Name: Key Date: February 25, 2020 Class Time:

Analytic Geometry & Calculus I | Tulsa Community College

Quiz#4: Differentiation – The Basic Rules, the Quotient Rule, and Product Rule

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use a scientific, nongraphing calculator on this assessment.

1. Find the first and second derivative of each of the following functions.

(a) $f(x) = 2x^4 - 5x^2 + 6x - 1$

$f'(x) = 2\cdot 4x^3 - 5\cdot 2x + 6 - 0$

$= \boxed{8x^3 - 10x + 6 = f'(x)}$

Compute f′. Simplify.

$f''(x) = 8\cdot 3x^2 - 10 + 0$

$= \boxed{24x^2 - 10 = f''(x)}$

Compute f″. Simplify.

(b) $g(x) = \frac{2}{x} + 4\sqrt[3]{x^4} + \frac{1}{3x^3}$

$= 2x^{-1} + 4x^{4/3} + \frac{1}{3}x^{-3}$

Rewrite g(x).

$g'(x) = 2(-1)x^{-2} + 4\left(\frac{4}{3}\right)x^{1/3} + \frac{1}{3}(-3)x^{-4}$

Compute g′.

$= \boxed{-2x^{-2} + \frac{16}{3}x^{1/3} - x^{-4} = g'(x)}$

Simplify

OR $\frac{-2}{x^2} + \frac{16}{3}\sqrt[3]{x} - \frac{1}{x^4} = g'(x)$.

$g''(x) = \frac{d}{dx}\left[-2x^{-2} + \frac{16}{3}x^{1/3} - x^{-4}\right]$

Compute g″.

$= -2(-2)x^{-3} + \frac{16}{3}\left(\frac{1}{3}\right)x^{-2/3} - (-4)x^{-5}$

Simplify.

$= \boxed{4x^{-3} + \frac{16}{9}x^{-2/3} + 4x^{-5} = g''(x)}$

OR $\frac{4}{x^3} + \frac{16}{9\sqrt[3]{x^2}} + \frac{4}{x^5} = g''(x)$.

(c) $h(t) = -\frac{3e^t}{2} + 3t + \frac{4}{5\sqrt{t}} - 2\ln t$

$= -\frac{3}{2}e^t + 3t + \frac{4}{5}t^{-1/2} - 2\ln t$ Rewrite h(t).

$h'(t) = -\frac{3}{2}e^t + 3 + \frac{4}{5}\left(-\frac{1}{2}\right)t^{-3/2} - \frac{2}{1}\cdot\frac{1}{t}$ Compute h'. Simplify.

$= \boxed{-\frac{3}{2}e^t + 3 - \frac{2}{5}t^{-3/2} - 2t^{-1} = h'(t)}$

OR $-\frac{3}{2}e^t + 3 - \frac{2}{5\sqrt{t^3}} - \frac{2}{t} = h'(t)$.

$h''(t) = -\frac{3}{2}e^t + 0 - \frac{2}{5}\left(-\frac{3}{2}\right)t^{-5/2} - 2(-1)t^{-2}$ Compute h''.

$= \boxed{-\frac{3}{2}e^t + \frac{3}{5}t^{-5/2} + 2t^{-2} = h''(t)}$ Simplify.

OR $-\frac{3}{2}e^t + \frac{3}{5\sqrt{t^5}} + \frac{2}{t^2} = h''(t)$.

2. Find the first derivative of the following function.

$y(x) = 3\sin x - 4\cos x + \frac{\tan x}{2} - \frac{3}{5}\sec x + 4\cot x - \frac{\csc x}{2}$ Rewrite y(x).

$= 3\sin x - 4\cos x + \frac{1}{2}\tan x - \frac{3}{5}\sec x + 4\cot x - \frac{1}{2}\csc x$.

Differentiate.

$y' = 3\cos x - 4(-\sin x) + \frac{1}{2}\sec^2 x - \frac{3}{5}\sec x\tan x + 4(-\csc^2 x)$
$-\frac{1}{2}(-\csc x\cot x)$

$\boxed{y' = 3\cos x + 4\sin x + \frac{1}{2}\sec^2 x - \frac{3}{5}\sec x\tan x - 4\csc^2 x + \frac{1}{2}\csc x\cot x.}$ Simplify.

Skill #: D11
Score:

2. Find the first derivative of the following functions. DO NOT expand and simplify your answer!

(a) $y(x) = \underbrace{\left(\frac{x^7}{14} - 5x^3\right)}_{\text{1st}} \underbrace{\left(\frac{1}{x} + 4e^x\right)}_{\text{2nd}}$

Product Rule

$$y' = \underbrace{\frac{d}{dx}\left[\frac{1}{14}x^7 - 5x^3\right]}_{\left(\frac{1}{14}\cdot 7x^6 - 5\cdot 3x^2\right)}\left(\frac{1}{x} + 4e^x\right) + \underbrace{\frac{d}{dx}\left[x^{-1} + 4e^x\right]}_{\left(-x^{-2} + 4e^x\right)}\left(\frac{x^7}{14} - 5x^3\right)$$

$$= \boxed{\left(\frac{1}{2}x^6 - 15x^2\right)\left(\frac{1}{x} + 4e^x\right) + \left(-\frac{1}{x^2} + 4e^x\right)\left(\frac{x^7}{14} - 5x^3\right) = y'}$$

Simplify.

Quotient Rule

(b) $f(x) = \frac{2x^3 - 3 + \cos x}{x - 4e^x + \ln x}$

$$f'(x) = \frac{(x - 4e^x + \ln x)\frac{d}{dx}[2x^3 - 3 + \cos x] - (2x^3 - 3 + \cos x)\frac{d}{dx}[x - 4e^x + \ln x]}{(x - 4e^x + \ln x)^2}$$

$$\Leftrightarrow \boxed{f'(x) = \frac{(x - 4e^x + \ln x)(6x^2 - \sin x) - (2x^3 - 3 + \cos x)\left(1 - 4e^x + \frac{1}{x}\right)}{(x - 4e^x + \ln x)^2}}$$

Simplify.

Please sign below.

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this quiz.

____

(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill #: G1
Score:

Skill #: D12
Score: