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Approaching mathematics through history and culture: A suggestion

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Abstract

This paper describes a multi-cultural and historical approach to the teaching of mathematics, created through the views of researchers and authors who support approaching school mathematics through its history and the various applications of mathematics in different cultures around the world. A specific design within the context of this new approach was implemented as a pilot course at a private school and the participant students were attending year 5 (out of 6) of high school. The selection of the material was based upon the syllabus taught at Greek Public schools, called Core Mathematics (in contrast to Advanced Mathematics), and it was enriched with historical and cultural elements. The detailed description of the material used aims into highlighting the idea for approachable and meaningful mathematics, without diminishing the value of more traditional exercises. A brief literature review is found at the beginning of this paper, along with some information on the methodology. Emphasis is given on the presentation of the material used and how the original ideas were changing while the course was evolving.

Introduction

A wider world view and the need for multicultural education are the main triggers for this pilot mathematics course, as described here. Authors, researchers, but also educators around the world have sought historical elements as well as cultural elements in order to enrich the teaching of various lessons, and therefore students' general pedagogy. This quest and its findings –whatever these may be – does not only target into enriching the teaching of the various lessons, but also in discovering facts and elements that can encourage a deeper understanding of the sciences, how did they evolve, why did they flourish, how society influenced their development, how the results influenced the society itself, and many more questions regarding the history of sciences For a more detailed description of the questions that the science historians may pose, one can refer to Gavroglou (2004).

Supporting the new approach

Ubitaran D'Ambrosio was the first to introduce the notion of Ethnomathematics. In his 1985 paper, he defined Ethnomathematics as the bridges that connect historians and anthropologists from one side and mathematicians from the other side, in order to identify the different kinds of mathematics that exists. This stance is not a widely

accepted one; A number of mathematicians and mathematics teachers cannot agree with the idea of the existence of different kinds of mathematics.

Zaslavsky (1973) discovered numerous interesting habits during her long research in African countries. These habits or perceptions were related to the mathematical tradition of the place, the methods they needed as well as strategy games children were playing at their leisure time. Shan and Bailey (1991) discuss the using of mathematics in our everyday lives and how mathematical textbooks may be biased towards specific groups of people. They believe that even racist behaviors can be challenged through the proper usage and teaching of mathematical notions.

The problems that emerge when such perceptions are expressed have been spotted and analyzed by Bishop (1991, 2001). In an article of his in Issues of Mathematics Teaching journal, titled "What values do you teach when you teach mathematics?" not all mathematics teachers would find his question valid. Do they teach any values at all to make it worth wondering what these values are? Derivatives are, in themselves, without any values and, therefore, the only value mathematics can carry is purely scientific. Nevertheless, there are educators who are convinced that mathematics is rich values that, although they may not be taught in the same immediate way as history, are such that mathematics conveys many more messages to the students than the teachers themselves would like to believe.

Joseph (1991) emphasizes the fact that mathematics is usually taught from a purely Euro-centric point of view, while contributions from other cultural groups, such as Egyptians, Babylonians, Arabs, Hindu, Chinese, are often omitted completely. This fact on its own focuses on the development of the sciences in Europe, as if the rest of the world never contributed, even though some non-European contributions are of central importance. Arabic numerals where created in India and the Arabic world and, if it was not for the Arab scientists, many scientific documents would have been lost forever during the medieval years. Cotton (2001) refers to another aspect of the options in the teaching of mathematics, regarding the ways in which mathematics can build or alter the understanding we have of our world. Mathematical tools provide a range of solutions that most people cannot understand.

This literature review can be extended further. On one hand the aim is to acknowledge the fact that Mathematical Education is a complicated issue and the approaches available are varied and rich in ways that move beyond traditional approaches. On the other hand, an aspect closely related to the first one is the recognition that the role of mathematics in education is not just to provide pupils with the scientific tools – which *are* important – but also to answer pupils' questions, such as "where does the mathematics from?", "why did it come about?", "how", "when" and so on, providing the students with a more understanding of mathematics as a science and as a human endeavor. This is how the role of a program for teaching mathematics through history and culture can be described in short.

Past researches on the role of the history of mathematics in the teaching of mathematics include teaching secondary school students and future maths teachers, but mainly the latter. Unfortunately, publications on using history of mathematics in secondary schools are sparse. Between 1994 and 1999, some students in England had the opportunity to select History of Mathematics as a unit towards the completion of their maths A level through the Nuffoeld Advanced Mathematics course (Neill, 1994). Lawrence (2007) has also been trialing historical elements for the secondary maths classroom and information, ideas and material from her project can be found online. Another example of research on approaching mathematics through history and culture in secondary schools is the doctoral dissertation by Pompeu (1992), which is nevertheless unpublished.

As far as using history of mathematics to train future teachers, Furinghetti (2007) describes a mathematical workshop for future teachers, where they studied history of mathematics, as well as historical recourses directly in order to gain a deeper understanding of the mathematical notions and where they came from and why did they survive up to today. Philippou and Christou (1998) noted that, after the completion of a three year program of teacher education using history of mathematics, the negative perceptions of the students towards mathematics were changing. An important factor influencing this change was probably the satisfaction students found when discovering the utility in mathematics.

The course described in this paper is based upon this very rationale: to disclose the utility in mathematics, where the maths came from, why it exists, and so on. The participating students were weak and had strongly negative attitudes towards mathematics. also In this paper, I focus on the description of the material and how decisions were made on changes during the year, since the methodology used was Action Research.

Action Research is the methodology used when the researcher is also the practitioner. When a problem has been located in a setting – in this case it is a school environment – actions are taken aimed at developing solutions to the problem. The practitioner brings in suggestions and modifies the actions according to the results or responses they receive (Gray, 2004). Since the researcher is also the teacher at this school setting, where a group of students has failed in the past or has very negative attitudes towards mathematics, Action Research was considered the most appropriate in terms of bringing suggestions for improving students' perceptions as well as achievement and understanding. Data collection methods included questionnaires, assessment material, interviews, observations, and journal keeping.

The participating students were aware of the fact that the course they attended was a pilot course and that they were not only allowed but also expected to express their opinions and sentiments regarding the material covered, what was most engrossing or

indifferent to them. The interaction between the teacher-researcher and her students was of great importance and it facilitated the receiving of feedback on what students considered to be relevant to them, helpful and important. This is how changes were decided, ending up with a proposition quite different from the original one, in terms of material and in ways of assessment. The material, the changes as well as the assessment methods suggested form the main part of this paper.

Detailed analysis of the original plan along with the changes

There is a rich bibliography to support such an endeavor and some indications of this are listed below. There are also many internet resources. Some of the ideas implemented in this program can also be found online at http://www2.warwick.ac.uk/fac/soc/wie/staff/staff_interests/hom/ioanna/, where students could find some information useful to them as well. The set of notes was prepared under the title "Mathematics through History and Culture".

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The program was implemented throughout school year 2007-08 and it took place at a private English-speaking school in Cyprus. Participating students were 16-17 years old and attended year 5 out of 6 before graduating. The material was based upon the respective material suggested by the Core Mathematics Curriculum of the Ministry of Education of Cyprus. The chapters were originally related to those in the book distributed by the Ministry, but approached in a different way. The origins of each notion are examined up to a point: why this notion emerged, how it was used in the various cultures and how it is applied in our own (or the students' own) current culture.

After the end of each chapter there was a small "chapter-break" that presented issues from a greater "mathematical" area.

The first chapter introduced the students to the idea behind the preparation of this course. The students were informed that the material to be taught largely coincides with the public schools' curriculum and that the approach was intended to be more helpful to them in terms of how approachable and how meaningful math can become through this course. The first activity was for the students to place some important historical facts on a number line. This action targeted the students acquiring a general perspective of history, an essential quality for a course heavily related to history. For example, many students were ignorant of how far back human civilization may go, and of how recent are the world wars, or even how we "measure" time.

The second chapter was a newspaper article written by a Mathematics Professor who aimed to demonstrate how mathematics is everywhere around us, without us realizing. The article is about Thanasis and his "adventures" from the time he wakes up, until he goes to his work. It was followed up by three questions open for discussion in class. These questions invited students to consider what the aim of the author was when writing this short story and if it was achieved, and to identify which mathematical fields were mentioned. The article was quite short but nevertheless rich; some students failed to see any purpose in it. The article can be found in the appendix.

Chapter three dealt with "Pythagoras, *his* theorem and the Pythagoreans". The respective chapter in the public school book included the theorem and some exercises involving it. Since it is a theorem students have met at an earlier stage, the approach of the public school book was used as the introduction to the chapter; that included a reminder of the theorem along with a few standard exercises. The next part dealt with "Pythagoras's theorem before Pythagoras". Students learned that the Babylonians dealt with finding the relation between the side of a square and its diagonal, which is a special case of Pythagoras's theorem, as it was named centuries after the Babylonians. They were also told that the Chinese used this theorem centuries before Pythagoras as well. This provides them with a sense of utility in math, since Babylonians and the Chinese used Pythagoras's theorem, not for the sake of mathematics, but because they needed it as a tool. Williams, J. (1993) and Gerdes, P.(1994) provide justifications as to why this kind of knowledge is important for the structuring of perceptions of the students.

Pythagoras's theorem was then examined as it existed in Ancient Greece, where it was proved, and it was no longer a mere tool. The Pythagoreans were also examined as a special society, which held specific beliefs and was heavily influenced by the strong perception that whole numbers may have special powers. These beliefs were diminished after the discovery of irrational numbers, with the use of Pythagoras's theorem. The hypotenuse of an isosceles right triangle with the two sides equal to 1cm is $\sqrt{2}$cm. After commenting on irrational numbers, the chapter closed with the introduction to students of a project that could include a biography, or Pythagoreans' practices or any other

ideas the students might have. Since most students for this project chose the "secure" option of writing a biography of Pythagoras, subsequent projects were designed to be more open-ended so as to make students use their imagination and abilities.

Chapter 4 was the "break-chapter". Students familiarized themselves with representing number using fingers. The discussion revolved around the fact that the current number system is not unique; other number systems were used in the past, and each system facilitated some needs. Students engaged during this lesson, and made comments regarding who might need this way of counting. What made the difference here was that the need for counting on fingers effectively was obvious; even though they probably wouldn't need it themselves, they did acknowledge the need for the existence of these alternative systems.

Chapter 5 introduced sequences through Fibonacci sequence. Fibonacci's problem of modelling a rabbit population initiated the discussion, which was extended through the application of Fibonacci sequence in nature and in art. Arithmetic and geometric sequences were then introduced through Malthus's assumption on population growth. Malthus, tried to predict what the results of the geometric growth of the population and the arithmetic growth of food production would be. He predicted that world would run out of food relatively early. The question of what Malthus meant by the terms "arithmetic" and "geometric" as well as why he was wrong, dominated a discussion where a lot of students joined and expressed their opinion. This resulted in students understanding of the sequences and how mathematical tools can be used to describe real-world facts. They ended up being more receptive when the discussion on sequences became more theoretical. An idea that emerged from the discussion in the classroom was about radioactive particles and how significant their "half-life" is. If we began with 200 radio-active particles after an explosion (such as the one in Chernobyl) and half of them give their radio activity every a given amount of time, how long will it take for the area to get clear? This is a brilliant example of a situation that may be modelled using a geometric sequence with common ratio equal to 0.5. In this chapter, students were assessed using a "traditional" test, which included general questions as well as exercises.

The next "break-chapter" consisted of part of a transcribed university lecture on mathematics. This lecture was about mathematical problems yet to be solved and the solution (or the proof that no solution exists) will turn the mathematician into a millionaire. The accomplishments of such a reading are multiple. Students find out that even mathematicians have trouble in solving some problems, that not all problems have solutions and that the "production" of new mathematics is an endless procedure. What was revealed was that students thought that professors of mathematics could become millionaires just by their profession.

In chapter 7, efforts were made to deal with trigonometry, mainly through astronomy. The plan was to have logarithms next, as the invention that facilitated the use of

trigonometry rules due to the fact the logarithms required less labour. However, trigonometry appeared to be too difficult and uninteresting to the students, and the chapter was eventually abandoned, as were the logarithms. The project that was planned for trigonometry was replaced by the *Professionals' Project*, an idea that arose from an informal interview with students. The instructions for this project were as follows:

> The aim of this project you are being assigned is to discover what kind of mathematics is used by different professionals.
>
> You must choose any three different professionals and interview them about the kind of mathematics they use when they work. Try to get details as well as some examples that will enrich your project and make your descriptions more vivid.
>
> You should write down the questions you asked and the answers you got as if you were writing for a journal. The minimum number of words is 500, but you should include all the information you manage to get, as there is no upper limit.

The results were very encouraging. Students could not find ready-made solutions online or anywhere else, so they had to use their imagination and communication skills to respond to this project. The professions chosen were quite varied and included an accountant, a house wife, a shoe sales person, an engineer, an architect, a computer programmer and more.

In the mean time, students were introduced to a board game taken from *SMILE* material (SMILE card number 0279), the HIGH JUMP GAME. The handout was double-sided and had the instructions on the one side and the board on the other. Students sat in pairs and small pieces of paper were used as counters. The winner of each pair would play against the winner of another pair until there was a winner. Student developed various strategies, some of them did not risk much – playing safe could not help them remain in the game for longer. Some more strategy games students learnt during this course were Tapatan from Kenya and Oware from Africa. Games helped them stay focused for longer and put sincere effort in devising an effective strategy.

The lessons then moved to Discussing Taxes. These lessons revolved around different kinds of taxes, how to calculate income tax, as well as calculating VAT using either the original value or the price paid at the till. The tables used to calculate these taxes can be seen below. Later came the calculation of VAT along with discounts. Students had to consider the question of which one should be calculated first and why. Does the decision make a real difference?

The following table can be used to calculate income tax, according to the laws in the Republic of Cyprus, in 2008.

Income	**Difference**	**Percentage**	**Tax**
€0 – €19 500	€	0%	
€19 501 – €28000	€	20%	
€28 001 – €36 300	€	25%	
€36 301 –	€	30%	
_____	_____	Total Sum:	

The table that follows helps in calculating VAT, as well as original value and final price for products that are subject to a 15% VAT.

Value	**VAT**	**Final Price**
100	15	115
x	*y*	*z*

Students' attitudes towards this chapter were significantly more positive than any of the previous chapters. They felt that general knowledge on taxes as well as the ability to calculate them was something relevant to their lives, consequently useful to them. They demonstrated good behavior, asked a lot of questions and also did quite well at their test.

The original plan for chapter 10 was to deal with some more consumer problems but, after covering taxes, a substantial skill for a consumer, the lesson moved towards statistics. The occasion that brought about the discussion of statistics was the recent election in Cyprus. Students had many questions related to the procedures, the exit polls, the blank votes and so on; consequently statistics was proved to be an area of interest to the students and mathematically rich as well. Newspaper or magazine articles which used statistics, were brought into the classroom in order to initiate discussions, about the issues they addressed and about the statistics they used. Data collection methods as well as presenting data were worked upon. The assessment for this chapter involved a project once more: students had the chance to choose any subject area they were interested in, devise a questionnaire, distribute it to at least ten people and present

their results along with a discussion. This project became the final part of the course, although the original plan had included more.
The parts left out regarded visual illusions, geometry and perspective as well the reading of "Flatland", by Abbot (1884) and discussions around mathematical novels. The idea about reading mathematical novels belongs to a Greek organization "Thales and Friends" who discusses approaching mathematics through fictional literature.

Conclusion

Opportunities to employ an annual program such as this are sparse. Such material is nevertheless an invaluable input that has been shown to help make mathematics meaningful and relevant to the students. The main inference made after the completion of this program and prior to any detailed analysis is that mathematics that is applicable in the students' own culture, or which carries social concerns, receives positive reactions from the students. Historical elements did not seem to be of any special interest to these specific students. Another issue raised is that the project is a wonderful assessment method, even for mathematics, and that original ideas that involved the students' personal experiences provided them with a great opportunity to demonstrate their understanding and skills.

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Appendix

An ordinary morning…
By Tefkros Michaelides
PhD Mathematics, Math's Teacher, Writer, Translator
Newspaper TA NEA 02/03/05

Thanasi's radio- alarm clock rang at seven. Due to the digital technology, which is based on numerical analysis and the binomial system, the sound filled the room, as if a whole orchestra had gathered next to his pillow. He got up. His fridge and microwave oven, functioning with fuzzy logic, a branch of the multi-value symbolic logic – which also was in charge for the secure function of the ABS of his car – had provided him with a huge breakfast in ten minutes time. At 7:40, he was typing the 4-digit code for the house alarm (according to the probability theory the potential burglar had only 1 chance in 10 000 to break in) and left for work, feeling safe. He used the tube; what a miracle! Tunnels, channels, supply nets, a whole underground city, designed according to Euler's Graph Theory. He got inside, made himself comfortable, and opened his newspaper: "12% reduction of accidents, after applying the alcohol tests – 27% of the drivers had already conformed to the new strict regulations". 12%, 27%! How on earth did they find that? He turned to the Sports section: Konstantinou scores a goal by sending Archimedes' b-type semi-regular 32-hedron (the football that is) into the net. At 8:30 he was entering his office. He switched on his computer (which was filled with integrated network based on Boolean Algebra, which Thanasis didn't know, nor he wanted to find out about it) and logged on to the internet. RSA code, based upon prime

numbers, provided him with a secure connection on the web. He opened his mailbox. Message from Maria – the you-know-who. She's a nice person he thought. Intelligent, polite, cheerful, smart, pretty. Her disadvantage was one and only; she was studying math. Couldn't she be studying something else? Something closer to real-life? Something useful anyway! These were Thanasi's thoughts when he saw the manager approaching, so he immediately signed out of his e-mail account…