

TRIGONOMETRY

IDENTITIES AND FORMULAS

Listen.
Learn.
Grow.

Mathematics is not a contemplative but
a creative subject-Hardy



Sean Douglas

Algebra Rules

Algebra Laws

	<i>Additive</i>	<i>Multiplicative</i>
Associative:	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$
Commutative:	$a + b = b + a$	$ab = ba$
Inverse:	$a + (-a) = 0$	$a \cdot \frac{1}{a} = \frac{a}{a} = 1$
Identity:	$a + 0 = a$	$a \cdot 1 = a$
Distributive:	$a(b + c) = ab + ac$	

Fractions

Common Dem: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ Fraction Mult: $\frac{a}{b} \frac{c}{d} = \frac{ac}{bd}$ Neg Exponent: $a^{-1} = \frac{1}{a}$

Basic rule: $\frac{ac}{b} = \frac{a}{b}c = \frac{c}{b}a$ Double fractions: $\frac{a/b}{c/d} = \frac{ad}{bc}$ Signs: $\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$

Exponent rules

Remember the logarithm and exponential are inverses of each other, thus the *only* way to get x by its self in e^x or $\ln(x)$ is to apply the inverse.

$$a^x b^x = (ab)^x \quad a^x a^y = a^{x+y} \quad a^{-x} = \frac{1}{a^x} \quad a^0 = 1$$

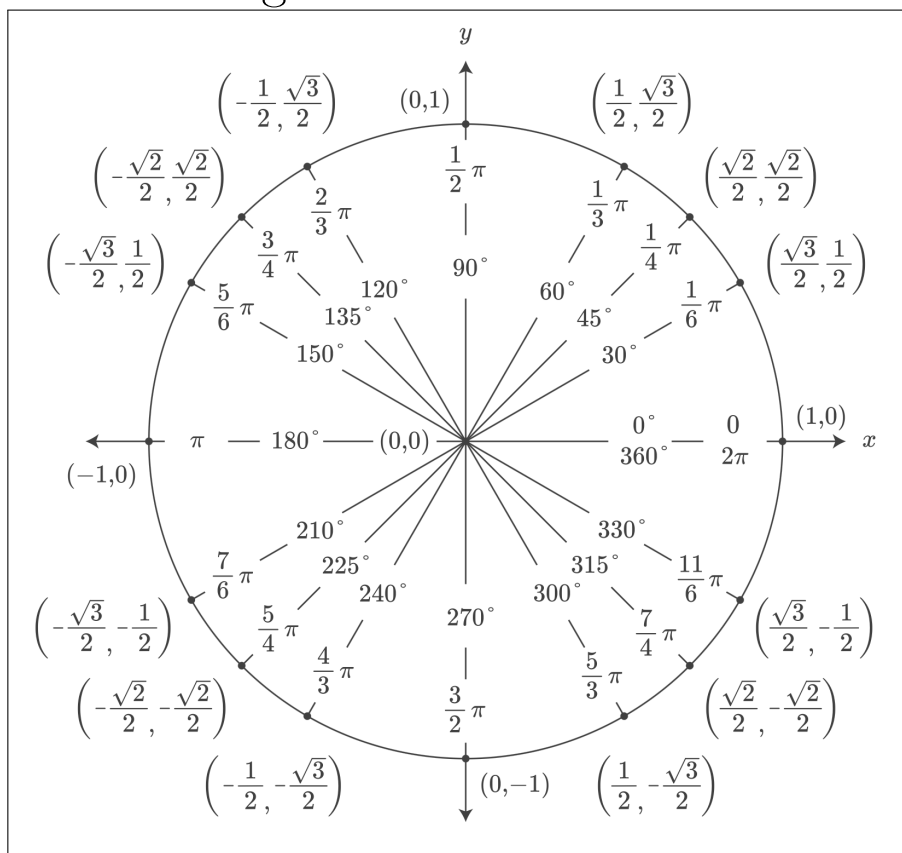
$$(a^x)^y = a^{xy} \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x} \quad \frac{a^x}{a^y} = a^{x-y} \quad a^x = e^{x \ln(a)}$$

Logarithms

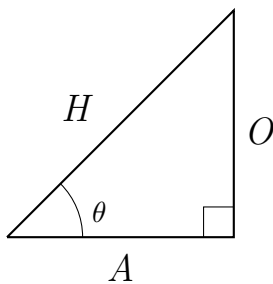
Remember the logarithm and exponential are inverses of each other, thus the *only* way to get x by its self in e^x or $\ln(x)$ is to do the opposite one.

$$\ln(ab) = \ln(a) + \ln(b) \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \quad \ln(a^x) = x \ln(a)$$

Trigonometric Identities



SOH-CAH-TOA



$$\sin(\theta) = \frac{O}{H}$$

$$\cos(\theta) = \frac{A}{H}$$

$$\tan(\theta) = \frac{O}{A}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{H}{O}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{H}{A}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{A}{O}$$

Co-function Identities

Divide by $\sin(\theta)$ and $\cos(\theta)$ to get the other Co-function Identities.

$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$$

Supplement Angle Identities

Take reciprocals on each side of the following to get the other supplement angle identities.

$$\begin{aligned}\sin(\pi - \theta) &= \sin(\theta) \\ \cos(\pi - \theta) &= -\cos(\theta) \\ \tan(\pi - \theta) &= -\tan(\theta)\end{aligned}$$

Negative angle Identities

Take reciprocals on each side of the following to get the other negative angle identities.

$$\begin{aligned}\sin(-\theta) &= -\sin(\theta) \\ \cos(-\theta) &= \cos(\theta) \\ \tan(-\theta) &= -\tan(\theta)\end{aligned}$$

Addition and Subtraction Identities

$$\begin{aligned}\sin(A + B) &= \sin(A)\cos(B) + \cos(A)\sin(B) & \sin(A - B) &= \sin(A)\cos(B) - \cos(A)\sin(B) \\ \cos(A + B) &= \cos(A)\cos(B) - \sin(A)\sin(B) & \cos(A - B) &= \cos(A)\cos(B) + \sin(A)\sin(B) \\ \tan(A + B) &= \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)} & \tan(A - B) &= \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}\end{aligned}$$

Useful Equations

$$\text{degrees} = \text{radians} \frac{180^\circ}{\pi} \qquad s = r\theta$$

Sum Identities

$$\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

Product Identities

$$\sin(A) \cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$$\cos(A) \cos(B) = \frac{1}{2}(\cos(A+B) + \cos(A-B))$$

$$\sin(A) \sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

Double Angle Identities

$$\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2 \cos^2(\theta) - 1 = 1 - 2 \sin^2(\theta)$$

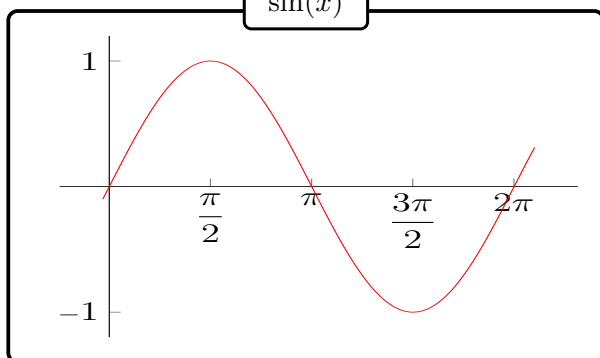
Half-Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

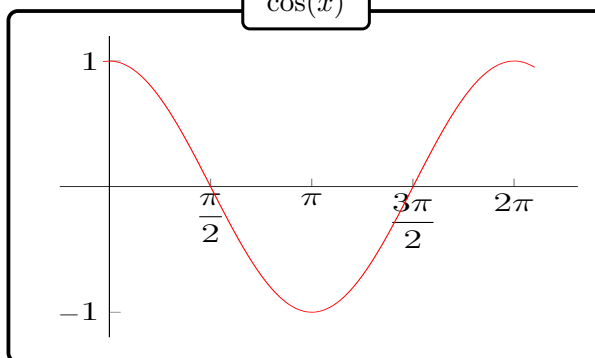
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan(\theta) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$

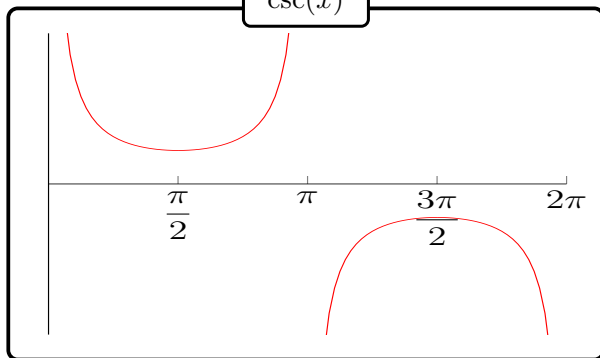
$\sin(x)$



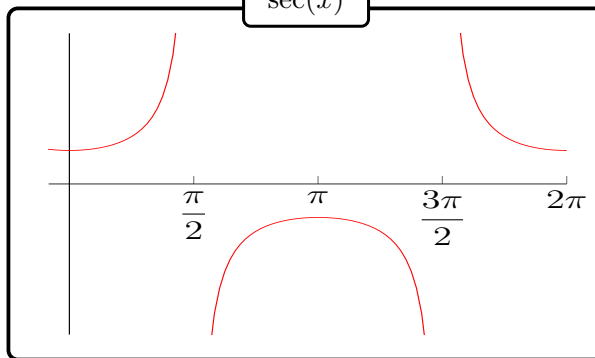
$\cos(x)$



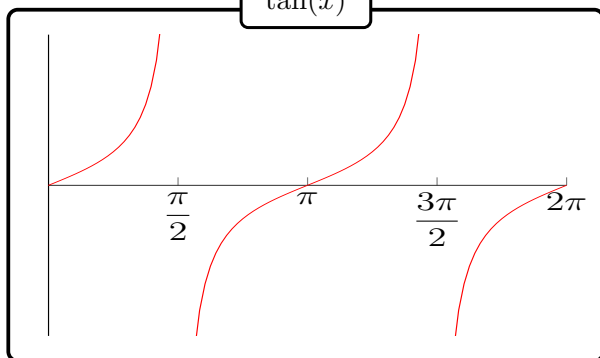
$\csc(x)$



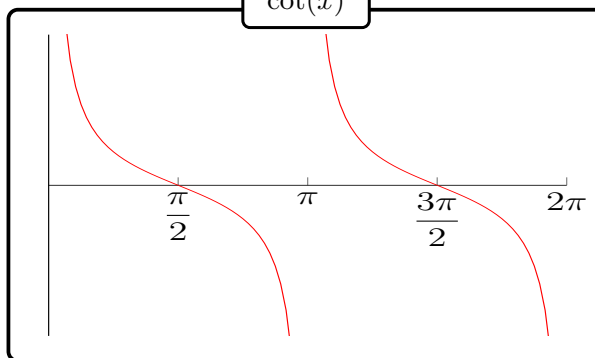
$\sec(x)$



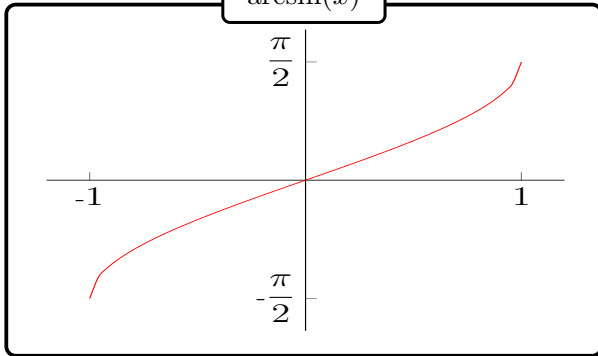
$\tan(x)$



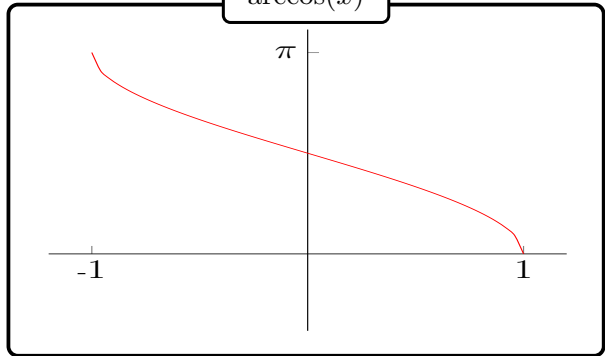
$\cot(x)$



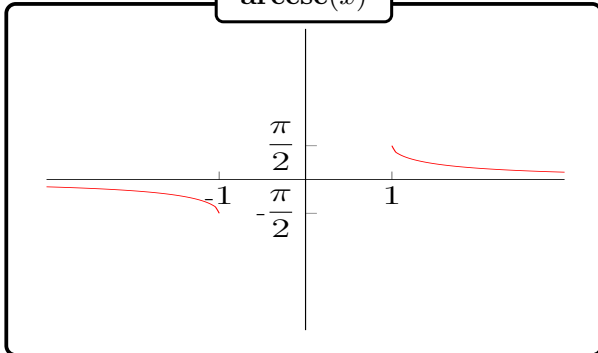
arcsin(x)



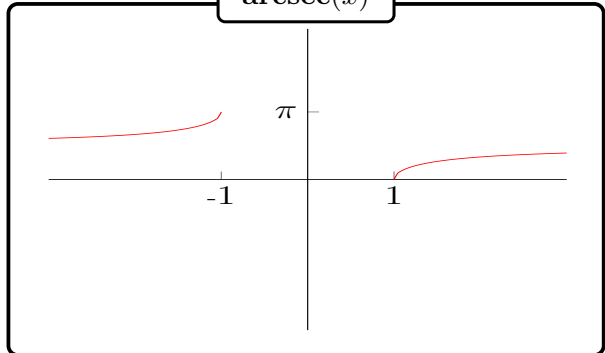
arccos(x)



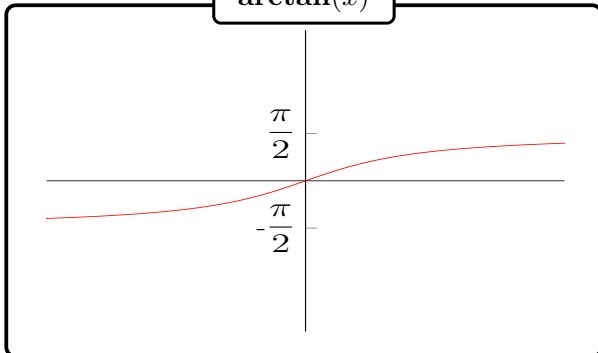
arccsc(x)



arcsec(x)



arctan(x)



arccot(x)

