Name: Key   Date: October 8, 2018   Class Time: ____________

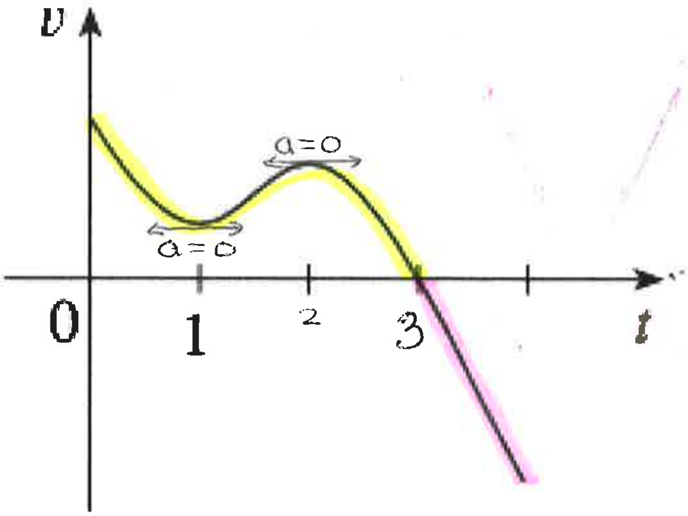
Analytic Geometry & Calculus I | Tulsa Community College

Quiz #5 Applications of Differentiation, The Chain Rule, and Implicit Differentiation

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use a scientific, nongraphing calculator on this assessment.

1. The graph of the velocity function $v(t)$ of a particle after t seconds is shown below. Use the graph of the function to answer the following questions. Be sure to justify your answers.



(a) When is the object moving forward? (Your answer should be a time interval or intervals.)

The object moves forward when $v(t) > 0$ $\Rightarrow$ The object moves forward when $0 \le t < 3$. $\Leftrightarrow [0, 3)$

(b) When is the object moving backward?

The object moves backward when $v(t) < 0$ $\Rightarrow$ The object moves backward for $t > 3$.
$\Leftrightarrow (3, \infty)$

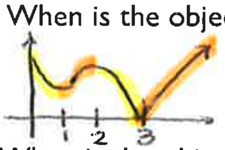
(c) When does the object change directions?

The object changes directions at $t = 3$.

(d) When is the acceleration of the object equal to zero?

Acceleration is zero when the slope of the tangent line is zero. This occurs at $t = 1$ and $t = 2$.

(e) When is the object speeding up?



speed

The object is speeding up when the speed is increasing $\Rightarrow$ $1 < t < 2$ and $t > 3$
or $(1, 2) \cup (3, \infty)$

(f) When is the object slowing down?

The object is slowing down when the speed is decreasing.
$\Rightarrow 0 \le t < 1$ and $2 < t < 3$
$\Leftrightarrow [0, 1) \cup (2, 3)$.

2. A particle has position function $s(t) = t^3 - 12t^2 + 36t$.

(a) Find the velocity function.

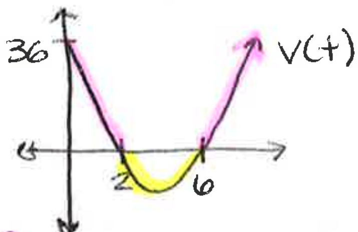
$$v(t) = s'(t) = \frac{d}{dt}[t^3 - 12t^2 + 36t]$$
$$= 3t^2 - 12 \cdot 2t + 36$$
$$= \boxed{3t^2 - 24t + 36}$$

(b) Find the acceleration function.

$$a(t) = v'(t) = \frac{d}{dt}[3t^2 - 24t + 36]$$
$$= 3 \cdot 2t - 24$$
$$= \boxed{6t - 24}$$

(c) When does the particle move forward (or in other words, in the positive direction)?

$$v(t) = 3t^2 - 24t + 36$$
$$= 3(t^2 - 8t + 12)$$
$$= 3(t-2)(t-6)$$



The particle moves forward when the velocity is positive

$\Rightarrow 0 \leq t < 2$ and $t > 6$ $\Leftrightarrow$ $[0, 2) \cup (6, \infty)$.

(d) Determine the t-values when the particles velocity is zero.

$v(t) = 0$ when $3(t-2)(t-6) = 0$

$t - 2 = 0$ OR $t - 6 = 0$

$+2 \quad +2$ $\qquad$ $+6 \quad +6$

$t = 2$ $\qquad$ $t = 6$

(e) Describe the motion of the particle immediately before and immediately after each of those t-values. In order to answer this question, you may want to graph the velocity function.

Just before $t=2$, the particle was slowing down in the forward direction. At $t=2$, the particle paused & turned around. Just after $t=2$, the particle began speeding up in the backwards (or negative) direction.

Just before $t=6$, the particle was slowing down in the negative (backward) direction. At $t=6$, the particle paused & turned around. Just after $t=6$, the particle began speeding up in the positive (or forward) direction.

Skill #: D13

Score:

3. Compute the derivative of the following functions. The chain rule and all of your basic derivative rules (product rule, quotient rule, exponent rule, power rule, etc) may be required. If the product or quotient rule is necessary, you do not have to simplify your final answer.

Chain Rule

(a) $y(x) = \sqrt{x^5 - 10x^2 + 4x^3 - 1}$ $= (\underbrace{x^5 - 10x^2 + 4x^3 - 1}_{\text{inside function}})^{1/2}$

$$y'(x) = \frac{1}{2}(x^5 - 10x^2 + 4x^3 - 1)^{-1/2} \frac{d}{dx}[x^5 - 10x^2 + 4x^3 - 1]$$

$$= \frac{1}{2}(x^5 - 10x^2 + 4x^3 - 1)^{-1/2}(5x^4 - 10 \cdot 2x + 4 \cdot 3x^2 - 0)$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x^5 - 10x^2 + 4x^3 - 1}} \cdot \frac{(5x^4 - 20x + 12x^2)}{1}$$

$$= \boxed{\frac{5x^4 - 20x + 12x^2}{2\sqrt{x^5 - 10x^2 + 4x^3 - 1}}}$$

Product Rule/ Chain Rule

(b) $f(x) = e^{5x}\sin(3x + 1)$

$$f'(x) = \frac{d}{dx}[e^{5x}] \cdot \sin(3x+1) + \frac{d}{dx}[\sin(3x+1)] \cdot e^{5x}$$

inside function   inside function

$$= e^{5x} \underbrace{\frac{d}{dx}[5x]}_{5} \sin(3x+1) + \cos(3x+1) \underbrace{\frac{d}{dx}[3x+1]}_{3} \cdot e^{5x}$$

$$= 5e^{5x}\sin(3x+1) + [3\cos(3x+1)]e^{5x}$$

$$= \boxed{5e^{5x}\sin(3x+1) + 3e^{5x}\cos(3x+1)}$$

Quotient Rule/ Chain Rule

(c) $g(x) = \frac{\ln x}{\tan(x^4)}$

$$g'(x) = \frac{\tan(x^4)\frac{d}{dx}[\ln x] - \ln x \frac{d}{dx}[\tan(x^4)]}{\tan^2(x^4)}$$

inside function

$$= \frac{\tan(x^4) \cdot \frac{1}{x} - \ln x \cdot \sec^2(x^4)\frac{d}{dx}[x^4]}{\tan^2(x^4)}$$

$$= \boxed{\frac{\frac{1}{x}\tan(x^4) - 4x^3 \ln x \sec^2(x^4)}{\tan^2(x^4)}} \cdot \frac{x}{x} \quad \text{OR}$$

$$= \boxed{\frac{\tan(x^4) - 4x^4 \ln x \sec^2(x^4)}{x\tan^2(x^4)}}$$

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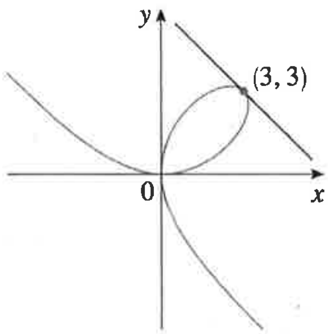
4. The graph of the relation given below, and the point $(x, y) = (3,3)$ are shown.



$$x^3 + y^3 = 6xy$$

Find the slope of the curve $\frac{dy}{dx}$ at the point (3,3) using **implicit differentiation**.

$$\frac{d}{dx}[x^3+y^3] = \frac{d}{dx}[\underbrace{6x}_{1st}\,\underbrace{y}_{2nd}]$$ Diff. wrt x.

$$3x^2 + 3y^2\frac{dy}{dx} = \frac{d}{dx}[6x]\cdot y + \frac{d}{dx}[y]\cdot 6x$$

$$3x^2+\cancel{3y^2\frac{dy}{dx}} = \cancel{6y} + 6x\frac{dy}{dx}$$

$$-6y - \cancel{3y^2\frac{dy}{dx}} \qquad \cancel{-6y} - 3y^2\frac{dy}{dx}$$

$$3x^2 - 6y = 6x\frac{dy}{dx} - 3y^2\frac{dy}{dx}$$ Move all $\frac{dy}{dx}$ terms to one side.

$$\frac{3x^2-6y}{6x-3y^2} = \frac{\frac{dy}{dx}\cancel{(6x-3y^2)}}{\cancel{6x-3y^2}}$$ Factor out $\frac{dy}{dx}$.

Divide to isolate y'.

$$\frac{dy}{dx} = \frac{3x^2-6y}{6x-3y^2} = \frac{\cancel{3}(x^2-2y)}{\cancel{3}(2x-y^2)} = \frac{x^2-2y}{2x-y^2}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(3,3)} = \frac{3^2-2\cdot3}{2\cdot3-3^2} = \frac{9-6}{6-9} = \frac{3}{-3} = \boxed{-1}$$ Substitute (3,3) into y'.

$\Rightarrow$ $\boxed{\text{The slope at } (x,y) = (3,3) \text{ is } -1}$.

5. Find $\frac{dy}{dx}$, given the equation of the relation below.

$$\sin(x+y) = y^2 \cos x$$

$$\frac{d}{dx}[\sin(\underbrace{x+y}_{\text{inside}})] = \frac{d}{dx}[\underbrace{y^2}_{\text{1st}}\underbrace{\cos x}_{\text{2nd}}]$$

Differentiate both sides wrt x.

$$\cos(x+y)\frac{d}{dx}[x+y] = \frac{d}{dx}[y^2]\cos x + \frac{d}{dx}[\cos x]\cdot y^2$$

$$\cos(x+y)\left(1+\frac{dy}{dx}\right) = 2y\frac{dy}{dx}\cos x - (\sin x)y^2$$

Distribute cos(x+y) on LHS.

$$\cos(x+y) + \cancel{\cos(x+y)\frac{dy}{dx}} = 2y\cos x\frac{dy}{dx} - \cancel{y^2\sin x}$$

$$+y^2\sin x \qquad \cancel{-\cos(x+y)\frac{dy}{dx}} \qquad -\cos(x+y)\frac{dy}{dx} \qquad \cancel{+y^2\sin x}$$

Move all $\frac{dy}{dx}$ terms to one side + all other terms to the other side.

$$\cos(x+y) + y^2\sin x = 2y\cos x\frac{dy}{dx} - \cos(x+y)\frac{dy}{dx}$$

$$\frac{\cos(x+y)+y^2\sin x}{2y\cos x-\cos(x+y)} = \frac{\frac{dy}{dx}[\cancel{2y\cos x-\cos(x+y)}]}{\cancel{2y\cos x-\cos(x+y)}}$$

Factor out $\frac{dy}{dx}$.

Divide by $2y\cos x - \cos(x+y)$ to isolate $\frac{dy}{dx}$.

$$\boxed{\frac{dy}{dx} = \frac{\cos(x+y)+y^2\sin x}{2y\cos x-\cos(x+y)}}$$

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam."

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(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill #: D15
Score:

Skill #: G1
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