function are $x = 1$ and $x = 3$.

Polynomials are continuous, so f must be continuous for $x < 1$ and $1 < x < 3$.

Transformations of cosine are continuous, so f must be continuous for $x > 3$.

The only possible discontinuities are x-values where we switch from branch to branch of the piecewise function, so $x = 1$ & $x = 3$ are the only possible discontinuities.

(b) Use the limit definition of continuity to determine if the function is continuous at $x = 1$.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2-2x+1) = (1)^2-2(1)+1 = 0.$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3x-2) = 3(1)-2 = 1.$$

Since $0 \neq 1$, $\lim_{x\to 1} f(x)$ d.n.e., so f is <u>not</u> continuous at $x=1$.

It has a jump discontinuity there.

(c) Use the limit definition of continuity to determine if the function is continuous at $x = 3$.

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (3x-2) = 3(3)-2 = 7.$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} [6\cos(x-3)+1] = 6\cos(3-3)+1 = 7.$$

$\Rightarrow \lim_{x\to 3} f(x) = 7$. Since $f(3) = 6\cos(3-3)+1 = 7$, we have $\lim_{x\to 3} f(x) = f(3)$. $\Rightarrow f$ is continuous at $x=3$.

3. Consider the function g.

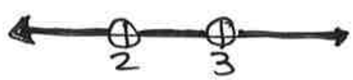
$$g(x) = \frac{(x-3)\sin x}{(x-3)(x-2)}$$

(a) Find the intervals of x-values where the function is continuous. Explain.

g is undefined when $(x-3)(x-2) = 0 \Rightarrow x=2$ \& $x=3$.

$\Rightarrow g$ is continuous for all x s.t. $x \neq 2$ \& $x \neq 3$.

$\Rightarrow g$ is continuous on $(-\infty, 2) \cup (2,3) \cup (3,\infty)$.

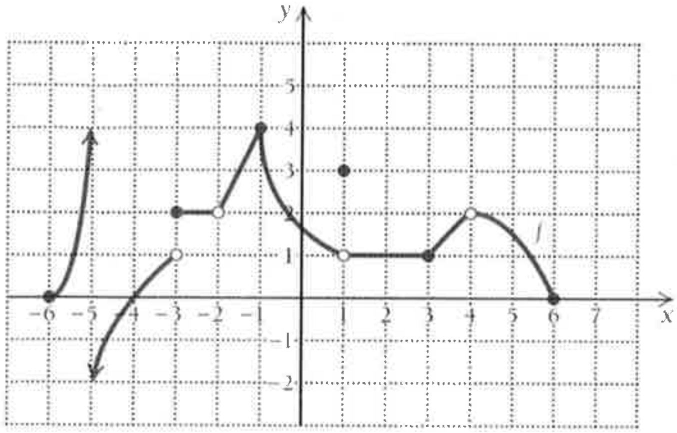


(b) Find the discontinuities of the function, and classify them by type (removable or nonremovable, jump, infinite, or oscillating.

$x=3$: Since the factor $(x-3)$ reduces, $x=3$ is a removable discontinuity. The graph has a hole at $x=3$.

$x=2$: Since substituting $x=2$ yields "$\frac{-\sin 2}{0}$", $x=2$ is a vertical asymptote. It is a nonremovable, infinite discontinuity.

4. Consider the function graphed below. This image is from the Bittinger Calculus text, 10th edition. State its discontinuities, and classify them by type.



$x = -5$: nonremovable, infinite discontinuity

$x = -3$: nonremovable, jump discontinuity

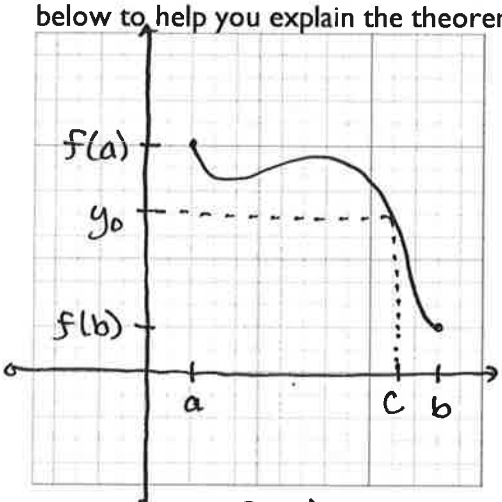
$x = -2$: removable disc.

$x = 1$: removable disc.

