Answers

1. (c)	2. (d)	3. (c)	4. (c)
5. (a)	6. (c)	7. (d)	8. (b)
9. (c)	10. (a)	11. (d)	12. (b)
13. (c)	14. (a)	15. (b)	

16. (c)	17. (c)	18. (b)	19. (a)
20. (b)	21. (a)	22. (b)	23. (a)
24. (c)	25. (d)	26. (a)	27. (c)
28. (b)	29. (d)	30. (c)	31. (c)
32. (a)	33. (a)	34. (a)	35. (d)
36. (c)	37. (c)	38. (b)	39. (d)
40. (a)	41. (d)	42. (a)	43. (c)
44. (b)	45. (c)	46. (d)	47. (c)
48. (b)	49. (a)	50. (c)	51. (a)
52. (c)	53. (a)	54. (b)	55. (c)
56. (c)	57. (d)	58. (a)	59. (c)
60. (a)	61. (b)	62. (b)	63. (c)
64. (b)	65. (c)	66. (c)	67. (d)
68. (a)	69. (c)	70. (a)	71. (c)
72. (b)	73. (d)	74. (c)	75. (d)
76. (b)	77. (b)	78. (b)	79. (d)
80. (c)	81. (a)	82. (a)	83. (b)
84. (c)	85. (a)	86. (c)	87. (d)
88. (c)	89. (a)	90. (b)	91. (c)
92. (c)	93. (d)	94. (a)	95. (c)
96. (d)	97. (c)	98. (d)	99. (b)
100. (a)	101. (c)	102. (a)	103. (d)
104. (c)			

Solutions

1. Since there no external forces, the centre of mass will continue to maintain its original motion. Hence the correct choice is (c).
2. The speed of the centre of mass of the system will remain unchanged (= v). The speed of the centre of mass of a system can change only if a net external force acts on it. The forces involved (such as the action and reaction and frictional forces), when the child runs on the trolley, are internal to the (trolley + child) system.
3. The only incorrect statement is (c). Since no external torque acts on the body even after the string is cut, the angular momentum will remain unchanged.
4. Since there is not external force acting on the gun–shot system, the centre of mass of the system continues to remain at rest. Hence the correct choice is (c).
5. Since the centre of mass moves through a distance x, the average force F is given by

$$Fx = Mgh$$

or
$$F = \frac{Mgh}{x}$$

Hence the correct choice is (a).
6. The (x, y) co-ordinates of the centre of mass are

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

and
$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

It is easy to show that $x = \frac{a}{2}$ and $y = \frac{2a}{3}$, which is choice (c).
7. The centre of mass of the whole carpet is originally at a height R above the floor. When the carpet unrolls itself and has a radius $R/2$, the centre of mass is at a height $R/2$ but the mass left over unrolled is $\frac{M(R/2)^2}{R^2} = \frac{M}{4}$. Hence the decrease in P.E. is

$$MgR - \frac{M}{4}g \cdot \frac{R}{2} = \frac{7}{8}\ MgR.$$

Hence the correct choice is (d).
8. Refer to Fig. 5.64. The moment of inertia about AD is

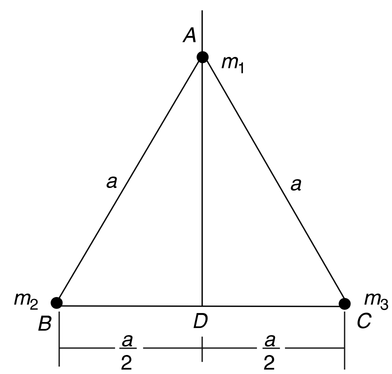

Fig. 5.64

$$I = m_1 \times (\text{distance of } m_1 \text{ from } AD)^2$$
$$+ m_2 \times (\text{distance of } m_2 \text{ from } AD)^2$$
$$+ m_3 \times (\text{distance of } m_3 \text{ from } AD)^2$$
$$= m_1 \times 0 + m_2 \times (BD)^2 + m_3 \times (CD)^2$$
$$= 0 + m_2 \times \left(\frac{a}{2}\right)^2 + m_3 \times \left(\frac{a}{2}\right)^2$$
$$= (m_2 + m_3)\,\frac{a^2}{4}$$

Hence the correct choice is (b).

9. Since the two atoms have the same mass, the centre of mass is at a distance of $a/2$ from each atom. Therefore, the moment of inertia of the molecule about its centre of mass is

$$I = m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{2}$$

Hence the correct choice is (c).

10. Kinetic energy is $k = \frac{1}{2} I\omega^2$, which gives

$$\omega = \sqrt{\frac{2k}{I}} = \sqrt{\frac{2k}{ma^2} \times \frac{2}{1}} = \frac{2}{a}\sqrt{\frac{k}{m}}$$

$\therefore$ Frequency $\nu = \dfrac{\omega}{2\pi} = \dfrac{1}{\pi a}\sqrt{\dfrac{k}{m}}$, which is choice (a).

11. Given

$$I_c = \frac{2}{5}\, MR^2$$

Using the parallel axes theorem, the moment of inertia about an axis tangential to the sphere will be

$$I = I_c + MR^2 = \frac{2}{5}\, MR^2 + MR^2$$
$$= \frac{7}{5}\, MR^2$$

Hence the correct choice is (d).

12. Let us consider two perpendicular diameters, one along the x-axis and the other along the y-axis. Then

$$I_x = I_y = \frac{1}{4}\, MR^2$$

According to the perpendicular axes theorem, the moment of inertia of the disc about an axis passing through the centre is

$$I_c = I_x + I_y = \frac{1}{4}\, MR^2 + \frac{1}{4}\, MR^2$$
$$= \frac{1}{2}\, MR^2$$

which is choice (b).

13. Since the disc is uniform, its centre of mass coincides with its centre. Therefore, the moment of inertia of the disc about an axis passing through its centre of mass and normal to its plane is

$$I_{\text{CM}} = I_C = \frac{1}{2}\, MR^2$$

According to the theorem of parallel axes, the moment of inertia of the disc about an axis passing through a point on its edge and normal to its plane is given by

$$I_e = I_{\text{CM}} + Mh^2$$
$$= \frac{1}{2}\, MR^2 + MR^2 \qquad (\because h = R)$$
$$= \frac{3}{2}\, MR^2$$

Hence the correct choice is (c).

14. Work done = increase in kinetic energy or

$$W = \frac{1}{2}\, I\omega_2^2 - \frac{1}{2}\, I\omega_1^2$$
$$= \frac{I}{2}\,(\omega_2^2 - \omega_1^2)$$
$$= 2\pi^2 I\,(\nu_2^2 - \nu_1^2) \qquad (\because \omega = 2\pi\nu)$$

or
$$I = \frac{W}{2\pi^2\left(\nu_2^2 - \nu_1^2\right)}$$

Hence the correct choice is (a).

15. The angular frequency of the composite system can be obtained by using the principle of conservation of angular momentum.

Total initial angular momentum of the two discs = $I_1\omega_1 + I_2\omega_2$

Since the two discs are brought into contact face to face (one on top of the other) and their axes of rotation coincide, the moment of inertia I_c of the composite system will be equal to the sum of their individual moments of inertia, i.e.

$$I_c = I_1 + I_2$$

If ω_c is the angular frequency of the composite system, the final angular momentum of the system is

$$I_c\omega_c = (I_1 + I_2)\omega_c$$

Since no external torque acts on the system,

Final angular momentum = Initial angular momentum

or $(I_1 + I_2)\omega_c = I_1\omega_1 + I_2\omega_2$

or
$$\omega_c = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

16. Let M be the mass of the cylinder and R be its radius. When it is at the top of the inclined plane of height h, its potential energy is Mgh (Fig. 5.65). As it rolls down the inclined plane, it moves along the plane and also rotates about an axis passing through its axis perpendicular to the plane of the figure, which shows its section in the plane of the paper. It thus acquires both kinetic energy of translation $\left(\frac{1}{2}Mv^2\right)$ and kinetic energy of rotation $\left(\frac{1}{2}I\omega^2\right)$ where v is its linear velocity and ω its angular velocity when it reaches the bottom of the plane. I is its moment of inertia about the axis mentioned above which is given by

$$I = \frac{1}{2}MR^2$$

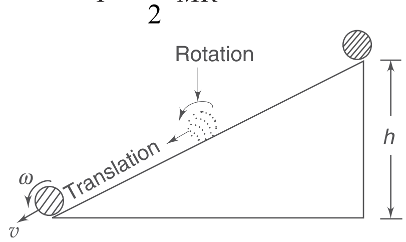

Fig. 5.65

From the law of conservation of energy, we have

Potential energy = Translational kinetic energy + Rotational kinetic energy

or $$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

or $$Mgh = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$(\because\ v = R\omega)$

$$= \frac{3}{4}MR^2\omega^2$$

or $$\omega^2 = \frac{4gh}{3R^2}$$

or $$\omega = \frac{2}{R}\sqrt{\frac{gh}{3}}$$, which is choice (c).

17. The rotational kinetic energy $= \frac{1}{2}I\omega^2$.

Substituting for ω^2 and I, we have

Rotational kinetic energy $= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{4gh}{3R^2} = \frac{Mgh}{3}$

Hence the correct choice is (c).

18. Let the mass of each dumb-bell be m. Then,

Total initial angular momentum $= mr_1^2\omega_1 + mr_1^2\omega_1$
$= 2mr_1^2\omega_1$

When the boy pulls the dumb-bells towards his chest, let the new value of period be T_2. It is given that

$$r_2 = 10 \text{ cm}$$

$$\omega_2 = \frac{2\pi}{T_2}$$

Final angular momentum $= 2mr_2^2\omega_2$

From the principle of conservation of angular momentum,

Initial angular momentum = Final angular momentum

or $$2mr_1^2\omega_1 = 2mr_2^2\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{r_2^2}{r_1^2}$$

Hence the correct choice is (b).

19. We first find the linear acceleration a by using the relation

$$s = ut + \frac{1}{2}at^2$$

or $$2 = 0 + \frac{1}{2} \times a \times (4)^2$$

which gives $a = \frac{1}{4}$ ms^{-2}. Now $R = 0.5$ m. The angular acceleration α is

$$\alpha = \frac{a}{R} = \frac{1}{4 \times 0.5} = \frac{1}{2} = 0.5 \text{ rad s}^{-2}$$

Hence the correct choice is (a).

20. The torque produced by the force of the falling weight is

$$\tau = mg \times \text{moment arm} = mgR$$

$$= 2 \times 10 \times 0.5 = 10 \text{ Nm}$$

Now $$I = \frac{\tau}{\alpha} = \frac{10}{0.5} = 20 \text{ kg m}^2$$

Hence the correct choice is (b).

21. Let M be the mass and R the initial radius of the earth. If ω is the angular velocity of the rotation of the earth, the duration T of the day is

$$T = \frac{2\pi}{\omega}$$

Let R' be the radius of the earth after contraction and ω' its angular velocity. From the conservation of angular momentum, we have

$$I\omega = I'\omega'$$

where $I\left(=\frac{2}{5}MR^2\right)$ and $I'\left(=\frac{2}{5}MR'^2\right)$ are the moments of inertia of the earth before and after contraction, respectively.

$$\therefore \quad \frac{2}{5}\ MR^2\omega = \frac{2}{5}\ MR'^2\omega'$$

$$\text{or} \qquad \omega' = \frac{R^2\omega}{R'^2} = 4\omega \qquad (\because R' = R/2)$$

The duration T' of the new day will be (since $T = 2\pi/\omega$)

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{4\omega} = \frac{T}{4}$$

$$T' = \frac{24 \text{ hours}}{4} = 6 \text{ hours}$$

22. $I_1 = \frac{1}{4}\ M_1R_1^2$ and $I_2 = \frac{1}{4}\ M_2R_2^2$. Therefore,

$$\frac{I_1}{I_2} = \frac{M_1}{M_2}\cdot\frac{R_1^2}{R_2^2} = \frac{1}{2}\times(2)^2 = 2$$

Hence the correct choice is (b).

23. From the law of conservation of angular momentum, we have

$$I\omega = I'\omega'$$

Here $I = MR^2$ and $I' = (M + 2m)\ R^2$. Therefore

$$\frac{\omega'}{\omega} = \frac{I}{I'} = \frac{M}{(M+2m)}$$

Hence the correct choice is (a).

24. For rolling :

$$Mgh = \frac{1}{2}\ Mv^2 + \frac{1}{2}\ I\omega^2$$

$$= \frac{1}{2}\ Mv^2 + \frac{1}{2}\times\left(\frac{2}{5}MR^2\right)\times\frac{v^2}{R^2}$$

$$= \frac{1}{2}\ Mv^2 + \frac{1}{5}\ Mv^2 = \frac{7}{10}\ Mv^2$$

$$\left(\because I = \frac{2}{5}MR^2 \text{ and } \omega = \frac{v}{R}\right)$$

For sliding :

$$Mgh = \frac{1}{2}\ Mv'^2. \text{ Therefore } \frac{1}{2}\ Mv'^2 = \frac{7}{10}\ Mv^2$$

$$\text{or} \qquad \frac{v'}{v} = \sqrt{\frac{7}{5}}, \text{ which is choice (c).}$$

25. Rotational kinetic energy is

$$(\text{KE})_r = \frac{1}{2}\ I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \frac{1}{4}\ MR^2\omega^2 \qquad (\because I = \frac{1}{2}\ MR^2)$$

where M is the mass of the disc and R its radius. Translational kinetic energy is

$$(\text{KE})_t = \frac{1}{2}\ Mv^2 = \frac{1}{2}\ M(R\omega)^2 = \frac{1}{2}\ MR^2\omega^2$$

$$(\because v = R\omega)$$

$$\text{Total energy, KE} = (\text{KE})_r + (\text{KE})_t = \frac{3}{4}\ MR^2\omega^2$$

$$\therefore \quad \frac{(\text{KE})_t}{\text{KE}} = \frac{2}{3}$$

Hence the correct choice is (d).

26. $(\text{KE})_r = \frac{1}{2}\ I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\ \omega^2 = \frac{1}{5}\ MR^2\omega^2$

$$(\because I = \frac{2}{5}\ MR^2)$$

$$(\text{KE})_t = \frac{1}{2}\ Mv^2 = \frac{1}{2}\ MR^2\omega^2$$

$$\text{Total KE} = \frac{7}{10}\ MR^2\omega^2$$

$$\therefore \quad \frac{(\text{KE})_r}{\text{KE}} = \frac{2}{7}$$

Hence the correct choice is (a).

27. Downward force $F = Mg \sin\theta$. The effective mass of the rolling disc is $M_{\text{eff}} = M + \frac{I_{\text{CM}}}{R^2}$, where I_{CM} is the moment of inertia about its centre of mass, which is

$$I_{\text{CM}} = \frac{1}{2}MR^2$$

$$\therefore \qquad M_{\text{eff}} = M + \frac{1}{2}\ \frac{MR^2}{R^2} = \frac{3M}{2}$$

$$\therefore \quad \text{Acceleration } a = \frac{F}{M_{\text{eff}}} = \frac{Mg\sin\theta}{3M/2}$$

$$= \frac{2}{3}\ g \sin 30° = \frac{g}{3}$$

Hence the correct choice is (c).

28. The acceleration of the block sliding down the plane is

$$a = g \sin\theta$$

where θ is the angle of inclination. If l is the length of the inclined plane, the velocity of the block on reaching the bottom is given by

$$v^2 = 2\,al = 2g \sin\theta \times l$$

or $$v = \sqrt{2gl\sin\theta}$$

The acceleration of the disc rolling down the plane is (as shown above)

$$a' = \frac{2}{3}\, g\sin\theta$$

Therefore, the velocity of the disc on reaching the bottom is given by

$$v'^2 = 2a'l = \frac{4}{3}\, gl\sin\theta \text{ or } v' = 2\sqrt{\frac{gl\sin\theta}{3}}$$

$$\therefore \quad \frac{v'}{v} = \sqrt{\frac{2}{3}}$$

Hence the correct choice is (b).

29. Let μ be the mass per unit length of the wire. The mass of loop A is $M_A = 2\pi R\mu$ and mass of loop B is $M_B = 4\,\pi R\mu$. Their moments of inertia respectively are

$$I_A = M_A R_A^2 = 2\,\pi R\mu \times R^2 = 2\pi\,\mu\,R^3$$

and $$I_B = M_B R_B^2 = 4\,\pi R\mu \times (2R)^2 = 16\,\pi\mu R^3$$

$$\therefore \quad \frac{I_A}{I_B} = \frac{1}{8}$$

Hence the correct choice is (d).

30. Here $M_A = 2\pi\mu R$ and $M_B = 2\pi n\mu R$. Their moments of inertia are

$$I_A = M_A R_A^2 = 2\pi\mu R \times R^2 = 2\pi\mu R^3$$

and $$I_B = M_B R_B^2 = 2\pi n\mu R \times (nR)^2 = 2\pi n^3 \mu R^3$$

$$\therefore \quad \frac{I_B}{I_A} = n^3;\ \text{but } \frac{I_B}{I_A} = m \text{ (given)}$$

Thus, $m = n^3$. Hence the correct choice is (c).

31. Let m be the mass of the coin. It will fly off when the centripetal force $mr\omega^2$ just exceeds the force of friction μmg. The minimum ω is given by

$$mr\omega^2 = \mu mg$$

or $$\omega = \sqrt{\frac{\mu g}{r}}$$

Hence the correct choice is (c).

32. The minimum angular frequency is independent of the mass. Hence the correct answer is still $\sqrt{\mu g/r}$ which is choice (a).

33. The (x, y) co-ordinates of the masses at O, A and B respectively are (refer to Fig. 5.40 on page 5.25)

$(x_1 = 0, y_1 = 0)$, $(x_2 = a, y_2 = 0)$ and $(x_3 = 0, y_3 = b)$

The (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m\times 0 + m\times a + m\times 0}{m+m+m} = \frac{a}{3}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m\times 0 + m\times 0 + m\times b}{m+m+m} = \frac{b}{3}$$

The position vector of the centre of mass is $x_{CM}\,\mathbf{i} + y_{CM}\,\mathbf{j}$

$$= \frac{a}{3}\,\mathbf{i} + \frac{b}{3}\,\mathbf{j} = \frac{1}{3}(a\mathbf{i} + b\mathbf{j}),$$

which is choice (a).

34. Refer to Fig. 5.41 on page 5.25. The (x, y) co-ordinates of the masses at O, A and B respectively are

$$(x_1 = 0, y_1 = 0),\ (x_2 = a, y_2 = 0),\ \left(x_3 = \frac{a}{2}, y_3 = \frac{a\sqrt{3}}{2}\right)$$

Therefore, the (x, y) co-ordinates of the centre of mass are

$$x_{CM} = \frac{m\times 0 + m\times a + m\times a/2}{m+m+m} = \frac{a}{2}$$

$$y_{CM} = \frac{m\times 0 + m\times a + m\times a\sqrt{3}/2}{m+m+m}$$

$$= \frac{a}{2\sqrt{3}}$$

$\therefore$ Position vector of centre of mass is $\frac{a}{2}\left(\mathbf{i} + \frac{\mathbf{j}}{\sqrt{3}}\right)$. Hence the correct choice is (a).

35. The kinetic energy (which is rotational) is $\frac{1}{2}I\omega^2$.

The moment of inertia $I = \frac{1}{2}mr^2$ and $\omega = \frac{v}{r}$.

Therefore, $\text{KE} = \frac{1}{2} \times \frac{1}{2}\, mr^2 \times \left(\frac{v}{r}\right)^2 = \frac{1}{4}\, mv^2$, which is choice (d).

36. The kinetic energy of a rolling disc consists of two parts: translational energy $= \frac{1}{2}\, mv^2$ and rotational energy $= \frac{1}{2}\, I\omega^2$.

$$\therefore \quad \text{KE} = \frac{1}{2}\, mv^2 + \frac{1}{2}\, I\omega^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} \times \left(\frac{1}{2}mr^2\right) \times \left(\frac{v}{r}\right)^2$$

$$(\because I = \frac{1}{2} mr^2)$$

$$= \frac{1}{2} mv^2 + \frac{1}{4} mv^2 = \frac{3}{4} mv^2$$

Hence the correct choice is (c).

37. Mass of sphere A, $M_A = \frac{4}{3} \pi R^3 \rho_A$, mass of sphere B, $M_B = \frac{4}{3} \pi R^3 \rho_B$. Now, $I_A = \frac{2}{5} M_A R^2$ and $I_B = \frac{2}{5} M_B R^2$. Therefore,

$$\frac{I_A}{I_B} = \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B}$$

Hence the correct choice is (c).

38. When the cylinder rolls without sliding, the acceleration down the plane is

$$a_r = \frac{2}{3} g \sin \theta$$

When the cylinder slides without rolling, the acceleration is

$$a_s = g \sin \theta$$

where θ is the inclination of the plane.

If h is the height of the inclined plane, their speeds on reaching the bottom are given by

$$v_r = \sqrt{2a_r h} \text{ and } v_s = \sqrt{2a_s h}$$

Since $a_s > a_r$, it follows that $v_s > v_r$, which is choice (b).

39. When the sphere rolls down the plane, its acceleration is given by

$$a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}}$$

Now, the moment of inertia of the sphere about its diameter is

$$I = \frac{2}{5} MR^2$$

Therefore, $$a = \frac{g \sin \theta}{1 + \frac{2}{5}} = \frac{5}{7} g \sin \theta \quad \text{(i)}$$

For rolling without sliding, the frictional force f provides the necessary torque τ which is given by

$$\tau = \text{force} \times \text{moment arm} = fR$$

But $\tau = I\alpha$, where α is the angular acceleration of the sphere. Thus, $I\alpha = fR$. Also, linear acceleration $a = \alpha R$. Therefore,

$$f = \frac{I\alpha}{R} = \frac{Ia}{R^2} = \frac{2}{5} Ma \left(\because I = \frac{2}{5} MR^2\right)$$

Now, force of friction = $\mu \times$ normal reaction = $\mu Mg \cos \theta$. Thus $\mu Mg \cos \theta = \frac{2}{5} Ma$

or $$a = \frac{5}{2} \mu g \cos \theta \quad \text{(ii)}$$

Equating (i) and (ii) we have

$$\frac{5}{7} g \sin \theta = \frac{5}{2} \mu g \cos \theta$$

or $$\mu = \frac{2}{7} \tan \theta$$

Hence the correct choice is (d).

40. Given $PQ = QR = RP = L$. The centre of mass is located at centroid C which cuts lines PS, QT and UR in the ratio $2:1$. Let $h = CS = CT = UC$. In ΔPQS, we have (see Fig. 5.66)

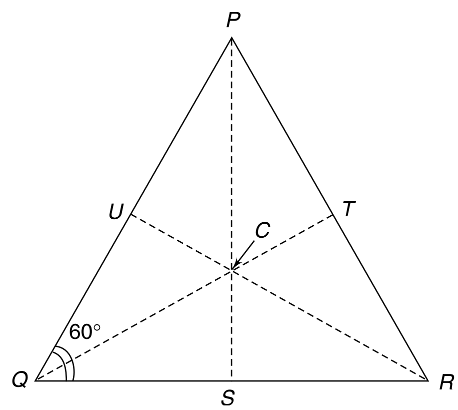

Fig. 5.66

$$PS = PQ \sin 60° = L \sin 60° = \frac{\sqrt{3}}{2} L.$$

$$\therefore \quad h = \frac{PS}{3} = \frac{1}{3} \times \frac{\sqrt{3}}{2} L = \frac{L}{2\sqrt{3}}$$

Since the structure consists of three identical rods, its moment of inertia about an axis passing through its centre of mass C and perpendicular to its plane is, from parallel axes theorem,

$$I_c = 3 (I + Mh^2)$$

where I is the moment of inertia of each rod about the axis passing through its centre and perpendicular to its length, which is given by

$$I = \frac{ML^2}{12}$$

Also $\quad Mh^2 = M\left(\frac{L}{2\sqrt{3}}\right)^2 = \frac{ML^2}{12}$

$\therefore \quad I_c = 3\left(\frac{ML^2}{12} + \frac{ML^2}{12}\right)$

$$= 3 \times \frac{ML^2}{6} = \frac{ML^2}{2}$$

Hence the correct choice is (a).

41. Refer to Fig. 5.43 on page 5.26. Moment of inertia is a scalar quantity. So the moment of inertia of the structure is the sum of the moments of inertia of the four rods about the specified axis of rotation, i.e.,

$$I = I_1 + I_2 + I_3 + I_4$$

where I_1 = moment of inertia of rod 1 about an axis passing through its centre E and perpendicular to its plane = $\frac{ML^2}{12}$,

I_2 = moment of inertia of rod 2 about an axis passing through its centre F and perpendicular to its plane = $\frac{ML^2}{12}$,

I_3 = moment of inertia of rod 3 about a parallel axis at a distance $\frac{L}{2}$ from it = $M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{4}$, and

I_4 = moment of inertia of rod 4 about a parallel axis at a distance $\frac{L}{2}$ from it = $\frac{ML^2}{4}$.

$\therefore \quad I = \frac{ML^2}{12} + \frac{ML^2}{12} + \frac{ML^2}{4} + \frac{ML^2}{4}$

$= \frac{2}{3} ML^2$, which is choice (d).

42. Refer to Fig. 5.67. It is clear from the figure that the moment of inertia of triangular sheet $ABC = \frac{1}{2} \times$ moment of inertia of a square sheet $ABCD$ about its diagonal AC or $I_t = \frac{1}{2} I_s$. Now, mass of square sheet $= M + M = 2M$. Therefore,

$$I_s = (2M)\frac{L^2}{12} = \frac{ML^2}{6}$$

$\therefore \quad I_t = \frac{I_s}{2} = \frac{ML^2}{12}$

Hence the correct choice is (a).

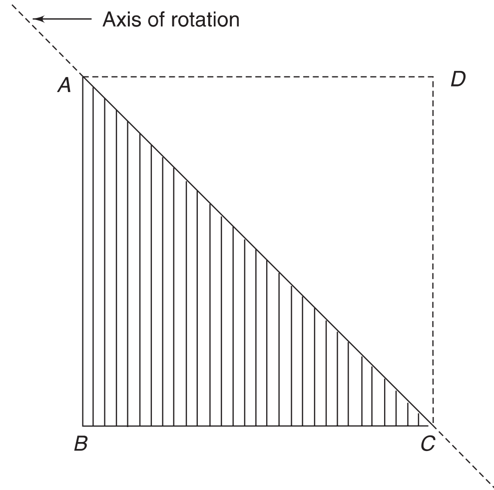

Fig. 5.67

43. $I_{\rm BC} = \frac{m(AB)^2}{3} = \frac{ma^2}{3}$

$$I_{\rm AB} = \frac{m(BC)^2}{3} = \frac{4}{3}ma^2$$

$$I_{\rm HF} = \frac{m(AB)^2}{12} = \frac{ma^2}{12}$$

$$I_{\rm EG} = \frac{m(BC)^2}{12} = \frac{ma^2}{3}$$

Thus, the moment of inertia about HF is the minimum, which is choice (c).

44. Let v be the speed of the sphere when it reaches B. Then, loss in PE = gain in translation KE + gain in rotational KE, i.e.

$$mg\,(6r - r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

or $\quad 5\,mgr = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{v^2}{r^2}$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

or $v = \sqrt{\frac{50gr}{7}}$, which is choice (b).

45. Horizontal force $= \frac{mv^2}{r} = \frac{m}{r} \times \frac{50gr}{7} = \frac{50mg}{7}$

Hence the correct choice is (c).

46. $I\omega = I_1\omega_1$. $I = \frac{2}{5}\ mr^2$, $I_1 = \frac{2}{5}\, m\left(\frac{r}{n}\right)^2 = \frac{I}{n^2}$. Hence

$$I\omega = \frac{I_1}{n^2}\ \omega_1$$

or $\omega_1 = n^2\omega$, which is choice (d).

47. $K = \frac{1}{2}\ I\omega^2$, $K_1 = \frac{1}{2}\ I_1\ \omega_1^2 = \frac{1}{2}\times\frac{I}{n^2}\ (n^2\omega)^2$

$$= \left(\frac{1}{2}\,I\omega^2\right)\ n^2 = n^2K.$$

Hence the correct choice is (c).

48. $V = \frac{4}{3}\ \pi r^3$ or $\log V = \log\left(\frac{4\pi}{3}\right) + 3\log r$.

Differentiating, we have

$$\frac{\delta V}{V} = 3\frac{\delta r}{r} \text{ or } \frac{\delta r}{r} = \frac{1}{3}\frac{\delta V}{V} = \frac{1}{3}\times 0.5\% = \frac{1}{6}\%$$

Since no external torque acts, $I\omega$ = constant or $\frac{2}{5}\ mr^2\omega$ = constant or $r^2\omega$ = constant (c)

or $2\log r + \log\omega = \log c$. Differentiating, we have

$$\frac{2\delta r}{r} + \frac{\delta\omega}{\omega} = 0$$

or $$\frac{\delta\omega}{\omega} = -2\,\frac{\delta r}{r} = -2\times\frac{1}{6}\% = -\frac{1}{3}\%$$

The negative sign indicates that ω decreases. Hence the correct choice is (b).

49. $I_1 = \frac{1}{2}\ MR^2$, $I_2 = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$

$$= M\left(\frac{R^2}{4} + \frac{(\sqrt{3}R)^2}{12}\right) = \frac{1}{2}\ MR^2$$

$\therefore \frac{I_1}{I_2} = 1$, which is choice (a).

50. In one full revolution the increase in PE = MgL, where M is the mass of the rod. Therefore,

$$MgL = \frac{1}{2}\,I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

or $\omega = \sqrt{\frac{6g}{L}}$. Now $v = L\omega = L\sqrt{\frac{6g}{L}} = \sqrt{6gL}$

Hence the correct choice is (c).

51. The cylinder will topple when the torque mgr equals the torque $ma\ \frac{h}{2}$ (see Fig. 5.68)

or $$a = \frac{2gr}{h} = \frac{g}{2} \qquad (\because\ h = 4r) \qquad \text{(i)}$$

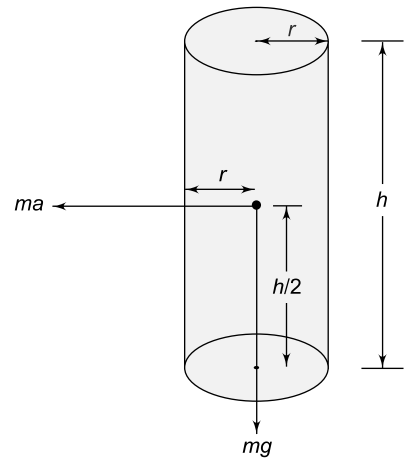

Fig. 5.68

Now $v = 2.45\ t^2$

$$\therefore \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(2.45t^2\right) = 4.9t \qquad \text{(ii)}$$

Equating (i) and (ii), we get $t = \frac{g}{2\times 4.9} = \frac{9.8}{9.8} = 1$ s.

Hence the correct choice is (a).

52. Loss in PE = gain in rotational KE. As the centre of mass of the rod falls through a distance $L/2$, the loss in PE = $\frac{MgL}{2}$. Gain in KE = $\frac{1}{2}\ I\omega^2 = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$

Equating the two, we have

$$\frac{MgL}{2} = \frac{ML^2\omega^2}{6}$$

or $\omega = \sqrt{\frac{3g}{L}}$, which is choice (c).

53. Using the parallel axes theorem, $I = I_{CM} + Mx^2$ where x is the distance of the axis of rotation from the C.M. (centre of mass) of the rod, which is $x = \frac{L}{2} - \frac{L}{4} = \frac{L}{4}$. Also $I_{C.M.} = \frac{ML^2}{12}$. Hence

$I = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$, which is choice (a).

54. PE at $\theta = 60°$ is $Mgh\ (1 - \cos\theta)$ where h is the distance between the axis of rotation and the centre of mass of the disc. Gain in KE when the disc reaches the equilibrium position $= \frac{1}{2}\ I\omega^2$ where $I = I_{\text{C.M.}} + Mx^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}\ MR^2$. Here x is the distance between the centre of mass and the axis of rotation, i.e. $x = R$.

Now PE = KE gives

$$MgR\ (1 - \cos 60°) = \frac{1}{2}I\omega^2 = \frac{3}{4}\ MR^2\ \omega^2$$

$(\because\ h = R)$

which gives $\omega = \sqrt{\dfrac{2g}{3R}}$, which is choice (b).

55. Loss in PE = gain in rotational KE. Thus

$$mgh = \frac{1}{2}\ (I + mR^2)\ \omega^2 = \frac{1}{2}\ (MR^2 + mR^2)\ \omega^2$$

$$= \frac{1}{2}\ R^2\ (M + m)\ \omega^2$$

or $\omega = \sqrt{\dfrac{2mgh}{(M+m)\,R^2}}$.

Hence the correct choice is (c).

56. Moment of inertia of complete disc about O is $I = \frac{1}{2}\ MR^2$. Mass of the cut-out part is $m = \left(\frac{M}{4}\right)$. The moment of inertia of the cut-out portion about its own centre $I_0 = \frac{1}{2}\ mr^2 = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{32}\ MR^2$ because $r = R/2$. From the parallel axes theorem, the moment of inertia of the cut out portion about O is

$$I_c = I_0 + mr^2 = \frac{1}{32}\ MR^2 + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$= \frac{3}{32}\ MR^2$$

$\therefore$ Moment of inertia of the shaded portion about O is $I_s = I - I_c = \frac{1}{2}\ MR^2 - \frac{3}{32}\ MR^2 = \frac{13}{32}\ MR^2$, which is choice (c).

57. We obtain the given hollow sphere as if a solid sphere of radius R has been removed from a solid sphere of radius $2R$. The mass of the given hollow sphere is (here ρ is the density of the material of the sphere)

$$M = M_1 - M_2$$

where $M_1 = \frac{4}{3}\pi\ (2R)^3\rho$ and $M_2 = \frac{4}{3}\ \pi R^3\rho$ are the masses of spheres of radii $2R$ and R respectively.

$$\therefore\qquad M = \frac{28}{3}\ \pi R^3\rho \qquad \text{(i)}$$

The moment of inertia of the given hollow sphere is

$$I = \frac{2}{5}\ M_1\ (2R)^2 - \frac{2}{5}\ M_2R^2$$

$$= \frac{2}{5}\times\frac{4}{3}\pi\ (2R)^3\ \rho(2R)^2 - \frac{2}{5}\times\frac{4}{3}\left(\pi R^3\rho\right)R^2$$

$$= \frac{2}{5}\ (32 - 1)\ \frac{4}{3}\ \pi R^5\rho \qquad \text{(ii)}$$

Using (i) in (ii), we get $I = \frac{62}{35}\ MR^2$, which is choice (d).

58. Let I_1 and ω_1 be the moment of inertia and angular frequency when his arms are outstretched and I_2 and ω_2 those when his arms are folded. Then

$$I_1\omega_1 = I_2\omega_2$$

Given $I_2 = \frac{3}{4}I_1$. Hence $I_1\omega_1 = \frac{3}{4}I_1\omega_2$ or $\omega_2 = \frac{4}{3}\ \omega_1$.

Initial KE is $K_1 = \frac{1}{2}I_1\omega_1^2$ and final KE is

$$K_2 = \frac{1}{2}\ I_2\ \omega_2^2 = \frac{1}{2}\times\frac{3I_1}{4}\times\left(\frac{4\omega_1}{3}\right)^2 = \frac{4}{3}\left(\frac{1}{2}\ I_1\omega_1^2\right)$$

$$= \frac{4}{3}K_1$$

$\therefore$ Percentage increase in KE

$$= \frac{K_2 - K_1}{K_1}\times 100 = \frac{\frac{4}{3}K_1 - K_1}{K_1}\times 100$$

$$= \frac{100}{3} = 33.3\%$$

Hence the correct choice is (a).

59. Refer to Fig. 5.69. When the masses are released, the torque is

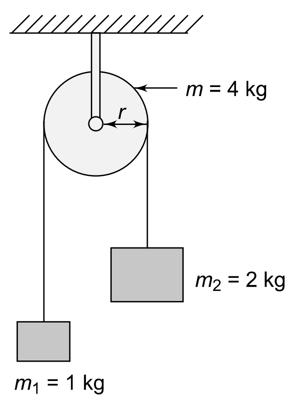

Fig. 5.69

$$\tau = (m_2 g - m_1 g) \times r$$
$$= (m_2 - m_1)\, gr \qquad \text{(i)}$$

But $\quad \tau = I\alpha = I\,\dfrac{a}{r} \qquad (\because a = r\alpha)$

where I = M.I. due to m_1 + M.I. due to m_2 + M.I. of pulley

$$= m_1 r^2 + m_2 r^2 + \frac{1}{2}\, mr^2$$
$$= \left(m_1 + m_2 + \frac{m}{2}\right) r^2$$

$\therefore \quad \tau = \left(m_1 + m_2 + \dfrac{m}{2}\right) ar \qquad \text{(ii)}$

Equating (i) and (ii), we get

$$a = \frac{(m_2 - m_1)\, g}{\left(m_1 + m_2 + \dfrac{m}{2}\right)}$$

Putting m_1 = 1 kg, m_2 = 2 kg and m = 4 kg, we get $a = \dfrac{g}{5}$. Hence the correct choice is (c).

60. Torque $I\alpha = F \times r$ or $10\alpha = (20t - 5t^2) \times 2$ or $\alpha = (4t - t^2)$ rad s^{-2}. But $\alpha = \dfrac{d\omega}{dt}$. Therefore

$$\frac{d\omega}{dt} = 4t - t^2$$

Integrating, we have

$$\omega = \left(2t^2 - \frac{t^3}{3}\right) = t^2\left(2 - \frac{t}{3}\right) \text{ rad s}^{-1}$$

The pulley reverses its direction when $\omega = 0$ momentarily, i.e. when $\left(2 - \dfrac{t}{3}\right) = 0$ or $t = 6$ s.

Now $\omega = \dfrac{d\theta}{dt}$. Thus

$$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

or $\quad \theta = \dfrac{2t^3}{3} - \dfrac{t^4}{12} = \dfrac{t^3}{3}\left(2 - \dfrac{t}{4}\right)$ rad

For $t = 6$ s, $\theta = \dfrac{(6)^3}{3}\left(2 - \dfrac{6}{4}\right) = 36$ rad

One full rotation corresponds to $\theta = 2\pi$ rad. Therefore, number of rotations

$$= \frac{36}{2\pi} = \frac{36}{2 \times 3.14} = 5.7$$

Thus the closest choice is (a).

61. Magnitude of torque = $Fr \sin\theta$ where θ is the angle between the force vector and the radius vector. The torques are (see Fig. 5.48 on page 5.29)

τ_1 = 8 N × 0.2 m × sin 30° = 0.8 Nm (clockwise)

τ_2 = 4 N × 0.2 m × sin 90° = 0.8 Nm (anticlockwise)

τ_3 = 9 N × 0.2 m × sin 90° = 1.8 Nm (clockwise)

τ_4 = 6 N × 0.2 m × sin 0° = 0

$\therefore$ Net torque = + 0.8 Nm – 0.8 Nm + 1.8 Nm + 0 = 1.8 Nm clockwise. Hence the correct choice is (b).

62. From the principle of conservation of angular momentum, $I_0\omega_0 = I\omega$, where I_0 and ω_0 are the moment of inertia and angular velocity when the beads are at the centre of the rod and I and ω those when the beads are at the ends of the rod.

$$I_0 = \frac{ML^2}{12} \text{ and } I = \frac{ML^2}{12} + \frac{mL^2}{4} + \frac{mL^2}{4} = \frac{L^2}{12}\,(M + 6m)$$

$$\therefore \quad \frac{ML^2}{12}\,\omega_0 = \frac{(M + 6m)\,\omega L^2}{12}$$

or $\quad \omega = \dfrac{M\omega_0}{(M + 6m)}$

Hence the correct choice is (b).

63. Because of symmetry about axes 1 and 2, $I_1 = I_2$. Similarly, $I_3 = I_4$. From perpendicular axes theorem, it follows that the moment of inertia of the plate about an axis passing through the centre and perpendicular to the plane of the plate is

$$I = I_1 + I_2 = I_3 + I_4 = 2I_1 = 2I_3$$
$$(\because I_1 = I_2,\ I_3 = I_4)$$

or $\quad I_1 = I_3$.

Thus $\quad I = I_1 + I_2 = I_3 + I_4 = I_1 + I_3$.

Hence the correct choice is (c).

64. Refer to Fig. 5.70. The angular momentum of the mass at point $P\,(x, y)$ about origin O is defined as

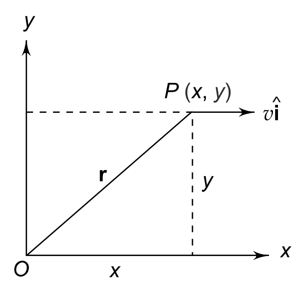

Fig. 5.70

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v} = m\,(x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \times (v\hat{\mathbf{i}})$$

$$= myv\,(-\hat{\mathbf{k}}) \qquad (\because \hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0 \text{ and } \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}})$$

Now m and v are constants. Also y remains constant as the mass moves parallel to the x-axis. Hence L remains constant. Thus the correct choice is (b).

65. Since there is no friction between the sphere and the horizontal surface and also between the spheres themselves, there will be no transfer of angular momentum from sphere A to sphere B due to the collision. Since the collision is elastic and the spheres have the same mass, the sphere A only transfers its linear velocity v to sphere B. Sphere A will continue to rotate with the same angular speed ω at a fixed location. Hence the correct choice is (c).

66. Refer to Fig. 5.71. Let $OC = R_C$ and let $\mathbf{v}_c$ be the velocity of the centre of mass of the disc. The linear momentum of the centre of mass is $\mathbf{p}_c = M\mathbf{v}_c$

If $\mathbf{L}_c$ is angular momentum of the disc about C, then the angular momentum about origin O is $\mathbf{L}_0 = \mathbf{L}_c + \mathbf{R}_c \times \mathbf{p}_c$

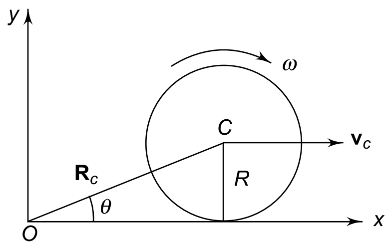

Fig. 5.71

$\therefore$ Magnitude $L_o = I_c\omega + R_c \times Mv_c \sin\theta$

$$= \frac{1}{2} MR^2\omega + MR_c v_c \sin\theta \quad \left(\because I_c = \frac{1}{2} MR^2\right)$$

$$= \frac{1}{2} MR^2\omega + MR \times R\omega \quad (\because R_c \sin\theta = R \text{ and } v_c = R\omega)$$

$$= \frac{3}{2} MR^2\omega$$

Hence the correct choice is (c).

67. Let m be the mass of the loop and r its radius. The moment of inertia of the loop about an axis passing through the centre O is (Fig. 5.72)

$$I_O = \frac{1}{2} mr^2$$

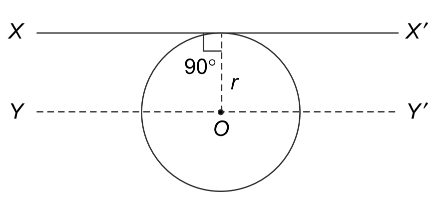

Fig. 5.72

From the parallel axes theorem, the moment of inertia about XX' is

$$I = I_O + mr^2 = \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2$$

The mass of the loop, $m = \rho L$ and radius $r = L/2\pi$. Hence

$$I = \frac{3}{2} \times \rho L \times \left(\frac{L}{2\pi}\right)^2 = \frac{3\rho L^3}{8\pi^2}$$

Thus the correct choice is (d).

68. The entire mass of the liquid can be regarded to be concentrated at the centre of mass of the tube which is at a distance of $r = \frac{L}{2}$ from the axis of revolution.

The force exerted by the liquid at the other end of the tube is the centripetal force of a mass M revolving in a circle of radius $r = \frac{L}{2}$. Thus

$$F_c = \frac{Mv^2}{r} = \frac{M(r\omega)^2}{r} = Mr\omega^2 = \frac{ML\omega^2}{2}$$

Hence the correct choice is (a).

69. Torque due to F about A is $\tau_1 = FL$

Since the weight mg acts through the centre of mass of the block (which is at a distance of $L/2$ from the side of the block) the torque due to weight mg about A is

$$\tau_2 = mg\left(\frac{L}{2}\right)$$

The minimum force required to topple the block is obtained when τ_1 is slightly greater than τ_2, i.e.

$$(\tau_1)_{\min} = \tau_2 \text{ or } F_{\min} L = mg\left(\frac{L}{2}\right) \text{ or } F_{\min} = \frac{mg}{2}$$

Hence the correct choice is (c).

70. Since no external force acts on the system, the centre of mass will remain at rest. Hence the correct choice is (a).

71. The velocity v_{CM} of the centre of mass can be obtained by using the principle of conservation of linear momentum,

$$MV = (M + m)\, v_{\text{CM}}$$

or $$v_{\text{CM}} = \frac{MV}{(M+m)} = \frac{10\text{ kg} \times 14\text{ ms}^{-1}}{(10+4)\text{ kg}}$$

$$= 10\text{ ms}^{-1}$$

Hence the correct choice is (c).

72. When a cylinder rolls up or down an inclined plane, its angular acceleration is always directed down the plane. Hence the frictional force acts up the inclined plane when the cylinder rolls up or down the plane. Thus, the correct choice is (b).

73. The correct choice is (d).

74. The magnitude of angular momentum of a rotating body is given by $L = I\omega$. If no torque acts, the angular momentum is conserved, i.e. $I\omega$ = constant. Hence $I_1\omega_1 = I_2\omega_2$. If K_1 and K_2 are the corresponding radii of gyration, then $I_1 = MK_1^2$ and $I_2 = MK_2^2$. Hence

$$MK_1^2\omega_1 = MK_2^2\omega_2$$

or $$\frac{K_1}{K_2} = \frac{\sqrt{\omega_2}}{\sqrt{\omega_1}}$$, which is choice (c).

75. Given: $l = 6R$. From parallel axes theorem, the moment of inertia about the given axis is given by

$$I = M\left(\frac{R^2}{4} + \frac{l^2}{3}\right)$$

$$= M\left[\frac{R^2}{4} + \frac{(6R)^2}{3}\right]$$

$$= M\left(\frac{R^2}{4} + \frac{36R^2}{3}\right) = \frac{49\,MR^2}{4}$$

Hence the correct choice is (d).

76. Areal velocity $A = \dfrac{\text{area swept by radius vector}}{\text{time taken}}$

Assuming that the orbit of the planet is a circle of radius R, then

$$A = \frac{\pi R^2}{T}$$

Now, time period $T = \dfrac{2\pi}{\omega}$. Hence

$$A = \frac{\pi R^2}{2\pi/\omega} = \frac{R^2\omega}{2}$$

or $$\omega = \frac{2A}{R^2}$$

Angular momentum $L = I\omega = (MR^2) \times \dfrac{2A}{R^2} = 2\,MA$

Hence the correct choice is (b).

77. The mass of the rod can be considered to be concentrated at its centre ($x = L/2$) where $x = 0$ is the origin. Hence

$$R_{CM} = \frac{M_1 \times L/2 + 0}{M_1 + M_2} = \frac{LM_1}{2(M_1 + M_2)}$$

Hence the correct choice is (b).

78. Let L cm be the original length of the spring and k be the spring constant. Then

$$m(L + x_1)\,\omega_1^2 = kx_1$$

and $$m(L + x_2)\,\omega_2^2 = kx_2$$

Dividing, we get

$$\left(\frac{L + x_1}{L + x_2}\right) \times \left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{x_1}{x_2} \qquad \text{(i)}$$

Given $x_1 = 1$ cm, $x_2 = 5$ cm and $\omega_2 = 2\omega_1$. Using these in (i) and solving, we get $L = 15$ cm, which is choice (b).

79. $L = I\omega = \dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}\omega} = \dfrac{2K}{\omega}$, where $K = \dfrac{1}{2}I\omega^2$ is the kinetic energy. If ω is doubled and K is halved, the value of L becomes one-fourth. Hence the correct choice is (d).

80. Rod POQ of length $l = 100$ cm is bent at its mid-point O so that $\angle POQ = 90°$ (see Fig. 5.73). The mass of part PO of length $l/2$ can be taken to be concentrated at its mid-point A whose coordinates are $(0, l/4)$ and of part OQ of length $l/2$ at its mid-point B whose coordinates are $(l/4, 0)$. The centre of mass of these two equal masses is at mid-point C between A and B. The coordinates of C are $(l/8, l/8)$.

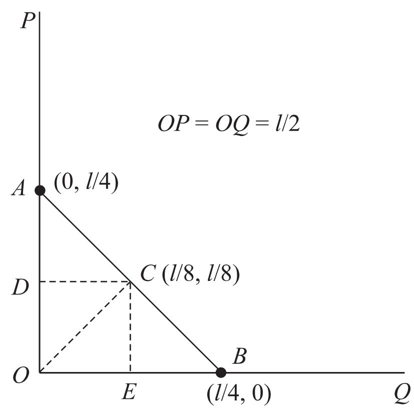

Fig. 5.73

$$\therefore \quad OC = \sqrt{(OE)^2 + (CE)^2} = \sqrt{\left(\frac{l}{8}\right)^2 + \left(\frac{l}{8}\right)^2}$$

$$= \frac{l}{\sqrt{32}} = \frac{100\text{ cm}}{\sqrt{32}} = 17.7\text{ cm},$$

which is choice (c).

81. $L = mr^2\,\omega$. For given m and ω, $L \propto r^2$. If r is halved, the angular momentum L becomes one-fourth. Hence the correct choice is (a).

82. Let M be the mass of the sphere. The mass of the disc will also be M. The moment of inertia of the sphere about its diameter is

$$I_s = \frac{2}{5} MR^2$$

The moment of inertia of the disc about its edge and perpendicular to its plane is (using parallel axes theorem)

$$I_d = I_{\text{cm}} + Mh^2 = \frac{1}{2} Mr^2 + Mr^2$$

$$= \frac{3}{2} Mr^2$$

Given $I_s = I_d$. Hence, we have

$$\frac{2}{5} MR^2 = \frac{3}{2} Mr^2$$

which gives $\dfrac{r}{R} = \dfrac{2}{\sqrt{15}}$, which is choice (a).

83. Consider a rod OP of length L lying along the x-axis with O as the origin (Fig. 5.74). Consider a small element AB of length dx at a distance x from O.

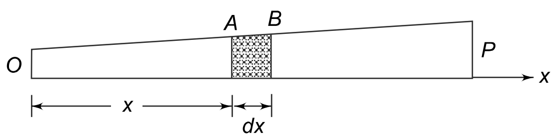

Fig. 5.74

Mass of element $AB\,(= dm) = mdx = \dfrac{k}{L}\,(xdx)$

The distance of the centre of mass from O is given by

$$x_{\text{CM}} = \frac{\int (dM)\,x}{\int (dM)} = \frac{\dfrac{k}{L}\displaystyle\int_0^L x^2\,dx}{\dfrac{k}{L}\displaystyle\int_0^L x\,dx}$$

$$= \frac{\left|\dfrac{x^3}{3}\right|_0^L}{\left|\dfrac{x^2}{2}\right|_0^L} = \frac{L^3/3}{L^2/2} = \frac{2L}{3}$$

The correct choice is (b).

84. Let ω be the angular velocity acquired by the system (rod + bullet) immediately after the collision. Since no external torque acts, the angular momentum of the system is conserved. Thus

$$mvL = I\omega \qquad (1)$$

where I is the moment of inertia of the system about an axis passing through O and perpendicular to the rod. Thus

I = M.I. of rod about O + M.I. of bullet stuck at its lower end about O

$$= \frac{1}{3}\,ML^2 + mL^2 = \frac{1}{3}\,(M + 3m)L^2 \qquad (2)$$

Using Eq. (1) in Eq. (2), we have

$$mvL = \frac{1}{3}\,(M + 3m)L^2\omega$$

or $$\omega = \frac{3mv}{L\,(M + 3m)}$$

Hence the correct choice is (c).

85. The initial angular momentum of the rotating record is

$$L = I\omega$$

where $$I = \frac{1}{2}\,MR^2.$$

Let ω' be the angular velocity of the record when the coin of mass m is placed on it at a distance r from its centre. The angular momentum of the system becomes

$$L' = (I + mr^2)\omega'$$

Since no external torque acts on the system, the angular momentum is conserved, i.e.

$$L' = L \text{ or } (I + mr^2)\omega' = I\omega$$

or $$\omega' = \frac{I\omega}{I + mr^2} = \frac{\frac{1}{2}MR^2\omega}{\frac{1}{2}MR^2 + mr^2}$$

or $$\omega' = \frac{\omega}{\left(1 + \dfrac{2mr^2}{MR^2}\right)}$$

Putting $r = R/2$, we find that the correct choice is (a).

86. Since the rod is uniform, its centre of mass is at its midpoint. So the entire mass $3M$ of the rod can be assumed to be concentrated at its centre (0, 0). Now refer to Fig. 5.75.

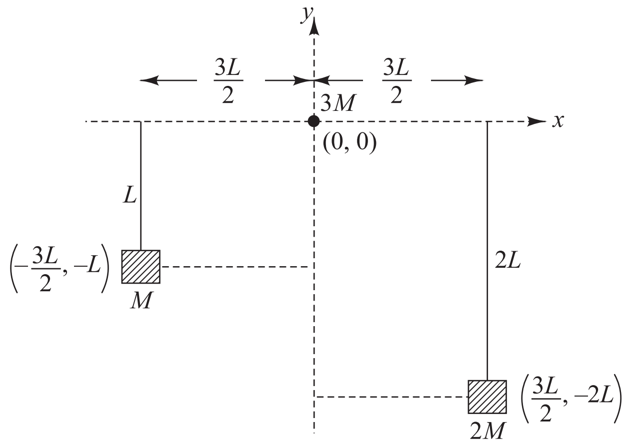

Fig. 5.75

The x and y coordinates of the centre of mass are

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{M \times (-3L/2) + 2M(3L/2) + 3M \times 0}{M + 2M + 3M}$$

$$= \frac{L}{4}$$

$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{M \times (-L) + 2M \times (-2L) + M \times 0}{M + 2M + 3M}$$

$$= -\frac{5L}{6}.$$

So the correct choice is (c).

87. Since no external force acts on the man-plank system, the centre of mass of the system cannot accelerate. Since the system is initially at rest, the centre of mass cannot move. Let the midpoint of the plank be at $x = 0$, which is also the location of the centre of mass of the plank (Fig. 5.76)

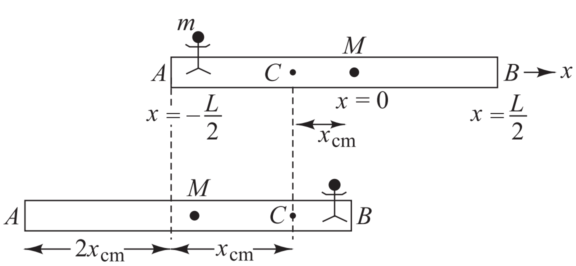

Fig. 5.76

The centre of mass C of the system when the man is at end A is

$$x_{\text{cm}} = \frac{m \times \left(-\frac{L}{2}\right) + M \times 0}{m + M} = -\frac{m}{2(m+M)}$$

The negative sign shows that the centre of mass is to the left of $x = 0$.

The centre of mass of the system when the man reaches end B is

$$x_{\text{cm}} = \frac{m \times \left(\frac{L}{2}\right) + M \times 0}{m + M} = \frac{mL}{2(m+m)}$$

to the right of $x = 0$.

Since the position of the centre of mass cannot change, this means that the plank itself must have moved through a distance $= 2x_{\text{cm}} = \frac{mL}{(m+M)}$ to the left. So the correct choice in (d).

88. The time taken by the man to move from A to B is

$$t = \frac{AB}{v} = \frac{L}{v}$$

Since the plank moves through a distance $d = \frac{mL}{(m+M)}$ to the left, the average speed of the plank will be

$$V = \frac{d}{t} = \frac{mv}{(m+M)} \text{ to the left.}$$

So the correct choice is (c).

89. Let the bar lie along the x-axis with its end A at $x = 0$ (see Fig. 5.77)

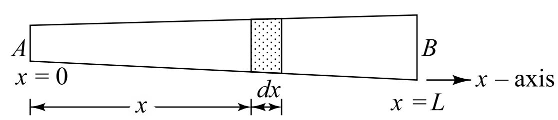

Fig. 5.77

Mass of the bar is

$$M \int dm = \int \lambda dx$$

$$= \int_0^L (4x+5)dx$$

$$= \left| \frac{4x^2}{2} + 5x \right|_0^L$$

$$= \frac{4L^2}{2} + 5L$$

$$= \frac{4(0.5)^2}{2} + 5 \times 0.5 \quad (\because L = 0.5\text{m})$$

$$= 3 \text{ kg}$$

The x coordinate of the centre of mass of the bar is determined from the equation

$$x_{\text{cm}} = \frac{1}{M}\int x\,dm$$

$$= \frac{1}{M}\int_0^L (4x+5)x\,dx$$

$$= \frac{1}{M}\left|\frac{4x^3}{3} + \frac{5x^2}{2}\right|_0^L$$

$$= \frac{1}{M}\left[\frac{4L^3}{3} + \frac{5L^2}{2}\right]_0^L$$

$$= \frac{1}{3}\left[\frac{4}{5}\times(0.5)^3 + \frac{5}{2}\times(0.5)^2\right]$$

$$= \frac{19}{72}\ \text{m} = 0.264\text{m} = 26.4\,\text{cm}$$

The x coordinate of the midpoint of the bar is $x_0 = 25.0$ cm.

$\therefore$ Distance of the centre of mass from the midpoint is $x_{\text{cm}} - x_0 = 26.4 - 25.0 = 1.4$ cm. Note that the centre of mass of the bar is to the right of its midpoint as expected. So the correct choice in (a).

90. The free body diagrams of the block and pulley are shown in Fig. 5.78.

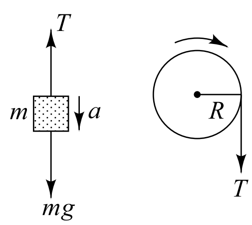

Fig. 5.78

The equation of motion of block is

$$mg - T = ma \qquad (1)$$

where T is the tension in the string. The torque acting on the pulley is

$$\tau = I\alpha$$

where $I = \frac{1}{2}MR^2$ and $\alpha = \frac{a}{R}$. Thus

$$\tau = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{1}{2}MRa$$

Now $\tau = RT$. Therefore

$$RT = \frac{1}{2}MRa$$

$$\Rightarrow \qquad T = \frac{1}{2}Ma \qquad (2)$$

Using (2) in (1), we have

$$mg - \frac{1}{2}Ma = ma$$

$$\Rightarrow \qquad a = \left(\frac{2m}{M+2m}\right)g$$

Putting $M = 4m$, we get $a = \frac{g}{3}$, which is choice (b).