Name: Key　　　　Date: November 19, 2018　　　　Class Time:__________

Analytic Geometry & Calculus I | Tulsa Community College

Quiz #9: Differential Equations and Initial Value Problems, and Riemann Sums

Remember to get full credit, you need to show all work, *clearly and neatly.* Remember, this isn't just about you getting the answer, but you *showing someone else* how you got the answer.

You may use the provided formula sheet for Riemann sums, as well as a scientific, nongraphing calculator.

1. Find a function $y(x)$ that satisfies the differential equation and the initial condition below.

$$\begin{cases} \dfrac{dy}{dx} = 6\sqrt{x} + 5\sin x \\ y(0) = 4 \end{cases}$$

$$y = \int \frac{dy}{dx}\,dx$$

$$= \int [6\sqrt{x} + 5\sin x]\,dx$$

$$= \int [6x^{1/2} + 5\sin x]\,dx$$

Write $\sqrt{x}$ as $x^{1/2}$, so that you can antidiff. using the power rule.

$$= 6\left(\tfrac{2}{3}\right)x^{3/2} + 5(-\cos x) + C$$

$$y = 4x^{3/2} - 5\cos x + C.$$

General Solution

Antidifferentiate & simplify.

$$y(0) = 4(0)^{3/2} - 5\cos 0 + C \overset{\text{set}}{=} 4$$

$$4 = 0 - 5\cdot 1 + C$$

$$4 = -5 + C$$

$$+5 \quad +5$$

$$9 = C$$

Substitute $x = 0$ & $y = 4$ & solve for C.

$$\Rightarrow \boxed{y = 4x^{3/2} - 5\cos x + 9}$$

Substitute $C = 9$ into the general solution to find the particular solution.

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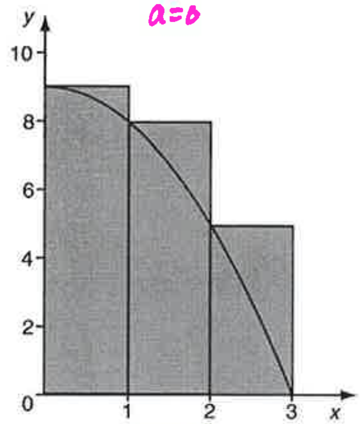
$b=3$ $f(x)$

2. Approximate $\int_0^3 (9-x^2)\,dx$ by computing the area of each rectangle and adding.

$a=0$



Δx
$= 1$

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{3} = 1$$

Compute or identify Δx.

$$f(x) = 9 - x^2$$

Identify $f(x)$. (It's the integrand.)

$$f(0) = 9 - 0^2 = 9$$
$$f(1) = 9 - 1^2 = 9 - 1 = 8$$
$$f(2) = 9 - 2^2 = 9 - 4 = 5$$

$f(0)$
$f(1)$
$f(2)$

$$\text{Area} \approx f(0)\Delta x + f(1)\Delta x + f(2)\Delta x$$
$$= 9\cdot 1 + 8\cdot 1 + 5\cdot 1$$
$$= \boxed{22 \text{ units}^2}$$

Compute the area of each rectangle & add the areas.

3. (a) **Show** that the right endpoint approximation of the integral of $f(x) = x^3 - 6x$, for $0 \le x \le 3$, using n rectangles of equal width is

$a=0$ $b=3$

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \sum_{i=1}^{n}\left[\frac{81i^3}{n^4} - \frac{54i}{n^2}\right].$$

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

Compute Δx.

$$x_i^* = a + i\Delta x = 0 + i\left(\frac{3}{n}\right) = \frac{3i}{n}.$$

Use the right endpt. formula to find x_i^*. Substitute $a=0$ & $\Delta x = 3/n$.

$$f(x_i^*) = \left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right) = \frac{3^3 i^3}{n^3} - \frac{6}{1}\cdot\frac{3i}{n} = \frac{27i^3}{n^3} - \frac{18i}{n}$$

Find $f(x_i^*)$ & simplify.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $(ab)^n = a^n b^n$

$\frac{A}{B}\cdot\frac{C}{D} = \frac{AC}{BD}$

$$\int_0^3 f(x)\,dx \approx \sum_{i=1}^{n} \boxed{f(x_i^*)}\,\boxed{\Delta x} = \sum_{i=1}^{n}\left[\boxed{\left(\frac{27i^3}{n^3} - \frac{18i}{n}\right)}\boxed{\left(\frac{3}{n}\right)}\right] = \boxed{\sum_{i=1}^{n}\left[\frac{81i^3}{n^4} - \frac{54i}{n^2}\right]}$$

Substitute $f(x_i^*)$ & Δx into Riemann sum.

Distribute.

(b) **Find the exact values** of he integral of $f(x) = x^3 - 6x$, for $0 \le x \le 3$, by evaluating

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x.$$

<u>Note:</u> If you were unable to complete (a), you can still use the fact that $\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \sum_{i=1}^{n}\left[\frac{81i^3}{n^4} - \frac{54i}{n^2}\right]$ to complete (b).

$$\int_0^3 f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n}\left[\frac{81i^3}{n^4} - \frac{54i}{n^2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{81}{n^4}\left(\sum_{i=1}^{n} i^3\right) - \frac{54}{n^2}\left(\sum_{i=1}^{n} i\right)\right]$$

Write the Riemann Sum as a difference of sums.

$$= \lim_{n\to\infty}\left[\frac{81}{n^4}\cdot\frac{n^2(n+1)^2}{4} - \frac{54}{n^2}\left(\frac{n(n+1)}{2}\right)\right]$$

Use the summation formulas for $\sum i$ & $\sum i^3$. (they are on the provided formula sheet.)

$$= \lim_{n\to\infty}\left[\frac{81}{4}\cdot\frac{(n+1)^2}{n^2}\cdot\frac{\cancel{n^2}}{\cancel{n^2}} - 27\,\frac{\cancel{n}}{\cancel{n}}\cdot\frac{(n+1)}{n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{81}{4}\left(\frac{n+1}{n}\right)^2 - 27\left(1+\frac{1}{n}\right)\right]$$

Simplify.

$$= \lim_{n\to\infty}\left[\frac{81}{4}\left(1+\underbrace{\frac{1}{n}}_{0}\right)^2 - 27\left(1+\underbrace{\frac{1}{n}}_{0}\right)\right]$$

Evaluate the limit.

$$= \frac{81}{4} - 27 = \boxed{\frac{-27}{4}} \text{ OR } \boxed{-6.75} = \int_0^3 f(x)\,dx$$

<u>Check:</u> $\int_0^3 [x^3 - 6x]\,dx$

$$= \left(\frac{1}{4}x^4 - \frac{6}{2}x^2\right)\Big|_0^3$$

$$= \left(\frac{1}{4}(3)^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right)$$

$$= \frac{81}{4} - 27 = \boxed{-\frac{27}{4}} \text{ OR } \boxed{-6.75}$$

Skill #: G1	Skill #: I27
Score:	Score:

INTEGRITY STATEMENT:

On my personal integrity, I have not given, nor received, nor witnessed any unauthorized assistance on this exam.

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(signature)

If you can't sign this in good conscience, please don't. Come speak to me.