$x = 4$: removable disc.

Skill #: F6
Score:

5. In your own words, state the Intermediate Value Theorem. While not required, you may use the grid below to help you explain the theorem.



$f(c) = y_0$
for c between
$x = a$ & $x = b$.

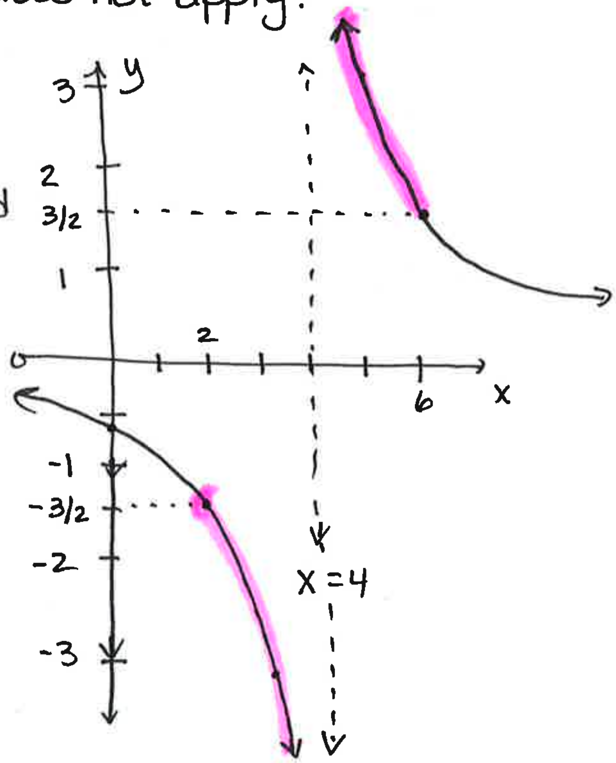
IVT: If f is continuous on $[a, b]$, and y_0 is between $f(a)$ & $f(b)$, then $f(x) = y_0$ has a solution between $x = a$ & $x = b$.

6. If possible, use the Intermediate Value Theorem to determine whether $f(x) = \frac{3}{x-4}$ is equal to 3 on the interval $2 \leq x \leq 6$. If it is not possible, explain.

f(x) has a vertical asymptote (or infinite discontinuity) at x = 4. Since 4 is in the interval [2,6], IVT does not apply.

$f(x) = \frac{3}{x-4}$

(The highlighted portion shows f for x in [2,6].)



Skill #: F7
Score:

7. This problem has multiple parts. First, you'll be asked to compute some limits and explain what they represent geometrically. Then, you'll be asked to graph the function. Keep the big picture in mind as you work through the following limits and interpretations.

(a) $\lim_{x \to 2} \frac{3(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)^2}$

$= \frac{3(2-2)}{(2+1)^2} = \frac{0}{9} = 0$

$\Rightarrow (x, y) = (2, 0)$ is on the graph of the function.

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ 2.
(b) The function has a removable discontinuity. The location of the hole in the graph is $(x, y) = (\_\_\_\_, \_\_\_\_)$.
(c) The function has a vertical asymptote, given by the equation ________.
(d) The function has a horizontal asymptote, given by the equation ________.
(e) The limit of the function does not exist, because ________________.

(b) $\lim_{x\to 3}\frac{3(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)^2}$

$= \frac{3(3-2)}{(3+1)^2} = \frac{3}{16}$

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ ____

(b) The function has a removable discontinuity. The location of the hole in the graph is $(x,y) = (\underline{3}, \underline{3/16})$.

(c) The function has a vertical asymptote, given by the equation ________________.

(d) The function has a horizontal asymptote, given by the equation ________________.

(e) The limit of the function does not exist, because ________________________________.

(c) $\lim_{x\to -1}\frac{3(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)^2}$

"$\frac{-9}{0}$" $\to \pm\infty$

$\lim_{x\to -1} 3(x-2) = 3(-1-2) = -9$ (neg #)

$\lim_{x\to -1} (x+1)^2 = 0$ (tiny pos #)

$\Rightarrow \lim_{x\to -1}\frac{3(x-2)(x-3)}{(x-3)(x+1)^2} = -\infty$

$\frac{\text{neg \#}}{\text{tiny pos. \#}} \longrightarrow -\infty.$

Choose one of the following geometric interpretations of the result you just found.

(a) The function is continuous at $x =$ ____

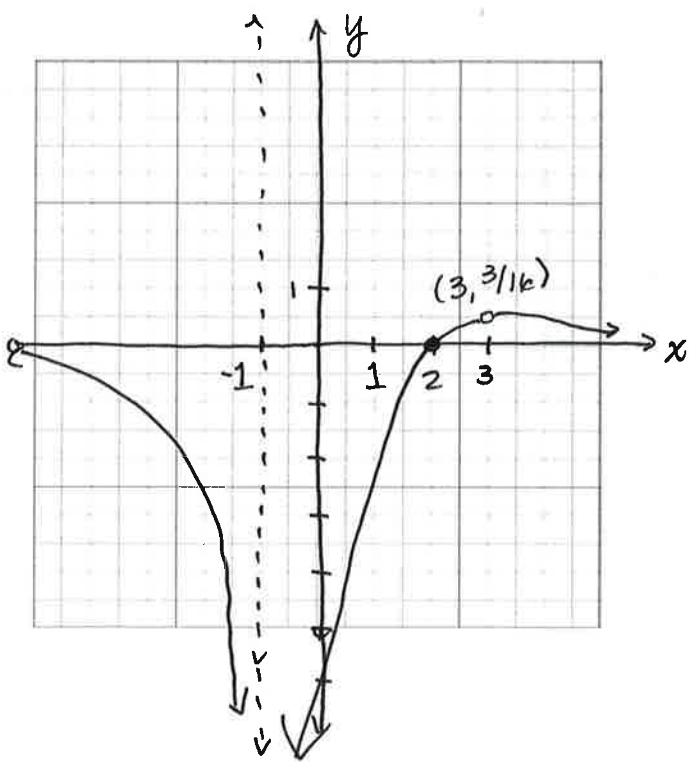
(b) The function has a removable discontinuity. The location of the hole in the graph is $(x,y) = (\_\_\_\_, \_\_\_\_)$.

(c) The function has a vertical asymptote, given by the equation $\underline{x = -1}$.

(d) The function has a horizontal asymptote, given by the equation ________________.

(e) The limit of the function does not exist, because ________________________________.

(d) Use the results of parts (a) through (c) to sketch a graph of the function.



* Note: If you reassess over this topic, you'll also compute

$\lim_{x\to\pm\infty} f(x)$

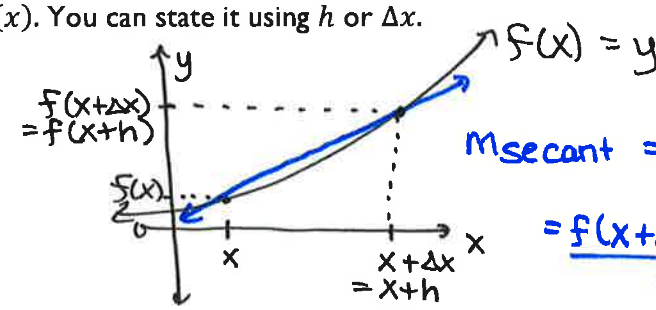
to determine the end behavior of the rational function.

Skill #: F8
Score:

8. State the limit definition of the derivative $f'(x)$. You can state it using h or Δx.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

or $$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{h}.$$



$$m_{secant} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

9. Use the limit definition from (8) to show that the derivative of $f(x) = 2x^2 - 3x + 5$ is $f'(x) = 4x - 3$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\boxed{f(x+\Delta x)} - \boxed{f(x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\boxed{2(x+\Delta x)^2 - 3(x+\Delta x) + 5} - (2x^2 - 3x + 5)}{\Delta x}$$ Substitute.

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)(x+\Delta x) \cancel{-3x} - 3\Delta x \cancel{+5} - 2x^2 \cancel{+3x} \cancel{-5}}{\Delta x}$$ Simplify.

$$= \lim_{\Delta x \to 0} \frac{2(x^2 + 2x\Delta x + (\Delta x)^2) - 3\Delta x - 2x^2}{\Delta x}$$ Distribute.

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + 2(\Delta x)^2 - 3(\Delta x) \cancel{-2x^2}}{\Delta x}$$ Simplify.

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(4x + 2\Delta x - 3)}{\cancel{\Delta x}}$$ Factor & reduce.

$$= 4x + 2(0) - 3$$ Evaluate the limit.

$$= \boxed{4x - 3}.$$

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam."

____
(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill #: G1
Score:

Skill #: D9
Score: