Question Booklet

Oscillations / SHM

With Detailed Solution

Question Booklet

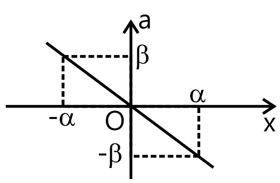
Exercise 1

Q.1 A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1:3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

Q.2 A body undergoing SHM about the origin has its equation is given by $x = 0.2\cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

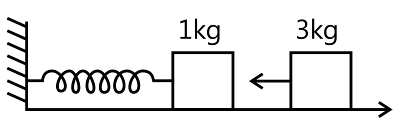
Q.3 Two particles A and B execute SHM along the same line with the same amplitude a, same frequency and same equilibrium position O. If the phase difference between them is $\phi = 2\sin^{-1}(0.9)$, then find the maximum distance between the two.

Q.4 The acceleration-displacement $(a - x)$ graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



Q.5 A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is $45°$.

Q.6 One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to x-axis. A block of mass m=1 kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is $x = 10 + 3\sin 10t$, is in second and x in cm. Another block of mass M=3kg, moving towards the origin with velocity 30cm/s collides with the block performing SHM at t=0 and gets stuck to it, calculate:

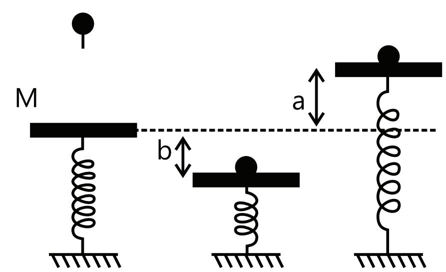


(i) New amplitude of oscillations.

(ii) New equation for position of the combined body.

(iii) Loss of energy during collision. Neglect friction.

Q.7 A mass M is in static equilibrium on a massless vertical spring as shown in the figure. A ball of mass m dropped from certain height sticks to the mass M after colliding with it. The oscillations they perform reach to height 'a' above the original level of scales & depth 'b' below it.

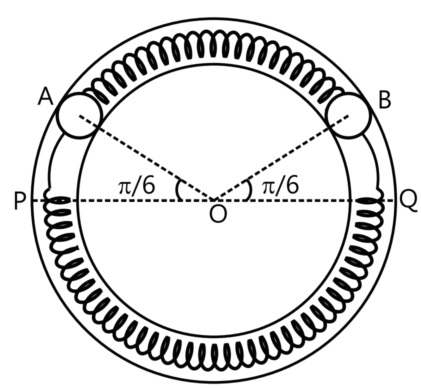


(a) Find the constant of force of the spring;

(b) Find the oscillation frequency.

(c) What is the height above the initial level from which the mass m was dropped?

Q.8 Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in figure. The pipe is fixed in a horizontal plane. The centers of the ball can move in a circle of radius 0.06m. Each spring has a natural length 0.06πm and force constant 0.1N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest

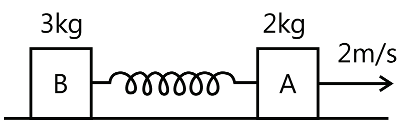


(a) Calculate the frequency of oscillation of the ball B.

(b) What is the total energy of the system?

(c) Find the speed of the ball A when A and B are at the two ends of the diameter PQ.

Q.9 Two blocks A(2kg) and B(3kg) rest up on a smooth horizontal surface are connected by a spring of stiffness 120 N/m. Initially the spring is unreformed. A is imparted a velocity of 2m/s along the line of the spring away from B. Find the displacement of A, t seconds later.



Q.10 A force $F = 10x + 2$ acts on a particle of mass 0.1 kg, where 'k' is in m and F in newton. If it is released from rest at $x = 0.2\text{m}$, find :

(a) Amplitude; (b) time period; (c) equation of motion.

Q.11 Potential Energy (U) of a body of unit mass moving in one-dimension conservative force field is given by, $U = (x^2 - 4x + 3)$. All units are in S.I.

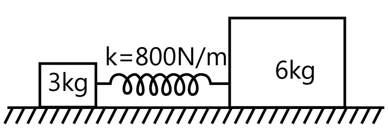
(i) Find the equilibrium position of the body.

(ii) Show that oscillations of the body about this equilibrium position are simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

Q.12 A body is executing SHM under the action of force whose maximum magnitude is 50N. Find the magnitude of force acting on the particle at the time when its energy is half kinetic and half potential.

Q.13 The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6cm and then released. Find



(a) Time period

(b) Amplitude of 3kg block

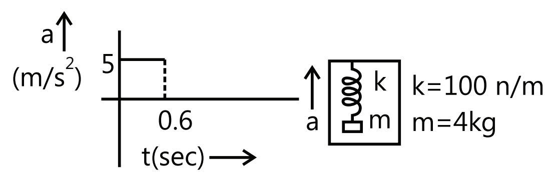
(c) Maximum momentum of 6kg block

Q.14 The resulting amplitude A' and the phase of the vibrations δ

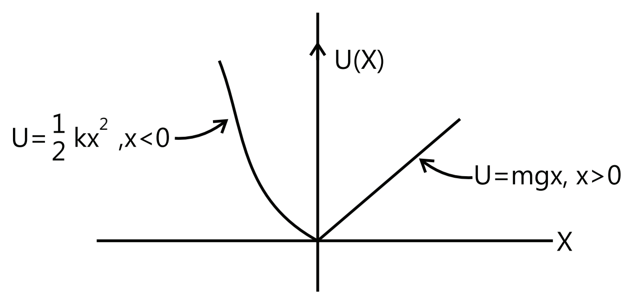
$$S = A\cos(\omega t) + \frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{A}{4}\cos(\omega t + \pi) + \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right) = A'\cos(\omega t + \delta)$$

are ___________ and __________ respectively.

Q.15 A spring block (force constant k=1000N/m and mass m=4kg) system is suspended from the ceiling of an elevator such that block is initially at rest. The elevator begins to move upwards at t=0. Acceleration time graph of the elevator is shown in the figure. Draw the displacement x (from its initial position taking upwards as positive) vs time graph of the block with respect to the elevator starting from t=0 to t=1 sec. Take $\pi^2 = 10$.

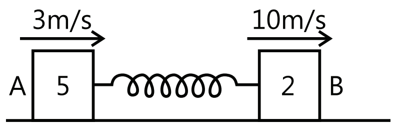


Q.16 A particle of mass m moves in the potential energy U shown below. Find the period of the motion when the particle has total energy E.



Q.17 The motion of a particle is described by x=30 $\sin(\pi t + \pi/6)$, where x is in cm and t in sec. Potential energy of the particle is twice of kinetic energy for the first time after t=0 when the particle is at position __________ after _______ time.

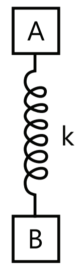
Q.18 Two blocks A (5kg) and B (2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



(a) Find the maximum extension of the spring.

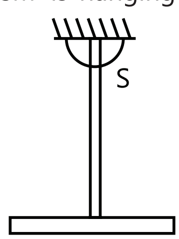
(b) When does the first maximum compression occurs after start.

Q.19 Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



Q.20 (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$ J / rad. The moment of inertia of rigid body is $10 kg\text{-}m^2$ about the axis of rotation.

(b) A simple pendulum of length $l = 0.5m$ is hanging from ceiling of a car. The car is kept on a horizontal plane The car starts accelerating on the horizontal road with acceleration of $5 m/s^2$. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.



Q.21 An object of mass 0.2kg executes SHM along the x-axis with frequency of $(25/\pi)$Hz. At the point $x = 0.04m$ the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ___________.

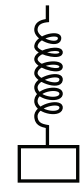
Q.22 A body of mass 1kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

Q.23 A body A of mass $m_1 = 1kg$ and a body B of mass m_2 = 4kg are attached to the ends of a spring. The body a performs vertical simple harmonic oscillations of amplitude a=1.6 cm and angular frequency $\omega = 25$ rad/s. Neglecting the mass of the spring determine the maximum and minimum values of force the system exerts on the surface on which it rests. [Take $g = 10 m/s^2$]

Q.24 A spring mass system is hanging from the ceiling of an elevator in equilibrium Elongation of spring is l. The elevator suddenly starts accelerating downwards with accelerating $g/3$ find

(a) The frequency and

(b) The amplitude of the resulting SHM.



Exercise 2

Single Correct Choice Type

Q.1 A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in second^{-1}) is:

(A) $2\pi\sqrt{3}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2\pi}$ (D) $\frac{1}{2\pi\sqrt{3}}$

Q.2 A particle executed SHM with time period T and amplitude A. The maximum possible average velocity in time $\frac{T}{4}$ is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$ (C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

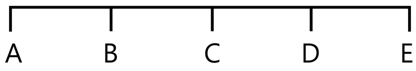
Q.3 A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(A) a/T (B) 2a/T (C) 3a/T (D) a/2T

Q.4 Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles

(A) 45° (B) 90° (C) 135° (D) 180°

Q.5 A body performs simple harmonic oscillations along the straight line ABCDE with C as the midpoint of AE. Its kinetic energies at B and D are each one fourth of its maximum value. If AE=2R, the distance between B and D is



(A) $\frac{\sqrt{3}R}{2}$ (B) $\frac{R}{\sqrt{2}}$

(C) $\sqrt{3}R$ (D) $\sqrt{2}R$

Q.6 In an elevator, a spring clock of time period T_s (mass attached to a spring) and a pendulum clock of time period T_p are kept. If the elevator accelerates upwards

(A) T_s well as T_p increases

(B) T_s remain same, T_p increases

(C) T_s remains same, T_p decreases

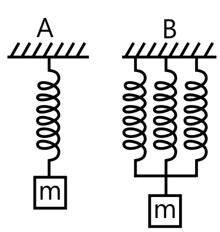
(D) T_s as well as T_p decreases

Q.7 Two bodies P & Q of equal mass are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the maximum velocities of them are equal during their motion, the ratio of amplitude of P to Q is:

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$

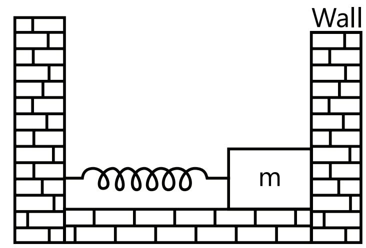
(C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

Q.8 The spring in figure. A and B are identical but length in A is three times each of that in B. the ratio of period T_A/T_B is



(A)$\sqrt{3}$ (B) 1/3 (C) 3 (D) $1/\sqrt{3}$

Q.9 In the figure the block of mass m, attached to the spring of stiffness k is in contact with the completely elastic wall, and the compression in the spring is 'e'. The spring is compressed further by 'e' by displacing the block towards left and is then released. If the collision between the block and the wall is completely elastic then the time period of oscillations of the block will be:



(A) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{k}}$

(C) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$

Q.10 A 2 kg block moving with 10 m/s strikes a spring of constant $\pi 2$ N/m attached to 2 Kg block at rest kept on a smooth floor. The time for which rear moving block remain in contact with spring will be

(A) $\sqrt{2}$ sec (B) $\frac{1}{\sqrt{2}}$ sec

(C) 1sec (D) $\frac{1}{2}$ sec

Q.11 In the above question, the velocity of the rear 2 kg block after it separates from the spring will be:

(A) 0 m/s (B) 5 m/s

(C) 10 m/s (D) 7.5 m/s

Q.12 A rod whose ends are A & B of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9:4 at points A and B. Find the distance of the center of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

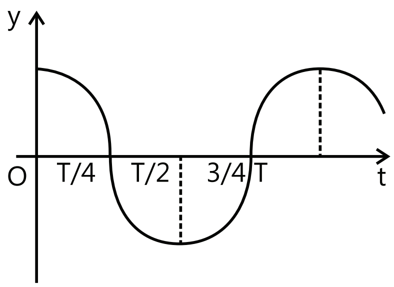
Q. 13 A circular disc has a tiny hole in it, at a distance z from its center. Its mass is M and radius R (R > z). A horizontal shaft is passed through the hole and held fixed so that the disc can freely swing in the vertical plane. For small disturbance, the disc performs SHM whose time period is the minimum for z =

(A) R/2 (B) R/3

(C) $R/\sqrt{2}$ (D) $R/\sqrt{3}$

Multiple Correct Choice Type

Q.14 The displacement-time graph of a particle executing SHM is shown which of the following statement is/are true?



(A) The velocity is maximum at t=T/2

(B) The acceleration is maximum at t=T

(C) The force is zero at t= 3T/4

(D) The potential energy equals the oscillation energy at t=T/2.

Q.15 The amplitude of a particle executing SHM about O is 10 cm. Then:

(A) When the K.E. is 0.64 of its max. K.E. its displacement is 6cm from O.

(B) When the displacement is 5cm from O its K.E.is 0.75 of its max. P.E.

(C) Its total energy at any point is equal to its maximum K.E.

(D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement.

Q.16 A particle of mass m performs SHM along a straight line with frequency f and amplitude A.

(A) The average kinetic energy of the particle is zero.

(B) The average potential energy is $m\pi 2f2A^2$.

(C) The frequency of oscillation of kinetic energy is 2f.

(D) Velocity function leads acceleration by $\pi / 2$

Q.17 A system is oscillating with undamped simple harmonic motion. Then the

(A) Average total energy per cycle of the motion is its maximum kinetic energy.

(B) Average total energy per cycle of the motion is $\frac{1}{\sqrt{2}}$ times its maximum kinetic energy.

(C) Root means square velocity $\frac{1}{\sqrt{2}}$ times its maximum velocity.

(D) Mean velocity is $\frac{1}{2}$ of maximum velocity.

Q.18 A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to ceiling. The block released from the position, where the spring has length 45cm.

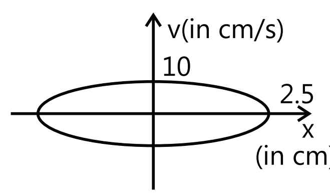
(A) The block will performs SHM of amplitude 5 cm.

(B) The block will have maximum velocity $30\sqrt{5}\,cm / sec$.

(C) The block will have maximum acceleration $15\,m / s^2$

(D) The minimum potential energy of the spring will be zero.

Q.19 The figure shows a graph between velocity and displacement (from mean position) of a particle performing SHM:



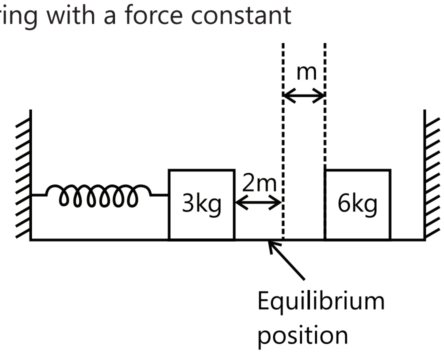
(A) The time period of the particle is 1.57s

(B) The maximum acceleration will be $40 cm / s^2$

(C) The velocity of particle is $2\sqrt{21} cm / s$ when it is at a distance 1 cm from the mean position.

(D) None of these

Q.20 Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to A Spring with a force constant



$k = 900 Nm^{-1}$ Which is compressed 2 m from beyond the equilibrium position. The 6 kg mass is at rest at 1m from mean position 3kg mass strikes the 6kg mass and the two stick together.

(A) Velocity of the combined masses immediately after the collision is $10 ms^{-1}$

(B) Velocity of the combined masses immediately after the collision is $5ms^{-1}$

(C) Amplitude of the resulting oscillations is $\sqrt{2}m$

(D) Amplitude of the resulting oscillation is $\sqrt{\frac{5}{2}}m$.

Q.21 A particle is executing SHM with amplitude A. time period T, maximum acceleration a_0 and maximum velocity v_0. Its starts from mean position at t-0 and at time t, it has the displacement A/2, acceleration a and velocity v then

(A) t=T/12 (B) $a = a_0 / 2$

(C) $v = v_0 / 2$ (D) t=T/8

Q.22 For a particle executing SHM, x=displacement from equilibrium position, v= velocity at any instant and a = acceleration at any instant, then

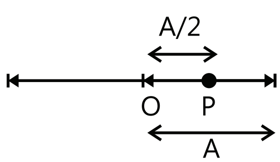
(A) v-x graph is a circle

(B) v-x graph is an ellipse

(C) a-x graph is a straight line

(D) a-v graph is an ellipse

Q.23 A particle starts from a point P at a distance of A/2 from the mean position O & travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of motion of particle is:



(A) $x = A\sin\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$ (B) $x = A\sin\left(\frac{2\pi}{T}t+\frac{5\pi}{6}\right)$

(C) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$ (D) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{3}\right)$

Q.24 Two particles execute SHM with amplitude A and 2A and angular frequency ω and 2ω respectively. At t=0 they starts with some initial phase difference. At, t= difference is: $\frac{2\pi}{3\omega}$. They are in same phase. Their initial phase

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) π

Q.25 A mass of 0.2 kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?

(A) In equilibrium, the spring will be stretched by 1cm.

(B) If the mass is raised till the spring is in not stretched state and then released, it will go down by 2 cm before moving upwards.

(C) The frequency of oscillation will be nearly 5 Hz.

(D) If the system is taken to moon, the frequency of oscillation will be the same as on the earth.

Q.26 The potential energy of particle of mass 0.1kg, moving along x-axis, is given by U=5x(x-4)J where x is in meters. It can be concluded that

(A) The particle is acted upon by a constant force.

(B) The speed of the particle is maximum at x=2m

(C) The particle executes simple harmonic motion

(D) The period of oscillation of the particle is π /5 s

Q.27 The displacement of a particle varies according to the relation x=3 sin 100t + $\cos^2$ 50t. Which of the following is/are correct about this motion.

(A) The motion of the particle is not SHM

(B) The amplitude of the SHM of the particle is 5 units

(C) The amplitude of the resultant SHM is $\sqrt{73}$ units.

(D) The maximum displacement of the particle from the origin is 9 units.

Q.28 The equation of motion for an oscillating particle is given by x=3sin (4πt) + 4cos (4πt), where x is in mm and t is in second

(A) The motion is simple harmonic

(B) The period of oscillation is 0.5 s

(C) The amplitude of oscillation is 5 mm

(D) The particle starts its motion from the equilibrium

Q.29 A linear harmonic oscillator of force constant $2\times10^6 Nm^{-1}$ and amplitude 0.01 m has a total mechanical energy of 160 J. Its

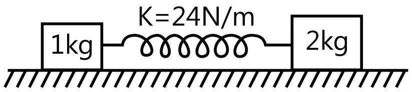
(A) Maximum potential energy is 100 J

(B) Maximum kinetic energy is 100 J

(C) Maximum potential energy is 160

(D) Minimum potential energy is zero.

Q.30 The two blocks shown here rest on a frictionless surface. If they are pulled apart by a small distance and released at t=0, the time when



1 kg block comes to rest can be

(A) $\frac{2\pi}{3}$ sec (B) π *sec.*

(C) $\frac{\pi}{2}$ sec (D) $\frac{\pi}{9}$ sec

Assertion Reasoning Type

Q.31 Statement-I: A particle is moving along x-axis. The resultant force F acting on it at position x is given by F=-ax-b. Where a and b are both positive constants. The motion of this particle is not SHM.

Statement-II: In SHM restoring force must be proportional to the displacement from mean position.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.32 Statement-I: For a particle performing SHM, its speed decreases as it goes away from the mean position.

Statement-II: In SHM, the acceleration is always opposite to the velocity of the particle.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and Statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.33 Statement-I: Motion of a ball bouncing elastically in vertical direction on a smooth horizontal floor is a periodic motion but not an SHM.

Statement-II: Motion is SHM when restoring force is proportional to displacement from mean position.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

Q.34 Statement-I: A particle, simultaneously subjected to two simple harmonic motions of same frequency and same amplitude, will perform SHM only if two SHM's are in the same direction

Statement-II: A particle, simultaneously subjected to two simple harmonic motions of same frequency and same amplitude, perpendicular to each other the particle can be in uniform circular motion.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.35 Statement-I: In case of oscillatory motion the average speed for any time interval is always greater than or equal to its average velocity.

Statement-II: Distance travelled by a particle cannot be less than its displacement.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I

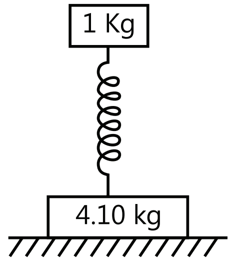
(B) Statement-I is true. statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Comprehension Type

Paragraph 1: When force acting on the particle is of nature $F = -kx$, motion of particle is SHM, Velocity at extreme is zero while at mean position it is maximum. In case of acceleration situation is just reverse. Maximum displacement of particle from mean position on both sides is same and is known as amplitude. Refer to figure One kg block performs vertical harmonic oscillations with amplitude 1.6 cm and frequency 25 rad s^{-1}.



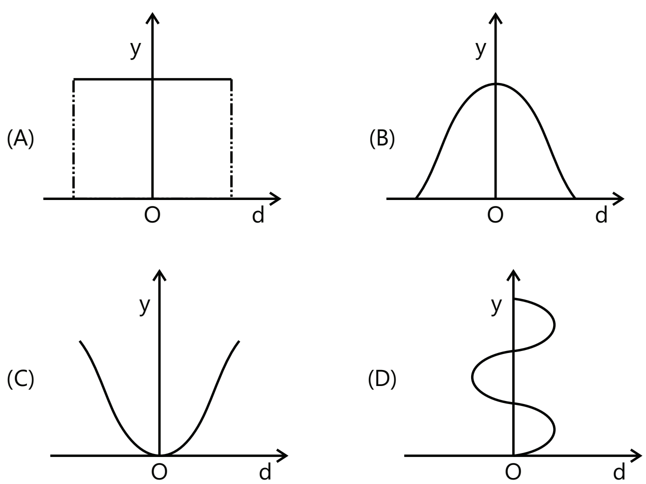
Q.36 The maximum value of the force that the system exerts on the surface is

(A) 20 N (B) 30 N (C) 40 N D) 60 N

Q.37 The minimum force is

(A) 20 N (B) 30 N (C) 40 N (D) 60 N

Paragraph 2: The graphs in figure show that a quantity y varies with displacement d in a system undergoing simple harmonic motion.



Which graphs best represents the relationship obtained when Y is

Q. 38 The total energy of the system

(A) I (B) II (C) III (D) IV

Q.39 The time

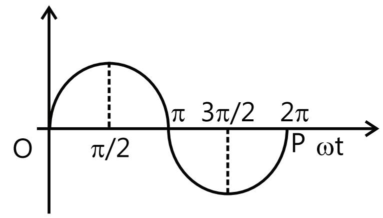
(A) I (B) II (C) III (D) IV

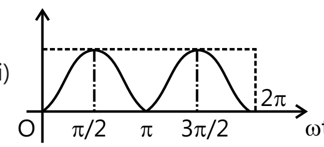
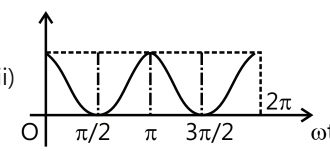
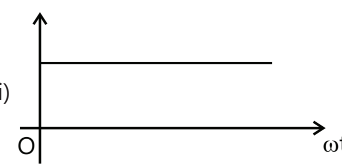
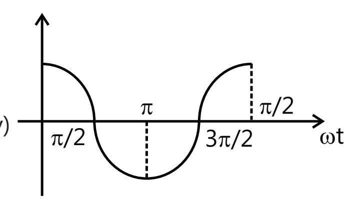
Q.40 The unbalanced force acting on the system

(A) I (B) II (C) III (D) None

Match the Columns

Q.41 The graph plotted between phase angle (ϕ) and displacement of a particle from equilibrium position (y) is a sinusoidal curve as shown below. Then the best matching is



Column A	Column B
(a) K.E. versus phase angle curve	(i) 
(b) P.E. versus phase angle curve	(ii) 
(c) T.E. versus phase angle curve	(iii) 
(d) Velocity versus phase angle curve	(iv) 

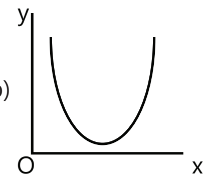
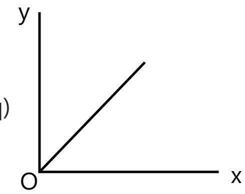
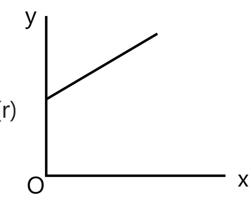
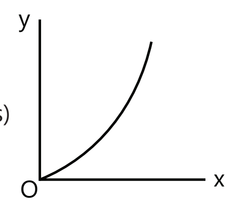
(A) (a)-(i), (b)-(ii), (c)-(iii) & (d)-(iv)

(B) (a)-(ii), (b)-(i), (c)-(iii) & (d)-(iv)

(C) (a)-(ii), (b)-(i), (c)-(iv) & (d) – (iii)

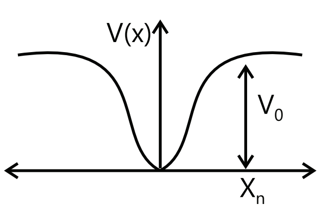
(D) (a)-(ii), (b)-(iii), (c)-(iv) & (d)-(i)

Q.42 Column I is a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 x 4 matrix given in the ORS.

Column I	Column II
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)	(p) 
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(q) 
(C) Range of projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle.	(r) 
(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)	(s) 

Previous Years' Questions

Paragraph 1: When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $v(x) = \alpha x^2 \, (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure below) ***(2010)***



Q.1 If the total energy of the particle is E, it will perform periodic motion only if

(A) E< 0 (B) E>0

(C) V_0 >E>0 (D) E> V_0

Q.2 For periodic motion of small amplitude A, the time period t of this particle is proportional to

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Q.3 The acceleration of this particle for $|x| > X_0$ is

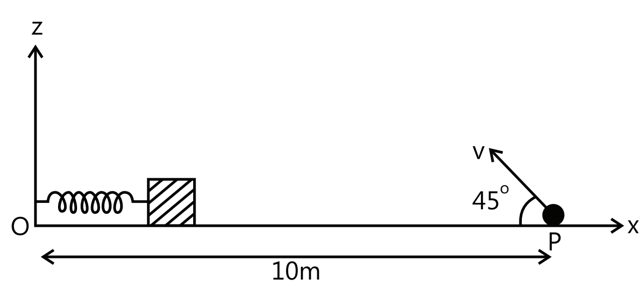
(A) Proportional to V_0

(B) Proportional to $\frac{V_0}{mX_0}$

(C) Proportional to $\sqrt{\frac{V_0}{mX_0}}$

(D) Zero

Q.4 A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v form point P at an angle of $45°$ as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2) ***(2012)***



(A) $\sqrt{50}$ m / s (B) $\sqrt{51}$ m / s

(C) $\sqrt{52}$ m / s (D) $\sqrt{53}$ m / s

Q.5 A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0. It collides elastically with a rigid wall. After this collision, ***(2013)***

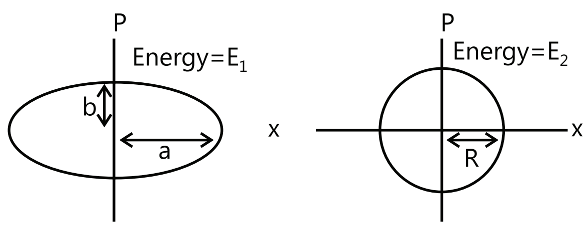
(A) The speed of the particle when it returns to its equilibrium position is u_0

(B) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

Q.6 Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are) ***(2015)***



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Q.7 A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (< M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? ***(2016)***

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases

Q.8 Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in column I with the characteristics in column II. ***(2007)***

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration α. The motion of the object is observed from the elevator during the period it maintain this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

Q.9 A linear harmonic oscillator or force constant 2×10^{6} N/m and amplitude 0.01m has a total mechanical energy of 160 J. Its ***(1989)***

(A) Maximum potential energy is 100 J

(B) Maximum kinetic energy is 100J

(C) Maximum potential energy is 160J

(D) Maximum potential energy is zero

Q.10 Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by $45°$, then ***(1999)***

(A) The resultant amplitude is $\left(1+\sqrt{2}\right)$a

(B) The phase of the resultant motion relative to the first is 90°

(C) The energy associated with the resulting motion is $\left(3+2\sqrt{2}\right)$ times the energy associated with any single motion

(D) The resulting motion is not simple harmonic

Q.11 Function x = A $\sin^2 \omega t$ + B $\cos^2 \omega t$ + C $\sin \omega t \cos \omega t$ represent SHM ***(2006)***

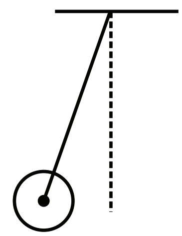
(A) For any value of A, B and C (except C=0)

(B) If A=-B, C=2B, amplitude= $\left|B\sqrt{2}\right|$

(C) If A=B; C=0

(D) If A=B; C=2B, amplitude= $\left|B\right|$

Q.12 A metal rod of length L and mass m is pivoted at one end. A thin disk of mass M and radius R $\left(<L\right)$ is attached at its center to the free end of the rod. Consider two ways the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true? ***(2011)***



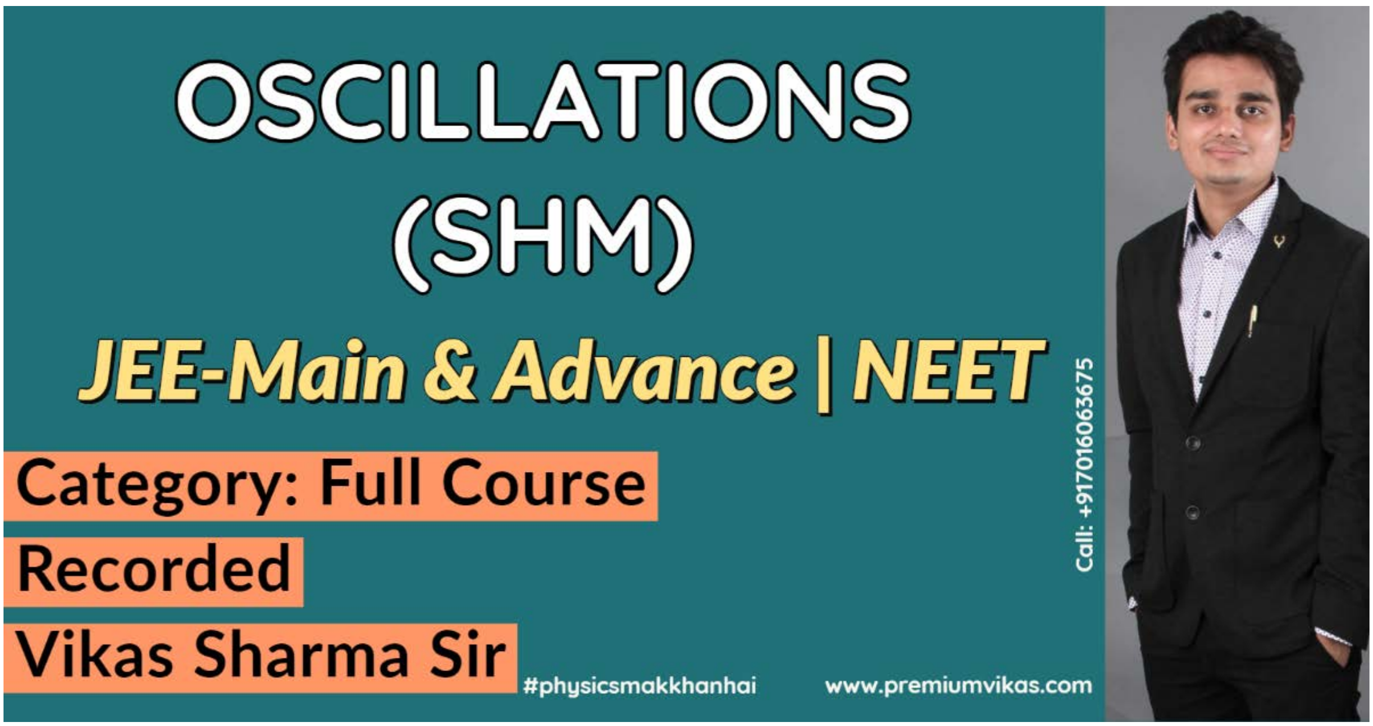
(A) Restoring torque in case A=Restoring torque in case B

(B) Restoring torque in case A<Restoring torque in case B

(C) Angular frequency for case A>Angular frequency for case B

(D) Angular frequency for case A<, angular frequency for case B

Answer Key



Question Booklet

Exercise 1

Q.1 $1/\sqrt{3}$

Q.2 2 m/s

Q.3 1.8a

Q.4 $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

Q.5 $x = 0.1\sin(4t + \pi/4)$

Q.6 3cm, $x = 10 - 3\sin 5t; \Delta E = 0.135J$

Q.7 (a) $k = \dfrac{2mg}{b-a}$: $(c)\dfrac{ab}{b-a}$, (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{2mg}{(b-a)(M+m)}}$

Q.8 $f = \dfrac{1}{\pi}; E = 4\pi^2 \times 10^{-5}\,J; v = 2\pi \times 10^{-2}\,m/s$

Q.9 $0.8t + 0.12\sin 10t$

Q.10 (a) 0.4 m, (b) $\dfrac{\pi}{5}$ sec., (c) x = 0.2 − 0.4 cosωt

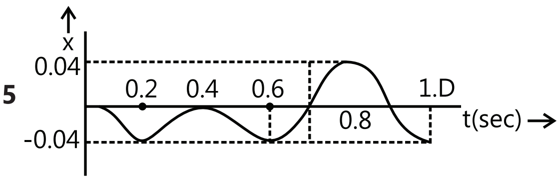
Q.11 (i) $x_0 = 2m$; (ii) $T = \sqrt{2}\pi$ sec.; (iii) $2\sqrt{3}$ m

Q.12 $25\sqrt{2}N$

Q.13 (a) $\dfrac{\pi}{10}$ sec, (b) 6cm (c) 2.40kgm/s.

Q.14 $\dfrac{3\sqrt{5}A}{8}\tan^{-1}\left(\dfrac{1}{2}\right)$

Q.15



Q.16 $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

Q.17 $10\sqrt{6}$cm, $\dfrac{1}{\pi}\sin^{-1}\left(\sqrt{\dfrac{2}{3}}\right) - \dfrac{1}{6}$ sec

Q.18 (a) 25cm, (b) $3\pi/56$ seconds

Q.19 $2\pi\sqrt{\dfrac{17L}{18g}}$

Q.20 (a) 2sec, (b) $T = \dfrac{2}{5^{1/4}}$ sec

Q.21 0.06m

Q.22 10πHz, $\dfrac{5\sqrt{37}}{6}$ cm

Q.23 60N, 40N

Q.24 (a) $\dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$, (b) $\dfrac{L}{3}$

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** D **Q.3** C **Q.4** C **Q.5** C **Q.6** C
Q.7 B **Q.8** C **Q.9** A **Q.10** C **Q.11** A **Q.12** D
Q.13 C

Multiple Correct Choice Type

Q.14 B, C, D **Q.15** A, B, C **Q.16** B, C **Q.17** A, C **Q.18** B, C, D **Q.19** A, B, C
Q.20 A, C **Q.21** A, B **Q.22** B, C, D **Q.23** B, D **Q.24** B, C **Q.25** A, B, C, D
Q.26 B, C, D **Q.27** B, D **Q.28** A, B, C **Q.29** B, C **Q.30** A, B, C

Assertion Reasoning Type

Q.31 D **Q.32** C **Q.33** A **Q.34** D **Q.35** A

Comprehension Type

Paragraph 1: **Q.36** D **Q.37** C

Paragraph 2: **Q.38** A **Q.39** D **Q.40** D

Match the Columns

Q.41 B **Q.42** A $\rightarrow$ p, s; B $\rightarrow$ q, r, s; C $\rightarrow$ s; D $\rightarrow$ q

Previous Years' Questions

Q.1 C	**Q.2** B	**Q.3** D	**Q.4** A
Q.5 A, D	**Q.6** B, D	**Q.7** A, B, D	**Q.8** A $\rightarrow$ p; B $\rightarrow$ q, r; C $\rightarrow$ p; D $\rightarrow$ r, q
Q.9 A	**Q.10** A, C	**Q.11** A, B, D	**Q.12** A, D

Solutions

Question Booklet

Exercise 1

Sol 1: $T \propto \frac{1}{k^{1/2}}$; $T = 2\pi\sqrt{\frac{m}{k}}$

$k_1 = 4k$; $k_2 = \frac{4k}{3}$

By $k_1\lambda_1 = k_2\lambda_2 = kl$

$T_1 = \frac{T}{2}$; $T_2 = \frac{T\sqrt{3}}{2}$

$\frac{T_1}{T_2} = \frac{1}{\sqrt{3}}$

Sol 2: $x = 0.2 \cos 5\pi t$

velocity $= \frac{dx}{dt} = -\pi \sin 5\pi t$

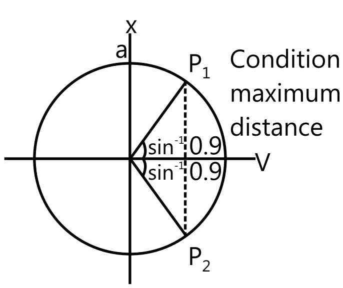
speed $= \pi\,|\sin 5\pi t|$

$v_{avg} = \frac{\pi\int_0^{0.7}|\sin 5\pi t|\,dt}{0.7}$

$= \frac{\pi}{0.7} \times 7 \times \int_0^{0.1} \sin 5\pi t\, dt = \frac{10\pi}{5\pi}\left[-\cos 5\pi t\right]_0^{0.1}$

$v_{avg} = 2$ m/s

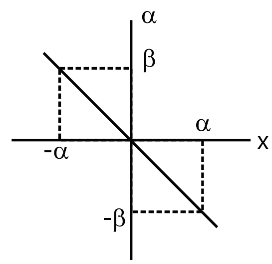
Sol 3: $\phi = 2 \sin^{-1}(0.9)$



P_1P_2 || y-axis

Max. Distance = 1.8 a

Sol 4:



$a = -\omega^2 x$

$-\omega^2 = \frac{\beta}{\alpha}$ = slope of a-x graph

$\omega = \sqrt{\frac{\beta}{\alpha}}$

Frequency $= \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

Sol 5: m = 0.1 kg

A = 0.1 m

$\frac{1}{2} \times m v_{max}^2 = 8 \times 10^{-3}$ J

$0.1 \times v_{max}^2 = 16 \times 10^{-3} \Rightarrow v_{max} = 0.4$ m/s

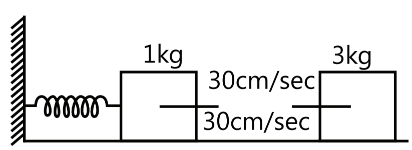
$A\omega = 0.4$

$0.1 \times \omega = 0.4 \Rightarrow \omega = 4$

$x = A \sin(\omega t + \phi)$

$x = 0.1 \sin(4t + \pi/4)$

Sol 6: (i)



$x = 10 + 3 \sin 10t$

At t = 0 s block 1 is at equilibrium position.

$v_1 = A\omega = 3 \times 10 = 30$ cm/s

$v_2 = 30$ cm/s

Conservation of momentum

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

$-1 \times 30 + 3 \times 30 = 4 \times v$

$v = 15$ cm/s

Final velocity is in opposite direction of initial velocity of block 1. This causes a phase change of π.

$\omega \propto m^{-1/2}$

$\omega' = 5$ rad/s

$A'\omega' = 15$; $A' = 3$ cm

New amplitude = 3 cm

(ii) New equation

$x = 10 + 3 \sin (5t + \pi)$

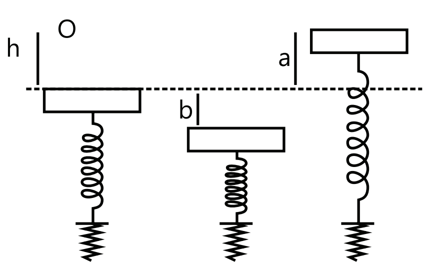
(iii) Loss of energy

$= \left(\frac{1}{2}\times 1\times 30^2 + \frac{1}{2}\times 3\times 30^2 - \frac{1}{2}\times 4\times 15^2\right) \times 10^{-4}$ J

$= \frac{1}{2}(900+2700-900)\times 10^{-4}$ J $= 1350 \times 10^{-4}$ J

$\Delta E_{loss} = 0.1350$ J

Sol 7:



(a) $mgh + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$

$= \frac{1}{2}k\left(\frac{Mg}{k}+b\right)^2 - (M+m)gb$

$= \frac{1}{2}k\left(\frac{Mg}{k}-a\right)^2 + (M+m)ga$

Equalising energies in 3 states

$\frac{1}{2}k\left(\frac{Mg}{k}+b\right)^2 - (M+m)gb$

$= \frac{1}{2}k\left(\frac{Mg}{k}-a\right)^2 + (M+m)ga$

$k\left(\frac{2mg}{k}(b+a)+b^2-a^2\right) = 2(M+m)g(a+b)$

$2Mg(b+a) + k(b^2-a^2) = 2(M+m)g(a+b)$ $k = \frac{2mg}{b-a}$

Constant of force of spring $= \frac{2mg}{b-a}$

(b) $\omega = \sqrt{\frac{k}{(M+m)}} = \sqrt{\frac{2mg}{(M+m)(b-a)}}$

$f = \frac{1}{2\pi}\sqrt{\frac{2mg}{(M+m)(b-a)}}$

(c) $mgh = \frac{1}{2}k\left[\left(\frac{Mg}{k}-a\right)^2 - \left(\frac{Mg}{k}\right)^2\right]$

$+ (M+m)ga$

$mgh = -\frac{1}{2}k\left[a\times\left(\frac{2Mg}{k}-a\right)\right]$

$+ (M+m)ga$

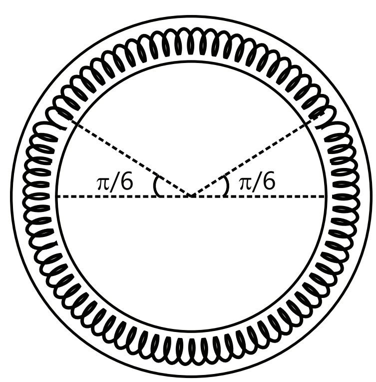
$mgh = \frac{-ka}{2}\left(\frac{2mg}{k}-a\right) + (M+m)ga$

$mgh = -Mga + \frac{ka^2}{2} + (M+m)ga$

$mgh = mga + \frac{2mga^2}{(b-a)2}$

$h = a + \frac{a^2}{(b-a)} = \frac{ab}{b-a}$

Sol 8:



(a) Frequency

Displace by $d\theta$

$\Delta x = 2R d\theta$

$d\alpha = -2 \dfrac{k}{m} \times \dfrac{2Rd\theta}{R}$

$d\alpha = -\omega^2 \, d\theta$

$\omega^2 = \dfrac{4k}{m}$ $\omega = 2\sqrt{\dfrac{k}{m}} = 2$

$f = \dfrac{2}{2\pi} = \dfrac{1}{\pi}$

(b) Total energy $= 2 \times \dfrac{1}{2} k \left(R\dfrac{\pi}{3}\right)^2$

$= 2 \times \dfrac{1}{2} \times 0.1 \times \left(\dfrac{0.06 \times \pi}{3}\right)^2$

$= 3.94 \times 10^{-4}$ J/$4\pi^2 \times 10^{-5}$ J

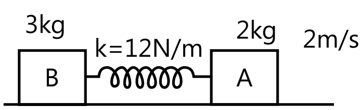
(c) $2 \times \dfrac{1}{2} mv^2 = 4\pi^2 \times 10^{-5}$

$v^2 = \dfrac{4\pi^2 \times 10^{-5}}{0.1}$

$v^2 = 4\pi^2 \times 10^{-4}$

$v = 2\pi \times 10^{-2} = 0.02\,\pi$ m/sec

Sol 9:



$V_{com} = \dfrac{2 \times 2 + 3 \times 0}{5} = 0.8$ m/s

$x_A = v_{com}\, t + A \sin \omega t$

At maximum expansion

$\dfrac{1}{2} \times 5 \times (0.8)^2 + \dfrac{1}{2}kx^2 = \dfrac{1}{2} \times 2 \times 2^2$

$kx^2 = 8 - 3.2 = 4.8$

$x = 0.2$

$A = \dfrac{3}{5}\, x = \dfrac{3}{5} \times 0.2 = 0.12$

$\mu = \dfrac{3 \times 2}{3+2} = 1.2$

$\omega = \sqrt{\dfrac{k}{\mu}} = \sqrt{\dfrac{120}{1.2}} = 10$

$x_A = 0.8\, t + 0.12 \sin 10\, t$

Sol 10: (a) m = 0.1 kg

$F = 10\,x + 2$

Only variable force causes SHM

(a) $F(x) = 10\,x + 2$

$a(x) = 100\,x + 20$

$v(x) = 50\,x^2 + 20\,x + c$

$v(0.2) = 0$

$50 \times 0.04 + 20 \times 0.2 + c = 0$

$c = -6$

$v = 50\,x^2 + 20\,x - 6$ $\begin{cases} x = 0.2 \\ x = -0.6 \end{cases}$

$A = \dfrac{0.2 - (-0.6)}{2} = 0.4$ m

Amplitude = 0.4 m

(b) $\omega = \sqrt{\dfrac{10}{0.1}} = 10$ rad/sec

$T = \dfrac{2\pi}{\omega} = \dfrac{\pi}{5}$ sec.

(c) $x = 0.2 - A \cos \omega t$

$x = 0.2 - 0.4 \cos \dfrac{5t}{\pi}$

Sol 11: $u = (x^2 - 4x + 3)$

(i) $F = -\dfrac{dU}{dx}$

$F = -2x + 4$

At equilibrium F = 0

$-2x + 4 = 0 \Rightarrow x = 2$ m

(ii) $dF = -2dx$ similar to $dF = -\omega^2 dx$ as in SHM

$2 = \dfrac{\omega^2}{m} = \omega^2$

$\omega = \sqrt{2}$

$T = \dfrac{2\pi}{\omega} = \sqrt{2}\,\pi$ sec

(iii) $A\omega = 2\sqrt{6}$

$A = \dfrac{2\sqrt{6}}{\sqrt{2}} \Rightarrow A = 2\sqrt{3}$ m

Sol 12: $F_{max} = m\omega^2 A$

P.E. $= \dfrac{1}{2}$ K.E.

$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2} \times \frac{1}{2}kA^2$

$\Rightarrow x = \frac{A}{\sqrt{2}}$

$F = m\omega^2 \frac{A}{\sqrt{2}} = \frac{F_{max}}{\sqrt{2}}$

$F = 25\sqrt{2}$ N

Sol 13: (a) $T = 2\pi\sqrt{\frac{\mu}{k}}$

$\mu = \frac{3\times6}{3+6} = \frac{18}{9} = 2$ kg

$T = 2\pi\sqrt{\frac{1}{400}} = \frac{\pi}{10}$ sec

(b) A = 6 cm

(c) $v_{cm} = 0$; $v_B = \frac{-1}{2}v_A$

$\frac{1}{2} \times k \times A^2 = \frac{1}{2} \times 3 \times (-2v_B)^2 + \frac{1}{2} \times 6 \times v_B^2$

$800 \times (0.06)^2 = 12v_B^2 + 6v_B^2$

$v_B^2 = \frac{8\times0.36}{18}$

$v_B = \frac{2\times0.6}{3}$

$V_{Bmax.} = 0.4$ m/s

$P_{Bmax} = 0.4 \times 6$

$P_{Bmax} = 2.4$ kg ms^{-1}

Sol 14: $s = \left(A - \frac{A}{4}\right)\cos\omega t - \left(\frac{A}{2} - \frac{A}{8}\right)\sin\omega t$

$s = \frac{3A}{4}\cos\omega t - \frac{3A}{8}\sin\omega t$

$s = \frac{3A}{8}(2\cos\omega t - \sin\omega t)$

$s = \frac{3\sqrt{5}}{8}A\left(\frac{2}{\sqrt{5}}\cos\omega t - \frac{1}{\sqrt{5}}\sin\omega t\right)$

$s = \frac{3\sqrt{5}}{8}A\cos\left(\omega t + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)$

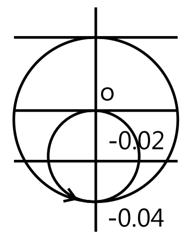
$A' = \frac{3\sqrt{5}}{8}A$; $\delta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Sol 15: $T = 2\pi\sqrt{\frac{m}{k}} = 0.4$ sec

$\omega = 5\pi$

For 0 < t < 0.6 sec

$x = -\frac{mg}{2k} + \frac{mg}{2k}\sin\left(5\pi t + \frac{\pi}{2}\right)$



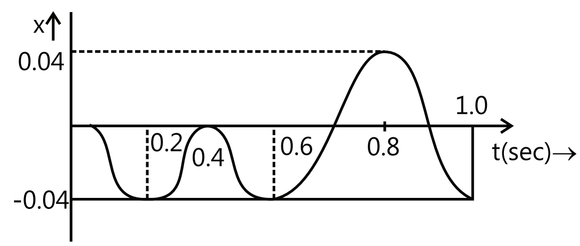
$\frac{mg}{2k} = \frac{4\times10}{2\times1000} = 0.02$ m

for 0 < t < 0.6 sec

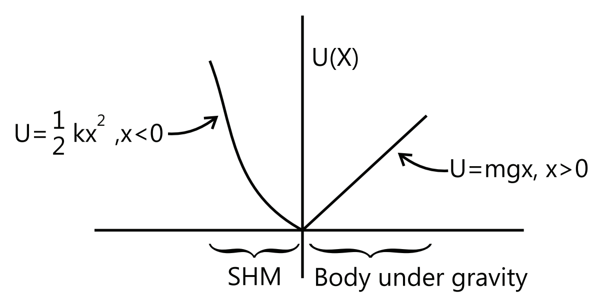
$x = -0.02 + 0.02\sin(5\pi t + \pi/2)$

for 0.6 < t 1 sec

$x = -0.04 + 0.04\sin(5\pi t)$



Sol 16:



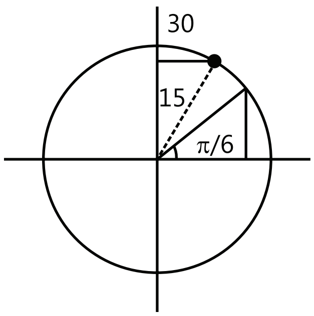
$T = \frac{1}{2} \times 2\pi\sqrt{\frac{m}{k}} + \frac{2v}{g}$

$\because E = \frac{1}{2}mv^2$; $T = \pi\sqrt{\frac{m}{k}} + \frac{2}{g}\sqrt{\frac{2E}{m}}$

$\because v = \sqrt{\frac{2E}{m}}$; $T = \pi\sqrt{\frac{m}{k}} + \frac{2\sqrt{2}}{g}\sqrt{\frac{E}{m}}$

Sol 17: $x = 30 \sin\left(\pi t + \frac{\pi}{6}\right)$

$T = \frac{2\pi}{\pi} = 2$



P.E. = 2 K.E.

P.E. = $\frac{2}{3}$ T.E.

$x = \sqrt{\frac{2}{3}}\ A$

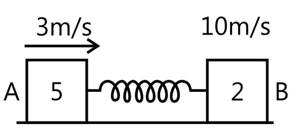
Position: $x = \sqrt{\frac{2}{3}} \times 30$

$x = 10\sqrt{6}$ cm

$$t = \frac{\left(\sin^{-1}\sqrt{\frac{2}{3}} - \frac{\pi}{6}\right)}{2\pi} \times 2 \text{ sec}$$

$$t = \frac{1}{\pi}\left(\sin^{-1}\sqrt{\frac{2}{3}} - \frac{\pi}{6}\right) \text{ sec}$$

Sol 18: (a)



$v_{cm} = \frac{5 \times 3 + 10 \times 2}{7} = 5ms^{-1}$

$\frac{1}{2}\ 5 \times 3^3 + \frac{1}{2} \times 2 \times 10^2$

$= \frac{1}{2}\ 7 \times 5^2 + \frac{1}{2}\ kx^2$

$45 + 200 = 175 + kx^2$

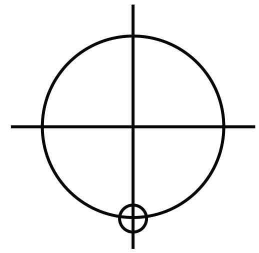
$kx^2 = 70$

$x^2 = \frac{70}{1120}$ $x = \frac{1}{4}$ m

Maximum extension = 0.25 m

(b) $t = \frac{3}{4}\ T$

$T = 2\pi\sqrt{\frac{\mu}{k}}$



$\mu = \frac{5 \times 2}{7} = \frac{10}{7}$

$T = 2\pi\sqrt{\frac{10}{7 \times 1120}} = \frac{2\pi}{28} = \frac{\pi}{14}$

Time for first maximum compression

$= \frac{3}{4} \times \frac{\pi}{14} = \frac{3\pi}{56}$ sec

Sol 19: $T = 2\pi\sqrt{\frac{I}{mgx}}$

$I = \frac{m\ell^2}{3} + \left(\frac{m\ell^2}{12} + m\ell^2\right)$

$x = \frac{mx\frac{1}{2} + mx\ell}{2m} = \frac{3\ell}{4} = \frac{4m\ell^2}{12} + \frac{13}{12}m\ell^2$

$I = \frac{17}{12}\ m\ell^2$

$T = 2\pi\sqrt{\frac{\frac{17}{12}m\ell^2}{2mg\frac{3\ell}{4}}}$;$T = 2\pi\sqrt{\frac{17\ell}{18g}}$

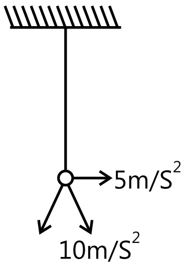
Sol 20: (a) $I\alpha = -k\theta$

$\alpha = -\frac{k}{I}\ \theta$

$\omega^2 = \frac{k}{2}$ $\omega = \sqrt{\frac{k}{I}} = \pi$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi}$;T = 2 sec

(b)



$g_{eff.} = \sqrt{10^2 + 5^2} = \sqrt{125}$

$g_{eff.} = 5\sqrt{5}$

$T = 2\pi \sqrt{\frac{\ell}{g}} = 2\pi \sqrt{\frac{0.5}{5\sqrt{5} \times 10}}$

$T = \frac{2}{5^{1/4}}$ sec

Sol 21: m = 0.2 kg $f = \frac{25}{\pi}$ Hz

P.E. = $\frac{4}{9}$ T.E.

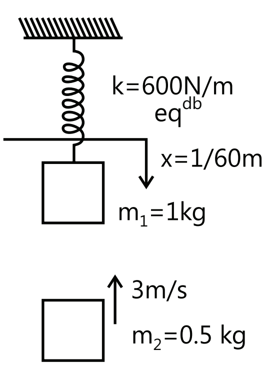
$\frac{1}{2} kx^2 = \frac{4}{9} \times \frac{1}{2} kA^2; x = \frac{2}{3} A$

$A = \frac{3}{2} x = \frac{3}{2} \times 0.04$

A = 0.06 m

Sol 22: 0.5 × 3 = 1.5 × v

v = 1 m/s



$\omega = \sqrt{\frac{600}{1.5}} = \sqrt{400} = 20$ rad./sec

$f = \frac{20}{2\pi} = \frac{10}{\pi}$ Hz

$\frac{1}{2} kx^2 + \frac{1.5 \times 1^2}{2}$

5/4

$= \frac{1}{2} kh^2 - 1.5 \times 10 \times \left(h - \frac{1}{60}\right)$

$\frac{1}{2} \times 600 \times \frac{1}{60^2} + \frac{1.5}{2}$

$= \frac{1}{2} \times 600 \times h^2 - 15 \times \left(h - \frac{1}{60}\right)$

$60 h^2 - 3h - 7/60 = 0$

$h = \frac{1.5}{60} + \frac{\sqrt{37}}{120}$ m

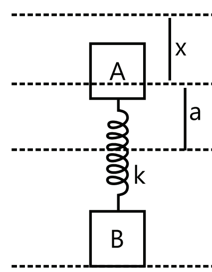
$A = h - \frac{1.5}{60} = \frac{\sqrt{37}}{120}$ m

$A = \frac{\sqrt{37}}{120} \times 100 \text{ cm} = \frac{5\sqrt{37}}{60}$ cm

Sol 23: m_1 = 1kg; m_2 = 4kg

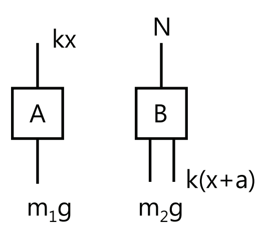
a = 1.6 cm

$kx = m_1 g$



$k = \omega^2 m_1 = 25^2 \times 1 = 625$ N/m

$N_{max} = m_2 g + k(x + a)$



$= (m_1 + m_2)g + ka = 50 + \frac{625 \times 1.6}{100}$

N_{max} = 60 N

$N_{min} = (M_1 + M_2)\, g - ka$

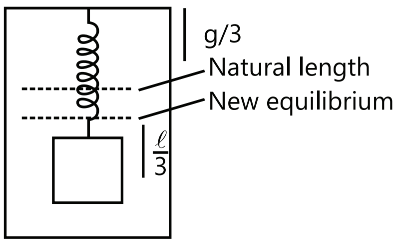
N_{min} = 40N

Sol 24: $k\ell = mg$; $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{9}{\ell}}$

$f = \frac{1}{2\pi}\left(\frac{g}{\ell}\right)^{1/2}$

$A = \ell/3$; $m \times \frac{2g}{3} = k \times x$

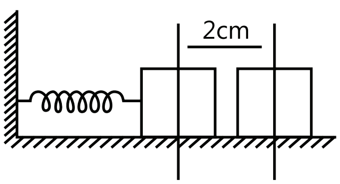
$x = \frac{2}{3}\frac{mg}{k} = \frac{2}{3}l$



Exercise 2

Single Correct Choice Type

Sol 1: (C)

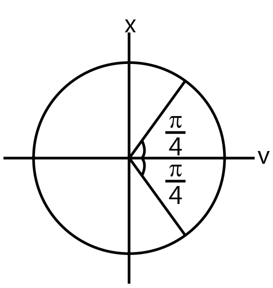


$\omega\sqrt{4-1} = \omega^2 \times 1$

$\omega = \sqrt{3}$

$F = \frac{\sqrt{3}}{2\pi}$ Hz

Sol 2: (D)

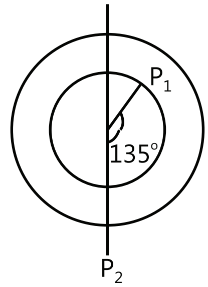


$v_{avg} = \frac{\text{displacement}}{\text{time}} = \frac{\sqrt{2}\times A}{T/4}$

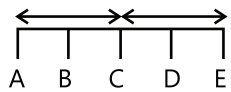
$v_{avg} = \frac{4\sqrt{2}A}{T}$

Sol 3: (C) $v_{mean} = \frac{a/2}{T/6} = \frac{3a}{T}$

Sol 4: (C)



Sol 5: (C)



$V_B^2 = \frac{1}{4}V_A^2$

$\frac{1}{4}R^2\omega^2 = \omega^2(R^2 - x^2) \Rightarrow R^2 = (R^2 - x^2)\,4$

$x = \frac{\sqrt{3}}{2}R$

$d_{BD} = 2x = \sqrt{3}R$

Sol 6: (C) $T_S = 2\pi\sqrt{\frac{m}{k}}$ T_s doesn't depend on g.

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$; $T_p \propto g^{-1/2}$

$\therefore T_p$ decreases

Sol 7: (B) $v_{max} = A\omega = \frac{A\sqrt{k}}{\sqrt{m}}$

$\frac{A_1\sqrt{k_1}}{\sqrt{m}} = \frac{A_2\sqrt{k_2}}{\sqrt{m}}$

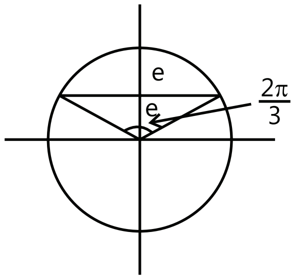
$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

Sol 8: (C) $k_A = k/3$; $k_B = 3k$

$T_A \propto k^{-1/2}$; $\frac{T_A}{T_B} = 3$

Sol 9: (A) $T = 2\pi\sqrt{\frac{m}{k}} \times \frac{2\pi/3}{2\pi}$

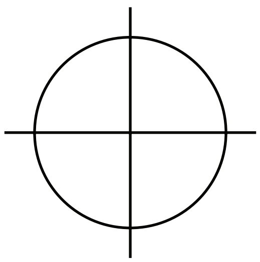
$T = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$



Sol 10: (C) $t = \frac{T}{4}$

$t = \frac{\pi}{2}\sqrt{\frac{\mu}{k}}$

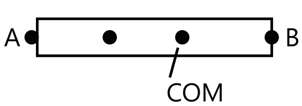
$t = \frac{\pi}{2}\sqrt{\frac{1}{\pi^2}}$; $t = \frac{1}{2}$ sec



Sol 11: (A) Both block have speed same as

$v_{cm} = 5m/s$

Sol 12: (D)



$T = 2\pi\sqrt{\frac{I}{mg\ell}}$

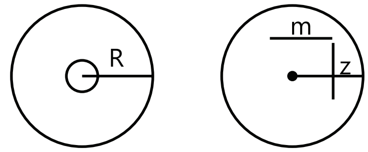
$\frac{T_A}{T_B} = \frac{3}{4} = \sqrt{\frac{9}{4}\times\frac{\ell_B}{\ell_A}}$

$\frac{3}{4} = \frac{3}{2}\sqrt{\frac{\ell_B}{\ell_A}}$

$\frac{\ell_B}{\ell_A} = \frac{1}{4}$

$\ell_A = \frac{4}{5} \times 25 = 20$ cm

Sol 13: (C)



$T = 2\pi\sqrt{\frac{I}{mgz}}$ $\Rightarrow I = \frac{mR^2 + 2mz^2}{2}$

$T = \sqrt{\frac{2\pi}{g}}\sqrt{m\frac{R^2}{2} + 2mz}$

$\frac{mR^2}{2} = 2mz$ for minimum T

$z = \frac{R}{\sqrt{2}}$

Multiple Correct Choice Type

Sol 14: (B, C, D) v = 0 at t = T/2

a is maximum at extremes

F = 0 at $t = \frac{3T}{4}$

K.E. = 0 at t = T/2

Sol 15: (A, B, C) K.E. = 0.64 $KE_{max.}$

$v = 0.8\, v_{max.}$

$\therefore$ x = 0.6 A = 6 cm

$x = \frac{A}{2}$ P.E. $= \frac{PE_{max}}{4}$ KE $= \frac{3}{4} PE_{max}$

KE_{max} = TE at mean position

$x = \frac{A}{2}$ $v = \frac{\sqrt{3}v_{max}}{2}$

Sol 16: (B, C) (A) KE_{avg} is never zero in SHM

(B) $PE_{avg} = \frac{1}{2}$ TE = $m\pi^2 t^2 A^2$

(C) Frequency of occurrence of mean position =2f

(D) Acceleration leads

Sol 17: (A, C) $v_{rms} = \sqrt{\frac{\int_0^T v^2 dt}{T}} = \frac{v}{\sqrt{2}}$

$$v_{mean} = \frac{\int_0^T v\,dt}{T} = \frac{\sqrt{8}}{\pi}\ V$$

Sol 18: (B, C, D) A = 3cm

$\frac{1}{2}\ m{v_m}^2 = \frac{1}{2} \times 500 \times 9$

$v_m = 3 \times 10\sqrt{5}$ cm/s ; $\omega = \sqrt{500} = 10\sqrt{5}$

$a_{max.} = \omega\ v_m$

$= 10\sqrt{5} \times 30\sqrt{5}$ cm/s^2

= 15 m/s^2

PE_{min} = 0 at mean position

Sol 19: (A, B, C) A = 2.5

$\omega = \frac{v_{max}}{A} = 4;\ T = \frac{2\pi}{4} = \frac{\pi}{2} = 1.57$ s

$v_{max} = 16 \times 2.5 = 40$ cm/s^2

$v = \omega\sqrt{A^2 - x^2} = 4\sqrt{2.5^2 - 1^2}$

$= 4\sqrt{5.25}$ cm/s $= 2\sqrt{21}$ cm/s

Sol 20: (A, C) Energy conservation:

$\frac{1}{2} \times 900 \times 2^2 = \frac{1}{2} \times 900 \times 1^2 + \frac{1}{2} \times 3\ {v_1}^2$

$2700 = 3 \times {v_1}^2$

${v_1}^2 = 900$

$v_1 = 30$ m/s

Conservation of momentum:-

3 × 30 + 6 × 0 = 9 × v

v = 10 ms^{-1}

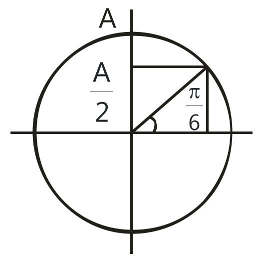
Energy conservation:-

$\frac{1}{2} \times 900 \times 1^2 + \frac{1}{2} \times 9 \times 10^2 = \frac{1}{2} \times 900 \times A^2$

$A^2 = 2;\quad A = \sqrt{2}$ m

Sol 21: (A, B) $t = \frac{\pi/6}{2\pi} \times T \Rightarrow t = \frac{T}{12}$

$v = \frac{\sqrt{3}}{2}\ V_0 \Rightarrow a \propto x \Rightarrow a = a_0/2$

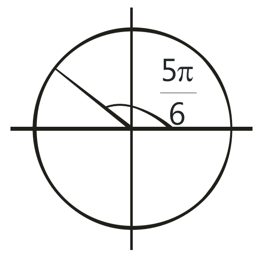


Sol 22: (B, C, D) $v^2 = \omega^2 (A^2 - x^2)$

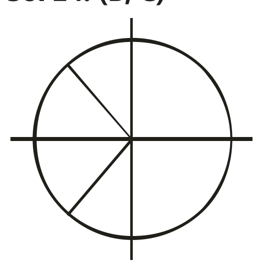
$a = -\omega^2 x$

$v^2 = \omega^2\left(A^2 - \frac{a^2}{\omega^4}\right)$

Sol 23: (B, D) $x = A \sin\left(\frac{2\pi t}{T} + \frac{5\pi}{6}\right) = A \cos\left(\frac{2\pi t}{T} + \frac{\pi}{3}\right)$



Sol 24: (B, C)



Initial phase difference = 0, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$

Sol 25: (A, B, C, D) $x = \frac{mg}{k} = \frac{0.2 \times 10}{200} = 1$ cm

Amplitude = 1 cm

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.2}} = 10\sqrt{10}$

$f = \frac{10\sqrt{10}}{2\pi} = \frac{5\sqrt{10}}{\pi} \cong 5$Hz

Amplitude changes, frequency remains the same.

Sol 26: (B, C, D) m = 0.1 kg

U = 5x(x – 4)

$F = -\frac{dU}{dx} = 20 - 10\,x$

P.E. minimum at x = 2 m

Force is linear function of x with negative slope.

$\omega^2 = \frac{10}{m}$

$\omega = \sqrt{\frac{10}{0.1}}$ = 10 rad/s

$T = \frac{2\pi}{10} = \frac{\pi}{5}$ sec

Sol 27: (B, D) $x = 3 \sin 100t + 8 \cos^2 50t$

$= 3 \sin 100t + 4 \cos 100t + 4$

$x = 5 \sin (100t + \sin^{-1} 4/5) + 4$

Sol 28: (A, B, C) $x = 5 \sin (4\pi t + \sin^{-1} 4/5)$ mm

$T = \frac{2\pi}{4\pi} = 0.5$ s

A = 5 mm

$\phi = \sin^{-1} (4/5)$

Sol 29: (B, C) $k = 2 \times 10^6$ Nm^{-1}

A = 0.01 m

T.E. = 160 J

PE_{max} = 160 J

when KE = 0 J

i.e. at equilibrium

$KE_{max} = \frac{1}{2} \times 2 \times 10^6 \times 10^{-4} = 100$ J

PE_{min} = 60 J

Sol 30: (A, B, C) $t = n\frac{T}{2}$

T = $2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{3 \times 24}} = \frac{\pi}{3}$

Assertion Reasoning Type

Sol 31: **(D)** The motion is SHM with $\omega = \sqrt{\frac{a}{m}}$

If the force is linear w.r.t. x and slope is negative. The motion is always SHM.

Sol 32: (C) When particle moves from extreme to mean position velocity and acceleration have same direction.

Sol 33: (A) Statement-II is the correct explanation.

Sol 34: (D) Phase remains same and SHMs are perpendicular**.**

Sol 35: (A) Statement-II is the correct explanation.

Comprehension Type

Paragraph 1:

Sol 36: (D) ω = 25 rad/s

$k = m\omega^2 = 1 \times 625 = 625$ Nm^{-1}

$F_{max} = 1 \times 9.8 + 625 \times \frac{16}{100 \times 10} + 4.1 \times 9.8$

$= 59.98$ N $\cong$ 60 N

$F_{min} = 5.1 \times 9.8 - 10 \cong 40$ N

Sol 37: (C) Minimum force on the surface = (50 – 10) N = 40 N

Sol 38: (A) TE of system is constant

Sol 39: (D) $d = A \sin (\omega t + \phi)$

Sol 40: (D) $F = -kx + c$

$k > 0$

Match the Columns

Sol 41: (B) (a) y = A sin (t)

v = A cos (t)

KE = c × $\cos^2$ (t)

(a) $\rightarrow$ (ii)

(b) $\rightarrow$ (i) PE + KE = const.

PE = c × $\sin^2$ t

(c) $\rightarrow$ (iii) TE constant always

(d) $\rightarrow$ (iv) v = A cos t

Sol 42: (A) PE $\propto x^2$ (A) $\rightarrow$ p, s

(B) $s = ut + \frac{1}{2}at^2$

q, r when a = 0 ; S when a $\neq$ 0

(C) Range = $\frac{v^2 \sin 2\theta}{g}$

(D) $T^2 = \frac{4\pi^2 \ell}{g}$

Previous Year's Questions

Sol 1: (C) If $E > V_B$, particle will escape. But simultaneously for oscillations, $E > 0$

Hence, the correct answer is $V_0 > E > 0$

Or the correct option is (c)

Sol 2: (B) $[\alpha] = \left[\frac{PE}{x^4}\right] = \left[\frac{ML^2T^{-2}}{L^4}\right] = [ML^{-2}T^{-2}]$

$\therefore \left[\frac{m}{\alpha}\right] = [L^2T^2]$; $\quad \therefore \left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = [T]$

As dimensions of amplitude A is [L]

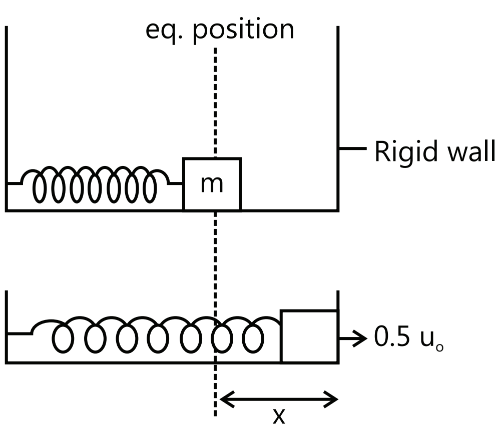
Sol 3: (D) For $|x| > x_0$, potential energy is constant. Hence, kinetic energy, speed or velocity will also remain constant.

$\therefore$ Acceleration will be zero

Sol 4: (A) $\frac{2v\sin 45°}{g} = 1$

$\therefore v = \sqrt{50}$ m / s

Sol 5: (A, D)



$$\frac{1}{2}mu_0^2 = \frac{1}{2}kx^2 + \frac{1}{2}\times m0.25\,u_0^2 \quad ....(i)$$

After elastic collision

Block speed is $0.5\,u_0$

So when it will come back to equilibrium point its speed will be u_0 as (A)

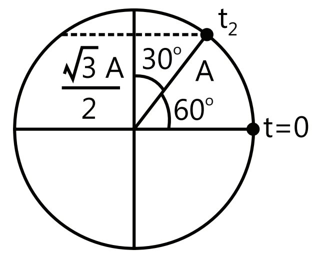
Amplitude $\frac{1}{2}mu_0^2 = \frac{1}{2}kA^2$

$A = \frac{u_0}{\sqrt{k}}$

Value of x from eq. (i)

$$\frac{3}{4}\times\frac{1}{2}mu_0^2 = \frac{1}{2}kx^2$$

$$x = \frac{\sqrt{3}u_0}{2}\sqrt{\frac{m}{k}}$$



Time to reach eq. position first time $\Rightarrow \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

Second time it will reach at time $\Rightarrow$

$$\frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{T}{2} \Rightarrow \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{2\pi\sqrt{m}}{\sqrt{k}\times 2} \Rightarrow \frac{5\pi}{3}\sqrt{\frac{m}{k}} \text{ as (D)}$$

For max. compression time is t_2

$$t_2 = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{T}{4}$$

$$= \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{2\pi}{\sqrt{k}}\frac{\sqrt{m}}{k} = \frac{7\pi}{6}\sqrt{\frac{m}{k}}$$

Sol 6: (B, D)

$$E_1 = \frac{1}{2}m\omega_1^2a^2 = \frac{b^2}{2m} \qquad \frac{a}{b} = \frac{1}{m\omega_1} = n^2 \quad ...(i)$$

$$E_2 = \frac{1}{2}m\omega_2^2R^2 = \frac{R^2}{2m} \qquad m\omega_2 = 1 \quad ...(ii)$$

From (i) and (ii) $\frac{\omega_2}{\omega_1} = n^2$

$$\frac{E_1}{E_2} = \left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{a}{R}\right)^2 = \frac{1}{n^2}\cdot\frac{\omega_1}{\omega_2}\cdot n^2 \Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Sol 7: (A, B, D)

(A) $\omega_i = \left(\frac{K}{M}\right)$ and $\omega_f = \left[\frac{K}{(M+m)}\right]$

Case I: $v_f = \frac{Mv_i}{M+m}\ \frac{1}{2}Mv^2 = \frac{1}{2}KA_i^2$

$$\Rightarrow A_i^2 = \frac{M}{K}v_1^2 \text{ and } \frac{1}{2}(M+m)v_f^2 = \frac{1}{2}KA_i^2$$

$$\Rightarrow A_f^2 = \frac{Mv^2}{K}\cdot\frac{M}{M+m} \Rightarrow \frac{A_f}{A_i} = \sqrt{\frac{M}{M+m}}$$

(B) $T_f = 2\pi\sqrt{\frac{M}{M+m}}$ for both

(C) $TE_{\text{case I}} = \frac{1}{2}(M+m)v_f^2 = \frac{1}{2}Mv^2\left(\frac{M}{M\ \ m}\right)$

$$TE_{\text{case II}} = \frac{1}{2}KA_f^2 = \frac{1}{2}KA_i^2$$

(D) VEP = $A_f\omega_f$: Decreases in both cases.

Sol 8: A → p; B → q, r; C → p; D → r, q

Sol 9: (A) The total mechanical energy = 160 J

The maximum PE will be 160 J at the instant when KE = 0

Sol 10: (A, C) By principle of superposition $y = y_1 + y_2 + y_3$

$= a\sin(\omega t + 45°) + a\sin\omega t + a\sin(\omega t - 45°)$

$= a\sin(\omega t + 45°) + a\sin(\omega t - 45°) + a\sin\omega t$

$= 2a\sin\omega t\cos 45° + a\sin\omega t$

$= \sqrt{2}a\sin\omega t + a\sin\omega t = \left(1+\sqrt{2}\right)a\sin\omega t$

∴ Amplitude of resultant motion = $\left(1+\sqrt{2}\right)$ a ...(i)

(b) The option is incorrect as the phase of the resultant motion relative to the first is 45°.

(c) Energy is SHM is proportional to $(\text{amplitude})^2$

$$\therefore \frac{E_R}{E_S} = \frac{\left(1+\sqrt{2}\right)^2 a^2}{a^2} \quad \therefore \frac{E_R}{E_S} = \frac{\left(1+2+2\sqrt{2}\right)}{1}$$

or $E_R = \left(3+2\sqrt{2}\right)E_S$

(d) Resultant motion is $y = \left(1+\sqrt{2}\right)a\sin\omega t$

Ii is SHM.

Sol 11: (A, B, D)

$$x = \frac{A}{2}(1-\cos 2\omega t) + \frac{B}{2}(1+\cos 2\omega t) + \frac{C}{2}\sin 2\omega t$$

For A = 0, B = 0

$$x = \frac{C}{2}\sin 2\omega t$$

A = -B and C = 2B

X = B cos 2ωt + B sin 2ωt

Amplitude = $\left|B\sqrt{2}\right|$

For A = B; C = 0

X = A,

Hence this is not correct option.

For A = B, C = 2B

X = B + B sin 2ωt

It is also represent SHM.

Sol 12: (A, D) Restoring torque is same in both cases

$$\alpha = \frac{T}{I} = -\omega^2\theta$$

In case A the moment of inertia is more as compared to B, so $\omega_B > \omega_A$