Name: Key　　　　Date: October 29, 2018　　　　Class Time:____________

Analytic Geometry & Calculus I | Tulsa Community College

Quiz #7: Linearization, the Mean Value Theorem, and Finding Absolute and Relative Extrema

Remember to get full credit, you need to show all work, clearly and neatly. Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use a scientific, nongraphing calculator on this assessment.

1. Given $y = f(x) = 2x^2 + 4x - 3$, and $x_0 = -2$, find the following.

(a) The differential of y.

$f'(x) = 4x + 4$

$dy = f'(x)\,dx$

$\boxed{dy = (4x+4)dx}$

Compute f'.

State formula for dy.

Substitute f' into formula.

(b) The linearization $L(x)$ of $y = f(x)$ at $x_0 = -2$.

$m = f'(-2) = 4(-2) + 4$
$= -8 + 4$
$= -4$

Compute $m = f'(-2)$.

$y_0 = f(-2) = 2(-2)^2 + 4(-2) - 3$
$= 2(4) - 8 - 3$
$= 8 - 8 - 3$
$= -3$

Compute $y_0 = f(-2)$.

$y - y_0 = m(x - x_0)$

$y - (-3) = -4(x - (-2))$

$y + 3 = -4(x+2)$

$y + 3 = -4x - 8$

$\quad -3 \qquad\quad -3$

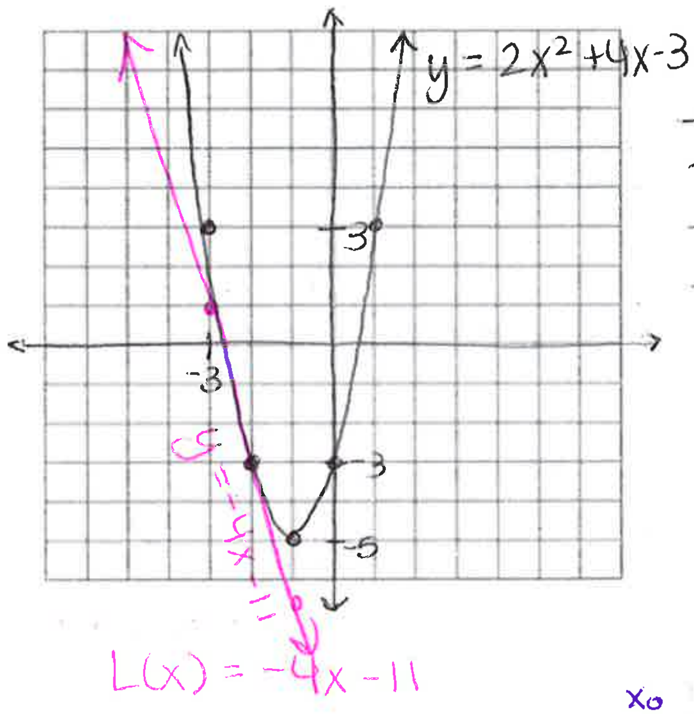
$y = -4x - 11 \implies \boxed{L(x) = -4x - 11}$

Point-Slope form

Substitute $(x_0, y_0) = (-2, -3)$ & $m = -4$ into formula.

Simplify.

(c) Graph the linearization $L(x)$ and the original function on the same set of coordinate axes. To graph the original function, you may want to create a table of values, using x-values $x = -3, -2, -1, 0, 1,$ and 2, and plotting the corresponding points.



x	$y = 2x^2 + 4x - 3$
-3	$2(-3)^2 + 4(-3) - 3 = 18 - 12 - 3 = 3$
-2	$2(-2)^2 + 4(-2) - 3 = 8 - 8 - 3 = -3$
-1	$2(-1)^2 + 4(-1) - 3 = 2 - 4 - 3 = -5$
0	$2(0)^2 + 4(0) - 3 = -3$
1	$2(1)^2 + 4(1) - 3 = 2 + 4 - 3 = 3$
2	$2(2)^2 + 4(2) - 3 = 8 + 8 - 3 = 13$

(d) What is the actual change in y as x changes from $\underset{x_0}{-2}$ to $\underset{x_{final}}{-1.95}$?

$$f(-1.95) = 2(-1.95)^2 + 4(-1.95) - 3$$
$$= -3.195$$

$$\Delta y = f(x_{final}) - f(x_{initial})$$
$$= f(-1.95) - f(-2)$$
$$= -3.195 - (-3) = \boxed{-0.195}$$

Actual change Δy

$\Delta y = y_{final} - y_{initial}$
$= f(x_{final}) - f(x_{initial})$

(e) What is the approximate change in y, using the tangent line as an approximation, as x changes from -2 to -1.95?

$$dy = f'(\underset{x_{initial}}{-2})\, dx$$
$$= -4 \cdot (0.05)$$
$$= \boxed{-0.2}$$

$dx = x_{final} - x_{initial}$
$= -1.95 - (-2)$
$= -1.95 + 2$
$= 0.05.$

Approx. change
$dy = f'(x_0)\, dx$
$dx = x_{final} - x_{initial}.$

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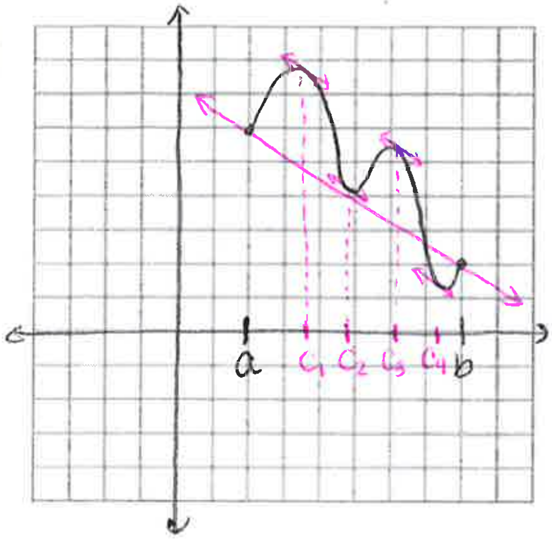
2. State the mean value theorem. If you want to, you can draw a graph to help illustrate your description.



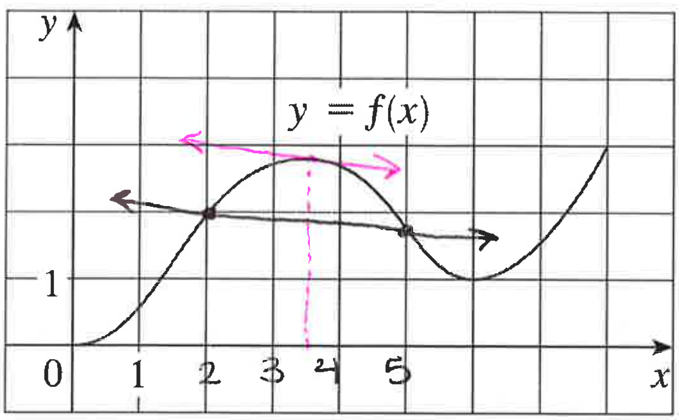
If ① f is continuous on $[a,b]$

& ② f is differentiable on (a,b),

then there is a c in (a,b) satisfying

$$\underbrace{f'(c)}_{\text{slope of tangent line}} = \underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{slope of secant through } (a, f(a)) \,\&\, (b, f(b))}.$$

3. Use the graph of f to estimate the values of c that satisfy the conclusion of the mean value theorem for the interval $[2, 5]$.



$c \approx 3.5$

The tangent line is parallel to the secant line through $x=2$ + $x=5$ at $c \approx 3.5$.

4. Determine whether the mean value theorem applies for the given function on the given interval. If the mean value theorem does not apply, explain why it does not apply. If the mean value theorem applies, DO NOT find the value of c guaranteed to exist by the mean value theorem.

(a) $f(x) = (x-2)^{2/5}, \quad [0,4]$

$$f'(x) = \frac{2}{5}(x-2)^{-3/5} \underbrace{\frac{d}{dx}[x-2]}_{1}$$

Use chain rule to compute f'.

$$f'(x) = \frac{2}{5\sqrt[5]{(x-2)^3}}$$

f is continuous on $[0,4]$, but f' is undefined at $x=2$, so MVT does not apply on the given interval.

f' undefined @ $x=2$ $\Rightarrow$ MVT does not apply.

(b) $g(x) = \frac{5}{2x+6}, \quad [-4,3]$

$$g(x) = \frac{5}{2(x+3)}$$

g is undefined at $x=-3$, so g is not continuous on $[-4,3]$ $\Rightarrow$ MVT does not apply on the given interval.

g undefined @ $x=-3$ $\Rightarrow$ MVT does not apply.

5. Verify that the function below satisfies the hypothesis of the mean value theorem on the given interval.

$$f(x) = x^2 - x - 12, \qquad [-2, 2]$$

find the value of c guaranteed to exist by the mean value theorem.

✓ f is continuous on $[-2, 2]$ b/c f is a polynomial,
✓ f is differentiable on $(-2, 2)$ b/c f is a polynomial
$\Rightarrow$ MVT applies.

Check/explain that f is continuous & differentiable.

$f(-2) = (-2)^2 - (-2) - 12 = 4 + 2 - 12 = -6$
$f(2) = (2)^2 - (2) - 12 = 4 - 2 - 12 = -10$
$\text{AROC} = \dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{-10 - (-6)}{2 + 2} = \dfrac{-10 + 6}{4} = \dfrac{-4}{4} = -1.$

Compute AROC.

$f'(x) = 2x - 1$

$2x - 1 \overset{\text{set}}{=} -1$
$\quad +1 \qquad +1$
$\dfrac{\cancel{2}x}{\cancel{2}} = \dfrac{0}{2} = 0$

$\boxed{c = 0}$

Compute f'.

Set $f' = \text{AROC}$ & solve for x.

Skill #: A21
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6. Find the absolute extrema of $y = 2x^3 - 2x^2$ on $[-1,2]$.

$y' = 2(3)x^2 - 2(2)x$
$= 6x^2 - 4x$
$= 2x(3x - 2) \overset{\text{set}}{=} 0$

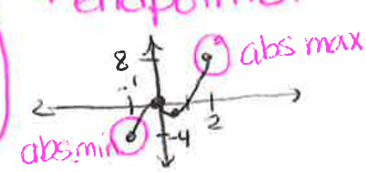
$\dfrac{\cancel{2}x}{\cancel{2}} = \dfrac{0}{2}$ OR $3x - 2 = 0$
$\qquad\qquad\qquad +2 \quad +2$
$x = 0 \qquad\qquad \dfrac{\cancel{3}x}{\cancel{3}} = \dfrac{2}{3}$

Compute y'.

Set $y' = 0$ + Solve for x.

$f(0) = 2(0)^3 - 2(0)^2 = 0$
$f(2/3) = 2\left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 = \frac{16}{27} - \frac{8}{9}\cdot\frac{3}{3} = \frac{-8}{27}$
$f(-1) = 2(-1)^3 - 2(-1)^2 = -4$ (abs min)
$f(2) = 2(2)^3 - 2(2)^2 = 8$ (abs max)

Find y-values at crit. values + endpoints.



7. Consider the function $y = -2x^3 - 3x^2 + 12x + 1$.

(a) Find the critical values of the function.

$$y' = -2(3)x^2 - 3(2)x + 12$$
$$= -6x^2 - 6x + 12$$
$$= -6(x^2 + x - 2)$$
$$= -6(x+2)(x-1)$$

Compute y'.

Factor y'.

Set $y' = 0$ & solve for x.

$$-6(x+2)(x-1) \overset{\text{set}}{=} 0$$

$$x + 2 = 0 \qquad x - 1 = 0$$
$$-2 \quad -2 \qquad +1 \quad +1$$
$$x = -2 \qquad x = 1$$

Critical values.

(b) Make a sign chart to determine the open intervals where the function is increasing or decreasing.

y'	$-\infty$ to -2	-2 to 1	1 to ∞
	↘ V	↗ ∩	↘
y'	$-$	$+$	$-$
-6	$-$	$-$	$-$
$(x+2)$	$-$	$+$	$+$
$(x-1)$	$-$	$-$	$+$

- List crit. values on # line.
- List factors of y' below # line & y' above # line.
- Determine the sign of each factor on each interval.
- multiply the signs to find the sign of y' on each interval.

Try $x = -3$: $-3+2 = -1$, $-3-1 = -4$

Try $x = 0$: $0+2 = 2$, $0-1 = -1$

Try $x = 2$: $2+2 = 4$, $2-1 = 1$

Increasing: $(-2, 1)$

Decreasing: $(-\infty, -2)$ & $(1, \infty)$

Conclusion

(c) Find the y-value at each critical value, and identify the function's relative minima and maxima.

$$y(-2) = -2(-2)^3 - 3(-2)^2 + 12(-2) + 1$$
$$= -2(-8) - 3(4) - 24 + 1$$
$$= 16 - 12 - 24 + 1$$
$$= -19$$

relative min of -19 @ $x = -2$.

$$y(1) = -2(1)^3 - 3(1)^2 + 12(1) + 1$$
$$= -2 - 3 + 12 + 1$$
$$= 8$$

rel. max of 8 @ $x = 1$.

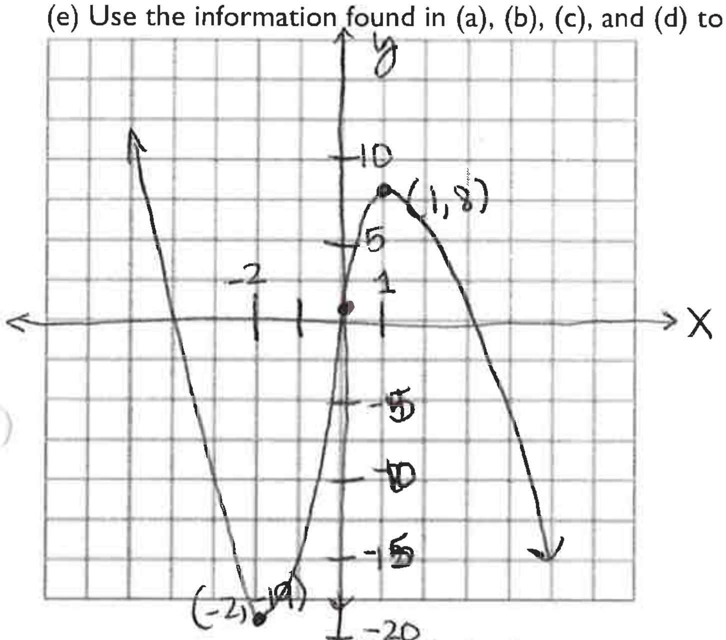
(d) Compute the second derivative at each critical value.

$$y'' = \frac{d}{dx}[-6x^2 - 6x + 12]$$
$$= -12x - 6$$

$y''(-2) = -12(-2) - 6 = 24 - 6 = 18 \Rightarrow \cup \Rightarrow f(-2)$ is rel. min

$y''(1) = -12(1) - 6 = -18 \Rightarrow \cap \Rightarrow f(1)$ is a rel. max

(e) Use the information found in (a), (b), (c), and (d) to graph the function.



Circle The Test used: First Derivative Test / Second Derivative Test / Both

The function is increasing on $(-2, 1)$.

The function is decreasing on $(-\infty, -2)$ & $(1, \infty)$

List each relative minimum value and relative maximum value of the function. You may not need all of the spaces below. Use as many as necessary.

Relative maximum of 8, occurring at $x =$ 1.

Relative maximum of ______, occurring at $x =$ ______.

Relative minimum of -19, occurring at $x =$ -2.

Relative minimum of ______, occurring at $x =$ ______.

Skill #: A22
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INTEGRITY STATEMENT:
"On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam."

____
(signature)

If you can't sign this in good conscience, please don't. Come speak to me.

Skill #: G1
Score: