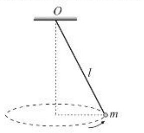
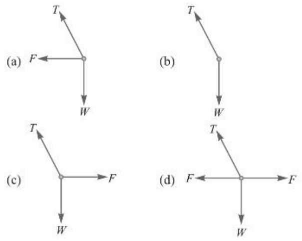
Detailed Solution to LLT Held On 2nd April During Live Streaming:

1.

A point mass m is suspended from a light thread of length l, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are





(c) In a frame attached to the mass, there are three forces on it, these are tension of string in the string and centrifugal force.

2.

One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v, the net force on the particle (directed towards the centre) is

(a) T (b) $T - \frac{mv^2}{l}$

(c) $T + \frac{mv^2}{l}$ (d) zero

(a) It has the centripetal force, which is equal to the tension in the string (T).

3.

A car is moving on a curved road with constant speed. If N_1 and N_2 are the reactions at A and B then:

(a) $N_1 < N_2$ (b) $N_1 > N_2$

(c) $N_1 = N_2$ (d) none

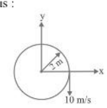
(a) $mg - N = \frac{mv^2}{R}$

or $N = mg - \frac{mv^2}{R}$

As $R_B > R_A$, and so $N_B > N_A$.

4.

A particle is moving along a circular path in the xy plane (see figure). When it crosses the x-axis, it has an acceleration along the path of 1.5 m/s^2, and is moving with a speed of 10 m/s in the negative y-direction. The total acceleration of the particle is :



(a) $50\,\hat{i} - 1.5\,\hat{j}$ m/s^2 (b) $-50\,\hat{i} - 1.5\,\hat{j}$ m/s^2

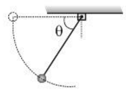
(c) $10\,\hat{i} - 1.5\,\hat{j}$ m/s^2 (d) $1.5\,\hat{i} - 50\,\hat{j}$ m/s^2

(b) $a_n = \frac{v^2}{r} = \frac{10^2}{2} = 50$ m/s^2, along negative x-axis.

so $\vec{a} = -50\hat{i} - 1.5\hat{j}$ m/s^2

5.

Figure shows a small mass connected to a string, which is attached to a vertical post. If the ball is released when the string is horizontal as shown, the magnitude of the total acceleration of the mass as a function of the angle θ is: **[KVPY–2011]**



(a) $g \sin\theta$ (b) $g\cos\theta$

(c) $g\sqrt{3\cos^2\theta + 1}$ (d) $g\sqrt{3\sin^2\theta + 1}$

$a_t = g\sin(90° - \theta) = g\cos\theta$

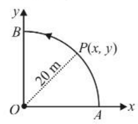
$v = \sqrt{2gh} = \sqrt{2gl\sin\theta}$

$\therefore\ a_c = \frac{v^2}{l} = \frac{2gl\sin\theta}{l} = 2g\sin\theta$

$a = \sqrt{{a_c}^2 + {a_t}^2} = g\sqrt{3\sin^2\theta + 1}$.

6.

A point P moves in circular-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. The acceleration of P when $t = 2s$ is nearly **[AIEEE -2010]**



(a) 14 m/s^2 (b) 13 m/s^2

(c) 12 m/s^2 (d) 7.2 m/s^2

$v = \frac{ds}{dt} = \frac{d}{dt}\left(t^3 + 5\right) = 3t^2$

At $t = 2s$, $v = 3(2)^2 = 12$ m/s

$a_n = \frac{v^2}{R} = \frac{12^2}{20} = 7.2$ m/s^2

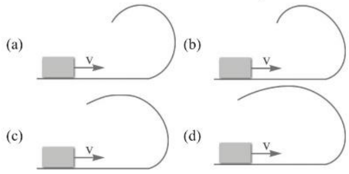
$a_t = \frac{dv}{dt} = \frac{d\left(3t^2\right)}{dt} = 6t$

$= 6 \times 2 = 12$ m/s^2

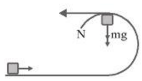
$\therefore\ a = \sqrt{a_n^2 + a_t^2} = \sqrt{(7.2)^2 + 12^2} = 14$ m/s^2

7.

A small block is shot into each of the four tracks as shown below. Each of the track rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in: **[IIT-JEE -2010]**



$N + mg = \frac{mv^2}{R}$

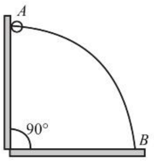


$\therefore\ N = \frac{mv^2}{R} - mg$.

As R is least in (a), so N is greatest in this case.

8.

A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is **[JEE Advance 2014]**

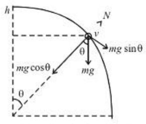


(a) always radially outwards

(b) always radially inwards

(c) radially outwards initially and radially inwards later

(d) radially inwards initially and radially outwards later



As the bead is moving in the circular path

$\therefore \; mg\cos\theta - N = \frac{mv^2}{R}$

$\therefore \; N = mg\cos\theta - \frac{mv^2}{R}$ ...(1)

By energy conservation, $\frac{1}{2}mv^2 = mg[R - R\cos\theta]$

$\therefore \; \frac{v^2}{R} = 2g(1-\cos\theta)$ ..(2)

From (1) and (2)

$N = mg\cos\theta - m[2g - 2g\cos\theta]$

$N = mg\cos\theta - 2mg + 2mg\cos\theta$

$N = 3mg\cos\theta - 2mg$

$\Rightarrow \; N = mg(3\cos\theta - 2)$

Clearly N is positive (acts radially outwards) when $\cos\theta > \frac{2}{3}$

Similarly, N acts radially inwards if $\cos\theta < \frac{2}{3}$

9.

Which of the following statements is false for a particle moving in a circle with a constant angular speed ?

(a) the velocity vector is tangent to the circle

(b) the acceleration vector is tangent to the circle

(c) the acceleration vector points to the centre of the circle

(d) the velocity and acceleration vectors are perpendicular to each other

For a particle with constant speed, the acceleration vector tends towards centre of the path.

10.

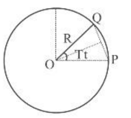
A particle moves along a circle of radius R with constant angular velocity ω. Its displacement magnitude in time t is :

(a) ωt (b) $2R\sin\omega t$

(c) $2R\cos\omega t$ (d) $2R\sin\frac{\omega t}{2}$

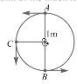
$PQ = \left|R\sin\frac{\omega t}{2}\right| \times 2$

$= 2R\sin\frac{\omega t}{2}$.



11.

A small block of mass 1 kg is connected to an inextinsible string of length 1 m and made to rotate in a vertical circle about the free end of the string. The tension at the highest point of the block is 10 N. Then match the two columns; ($g = 10\ m/s^2$)



	Column - I		Column - II
A.	speed of block at A	(p)	$\sqrt{60}$ m/s
B.	speed of block at B	(q)	$\sqrt{20}$ m/s
C.	tension in string at B	(r)	40 N
D.	tension in string at C	(s)	70 N

At A; $mg + T = \frac{mv_A^2}{r}$

or $1\times 10 + 10 = \frac{1\times v_A^2}{1}$

$\therefore \; v_A = \sqrt{20}$ m/s

Now $v_c^2 = v_A^2 + 2g\times 1$

$= 20 + 2\times 10$

$\therefore \; v_c = \sqrt{40}$ m/s

$T_c = \frac{mv_c^2}{r} = \frac{1\times 40}{1} = 40$ N

At B;

$v_B^2 = v_A^2 + 2g\times 2$

$= 20 + 2\times 10\times 2$

$\therefore \; v_B = \sqrt{60}$ m/s.

$T_B = mg + \frac{mv_B^2}{r}$

$= 1\times 10 + \frac{1\times 60}{1} = 70$ N

12.

A particle of 500 gm mass moves along a horizontal circle of radius 16 m such that normal acceleration of particle varies with time as $a_n = 9t^2$

	Column I		Column II
A.	Tangential force on particle at t = 1 second (in newton)	(p)	72
B.	Total force on particle at t = 1 second (in newton)	(q)	36
C.	Power delivered by total force at t = 1 sec (in watt)	(r)	7.5
D.	Average power developed by total force over first one second (in watt)	(s)	6

$a_n = \frac{v^2}{16} = 9t^2 \Rightarrow v = 12\,t$ and $\frac{dv}{dt} = 12$

Tangential force $m.\frac{dv}{dt} = \frac{3}{2}\sqrt{16} = 6N$

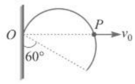
Total force $= \sqrt{6^2 + \left(\frac{mv^2}{R}\right)^2} = \sqrt{6^2 + \left(\frac{9}{2}\right)^2} = 7.5N.$

Power $= F_T.v = 6 \times 3\sqrt{16} = 72$ watt.

Average power $= \frac{72}{2} = 36$ watt.

13.

A thin but rigid semicircular wire frame of radius r is hinged at O and can rotate in its own vertical plane. A smooth peg P starts from O and moves horizontally with constant speed v_0, lifting the frame upward as shown in figure.



Find the angular velocity ω of the frame when its diameter makes an angle of 60° with the vertical :

(a) v_0/r (b) $v_0/2r$

(c) $2v_0/r$ (d) none

(a) $\frac{x}{\sin 2\theta} = \frac{r}{\sin(90-\theta)}$

$\Rightarrow \; x = 2r\sin\theta$

$\therefore \; \frac{dx}{dt} = 2r\cos\theta \times \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{dx/dt}{2r\cos\theta} = \frac{v_0}{2r\cos 60°} = \frac{v_0}{r}$

14.

Read the two statements carefully to mark the correct option out of the options given below:

(a) **Statement - 1** is true, **Statement - 2** is true; **Statement - 2** is correct explanation for **Statement - 1**.
(b) **Statement -1** is true, **Statement - 2** is true; **Statement - 2** is not correct explanation for **Statement - 1**.
(c) **Statement - 1** is true, **Statement - 2** is false.
(d) **Statement - 1** is false, **Statement - 2** is true

Statement - 1

As the frictional force increases, the safe velocity limit for taking a turn on an unbanked road also increases.

Statement - 2

Banking of roads will increase the value of limiting velocity.

(b) $v = \sqrt{mrg}$, if m increases, v also increases.

Also $\tan q = \frac{v^2}{rg}$

15.

Statement - 1

Force required to move a body uniformly along straight line is zero.

Statement - 2

The force required to move a body uniformly along a circle is zero.

(c) Body moving along a straight line, will have zero acceleration, and so force needed, $F = 0$. But body moving along a circular path, will have centripetal acceleration, and so $F_n = \frac{mv^2}{r}$.

16.

A particle is moving with velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation of its path is **[AIEEE 2010]**

(a) $y^2 = x^2 +$ constant (b) $y = x^2 +$ constant
(c) $y^2 = x +$ constant (d) $xy =$ constant

(a) On comparing with, $\vec{v} = v_x\hat{i} + v_y\hat{j}$, we get

$$v_x = \frac{dx}{dt} = ky$$

and $$v_y = \frac{dy}{dt} = kx$$

Dividing equation (ii) by (i), we get

$$ydy = xdx$$

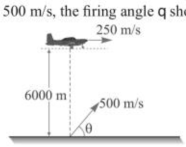
Now $\int_0^y y\,dy = \int_0^x x\,dx$

or $\frac{y^2}{2} = \frac{x^2}{2} + k$

or $y^2 = x^2 +$ constant

17.

An aircraft moving with a speed of 250 m/s is at a height of 6000 m, just overhead of an anti aircraft gun. If the muzzle velocity is 500 m/s, the firing angle q should be:



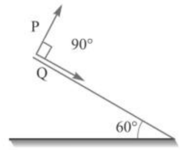
(a) 30° (b) 45°
(c) 60° (d) none of these.

(c) $500 \cos q = 250 \quad Ω \cos q = \frac{1}{2}$
or $q = 60°$.

18.

A particle P is projected from a point on the surface of long smooth inclined plane and Q starts moving down the plane from the same position. P and Q collide after 4 second. The speed of projection of P is : ($g = 10$ m/s^2)

(a) 5 m/s (b) 10 m/s
(c) 15 m/s (d) 20 m/s



Particle will collide when P hits the inclined plane. So time of flight of P,

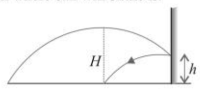
$$0 = ut - \frac{1}{2}(g \cos 60°)\,T^2$$

or $$T = \frac{2u}{g \cos 60è}$$

or $$4 = \frac{2u}{g ì \frac{1}{2}} \quad Ω \ u = g.$$

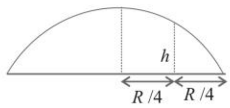
19.

A stone is projected from a horizontal plane. It attains maximum height H and strikes a stationary smooth wall and falls on the ground vertically below the maximum height. Assuming the collision to be elastic, the height of the point on the wall where ball will strike is:



(a) $\frac{H}{4}$ (b) $\frac{H}{2}$
(c) $\frac{3H}{4}$ (d) $\frac{7H}{8}$

(a) $R = \frac{u^2 \sin 2q}{g}$ and $\frac{R}{4} = u \cos q ì \sqrt{\frac{2b}{g}}$



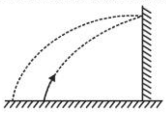
Also, $H = \frac{u^2 \sin^2 q}{2g}$; solving above equations

We get $\frac{H}{4} = h$.

20.

Statement - 1

A projectile, launched from ground, collides with a smooth vertical wall and returns to the ground. The total time of flight is the same had there been no collision.



Statement - 2

The collision changes only the horizontal component of velocity.

(a) The time of flight depends only on the vertical component of velocity which remains unchanged in collision with a vertical wall.