Name: Key Date: March 3, 2020 Class Time:____________

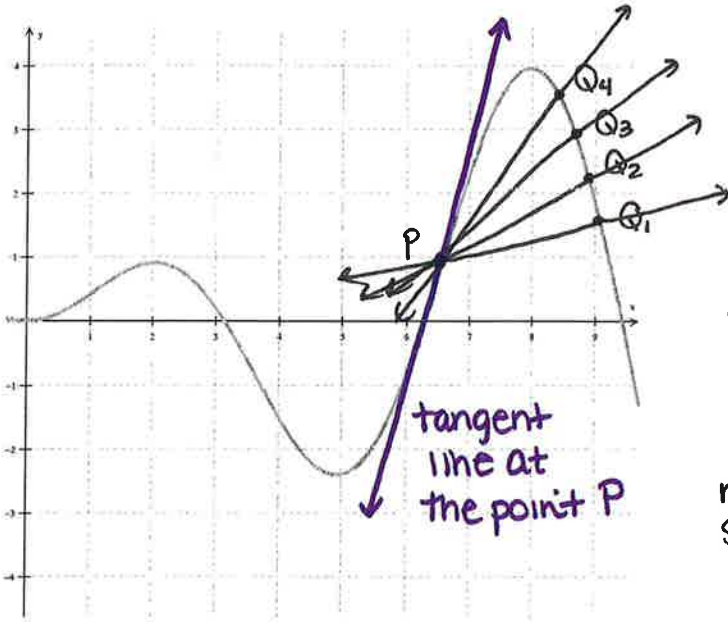
Analytic Geometry & Calculus I | Tulsa Community College

Quiz #5: Conceptual Understanding of the Derivative, and Applications of Differentiation

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use a scientific, nongraphing calculator on this assessment.

1. Write down the limit definition of the derivative. Then explain – you may use the graph below to help you illustrate – where that definition comes from. Be sure to mention h or Δx, secant, tangent, and slope.



$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

OR $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

The difference quotient

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

represents the slope of the secant line through P & Q.

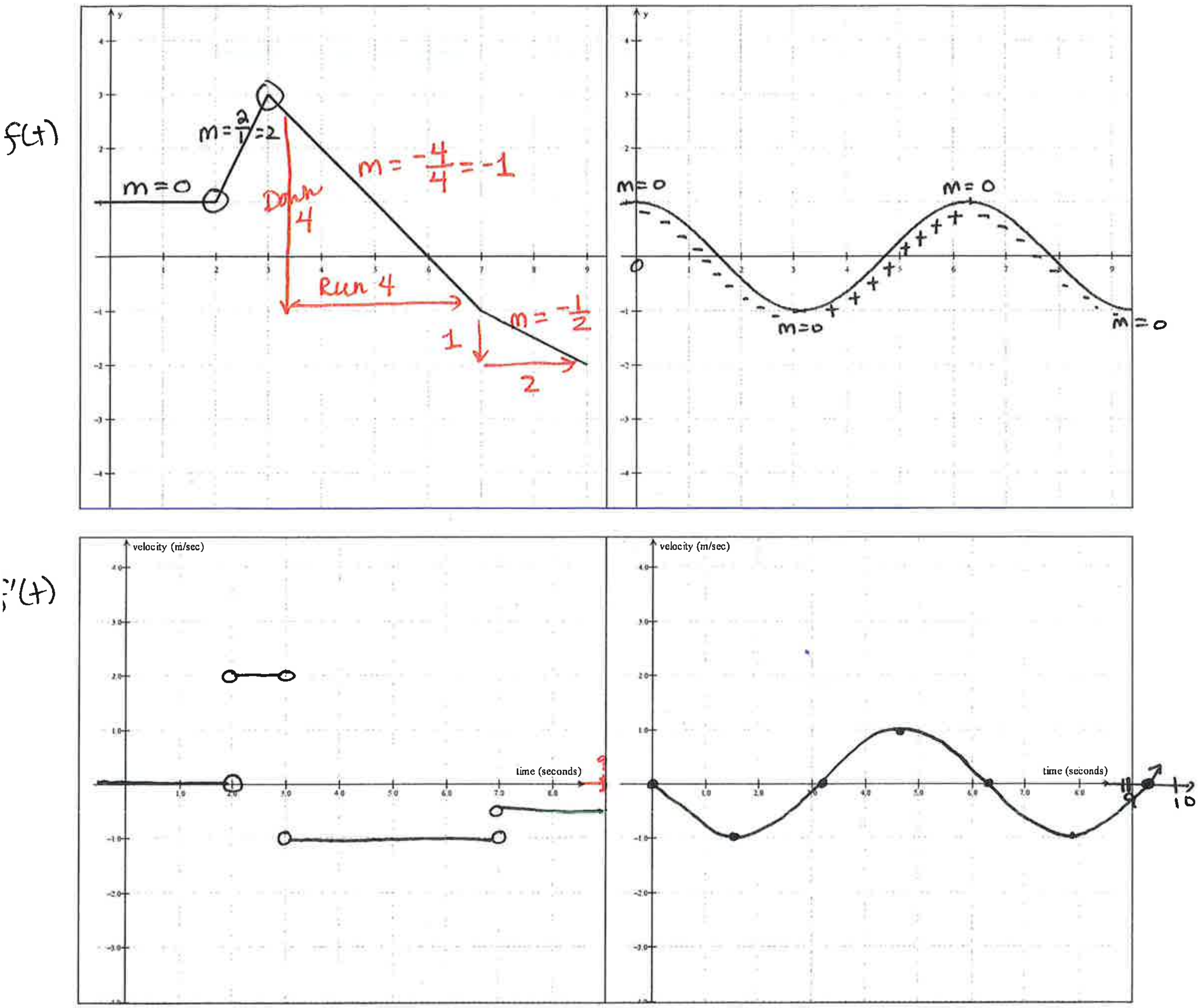
The slope of the tangent at P is given by the derivative:

$$\boxed{m_{tangent}} = \boxed{f'(x)} = \lim_{\Delta x \to 0} \boxed{m_{secant}}$$

$$= \lim_{\Delta x \to 0} \boxed{\frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

OR $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

2. Given the following graphs of position functions $f(t)$ and $g(t)$, sketch a graphs of the velocity functions $f'(t)$ *and* $g'(t)$ below.

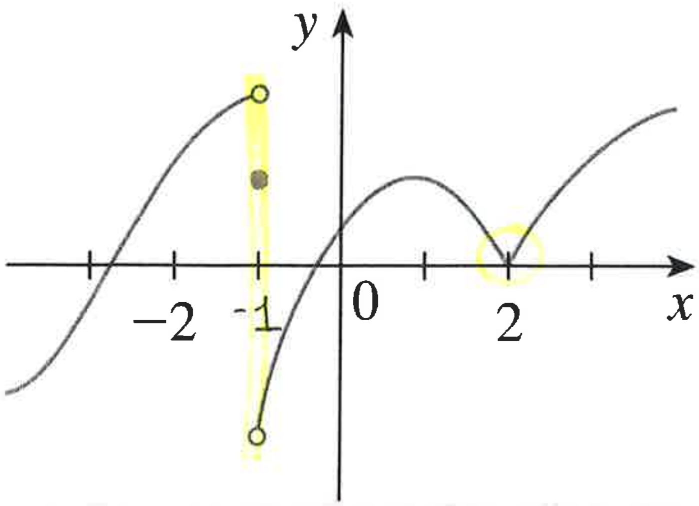


3. If $C(x)$ represents the cost a company incurs by producing x calculus books, what does $\frac{dC}{dx}$ represent? Use real world terms – not geometric terms, such as slope of the tangent line.

$$\frac{dC}{dx} = \lim_{\Delta x \to 0} \frac{\Delta C}{\Delta x}$$: instantaneous change in cost per book at given x-value.

4. State the x-values at which the function below is NOT differentiable. Explain your reasoning.

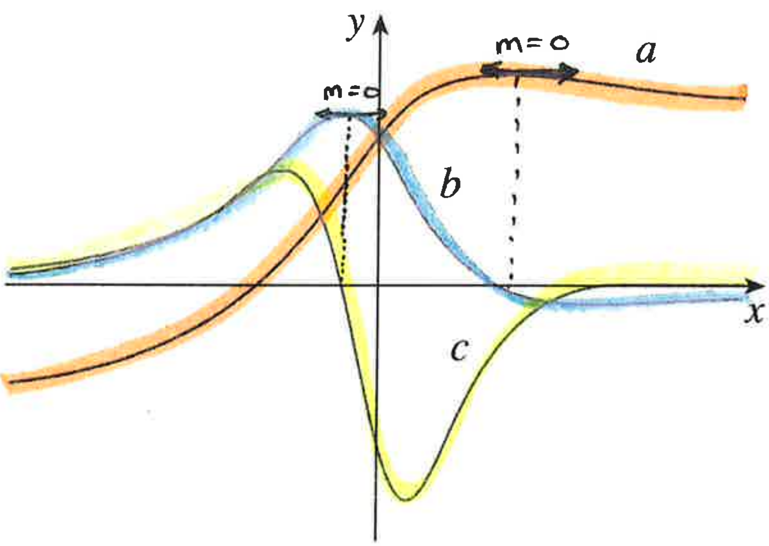
x-values where $f'(x)$ d.n.e.



Look for
- discontinuities
- sharp corner or cusp
- vertical tangent line

$x = -1$: discontinuity
$x = 2$: sharp corner or cusp.

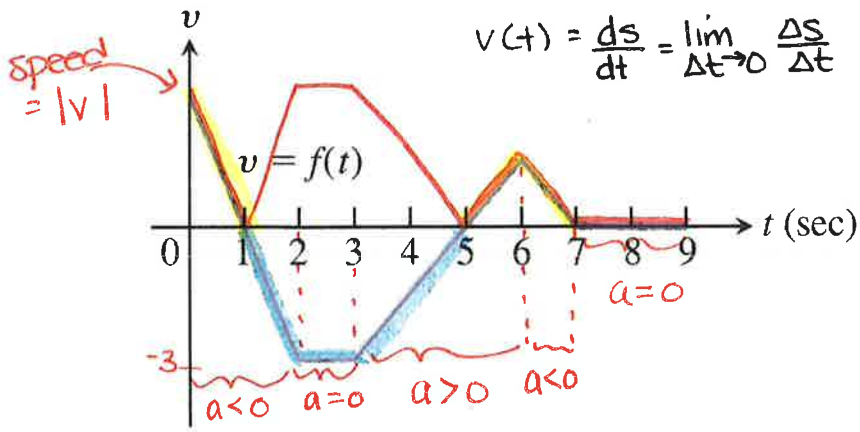
5. The graphs of f, its derivative f', and its derivative's derivative f'' are shown below. Identify each curve and give reasons for your choices.



Graph a : f
Graph b : f'
Graph c : f''

Skill #: D10
Score:

6. The graph of the velocity function $v(t)$ of a particle after t seconds is shown below. Use the graph of the function to answer the following questions. Be sure to justify your answers.



$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

(a) When is the object moving forward? (Your answer should be a time interval or intervals.)

(0, 1) and (5, 7)

(b) When is the object moving backward?

(1, 5)

(c) When does the object change directions?

$t = 1$, $t = 5$

(d) When is the acceleration of the object equal to zero? When is the acceleration positive? When is the acceleration negative? $a = 0$ when $2 < t < 3$ & $7 < t < 9$.

$a > 0$ when $3 < t < 6$

$a < 0$ when $0 < t < 2$ & $6 < t < 7$.

(e) When is the object speeding up?

(1, 2) & (5, 6)

(f) When is the object slowing down?

(0, 1), & (3, 5), & (6, 7)

7. Let $D(t)$ be the US national debt at time t. The table below gives the approximate values of this function by providing end-of-year estimates, in billions of dollars, from 1980 to 2005. Interpret and estimate the value of $D'(1990)$.

t	$D(t)$
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2
2005	7932.7

$D'(1990) \approx \frac{\Delta D}{\Delta t} = \frac{3233.3 - 1945.9}{1990 - 1985} \approx \frac{\$257.48}{1 \text{ year}}$

⇒ National debt was increasing at ≈ \$257.48 billion per year in 1990

$D'(1990) \approx \frac{\Delta D}{\Delta t} = \frac{4974.0 - 3233.3}{1995 - 1990} \approx \frac{\$348.14}{1 \text{ yr}}$

⇒ National debt was increasing at ≈ \$348.14 billion / yr in 1990

8. A particle has position function $s(t) = 4 - 9t + 6t^2 - t^3$

(a) Find the velocity function.

$$s'(t) = \frac{d}{dt}[4 - 9t + 6t^2 - t^3]$$
$$= 0 - 9 + 6 \cdot 2t - 3t^2$$
$$= \boxed{-9 + 12t - 3t^2}$$

$$v(t) = -3t^2 + 12t - 9$$
$$= -3(t^2 - 4t + 3)$$
$$= -3(t-3)(t-1)$$

$a = -3 \Rightarrow$

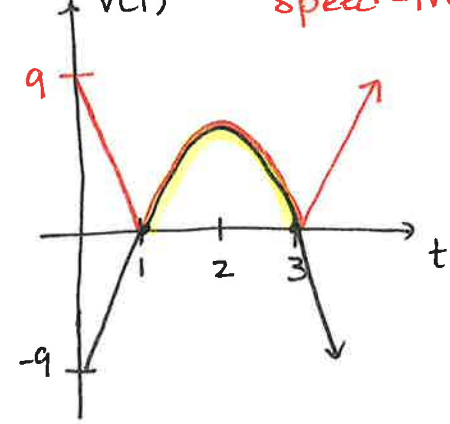
(b) Find the acceleration function.

$$s''(t) = \frac{d}{dt}[-9 + 12t - 3t^2]$$
$$= 0 + 12 - 3 \cdot 2t^1$$
$$= 12 - 6t$$

(c) When does the particle move forward (or in other words, in the positive direction)? While its not necessary, it may help to graph the velocity.

speed $= |v(t)|$

The particle moves forward when $v(t) > 0 \iff 1 < t < 3$.



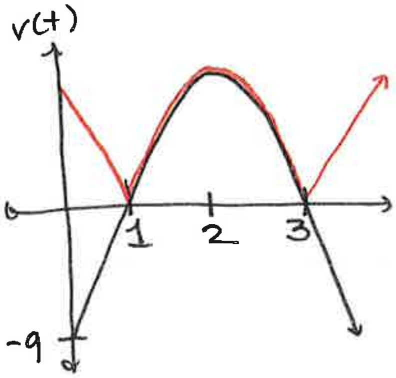
(d) Determine the t-values when the particle is speeding up.

The particle's speed increases when $1 < t < 2$ and $t > 3$.

(e) Determine the t-values when the particle is slowing down.

The particle is slowing down when $0 < t < 1$ and $2 < t < 3$.

(f) Describe the motion of the particle immediately before and immediately after each of those t-values. In order to answer this question, you may want to graph the velocity function.



Speed = $|v(t)|$

Initially, the particle is moving in the negative direction while slowing down. At $t = 1$, the particle stops momentarily, turns around & begins gaining speed as it moves in the positive direction. At $t = 2$, the particle begins slowing down. At $t = 3$, the particle stops momentarily & begins moving in the negative direction, while increasing it speed.

INTEGRITY STATEMENT: Please sign below.

"On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this quiz."

____
(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill #: D13
Score:

Skill #: G1
Score: