

## Unit 1 Lesson 3 Lesson Plan: The Formal Definition of a Limit

### Purpose

We've discussed the intuitive definition of a limit, and how to compute it graphically, numerically, and algebraically. We said that "the limit of  $f(x)$  as  $x$  approaches  $a$  is  $L$ " means that as  $x$  gets arbitrary close to  $a$ , the function values  $y = f(x)$  get arbitrarily close to  $L$ . If you wondered, "What do you mean by close?" then this lesson is for you. We'll also look at a little of the history in an article called "Who Gave You the Epsilon? Cauchy and the Origins of Rigorous Calculus" by Judith Grabiner from the *American Mathematical Monthly*.

### Lesson Outcomes

By the end of this lesson, you will be able to

- State the epsilon-delta definition of a limit, and explain it in your own words,
- Find a  $\delta$  for a given  $\varepsilon$ , given a limit,
- Prove that the limit of a given function as  $x$  approaches  $a$  is  $L$ , using the epsilon-delta definition of a limit.

### Materials That You'll Need

Before you get started, read the following handouts.

- U1 L3 Lesson Notes – The Formal Definition of a Limit
- U1 L3 Lesson Notes and Practice Problems – The Formal Definition of a Limit
- "Who Gave You The Epsilon? Cauchy and the Origins of Rigorous Calculus" by Judith Grabiner,
- A printable version of this lesson plan.

You'll also need access to HW #1. You may also want to read the section about the formal definition of a limit in Section 2.2 of our text.

### Steps to Complete the Task

#### Step 1: Learn about the formal definition of a limit.

Listen and take notes during lecture, read the lesson notes, and the article about the origins of rigorous calculus.

#### Step 2: Complete the related textbook problems and conceptual questions.

### Criteria for Success

You know that you've succeeded in mastering this material if you can

- State the epsilon-delta definition of a limit, and explain it in your own words,
- Find a  $\delta$  for a given  $\varepsilon$ , given a limit,
- Prove that the limit of a given function as  $x$  approaches  $a$  is  $L$ , using the epsilon-delta definition of a limit.